Follow the light: Relativity and Simultaneity

Richard Kaufman (rdkaufman01 at gmail dot com)

Abstract

The acclaimed Caltech series, “The Mechanical Universe”, discusses the speed of light and relativity. Here we elaborate on their demonstration to show how one observer will determine that another observer will see that an event is simultaneous if light is followed back to the observer’s eyes.

Discussion

Caltech’s acclaimed series, “The Mechanical Universe…And Beyond”, was released in the mid 1980s. It has 52 episodes for introductory college physics.

Episode 42 is titled “The Lorentz Transformation”[1] and won the 1987 Japan Prize for educational content.[2] The subsequent episode is titled ”Velocity and Time”[3] and continues the discussion into Einstein’s relativity. The insightful videos and animations show:

- The speed of light is the same for all inertial (non-accelerating) observers, no matter their speed.
- Space (length) and time are relative measures.
- Observers in uniform relative motion to each other will disagree on whether events occur simultaneously.

Both episodes show Figures 1 and 2 which capture a sequence of moving animations. Henry Lorentz is on a train moving to the right at 0.4*c* (*i.e.*, 0.4 the speed of light) relative to Albert Einstein, who stands on the ground. The following text is from Episode 42 (starting at 8:24):

> … at the exact place and time they pass each other, they observe a flash of light. A sphere of light expands outward from that point. Since each measures the speed of light relative to himself, each believes correctly that he is always at the center of that expanding sphere - even though they themselves move farther and farther apart. How can two people in different places both be at the center of the same sphere?
>
> To confirm his perception, each sets up light detectors an equal distance apart. However, while Albert’s detectors register the light arriving simultaneously, he believes the light strikes Henry’s detectors at two different times. Meanwhile, Henry sees the same thing - in reverse. They agree on the speed of light, but they disagree on whether events happen simultaneously or at different times.

Later, 3D like space-time diagrams are used (and offset) to show how each observer views their own light detectors to flash simultaneously. However, we will focus on Figures 1 and 2 here.

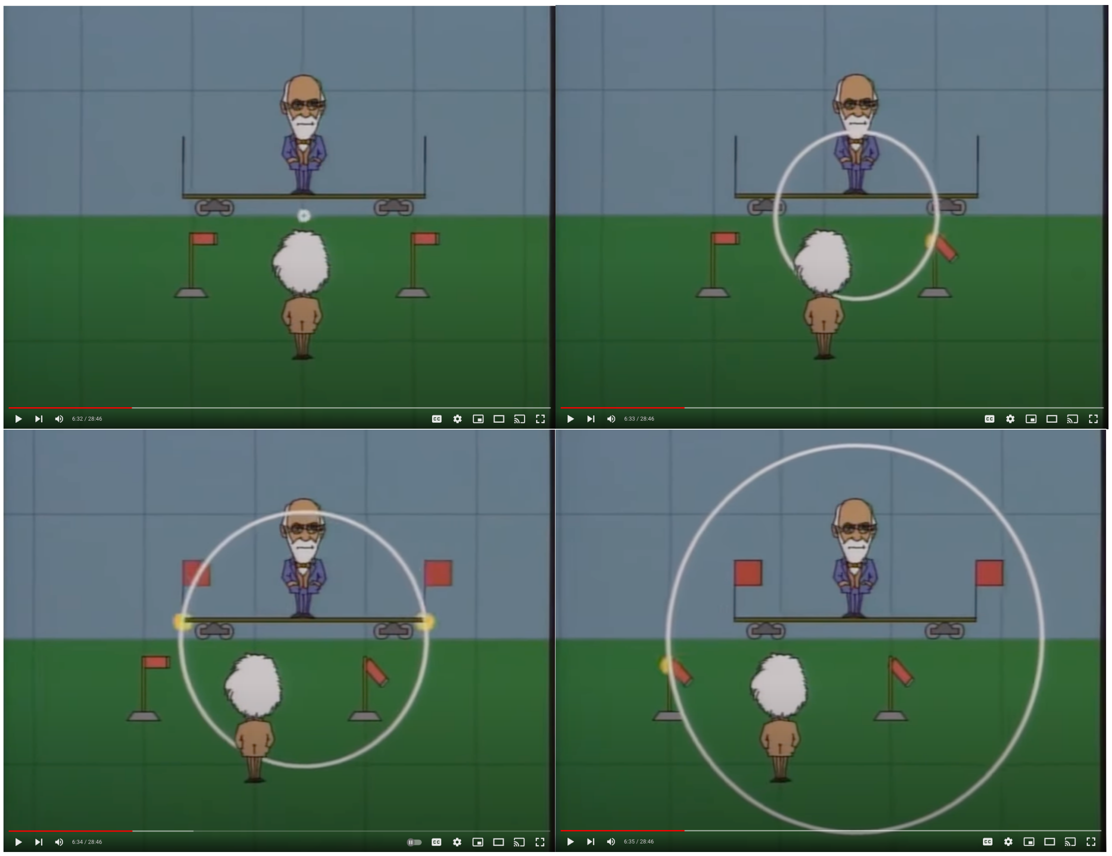

Figure 1: Henry's observation that he is at the center of the expanding sphere of light while riding on a train at a speed of 0.4*c* relative to the ground. His light detectors flash simultaneously, while those on the ground flash at different times.

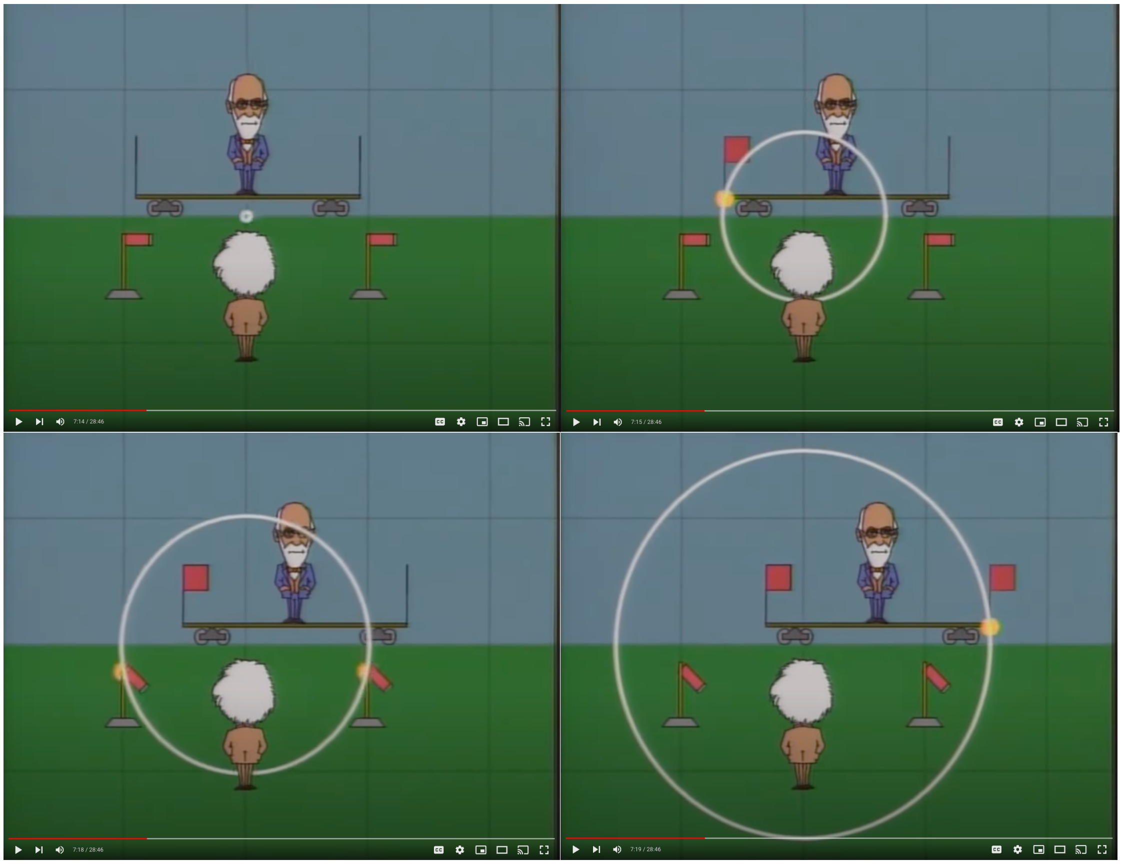

Figure 2: Albert's observation that he is at the center of the expanding sphere of light on the ground. His light detectors flash simultaneously, while those on the train flash at different times.

This demonstration must use an exceptionally long train (and exaggerated dimensions for each man) since the speed of light is shown. Although the videos for Figures 1 and 2 show when the light detectors are triggered, they do not show when each observer sees the detector triggered. So, while watching the videos, we might ask when the light from each light detector actually reaches an observer.

Specifically, we will consider Figure 2 where the sphere of light expands in Albert's rest frame. (A similar situation would arise if we considered Henry's frame). In his own frame, Albert can trace the light from Henry's detectors to show that Henry sees them flash simultaneously (just as Henry sees in his own reference frame).

At the exact time that Henry and Albert pass each other, the flash of light starts, and the time is set equal to 0. In Albert's frame, the expanding light sphere hits the left light detector, $D1$, on the train first, and give off Flash 1 (shown in the upper-right of Figure 2). When the light sphere hits the right light detector, $D2$, on the train, it gives off Flash 2 (shown in the lower-right of Figure 2). In order to quantify how long it takes for these light flashes to reach the center of the train where Henry is located (at a scale where he would appear to be microscopic relative to the wide length of the train) we must do some simple algebra for Albert's frame:

c is the speed of light
v is the speed of the train to the right (0.4c in the video)
l is half the train's length
t_1 is the time for the expanding sphere of light to hit $D1$, causing Flash 1
t_2 is the time it takes for the light from Flash 1 to reach the center of the train
t_3 is the time for the expanding sphere of light to hit $D2$, causing Flash 2
t_4 is the time it takes for the light from Flash 2 to reach the center of the train

The following calculations are based on the speed of light c, the velocity of the train v, and the distance formula: $distance$ equals $velocity$ times $time$.

The time t_1, from the start until Flash 1, can be found from the distance formula:

$$ct_1 = l - vt_1, \quad \text{so } t_1 = \frac{l}{c+v}$$

The time t_2, for light Flash 1 to reach the center of the train, can be found from the distance formula:

$$ct_2 = l + vt_2, \quad \text{so } t_2 = \frac{l}{c-v}$$

The time t_3, from the start until Flash 2, can be found from the distance formula:

$$ct_3 = l + vt_3, \quad \text{so } t_3 = \frac{l}{c-v}$$

The time t_4 for light Flash 2 to reach the center of the train, can be found from the distance formula:

$$l = vt_4 + ct_4, \quad \text{so } t_4 = \frac{l}{c+v}$$

Therefore, the time for the initial flash of light to hit $D1$, and then the subsequent light from Flash 1 to reach the center of the train is:

$$t_1 + t_2 = \frac{l}{c+v} + \frac{l}{c-v}$$

The time for the initial flash of light to hit $D2$, and then for the subsequent light from Flash 2 to reach the center of the train is:

$$t_3 + t_4 = \frac{l}{c-v} + \frac{l}{c+v}$$

So $t_1 + t_2 = t_3 + t_4$.
Therefore, Henry sees that both Flash 1 and Flash 2 are triggered simultaneously in Albert's frame. This makes sense after all, but it just required a little more thought, since it was not explicitly stated or shown in the video. Perhaps there is more agreement between observers after all.

[1] Caltech "Episode 42: The Lorentz Transformation - the Mechanical Universe." YouTube, YouTube, 19 Dec. 2016, https://www.youtube.com/watch?v=feBT0Anpg4A&list=PL8_xPU5epJddRABXqJ5h5G0dk-XGtA5cZ&index=42.

[2] "About the Japan Prize the Grand Prix Winners." *Japan Prize International Contest For Educational Media*, http://www.nhk.or.jp/jp-prize/english/about/grandprix_winners.html.

[3] Caltech "Episode 43: Velocity and Time - the Mechanical Universe." *YouTube*, YouTube, 19 Dec. 2016, https://www.youtube.com/watch?v=BFLUa0ciMjw&list=PPSV.