Tau- and M-neutrino and All Massive Leptons

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Abstract

The mass eigenstates of the electron and muon neutrinos are built from the existing left -handed neutrino wave functions and these mass eigen states acquire mass through their interaction with the same Higgs field as their electrically charged partners. The above scheme requires, existence of another massive M- neutrino along -with an electrically charged massive M-Lepton to account for the tau family.

1 INTRODUCTION

The neutrino fields are $\psi^{\nu}_{(L/R)} = \Gamma_{(L/R)}\psi^{\nu}$, where the projectors are $\Gamma_{(L/R)} = \frac{1}{2}\,(1 \mp \gamma^5)$. The projectors ensure that the massless fields $\psi^{\nu}_{(L/R)}$ have just two components. To begin with we define mass eigenstates for the electron and muon neutrinos using the existing left-handed fields only. To this end let the mass eigenstate of the electron -neutrino be ν_1 such that $\Gamma_L \nu_1 = \psi_L^{\nu_e}$. The charge conjugate spinor ψ_L^c of $\psi_L^{\nu_\mu}$ is C $\left[\overline{\psi_L^{\nu_\mu}}\right]^T$ of the muon neutrino where C= $i\gamma^0\gamma^2$.This charge conjugate ψ_L^c is of opposite chirality to $\psi_L^{\nu_\mu}$ .In our scheme ,the electron-neutrino Dirac mass eigen state is

$$\nu_1 = \psi_L^{\nu_e} + C\left[\overline{\psi_L^{\nu_\mu}}\right]^T \quad . \tag{1}$$

The above wave function is a four component wave function and we consider this as the electron neutrino mass eigen state and it is the Left-handed part of ν_1 that participates in Standard model electroweak Interactions. In a similar way the muon neutrino Dirac mass eigenstate Is ν_2 , where ,

$$\nu_2 = \psi_L^{\nu_\mu} + C\left[\overline{\psi_L^{\nu_e}}\right]^T \quad . \tag{2}$$

The four component states acquire mass through their interaction with the Higgs field like the charged leptons. There is no reason to suggest that they should have the same mass generating yukawa constant with the HIGGS field. Because of similar build -up, the masses of neutrinos however appear to be nearly equal. Standard model ,SM [1] does not account for the origin of the small, non-zero neutrino masses seen in neutrino oscillation experiments [2].SM extensions to incorporate non-zero masses typically require right handed neutrinos. The above definitions of the four component Dirac mass eigen state is an attempt in this direction.

2. First Category Leptons And Their Masses.

Gauge bosons acquire their masses through spontaneous symmetry breakdown. It has been a dream ever since to explain and to derive the masses of all fermions through the same process with of course the same Higgs field ϕ with the VEV, $V_0 = 246.22\ GeV$. The first category of leptons are ,electron & electron-neutrino, muon & muon neutrino. The SM Higgs field couples to all these leptons through the following LAGRANGIAN :

$$L = -h_1\overline{\nu_1}\,\nu_1\phi - h_1\bar{e}e\,\phi - \mathrm{i}a_1\bar{e}\gamma^5 e\phi - h_2\overline{\nu_2}\nu_2\phi - h_2\bar{\mu}\mu\phi - ia_2\bar{\mu}\gamma^5\mu\phi\ . \quad (3)$$

In the above Lagrangian the neutrino fields are four component fields as defined earlier. After symmetry breaking , $\phi = \phi_0 + V_0$,we have,

$$L = -h_1\overline{\nu_1}\nu_1 V_0 - h_1\overline{\nu_1}\nu_1\phi_0 - h_1\bar{e}eV_0 - h_1\bar{e}e\phi_0 - \mathrm{i}\,a_1\bar{e}\gamma^5 eV_0 - ia_1\bar{e}\gamma^5 e\phi_0 - h_2\overline{\nu_2}\nu_2 V_0 - h_2\overline{\nu_2}\nu_2\phi_0 - ia_2\bar{\mu}\gamma^5\mu V_0 - ia_2\bar{\mu}\gamma^5\mu\phi_0 - h_2\bar{\mu}\mu V_0 - h_2\bar{\mu}\mu\phi_0\ . \quad (4)$$

From the above we observe that the electron-neutrino and the muon neutrino acquire the following Dirac masses :

electron- neutrino mass $m_1 = h_1 V_0$ and (5)

muon -neutrino mass $m_2 = h_2 V_0$. (6)

The electron acquires the same mass m_1 as its neutrino if $a_1 = 0$. Similarly the muon acquires the same mass m_2 as its neutrino if a_2 is zero. The mass giving part of the Lagrangian for the charged leptons is

$$L = -m_1\bar{e}e - h_1\bar{e}e\phi_0 - ia_1\bar{e}\gamma^5 eV_0 - ia_1\bar{e}\gamma^5 e\phi_0 - m_2\bar{\mu}\mu - h_2\bar{\mu}\mu\phi_0 - \mathrm{i}a_2\bar{\mu}\,\gamma^5\mu V_0 - ia_2\bar{\mu}\gamma^5\mu\phi_0\ . \quad (7)$$

We consider the following transformations :

$$e = \exp\left(-\frac{1}{2} i\alpha_1\gamma^5\right)e' \quad \text{and} \tag{8}$$

$$\mu = \exp\left(-\frac{1}{2} i\alpha_2\gamma^5\right)\mu' \quad , \tag{9}$$

Where α_1 and α_2 are real parameters. The Vector and axial vector Interactions are not affected by these transformations. We choose α_1 and $\alpha_2$in a way so that the constant coefficients of $\overline{e'}\gamma^5 e'$ and $\overline{\mu'}\gamma^5\mu'$ are zero,

$$-L = (-im_1 sin\alpha_1 + a_1 V_0 cos\alpha_1)\overline{e'}\gamma^5 e' + (m_1 cos\alpha_1 + a_1 V_0 sin\alpha_1)\overline{e'}e' + a_1\overline{e'}[sin\alpha_1 + i\gamma^5 cos\alpha_1]e'\phi_0 + h_1\overline{e'}[cos\alpha_1 - i\gamma^5 sin\alpha_1]\, e'\phi_0 \text{ plus similar terms for the muon} \tag{10}$$

When we set the constant coefficient of the first term zero ,it gives,

$$tan\alpha_1 = \frac{a_1 V_0}{m_1}\,. \tag{11}$$

Similar process for the muon also gives,

$$tan\alpha_2 = \frac{a_2 V_0}{m_2}\,. \tag{12}$$

The masses are given by, $m_e = m_1 sec\alpha_1$ and $m_\mu = m_2 sec\alpha_2$. (13)

3.Electron- Muon Mass Ratio And The Neutrino Masses.

The mass of the electron is given by,

$$m_e^2 = m_1^2 sec^2\alpha_1 = m_1^2[1 + tan^2\alpha_1] = m_1^2\left[1 + \frac{a_1^2 V_0^2}{m_1^2}\right]\,. \tag{14}$$

The above relation can still be arranged in a very transparent way:

$$m_e^2 = m_1 V_0\left[\frac{a_1^2}{h_1} + h_1\right] = m_1 V_0 q_1\,, \tag{15}$$

Where $q_1 = \left[\frac{a_1^2}{h_1} + h_1\right]$. (16)

From, Eq.(15& 5) we note that the square of the electron mass is proportional to the square of V_0^2 where V_0 is the VEV .

In an exactly similar way we note that,

$$m_\mu^2 = m_2 V_0 q_2, \qquad \text{where} \tag{17}$$

$$q_2 = \left[\frac{a_2^2}{h_2} + h_2\right]. \tag{18}$$

The electron-muon mass ratio is still an unknown unsolved problem of the standard model. The mass of the electron is lesser than the mass of the muon. Let us assume that the two masses are given by,

$$m_e^2 = m_1 V_0 K(A - B)^2 \quad , \tag{19}$$

$$m_\mu^2 = m_2 V_0 K(A + B)^2 \ . \tag{20}$$

We believe that the three constants K,A and B are related to the gauge constants $g\ and g'$ and Weinberg mixing parameter of the SM.From $q_1 and q_2$ also we infer this idea. The electron and muon acquire mass through their interaction with the HIGGS field. The factors K, A, and B must depend only on the gauge constants or functions of those constants. To this end , we found, [3],

$$m_e^2 = m_1 V_0 \frac{1}{2}\frac{g_A^2}{g_V^4}\left[(g_A^2 + g_V^2)^{1/2} - (g_A^2 - g_V^2)^{1/2}\right]^2 \quad , \tag{21}$$

$$m_\mu^2 = m_2 V_0 \frac{1}{2}\frac{g_A^2}{g_V^4}\left[(g_A^2 + g_V^2)^{1/2} + (g_A^2 - g_V^2)^{1/2}\right]^2 \quad . \tag{22}$$

In the above , $g_V\ and g_A$ are the vector and axial vector coupling constants of e or μ leptons with the Z-boson of the SM.These can still be recast so that the factors $q_1 and q_2$ are readily apparent.

$$m_e^2 = m_1 V_0 \frac{g_A^4}{g_V^4}\left[1-\left(1-\frac{g_V^4}{g_A^4}\right)^{1/2}\right] = m_1 V_0 q_1 \quad . \tag{23}$$

$$m_\mu^2 = m_2 V_0 \frac{g_A^4}{g_V^4}\left[1+\left(1-\frac{g_V^4}{g_A^4}\right)^{1/2}\right] = m_2 V_0 q_2 \quad . \tag{24}$$

From the above expressions as and when $m_1 = m_2$ it just follows that

$$\frac{2m_e m_\mu}{m_e^2+m_\mu^2} = \left(\frac{g_V}{g_A}\right)^2 = [-1+4sin^2\vartheta_W]^2 \quad . \tag{25}$$

The exact masses of the electron and muon are known, $m_e = 0.51099MeV$,$m_\mu = 105.65839MeV$,And LHS= 0.009672 .On the Other-hand the RHS of Eq.(25) is 0.009672 if $sin^2\vartheta_W = 0.225413$ or 0.274587. The Weinberg mixing parameter of the electroweak model Is known experimentally. It is about 0.23 .The conclusion we draw from this is that whenever the electron neutrino Dirac mass is exactly equal to the muon-neutrino mass ,Eq(25) is exact and the mixing parameter is 0.225413, which is very close to the experimental value. From Eq.(25), We note that,

$$\frac{m_e}{m_\mu} \approx \frac{1}{2}\left(\frac{g_V}{g_A}\right)^2 = \frac{1}{2}\,[-1+4sin^2\vartheta_W]^2 \quad . \tag{26}$$

The above relation solves the famous electron-muon mass ratio , while the electron and muon acquired their masses through the Higgs field. The mass of the electron -neutrino can now be theoretically found from the relation ,Eq.(23) ,assuming that, $\left(\frac{g_V}{g_A}\right)^4 \ll 1.$

$$m_1 = 2\frac{m_e^2}{V_0} = 2.120957eV \quad . \tag{27}$$

The above mass is the Dirac mass of the electron-neutrino and it does not depend on the Weinberg mixing parameter. From Eq.(24) we note

that, $m_2 = \frac{m_\mu^2}{2V_0}\left(\frac{g_V}{g_A}\right)^4$ =2.130995 eV. (28)

The muon-neutrino mass above is estimated for the Weinberg mixing parameter $sin^2\vartheta_W = 0.2254$. For $sin^2\vartheta_W = 0.23$,

$$m_2 = \frac{m_\mu^2}{2V_0}\left(\frac{g_V}{g_A}\right)^4 = 0.929477 \text{ eV}. \quad (29)$$

The exact values of m_1 and m_2 are estimated below for0.2254.

$m_1 = 2.120957\ eV$ and $m_2 = 2.131045\ eV$. (30)

It should be very clear now that the electron-neutrino and muon-neutrino have different but very nearly equal Dirac masses.

3.Another Category of Leptons And Their Masses.

The charged Tau lepton and its neutrino are already experimentally observed. The mass of the charged Tau lepton is,

$$m_\tau = 1.777GeV\ . \quad (31)$$

The Tau-neutrino is very massive . Its mass is not known exactly. It is supposed to have a mass of about 18 MeV or more. This value is not confirmed experimentally. The left chiral state $\psi_L^{\nu_\tau}$ of this neutrino along-with the charged Tau lepton participates in the electro weak SM model like the electron and it neutrino. This chiral state is massless. Let there be another M-neutrino with the left chiral state $\psi_L^{\nu_M}$ similar to the left-handed μ -neutrino. This $\psi_L^{\nu_M}$ along with the charged M-lepton participate in the Electroweak SM model much like the μ lepton and its neutrino. The Dirac- mass eigen states of the Tau-neutrino and M-neutrino are ν_3 and ν_4 .These are four component mass-eigen states.

$$\nu_3 = \psi_L^{\nu_\tau} + C\left[\overline{\psi_L^{\nu_M}}\right]^T \quad . \tag{32}$$

$$\nu_4 = \psi_L^{\nu_M} + C\left[\overline{\psi_L^{\nu_\tau}}\right]^T \quad . \tag{33}$$

It is these mass-eigen states which acquire mass by their interaction with the same Higgs field through which all other particles acquire their mass. The Lagrangian that gives mass is,

$$\mathrm{L} = -h_3\overline{\nu_3}\,\nu_3\phi - h_3\bar{\tau}\tau\,\phi - \mathrm{i}a_3\bar{\tau}\gamma^5\tau\phi - h_4\overline{\nu_4}\nu_4\phi - h_4\bar{M}M\phi - ia_4\bar{M}\gamma^5 M\phi \ . \tag{34}$$

After symmetry breaking the Tau neutrino and the M-neutrino acquire The following masses:

$$m_3 = h_3 V_0 \quad \text{, and} \tag{35}$$

$$m_4 = h_4 V_0 \quad . \tag{36}$$

We follow the same procedure as in the case of electron and muon to obtain the masses of the charged tau lepton and the charged M-lepton and obtain,

$$m_\tau^2 = m_3 V_0 \frac{g_A^4}{g_V^4}\left[1 - \left(1 - \frac{g_V^4}{g_A^4}\right)^{1/2}\right] = m_3 V_0 q_3 \ . \tag{37}$$

$$m_M^2 = m_4 V_0 \frac{g_A^4}{g_V^4}\left[1 + \left(1 - \frac{g_V^4}{g_A^4}\right)^{1/2}\right] = m_4 V_0 q_4 \quad . \tag{38}$$

In Eq.(37), m_τ is the mass of the charged Tau -lepton and $g_V and g_A$ are the vector and axial vector coupling constants of the charged tau

Lepton with the Z-boson of the SM, and m_3 is the Dirac mass of the Tau-neutrino. Similarly m_M is the mass of the charged lepton yet to be discovered and m_4 is the Dirac mass of the M-neutrino. From Eq.(37) It just follows that,

$$m_3 = 2\frac{m_\tau^2}{V_0} = 25.65\ MeV. \qquad (39)$$

Direct bounds on the Tau-neutrino mass come from reconstruction of τ multi-hadronic decays. The best limits come from the Aleph experiment at LEP studying the reactions, $\tau^- \rightarrow 2\pi^- + \pi^+ + \nu_\tau$ .They set a limit of $m_3 < 22.3\ MeV$ from a total of 2939 events [4,5,6] the mass in Eq.(39) coincides with this experimental estimates of the tau neutrino mass. The electron-neutrino and muon-neutrino have almost equal mass.The Tau-neutrino and the M-neutrino must as well have almost equal mass.

Let , $m_4 = 25.77\ MeV$. (40)

The above mass can be used to find the mass of the charged M-lepton, From Eq.(38).This gives,

$$m_M = 367.44 GeV \qquad . \qquad (41)$$

The above shows that this charged lepton is very massive about $4m_Z$, where m_Z is the mass of the Standard Z boson.

4. NEUTRINO OSCILLATIONS

The particular mass states of the neutrinos are not identical to the eigen states of the weak force. This would lead to an oscillation between different neutrino types as a beam of neutrinos propagate through space .To estimate this we follow two flavor mixing like Cabibbo type of mixing of quarks. For obtaining the electron-neutrino & muon-neutrino mixing we proceed in the following [7] way :

Let the electron and its neutrino mass matrix be given by, M_e, where,

$$M_e = \begin{pmatrix} 0 & \sqrt{m_e m_1} \\ \sqrt{m_e m_1} & m_e - m_1 \end{pmatrix}. \quad (42)$$

This mass matrix is diagonalized by an orthogonal matrix , O_e, where,

$$O_e = \begin{pmatrix} cos\phi_1 & -sin\phi_1 \\ sin\phi_1 & cos\phi_1 \end{pmatrix}, \quad (43)$$

Where , $tan\phi_1 = \sqrt{\frac{m_1}{m_e}} = \sqrt{\frac{2.120957}{0.51099 \times 10^6}} = 0.002037$, (44)

From the above we note that, $\phi_1 = 0.116730\ degree$. (45)

Let the mass matrix for the muon and its neutrino be given by the matrix, M_μ where,

$$M_\mu = \begin{pmatrix} 0 & \sqrt{m_2 m_\mu} \\ \sqrt{m_2 m_\mu} & m_\mu - m_2 \end{pmatrix}. \quad (46)$$

The above mass matrix is diagonalized by an orthogonal matrix $O_\mu(\phi_2)$,

$$\tan\phi_2 = \sqrt{\frac{m_2}{m_\mu}} = \sqrt{\frac{2.131045}{105.65839 \times 10^6}} = 0.000142 \ . \quad (47)$$

The angle $\phi_2 = 0.008137$ degree. (48)

The absolute mass eigen -state of the electron-neutrino ν_1 and the absolute mass eigen-state of the muon -neutrino ,ν_2 mix together in the following way while propagating:

$$\nu_e = \nu_1 cos\vartheta_1 - \nu_2 sin\vartheta_1$$

$$\nu_\mu = \nu_1 sin\vartheta_{\ 1} + \nu_2 cos\vartheta_1 \ , \tag{49}$$

Where, $\vartheta_1 = \ \phi_1 - \phi_2 = 0.108593$ degree . (50)

In view of the mixing of $\nu_e and \nu_\mu \ with$ the mixing angle ϑ_1 the relative Phase of $\nu_e \ and \ \nu_\mu$ changes because of the mass difference so that a neutrino originating as ν_e has a non-zero probability of being detected as ν_μ .If an electron-type of neutrino is propagating with momentumP_e at time t=0,it will have a probability of oscillation $P_1 = \ P_{vetov\mu}$, where,

$$P_1 = \ sin^2 2\vartheta_1 sin^2 \left[\frac{1.27\Delta m^2 L}{E_e}\right] . \tag{51}$$

In the above, ϑ_1 is given by Eq. (50), and ,

$$\Delta m^2 = m_2^2 - m_1{}^2 = (2.131045eV)^2 - (2.120957eV)^2 \ . \tag{52}$$

Moreover $E\ _e$is the initial energy of the electron- neutrino in GeV and L is in km.[6].

5. Tau-neutrino Mixing

We consider exactly a similar mass matrix for the Tau and its neutrino:

$$M_\tau = \begin{pmatrix} 0 & \sqrt{m_3 m_\tau} \\ \sqrt{m_3 m_\tau} & m_\tau - m_3 \end{pmatrix} . \tag{53}$$

The above mass matrix is diagonalized by the orthogonal matrix O_τ,with

$$O_\tau = \begin{pmatrix} cos\phi_3 & -sin\phi_3 \\ sin\phi_3 & cos\phi_3 \end{pmatrix} \quad . \tag{54}$$

$$\tan\phi_3 = \sqrt{\frac{m_3}{m_\tau}} = \sqrt{\frac{25.65}{1777}} = \quad 0.120143 \quad . \tag{55}$$

And $\phi_3 = 6.850874\ degree$. (56)

The absolute mass eigen -state of the electron-neutrino ,ν_1 and the absolute mass eigen-state of the Tau -neutrino ,ν_3 mix together in the following way while propagating:

$$\nu_e = \nu_1 cos\vartheta_2 - \nu_3 sin\vartheta_2$$

$$\nu_\tau = \nu_1 sin\vartheta_2 + \nu_3 cos\vartheta_2 \ , \tag{57}$$

where, $\vartheta_2 = \ \phi_3 - \phi_1 = 6.734144$ degree . (58)

In view of the mixing of $\nu_e and \nu_\tau\ with$ the mixing angle ϑ_2 the relative Phase of $\nu_e\ and\ \nu_\tau$ changes because of the mass difference so that a neutrino originating as ν_e has a non-zero probability of being detected as ν_τ .If an electron-type of neutrino is propagating with momentumP_e at time t=0,it will have a probability of oscillation $P_2 = \ P_{\nu eto\nu\tau}$, where,

$$P_2 = \ sin^2 2\vartheta_2 sin^2\left[\frac{1.27\Delta m^2 L}{E_e}\right] . \tag{59}$$

where , ϑ_2 is given by Eq. (58), and ,

$$\Delta m^2 = m_3^2 - {m_1}^2 = (25.65 \times 10^6 eV)^2 - (2.120957eV)^2 \ . \tag{60}$$

Moreover $E\ _e$is the initial energy of the electron- neutrino in GeV and L is in km.[6].

In a similar way ,a muon-neutrino ,originating with an initial energy E_μ GeV will have a probability of oscillation $P_3 = P_{\nu\mu to\nu\tau}$, where

$P_3 = \ sin^2 2\vartheta_3 sin^2 \left[\frac{1.27\Delta m^2 L}{E_\mu}\right]$. (61)

Here, $\vartheta_3 = \ \phi_3 - \phi_2 = 6.842737 \ degree,$ (62)

And, $\Delta m^2 = m_3^2 - {m_2}^2 = (25.65 \times 10^6 eV)^2 - (2.31045 eV)^2$

It will be noticed that $P_2 \approx P_3$ whenever the initial energy and L are of equal values .

6. M-neutrino Mixing

We consider a mass matrix exactly similar to Eq.(53) with the mass of the M-neutrino (25.77 MeV), and the mass of the charged M-lepton(367.44Gev).This mass matrix is diagonalized by the orthogonal matrix O_4, where ,

$$tan\phi_4 = \sqrt{\frac{25.44\times10^6}{364.44\times10^9}} = 0.008375 . \quad (63)$$

$$\phi_4 = 0.479841 \ degree . \quad (64)$$

A Tau- neutrino with an initial energy E_τ GeV , will have a probability of oscillation into a M-neutrino , $P_4 = P_{\nu\tau to \nu M}$, where,

$$\mathrm{P}_4 = \ \sin^2 2\vartheta_4 \sin^2 \left[\frac{1.27\Delta \mathrm{m}^2 \mathrm{L}}{\mathrm{E}_\tau}\right] . \quad (65)$$

where, $\vartheta_4 = \phi_3 - \phi_4 = 6.371033$ degrees . (66)

$\Delta m^2 = m_3^2 - {m_2}^2 = (25.77 \times 10^6 eV)^2 - (25.65 \times 10^6)^2.$ (67)

Similarly an electron neutrino originating with an initial energy E_e GeV, at a distance of L km from the observation location has a probability , $P_5 = P_{\nu e to \nu M}$,

$P_5 = sin^2 2\vartheta_5 sin^2 \left[\frac{1.27\Delta m^2 L}{E_e}\right]$,

(68)

where, $\vartheta_5 = \phi_4 - \phi_1 = 0.363111$ degrees . (69)

with $\Delta m^2 = (25.77 \times 10^6)^2 - (2.120957)^2$. (70)

The muon-neutrino like-wise oscillates into a M-neutrino ,$P_6 = P_{\nu\mu to\nu M}$, with $\vartheta_6 = \phi_4 - \phi_2 = 0.471704$ degrees , and

with $\Delta m^2 = (25.77 \times 10^6)^2 - (2.1301045)^2$, giving,

$P_6 = sin^2 2\vartheta_6 sin^2 \left[\frac{1.27\Delta m^2 L}{E_\mu}\right]$. (71)

Experiment will definitely confirm all that is derived here.

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