Algebraic-Geometry Tools for Particle Physics

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Abstract

The TOI Platonic groups already improved the SM allowing to explain what fermion generations, quark flavors and the CKM and PMNS mixing matrices are and how to compute them theoretically. After the unification of the fundamental forces, including Gravity,the next step is to remodel Electroweak Theory as a theory of transitions of states. The tools needed to extend the Membrane Theory from Platonic and Archimedian Klein geometries, to account for the nuclear shells, baryon spectrum etc.: modular curves, Belyi pairs, dessin d'enfant and Belyi morphisms.

This approach allows to interpret mass as monodromy (curvature of quark fields connection of EM type), introducing many other finite groups: Galois groups and associated tools (Riemann surfaces, divisors, periods, tessentalions, Fuchsian groups etc.).

In this way an intrinsic" String Theory touches base with the Standard Model.

Preamble

Some of the main mathematical frameworks for fundamental Physics have evolved on three main "Mathematics Channels": 1) Lagrangian Mechanics, Hamiltonian Mechanics, Semi-Riemannian Geometry (Einstein's GR), Cartan's Moving Frame (Diff. Geometry "upgrade" of GR); 2) Weyl's Gauge Theory (EM: scalar, spinorial and QED), Yang-Mills; 3) String Theory, M-Theory... with links between them: Kaluza-Klein (GR, Gauge Theory, ST); universal language: Quantum Computing (Quantum Turing Machine); Cartan Geometry (GR and SM).

The connection between Classical and Quantum Physics relies on a gauge theory / Cartan Geometry parallel, to obtain Space and Time as emerging from "internal" quantum description of phenomena.

The "bridge" between SM and String Theory methods rely on introducing Belyi maps and morphisms, together with finite groups (analog to "lattice models", incorporating symmetries and their representations).

An inspection of the phenomenology in Elementary Particle Physics shows the adequacy of using "old" mathematical frameworks (especially Cartan Geometry and Riemann Surfaces, in the context of Belyi Theorem), yet "post-gauge theory", to understand Particle Physics (SM), especially states, transitions, mass and lifetimes.

This allows to relate with the Network Model of Quantum Computing and Quantum Information Dynamics, at a software and hardware level (e.g. superconducting Q-circuits technology for Q-Computers).

The article attempts to better understand the relation between lepton masses, j-invariant and the Monster Group, as briefly mentioned in [7]; see also [35] for a preliminary discussion.

Further considerations and speculations are recorded, for future investigations.

1 Introduction

Recall that new developments of the SM were obtained during last couple of decades[1]: A) finite gauge groups (3rd quantization), extend and reflect the quantum aspects of our Universe [4, 5, 3], allowing to compute CKM and PMNS mixing flavor matrices and the Weinberg angle; B) The Theory of Gravity

[1] Not yet officially" reported and integrated within The Standard Model.

of a quantum origin, part of the divergence component of the quark fields of nucleons (of EM type) [11, 9], spin direction dependent, hence controllable [10, 12], as the observation [21] and experiments have demonstrated [13, 14]; C) Unification of Electroweak Theory and QCD, by recognizing that quarks are not independent, yet interaction particles, but rather spin directions of a nucleon, with fractional electric and magnetic charge, hence $SU(3)$ is the symmetry group of a Cartan moving frame associated to the $U(1) \to SU(2)$ gauge theory of EWT [15]; D) Emergence of time from quantum phase and space from color quark fields [36, 16], in the gauge theory / Cartan Geometry of Moving Frame framework, *relating Quantum Theory from Classical Theory*, not duplicating it like in String or M-Theory, with extra external dimensions.

Further observations and "pieces of this puzzle" that need to be put together will be also reported below.

1.1 Quantum-Classical Duality

The combination of gauge theory approach with Cartan Geometry approach using the moving frame (RGB quarks and T-leptons as EM $SU(2)$-connections) provides a powerful tool to model and understand both aspects of our world: Quantum and Classical. We emphasize that the principal bundle approach allows to relate the Quantum internal space modeling and the Classical external emerging Space-Time theory, in a duality transporting the uniqueness at the quantum description (Universal Quantum Computing, together with uniqueness of bi-invariant connections), to the classical models, via *soldering*.

This duality is a point dependent, differential geometry version of Poincare duality (e.g. $Z \to R \to S^1$), and reflected in several Fourier Transform theory theorems with physical meaning.

1.2 Finite Groups

Moreover the Platonic finite group approach to flavors and generations leads to an understanding of the Weak Force as modeling transitions of geometry and QCD those of vibrational modes (Quantum Flavor Dynamics joins QCD).

The introduction of finite subgroups of $SU(2)$ allowed to compute the CKM and PMNS mixing matrices, together with the Weinberg angle of the SM, as a confirmation for the correctness of the "3rd quantization" postulate, introduced by the other [3], and bringing Platonic solids yet again to forefront of Science.

Previous work explained what quarks are, that colors are labels of a Cartan geometry frame, and also suggested how to unify the fundamental interactions, including Gravity, and having as a consequence the emergence of relativistic time and space etc.

1.3 Next step

The next step needed, is to understand what mass is and to redesign ElectroWeak Theory, from a theory of the Weak Force, previously called Quantum Flavor Dynamics, into a theory of transitions of baryonic states with mesonic bonds (Nuclear version of Chemistry reactions) and to reformulate QCD as a theory of modes of "vibration" of nucleons, similar to the theory of electron states (orbitals) and transitions, in an atom, and overlapping significantly with EWT at the level of W, Z-bosons Weak Force interaction and One Boson Exchange model of Nuclear Force via flavor mesons (pions etc.) and gluons exchange.

1.4 Mathematics involved

In [5] it was suggested that the Monster Group governs all physics, starting from the observation that lepton masses for the three TOI generations correspond to normalization constants of Klein's j-invariant associated to Platonic solids. In a preliminary presentation [35] it is argued that these constants are rather related to modular curves and congruence subgroups $PSL_2(F_p)$ series, associated to the modular curves as models for excited states of baryons.

It is no surprise that the modular group $PSL_2(Z)$ (integral Mobius transformations, a.k.a. Lotentz transformations) is central here (lattices $L \to C$, elliptic curves etc.) and that the "faces" of a baryon, as a "membrane", vibrates at $Z/n \to U(1)$ "frequencies" (see [36] for the emergence of relativistic time from EM quantum phase).

1.5 From "Force Physics" to "Symmetry Physics" and Geometrization

In this way many other finite groups enter the Elementary Particle Physics arena[2], beyond our basic TOIs (related to generators of $SU(2)$) and we may rephrase saying that Algebraic-Geometric-Number Theory is the Ultimate Physics Theory.

In a context where everything is quantized, the "continuum curvature" of a classical connection gets quantized to points of ramification and monodromy; this explains how Group Theory and their representations, Topology and Geometry starts to the replace and dominate the traditional Differential-Integral approach to modeling.

The intermediary link from SM to the proposed algebraic-geometric model is provided by Cartan Geometry (connections and moving frames), where the local Klein Geometry model is enriched with the Belyi map context (discrete data, electric and magnetic poles from the quark model etc.).

1.6 ... and Quantum Computing

This is quite natural when thinking of the Universe as a Quantum Turing machine, as introduced by Paul Benioff in 1980 [17].

On the other hand the power of braided categories as models for QC is again inviting towards algebrizing the Quantum Physics Models and shifting towards the Network Model for a Quantum System.

1.7 Masses, Topological degree and "oscillating dimensions"

An effective theory of masses of elementary particles is already in place [19, 20]. The masses are computed exhibiting a hierarchy in terms of "powers of alpha" α^D, the finite structure constant and what was interpreted as an "oscillating" number of compactified dimensions in String Theory context.

1.8 ... and Fine Structure Constant

This should be related to ramification indexes and topological degree of the Belyi map, each monodromy involving a quark field of EM type (for each color) and hence the ratio between the electric and magnetic charge (fluxon) $\alpha = (e/c)/(h/e)$.

Why 1/137 could be related to the Monster Group, maybe (Ogg Th. and $PSL_2(F_p)$ [35]); or just how the RGB and T-connections (quarks and electron, $U(1) \to SU(2)$) relate in TOI Cartan Geometry local model (via ADE-correspondence), with a dependence on p accounting for the "running constant" feature.

1.8.1 Universal constant "c"

The "speed of light" c reflects Hodge duality (related to $E = \epsilon D, B = \mu H$), and perhaps should be understood in terms of Selberg-Witten self-dual equations, before investigating how it determines the emergent Space and Time split $1/c^2 = \epsilon \cdot \vec{\mu}$ in EM-type of connections (RGB and T).

Recall that $B = \nabla \times \vec{a}A$ measures the monodromy of quantum phase (quantized fluxon $g_M = h/e$ as an *AG-period*):

$$\Delta\phi = \frac{1}{h/e} \oint \vec{A} d\vec{r},$$

and hence emergent local proper time. How this is related with $\oint \phi dt$ is a "de Broglie-Feynman" related interpretation (*exp*($i\omega t$ "frequency" of the EM-connection?), not clear at this time.

[2]Their role is more important: gauge-groups / monodromy, Galois groups / algebraic fundamental groups etc..

1.8.2 c and Planck's constant

Planck constant h is the deformation parameter (Heisenberg group and CCR) used to quantize classical formalism, e.g. in deformation quantization. It "deforms" the symplectic structure, associated with canonical external coordinates q and p.

The "speed" of light c is in fact another *deformation parameter*[3], corresponding to the *central extension* $R \to H \to (R^3, \times)$ which define quaternions as a Lie algebra structure, with applications to Lorentz transformations $H = \{ct + xi + yj + zk | x, y, z, t \in R\}$.

The correspondence $C \times C \cong H \cong R^{1,3}$ is important, but possibly misleading, as being just the tip of the iceberg relating $SU(2)$ (qubits in QC), $SL_2(C)$ / Mobius transformations and Lorentz transformations $SO(3,1)$, including space rotations [18].

While Heisenberg's use of h quantizes T^*R^3 (symplectic structure with 3+3 dimensions), $1/c$ deforms also Galilei's group into Lorentz transformations ($c \to \infty$ limit), for Space-Time (3+1 dimensions). Hodge structure in 3+1 EM is a related duality (differential forms / currents). Then the fine structure constant $he^{-1}\alpha = ec^{-1}$ seems to relate the two quantization-deformations, related to electric field (divergence and time generator T) and magnetic field (curl and magnetic charges, the three RGB space generators).

The framework suggests the presence of duality (e.g. Hodge) and a bi-algebra deformation-quantization framework (quantum groups).

Why the ratio of $g_e = e/c$ and g_M is approx. 1/137 is the question which may be now closer to be answered. It requires a better understanding of mass in terms of curvature of the EM connection and magnetic vector potential $P = p - e/cA$, as part also of the generalized momentum for particle (source) and field (interaction).

1.9 ... and lifetimes

In addition, the lifetime of such a state, sort of a quality factor (energy "loss" / energy level quantum jump, per quantum phase period / proper time quanta) should be correlated with the initial-final state transition, with its change in geometry (Galois group correspondence; Riemann-Hurwitz Theorem).

How these relate to the current computations in the SM, remains to be determined.

1.10 Electric Charge Asymmetry

Not only the electron's charge e ($U(1)$-period for "T-quark" e as a generator of $U(1)$), taken as negative is essentially different from proton's p^+ positive total charge, but also the way it breaks into fractional charges of the RGB quark fields $diag[+2, +2, -1]$, with $+2/3$ vs. $-1/3$ shows an asymmetry which can be traced back to the isoclinic embedding of $U(1) \to SU(2)$ (square roots: ± 1; Galois group of $Z[i]$) and center of $SU(3)$ (symmetry group of $SU(2)$-Cartan moving frame of RGB quarks / simple roots with Cartan matrix), i.e. cubic roots $1, \omega, \omega^2$ (part of the cyclotomic units of $Z[\omega]$).

How this relates to Platonic rank 3 Lie algebras $(A, B/C, H)$ and Weinberg angle, which "should be" $\pi/6$ [8]?

Does this help understand 1/137?

1.11 ... and Neutron

It seams that the neutron is a "super-symmetric" state for baryons (matter nodes) and should be modeled as Hopf bundle / Klein Geometry local model for Cartan (moving frame) Geometry.

An interaction (Gravitational!?) via a neutrino (fermion?) breaks the symmetry. This is currently modeled as a beta decay in EWT, via W boson / weak force.

But the "coincidence" of its electric charge structure $[2, -1, -1]$ (Compare with Cartan matrix) is inviting for a deeper math model. The relation with the above charge asymmetry is also important.

But there is a need for "new mathematical tools", which will be briefly mentioned in this article, together with the intended use and ideas envisioned, in order that Particle Physicists to develop the SM.

[3] A central extension is an infinitesimal deformation.

2 Algebraic-Geometric Approach to SM

2.1 Background

For a basic introduction of the main concepts: modular group, modular curves, dessins d'Enfant, Belyi maps/functions, j-invariant see Wikipedia.

For the connection between Klein's syzygy and the j-invariant [34] see [1].

Moreover, for how the tetrahedron is realized as a dessins d'enfants, via a Belyi map whose j-invariant involving the Hessian, covariant and Discriminant yield the syzygy where 1728 (normalization of j-invariant) enters the picture (the mass of the taon [5]), see [1].

To see why the Monster is relevant, see [2]; the prime factors of its size are the primes for which the modular curves are spheres (Ogg's Th.). But there is much more to this, via Belyi pairs and morphisms, from a physics point of view, as explained in a preliminary presentation [35].

2.2 Invariant Theory

The preliminary alanysis shows that the lepton masses as coefficients are rather related to the Invariant Theory of the corresponding symmetry groups. In other wors, given a finite group, typically a Galois group of a Belyi function for a regular dessin d'enfant, its invariant algebra is a polynomial algebra [22] of its states (vector space), with a set of generators, e.g. discriminant, hessian and Jacobian for cubic case, which satisfy a syzygy with some coefficients.

2.3 ... and mass

It is expected that the Galois group is related to the mass of the state (or change of it in decays / Belyi morphisms), since the ramified covering map as a flat connection has its "charges" (electric and magnetic) concentrated at these monodromy points: fractional electric and magnetic charges, called "quarks".

2.4 From Three to Several "Quarks"

The 3D dimensionality of a baryon (space of qubits S^3) in its normal excited states implies that the Lie algebra is of rank 3 ($A, B/C, H$ with root systems having Platonic symmetry [24, 23].

yet higher energy processes have experimentally detected the presence of several quarks and anti-quarks in a proton [25]. It is natural to associate these centers with a *divisor* on a Riemann Surface, and relate Feynman Diagrams approach with the Theory of Periods [26], in the context of these Belyi functions / morphisms, as models of "weak force decays".

2.5 Colors vs EM-Charges

Note that we still have only three colors, RGB, corresponding to the 3 generators of $SU(2)$ and 3D-emerging space, with time emerging from $U(1)$; but the distribution of EM-charges ((A_C, ϕ_C)- potentials for $C = R, G, B$) may be more complex, involving several "quark and anti-quark fields" in a baryon, but with a total baryon number 1 (3 basic quarks, as divisor in Belyi Theorem).

3 Modular Curves for Baryons and Decays Model Building

The role of Platonic solids and their symmetry groups as models for fermion generations and quark flavors was recognized since the 1990s [4]. That "everything is quantized" (locally finite) was coined as the "3rd quantization" in [3], justifying the quantization prescriptions, including why the angular momentum is quantized.

Recently it become clear that the "particle zoo" for baryons is just the result of the possible geometries for nucleon, together with its "vibration modes". It remains to provide a concrete model for these Klein geometries and excited states. The mesons as bonds between baryons will follow suit.

We will briefly mention the main idea of using Belyi maps to model baryons, their flavor geometry and QCD resonant modes, based on the preliminary work "The Beauty and The Beast" [35] (for more regarding the connection with the Monster Group and genus zero modular curves, for its prime divisors - Ogg's Theorem - see loc. cit.).

We will then point at the possible use of String Theory, without super-symmetry, to develop the theory of decays and formation of higher energy states for baryons, the corresponding processes of interaction and the modes of "vibration" we call resonances.

3.1 Baryon Geometry and Modes of "Vibration"

The main concepts are introduced, without further developments at this time.

3.1.1 Belyi Maps and Platonic Solids

We will proceed by example and references, following [49]; for further details see also [47]; the EC case is presented in [48] [4].

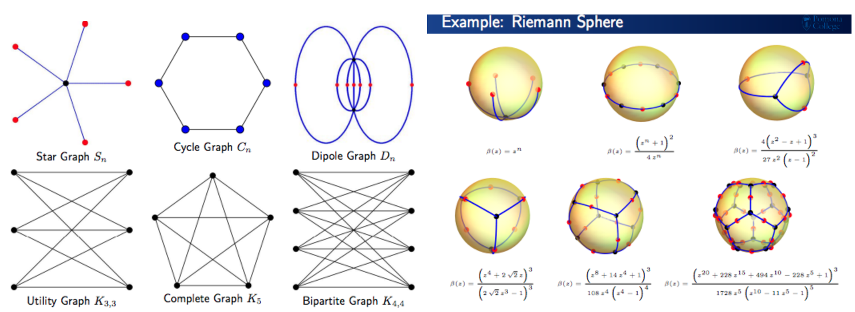
Definition 3.1 *A Belyi map associated to the finite subgroup $G \subset Aut(P^1(C))$ is a function $\beta : P^1(C) \to P^1(C)$ satisfying the following [49]:*
1) It is rational $\beta(z) = p(z)/q(z)$;
2) Has at most three critical points within $\{0, 1, \infty\}$;
3) It is invariant precisely under the G-action.

• $Aut(P^1(C)$ is the group of Mobius transformations of the sphere (conformal: preserve angles). This consists in fractional transformations $az + b)/(cz + d)$ of determinant one ($PSL_2(C)$; also physicists group of Lorentz transformations).

• The "integral part" of it $SL_2(Z)$ is the modular group! Here we focus on finite subgroups, not *congruence subgroups*, of finite index (dual?!).

3.1.2 Dessins d'Enfants

Definition 3.2 *Given a Belyi map $\beta : P^1(C) \to P^1(C)$, a Dessin d'Enfants is a connected, bipartite, planar graph $\Delta_\beta : B \to W$ with the following properties:*
1) The "Black" vertices are $B = \beta^{-1}(0)$;
2) The "white" vertices are $W = \beta^{-1}(1)$;
3) The edges are $E = \beta^{-1}([0, 1])$;
4) The midpoints of faces are $F = \beta^{-1}(\infty)$.



3.1.3 Belyi Theorem

General Belyi maps (Belyi functions) are defined on Riemann surfaces $\beta : X \to P^1(C)$ ("functionals" / "dual to R.S.).

Belyi Theorem (1979) 1 *Any non-singular algebraic curve X, defined by algebraic number coefficients, represents a compact Riemann surface which is a ramified covering of the Riemann sphere, ramified at three points only.*

[4] The author is grateful for the beautiful pictures available in [49], which contains the corresponding technical aspects.

Definition 3.3 *Such a ramified cover is called a Belyi function and (X, f) a Belyi pair.*

• A *morphism of Belyi pairs* is a morphism of ramified covers as fibrations, over the identity map of S^1.

• The 3 marked points can be interpreted as quarks, e.g. $\{1, \omega, \omega^2\}$ the cubic roots as "fractional charges".

3.1.4 Regular Solids as Dessins d'Enfant: The Tetrahedron

• Map the points of the tetrahedron B on the Riemann sphere $\sigma(B) = \{\infty, 1, \omega, \omega^2\}$ (cubic roots of 1), using the stereographic projection (see [49]).

• Define a *homogeneous polynomial* which vanishes on those points (projective coordinates (τ_1, τ_0)):

$$Y = \frac{X^3 - 1}{X} \quad \leftrightarrow \quad \delta(\tau_1, \tau_0) = 3\tau_0(\tau_1^3 - \tau_0^3).$$

• Use invariant theory to find 3 more homogeneous polynomials, and a relation between them:

$$c_4 = Hessian(\tau_1, \tau_0), c_6 = Jacobian(\delta, c_4), \Delta = Disc(\delta).$$

• The relation between them (syzygy):

$$c_4^3 - c_6^2 = 1728\Delta.$$

Remark 3.1 • *Klein's approach essentially uses dessins d'enfant and yields a polynomial which looks like the inverse of the j-invariant for an EC:*

$$\beta(z) = (c_4^3 - c_6^2)/c_4^3 \quad <-> \quad j(\tau) = 1/\beta(\tau).$$

It gives a way to relate Belyi pairs $\beta : EC \to P^1(C)$ and theory of elliptic curves (see [48]).

• *The relation between discriminant, hessian and "cov" [49]:*

a) Discriminant $\delta = \prod_{i<j}(r_i - r_j)^2$ can be written in terms of the coefficients (Viete's relation and Fundamental Theorem of Symmetric Functions), here:

$$Disc = -4A^3 - 3B^2.$$

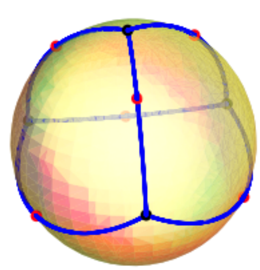
b) The Hessian matrix is $Jacobian(grad(f))$ and $det(Hess(f)) = \Delta(f)$.

c) The covariant "cov" above is a Jacobian of $(f, Hess(f)) : C^2 \to C^2$.

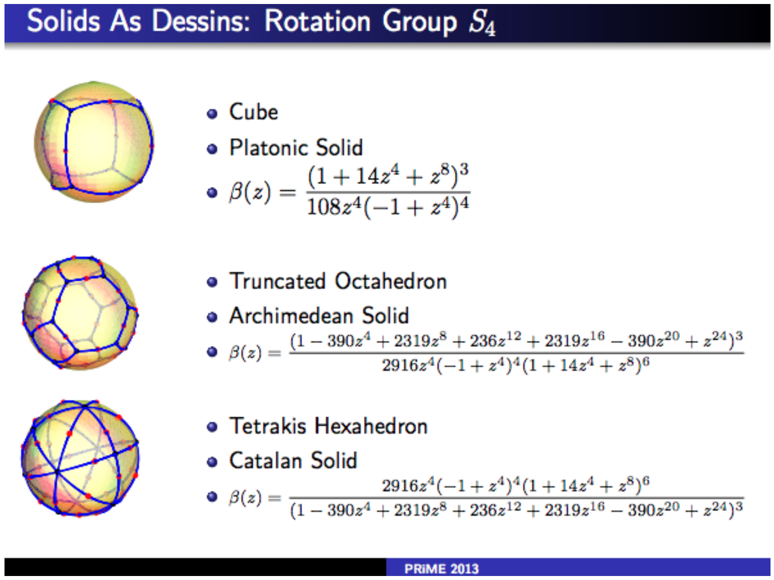
• *Problem: what is the syzygy of the three polynomials and why is it related with the j-invariant?*

Exercise: check that the Belyi map for the tetrahedron is [49]:

$$\beta(z) = \frac{c_4(\tau_1, \tau_0)^3 - c_6(\tau_1, \tau_0)^2}{c_4(\tau_1, \tau_0)^3} = \frac{64(z^3-1)^3}{z^3(z^3+8)^3} \quad \text{where} \quad z = \frac{\tau_1}{\tau_0}$$



Exercise • Compute the Belyi maps with symmetry group $O = S^4$ [49]:



3.1.5 Comparing with Lepton Masses

Pros

• The normalization coefficients include 108. Masses of course are not absolute, and both 108 and 1728 should be compared with mass ratios of leptons ($m_e \approx 1$?).

• Plotting both the coefficients $1, 108, 1728$ and the experimental data $0.511, 103.5, 1771$ gives a reasonable match (electron mass is small - exceptional?).

• Fun fact: $1729 = 1 + 1728$ is Ramanujan's taxi number [5], with many interesting properties (see Wiki: sum of cubes, Carmichael number, Loeschian norm of four 1st quadrant Eisenstein integers etc.)

... and **Cons**

• But why 1 is not there, $1/64$ occurs for the tetrahedron and 2916 for truncated octahedron? Also dual geometries, which in this author's opinion correspond to the same weak isospin (u/d-type per generation), have inverse rational functions, hence coefficients!?

3.1.6 Modular Curves, Belyi Ramified Covers and Galois Group

With modular curves $X_0^+(p) = \Gamma_0(p)/\mathcal{H}^*$ part of Belyi pairs [47], e.g. sphere, elliptic curve (with additional structure) algebraic theory meets Topology; a few aspects need be better understood in Physics Applications (SM) ...

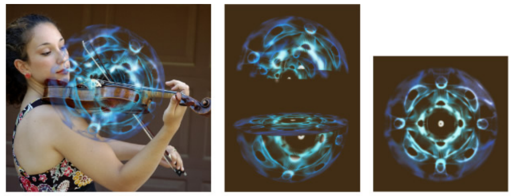
• A Belyi map / function is a ramified cover, with a group of covering transformations called *deck transformations*; when it is Galois, corresponding to the algebraic extension ("function fields") then the modular curve is called *quasi-Platonic* (regular dessin d'enfent), with a maximum number of automorphisms $84(g-1)$ [47], p.17.

• Question: is the Galois group the automorphisms group of the corresponding dessin d'enfant? (e.g. Platonic / Archemidian / Johnson polyhedron)

• How does this relate with tesselations of Riemann surfaces?

More regular solids: There are many more such 3D-cymatics modes: Archimedian, Jonson solids etc. "upgrade" of String Theory:

[5] Thank you Sunil!



Mass and Energy Levels: Think of these covering maps as *transitions between baryon states*, in analogy with electron transitions between orbitals: s,p,d, f ... (2D-cymatics / drums: see Wiki: atomic orbitals).

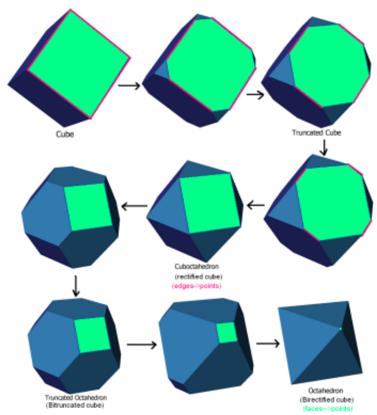
- Compute the Galois groups
- Compare with masses of baryons

Problem:

Search for a relation between mesons (transition bonds between baryons) and morphisms of Belyi maps / functions, and their Galois groups.

3.1.7 Operations on Regular Solids

13 Archimedean solids and 13 Catalan solids can be obtained from Platonic solids using seven geometric operations; example:



These operations can be algebraically recognized as Belyi maps.

3.1.8 Algebraic-Geometric Morphisms

Associate hypermaps and Belyi maps to geometric operations [49, 50]:

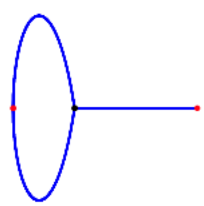
1. Hypermap of Truncation
2. Corresponding Belyĭ map

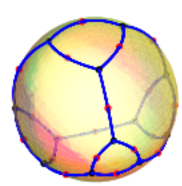
$$\phi_{\text{truncation}}(w) = \frac{(4\,w-1)^3}{27\,w}$$

3. Truncated Tetrahedron Belyĭ map

$$\beta = \phi_{\text{truncation}} \circ \beta_{\text{tetrahedron}}$$

$$\beta(z) = \frac{(1-232z^3+960z^6-256z^9+256z^{12})^3}{1728z^3(z^3-1)^3(8z^3+1)^6}$$

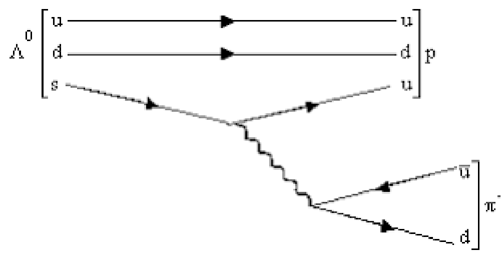




• Math: Study the corresponding Category of Belyi pairs and morphisms.

• Physics: Model Weak Force decays in this way.

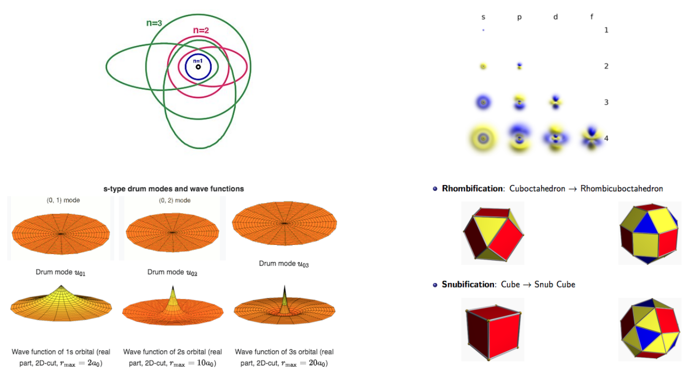
Example of weak force decay $\Lambda^0 \to p + \pi^-$, using Quark Line Diagrams:



Lambda $\Lambda(uds)$ and proton $p = (udd)$ are baryons (3-points Riemann sphere) and π^- is a meson (quark-antiquark nuclear "bond").

3.1.9 Theory of Electron Orbitals vs. Theory of Nucleon States

The theory of baryons, from the **generic** $SU(3)$ symmetry to **specific finite groups** (Platonic, Galois etc.; 3D-Modes/cymatics), is similar to the theory of the Electronic Orbitals: after Bohr-Sommerfeld Model and BEFORE the Schrodinger Equation (2D-drum modes):



3.2 On String Theory and Supper-symmetry

String Theory and its unified approach via M-Theory, developed the $U(1)$-quantum phase aspect towards an S^3-Hopf bundle version of SM's gauge theory.

Supersymmetry should rather be realized via the Network Model, incorporating topological quantum spaces together with local gauge theory (monodromy) aspects.

The total lack of experimental evidence suggests a reformulation consistent with the SM and QC.

3.2.1 String Theory

String Theory is stuck in its Kaluza-Klein format of Space-Time external dimensions $M^{3,1}$ together with compactified dimensions $CY \times U(1)$ (if 11 dimensions). The extra" dimensions are internal" i.e. true quantum dimensions, as in the principal / associated bundles of gauge theory in the SM, which means that the *base Space-Time manifold*, with its structure is emergent from these internal dimensions. The fibration approach is as close as possible to relate Quantum" Network with the continuum model of classical physics. Fortunately, this attachment to classical physics can be easily be corrected; the result is a uniqueness of connections in the bundle approach vs. the unidentified" landscape plaguing the current theory.

Claiming that it provides a theory of Quantum Gravity (quantizing GR) is not a justification: Gravity is a natural part of the SM, if the theory is developed based on observation, experiment and unification of EWT and QCD [13, 10].

3.2.2 SUSY

Super-symmetry postulates a unification of fermions and bosons, at the level of classical elementary particle approach (old paradigm). Fermions and gauge bosons are coupled in the Network Model [37] in a natural way. At the level of baryons, as nodes in the Network, Poincare duality is natural, implementing the vibration of the baryon with its mode corresponding to a dessin d'enfant (e.g. Platonic) associated to a Belyi map. Thus super-symmetry is in fact a duality of geometries / networks with underlying graphs.

The asymmetry object-morphism, parts of system and how they interact $f : A \to B$ (Category Theory) is an additional essential structure for understanding / modeling change (transitions). Forcing the interchange between fermions (sources/ models for nodes of the network) and gauge bosons (channels) is the wrong direction from the universal natural model (paradigm).

4 Geometry of "Elementary Particles"

As an analogy, *abstract groups* are built out of *simple groups*, sometimes called "elementary", which have *structure* that can be *presented* using *generators* ...

The Universe is made of *matter* and "functions" via *interactions*. The "elementary particles" of matter are baryons and leptons interacting via mesons, photons etc.

4.1 What is "Elementary" ...

We consider quarks and leptons as *gauge fermions* (a slight duplication of role for electron and alike), to emphasize their relation with *gauge bosons*, like photon $U(1)$, massive $W^{\pm}, Z^0$ ($SU(2)$), gluons ($SU(3)$) and Higgs boson. In our approach, in order to unify EWT and QCD, massive bosons $W's$ and Z are a duplication of pions $\pi^{\pm}, \pi^0$, while gluons have a structure $g_{R\bar{g}} = (g_R \bar{G_G})$. Perhaps is more natural to look for a Theory where gauge fermions have gauge bosons partners, e.g. electron and photon, and introduce as needed, e.g. quarks and color bosons (carriers of quark fields of EM type, for each "color"), instead of "doubling everything" like in SUSY approach to unification of (gauge) fermions and bosons currently present in the SM Table of Elementary Particles.

4.2 Geometry of Baryons

Baryons are "nodes" of the quantum Network and mesons are Strong Bonds between them.

QCD models processes that do not change the "flavor" of quarks, i.e. do not change the Geometry / Group of symmetry.

Hence Belyi maps should probably model the $U(1)$-vibration / $SU(2)$ spin / angular momentum theory of their "vibration modes" (strong orbitals / shells).

4.3 Geometry of Mesons

Mesons are channels / nuclear bonds, not just in nuclei, but also in EPP-processes. This is apparent from QLD and theory of Weak Nuclear Force (OBE model).

4.4 Geometry of Leptons

The measurement of a lepton (a leptonic interaction) appears as pointwise, but the Network Model and complexity of their structure (magnetic moment, decays of mesons or baryons) are indication of their internal structure, as a requirement for better models.

The electron has a structure showing correlation with baryon's structure, that is in terms of geometry of quark systems.

Hence we speculate that being correlated with baryons in EP-processes, they have a structure perhaps due to the decomposition of a 4D-polytope in 3+1 dimensions (Space structure: quark frame; Time direction and associated structure).

4.5 Decays as Quantum Processes

The Weak Interaction changes their geometry / group of symmetry, hence the Dessin d'enfant, mass and number of nodes and mesonic bonds; can this processes be modeled by Belyi morphisms?

Note that the role of massive bosons W, Z are quite similar to that of pions (and heavier mesons) in modeling Weak Nuclear Force in QCD: a merger is inviting (we provided further insight into this elsewhere).

4.6 Feynman Diagrams and Dessins d'Enfant

The correspondence between Feynman Diagrams and dessins d'enfant (DdE) [44], with their associated Belyi maps, allows to upgrade" QFT (1D TQFT) to a 2D-TQFT (String Theory but not via embeddings: rather principle bundles), based on Belyi morphisms.

What is the equivalent of alpha at the level of RS?

5 Conclusions

There are three main frameworks for EPP: Point Form QFT (Weinberg) and Feynman diagrams; 2) String THeory (S^1), Riemann surfaces and CFT; 3) SM (Quanrk Model) and Quantum Computing (Qubit Model).

The correspondence between them: Feynman diagrams as fixed points of a moduli space, representable as dessins d'enfants and Belyi maps as models for baryon states.

5.1 Finite within Continuous groups

Breaking the continuum symmetry" is a natural quantization (e.g. Pontryagin duality). It comes with the power of Hodge structures and lattice models (modular curves, Galois Theory etc.).

This allows to relate the finite groups picture" (horizontal gauge groups" / new symmetries" in Gauge Theory) and traditional approach (SM, lagrangians, Noether Th. etc.).

Comparing with the QC software and hardware, especially based on superconducting circuits (Josephson junction etc.) allows (perhaps surprisingly) to relate LEP / solid state physics and HEP / Elementary Particles Physics.

5.2 The Universal (not elementary) Particle"

The key to understand all this is a discret model for neutron (Hopf bundle, Belyi maps and Cartan connection / quark fields of EM type, with relativistic time emerging from quantum phase: unitary tangent bundle of S^2, as a local quantum model).

The continuous / finite " subgroups approach also allows to include global symmetries accounting for topological sources (charges), as monodromy subgroups. This allows to relate with the local model of a baryon as a Belyi map (ramified cover / flat connection / deck transformations as a Galois group / Klein geometry).

5.3 ... and Gravity

Gravity is expected to come "for free", as a "deformation" of EM, when resolving the pointwise charge with a magnetic dipole via RGB-T quark fields, to match observations and experiments which confirmed gravitational charge is spin direction dependent, hence not just attractive, and can be controlled.

5.4 Unification of "Fundamental" Interactions

EWT and QCD are separated theories because they are "dedicated" frameworks, responsible of explaining two classes of experiments (flavor changing and vibration modes; explaining "confinement" is what keeps them apart).

They cannot be unified as "gauge theories" per se, since there is need for more structure: Hopf bundle/ Klein geometry local model, Cartan geometry with $U(3)$ reinterpreted as a group of symmetries of the vierbein for the $SU(2)$-connection principal bundle etc. The finite subgroups (monodromy, ramifications / fractional charges etc.) come as additional structure and theoretical constructs.

5.5 ... and *Z/n*

2D finite groups (regular polygons!) are associated with finite frequencies (de Broglie etc.), e.g. of light / EM wave for defining time in terms of quantum phase via the vector potential.

Resonances also correspond to various frequencies of the faces of Archimedian solids (Johnson), simply modeled as "Bohr orbits" (or using Invariant Theory: see Baez, Dodecahedron and QM).

A Appendix: Material for Further Research

The author, rather an application oriented designer, "feels" that Feynman diagrams and periods, TQFTs and Intrinsic String Theory as a more geometric approach to modeling is closer to Nature's Mathematical Universe and SM ...

A.1 Origin of the Monster Group

Ogg's Theorem [35] seams to be closer to its existence: modular curves and Belyi functions are closer to model finite modes of vibration of baryons.

The finite Mobius groups $PSL_2(F_p)$ (rational fractional transformations $SL_2(Z) = Aut(P^1Q)$) are related to Galois groups for regular Belyi pairs. Perhaps their Category for a fixed prime p (localization) is a groupoid which happens to be a group when p is exceptional, i.e. $p||M|$, as in Ogg's Theorem (modular curve is a sphere: "principal" function field case / Hauptmodule).

It is inviting to match this cases with the "elementary baryons" and QLD as processes via meson "bonds", which are related to Intrinsic String Theory of Belyi morphisms ...

The Space-Time emerges from the Q-Theory anyways (Gauge - Cartan Geometry), and an analog of Whitney Emedeing Theorem should relate with String Theory for various backgrounds and CY from the Landscape.

A.2 Beyond Normal subgroups

Jordan-Holder Theorem is a nice general theory of the structure in the "ker-coker Theory" (good case for Category Theory).

For general Klein Geometry $H \to G$ (Cartan Theory for Lie Theory) the bundle approach is needed: a connection type of geometry, rather then algebraic (normal subgroup case).

The *core* of a Klein geometry $K \to H$ is the largest normal subgroup in G contained in H. The "fundamental interaction" case is that of an *effective Klein geometry*, with $K = 1$. It generalizes the simple group case, and leads to groupoids, an analog of *algebraic fundamental group* in Algebraic Geometry, which is an AG-version of Galois group.

In other words, Galois Theory needs to be developed beyond the "normal" (still separable) case.

For example, ramified covers which are not Galois are a good laboratory for studying "quantum" flat connections, in the spirit of gauge theory.

Then the "sphere" case for modular curves and Belyi functions should still provide "groups", as a gauge theory over bases with trivial (topological) fundamental groupoids (local quantum models for a Cartan Geometry over general Space-Time manifolds).

A.3 Relation with SM

It is a break of symmetry between left and right actions, useful in SM, where $U(1)^L_{EM}$ acts on the left and $SU(2)^R$ on the right (e.g. quaternion multiplication [6]), perhaps the origin of Weinberg angle.

This should also justify the Space-Time structure of the base manifold, with time emerging from $U(1)$-quantum phase and space emerging from $SU(2)$ rotations ($SO(3)$ fermions and bosons). Spin 1/2 and integer cases are unified via $SU(2)$ action; the 2:1 relation to Mobius-Lorentz transformations are a consequence of the $Z/2$ kernel and corresponds to a central extension (infinitesimal deformation: Heisenberg h due to "space alignment" via $SU(2)$-connection; Einstein c, resulting from time synchronization with light / $U(1)$-quantum phase; Maslov index and Witt / Virasoro algebra [38, 39]).

A.4 ... and QM, ST

There are many possibilities to be relevant to the above interpretations: $SU(2) \cong Sp_1(C)$ (unit quaternions as qubits $H = CxC$, with p, q symplectic Darbeaux coordinates and $\theta = pdq$ Poincare-Liuville 1-form / symplectic potential / tautological one form, and time / quantum phase); extension of $SL_2(Z)$ as the 3-strands braid group (3 quarks in QLD and 3-marked points Bloch sphere / quarks, in Belyi Theorem etc.).

Topologically, $SU(2) \cong S^3$ defines the Hopf fibration or alternatively Klein Geometry $U(1) \to SU(2) \to S^2$ local model of Cartan Geometry (connections and moving frame approach, enreaching the gauge theory framework of the SM, and allowing to unify $SU(2)$ and $SU(3)$, but not as in GUT, rather in a functional way, reducing the complexity by understanding their roles).

Note also that when considering $SU(2)$-Cartan moving frame we have 3 such $SU(2) \cong Sp(1)$, one per quark color RGB, related to $Sp(3)$, the hyper-unitary group in 3D, related to hyperkhaler manifolds, Calabi-Yau and String Theory.

A.5 Back to Sporadic Groups ...

Jordan-Holder property fails for fibrations [40]. The example presented is relevant to our modular curves and Belyi ramified covers, not necessarily Galois. The structure of the Category of such fibrations, with towers of fibrations as paths" of morphisms (Klein geometry), has a non-trivial fundamental groupoid" structure.

Now how the sporadic groups fit in all this, is a natural question for the Working Mathematician.

The prime factors of sporadic groups are the primes between 2 and 71, except for 59 and 61 [41].

A lead in this direction is [42]: the sporadic case is characterized as the disconnected case in some sense (existence of subgroups of order pq); not a connected groupoid case.

A.6 ... and Powers of alpha Hierarchy of Masses

The elementary particle masses data shows an alpha power hierarchy [19], *graded by dimensions of circle fibrations* interpreted as an oscillation of dimensions in String Theory [20].

This may be related to various related aspects, in the above context: Liouville action-angle variables for integrable systems, Jacobian tori of modular curves (Abel-Jacobi Theorem and period matrix) and Galois groups for Belyi functions (divisors and algebraic fundamental group) etc.

The relevance of alpha (ratio between E and M charges of quark fields of EM type, as periods) may be also related to the Laurent series of Belyi rational functions and Vir algebra (central charge and alpha?).

Recall that mass seams to be of magnetic origin": monodromy (quantum phase period $e/h \cdot \oint Adr$ / quantized magnetic field) as localized curvature (flat connection), reflected in the Galois group and associated Klein Geometry (discrete gauge theory group).

But not just the sporadic groups are relevant (genus zero): also congruence groups and modular subgroups, including $PSL_2(F_p)$.

A.7 What is Mass?

Einstein's famous equation $E = mc^2$, or rather relativistic norm $E^2 - (pc)^2 = (m_0c^2)^2$ seams incomplete in the context of the need to complete momentum to canonical momentum

$$P = p - e/c\,A \leftrightarrow cP = p - \frac{e}{c}A \quad \leftrightarrow \quad cP = cp - eA, \quad (cP)^2 = (cp)^2 + e^2A^2,$$

$$cP = cp - eA, \quad (cP)^2 = (cp)^2 + e^2A^2,$$

for gauge invariance [32], by including the field / EM-connection.

Now Space is defined by three other connections of EM type (RGB quark fields). Hence mass should be somehow related to the internal vector potentials A_R, A_G, A_B (3D-elastic body theory):

$$c^4 m_0^2 = e^2(|A_R|^2 + |A_G|^2 + |A_B|^2), \quad E^2 - (pc)^2 = e^2\|\mathbf{A}\|^2,$$

with $\mathbf{A} = (A_R, A_G, A_B)$ a 2-tensor most likely related to meson bonds, e.g. $\pi(u\bar{d})_{R\bar{G}}$ (EWT as Klein Platonic-Galois Geometry within Cartan moving frame geometry) and gluon theory of QCD, e.g. $g_{R\bar{G}}$ as gauge bosons associated to the mesons as fermionic channels / bonds (elasticity theory; not quark confinement) and quarks as sources with color charges" (space generators).

Here $1/c$ is the deformation parameter for Space-Time transformations, $g_M = h/e$ the fluxon is the unit of magnetic charge ($e/h \oint Adr$ is the quantum phase holonomy as a period). Alpha starts to crop in:

$$\alpha c = e \cdot \frac{e}{h}.$$

A.7.1 Dimensions as Generators

The power of alpha from [19] was interpreted in [20] as "oscillating dimensions" coming from String Theory (see principal and auxiliary dimensions "orbitalls").

The various mechanisms involved in Particle Physics processes are due to the gauge groups and finite structure ($Z/n \to U(1)$ and TOI in $SU(2)$, with their geometries: Platonic (3), Archimedian (Belyi) etc.

The dependence of mass and lifetime (resonance width) of "elementary particles" and channels (mesons) on a power of alpha corresponds to the number of generators involved (e.g. 3 per quark field direction). In [20], the dimension numbers d and D correspond to decomposing $11D$ in essentially $4 + 6 + 1$ due to $M^{3,1} \times Calabi - Yau \times U(1)$ structure of the "landscape".

A.7.2 Slopes and Forms: Newton vs. Leibnitz

c is considered a "speed" but it is really a structure (Hodge: internally) / light-cone distribution (externally; Lagrangian subspace). The corresponding form is, in EM Space-Time variables $Adr - \phi dt$ or in symplectic form PdQ, with 4-vector format $pdr - (E/c)(cdt)$ (or $(cp)\, dr - E\, d(ct)$).

h is a symplectic pairing (an integral): unit of action $S = \oint Ldt$ (Langrage / Hamilton) or $S = \oint pdq$ (reduced action / Maupertuis).

$e^2/4\pi\epsilon$ is the strength of E-interaction of two elementary charges ($1/r$ harmonic is universal: Poisson fundamental solution).

So, how to regroup factors in α and reformulate it in a homogeneous way, to reveal its message: what is the structure it refers to.

B On Fine Structure Constant ... and "everything else"

The RH is probably the most famous and important problem in Mathematics. In Physics, what the fine structure constant is, is arguibly the most important problem: it relates so many apparently disparated theories, from HEP to Solid State Physics ... Maybe they are related!?

Since quantum searcing algorithms are most powerful, let's try some "brain storming" speculations (or Google Searching!?).

B.1 The Structure of alpha

In a few previous articles the "internal" (quantum) description of fundamental physics was related to the "external" (classical) models: how time emerges from quantum phase, space from quark structure (colors and Cartan moving frames) etc. It attempts to relate the gauge theory bundle approach of SM with the Space-Time (base manifold) structures and theory (EM etc.).

It is natural to "separate" c the universal "coupling constant" (central extension) of Space-Time and QC / SM parameters.

hence:

$$\alpha = e^2/ch = \frac{1}{c} \cdot \frac{e^2}{h}.$$

This separates c, pertaining to Space-Time "structure" (Lorentz transformations etc.) and Hall effect resistance:

$$R_H = \frac{e^2}{h}.$$

B.2 ... and Rational Transformations

The later is related to fractional QHE and fractions $Q \to P^1Z$, to be related to the rational Riemann sphere, and of course, the modular group $SL_2(Z) \to SL_2(C)$ its automorphisms (rational fractional Mobius transformations).

These yield the sequence of simple finite groups $SL_2(F_p)$ as kernels of reduction of coefficients for SL_2 (projective transformations; conformal etc.).

In a previous presentation baryons states were conjecturally related to Belyi functions (ramified covers) realizing Platonic / Archimedian / Johnson "modes of vibration" of baryons via dessins d'enfant [35].

B.3 A "Maybe Conjecture"

This is an invitation to a study towards formulating a meaningful conjecture ...

If the fractions from QHE are finite analogues of alpha, then maybe the FSC is a syzygy between the relevant invariants, relating the Q/Classical levels of description ...

For example, lepton masses seem to be related to the syzygy between discriminant, Hessian and Jacobian for cubic equations (EC); the coefficients $1, 108, 1728$ appear when realizing dual Platonic solids, for the TOI groups, as finite symmetry groups which are the geometries of three generations and six (isospin $SU(2)$) dual quark flavors [7] (see also [35] for an alternative interpretation in terms of Belyi functions).

B.4 ... and masses

This is supported by the "power of alpha" grading of masses and lifetimes [19] (conjugate to energy widths), which can be interpreted as *impedance and Quality factors* in a Quantum Resonance analog of RLC-circuits / Q-harmonic oscillator framework.

Alternatively, [20] used electron mass (and charge e) with mass of Z^0 to compute masses as powers of alpha at exponents interpreted as linked circles $U(1)$, in a String Theory approach (Belyi functions) with "oscialting" number of dimensions (grading).

B.5 Partial conclusions

So, understanding c is still needed (Hodge structure in EM), ϵ and μ corresponding to C and L (impedance and $\frac{L}{C}$), of one hand (base manifolds, Lorentz transformations); and e^2/h on the other.

How quantization defines "fractional" ratio ($SL_2(Z)$, Farey fractions etc.) maybe the way to derive α?

Note that Shannon entropy (partition functions, Boltzmann law etc.) is just a probabilistic average of quantity of information $\log p$; and that rational numbers are a free exponential version of a Lie algebra of primes [27]:

$$exp : P \to (Q, \cdot)$$

B.6 RM and Primes

This (unproved) duality (yet documented [28]) is a Fourier Transforma correspondence which also underpins Heisenberg undertainty law, as a quantization (central extension).

B.7 Quantization and Bialgebras

Recall that the byproduct of quantization is a duality (bialgebra quantization etc.), leading to a Hopf algebra structure which binds products (creation) and coproducts (decays) of processes in general.

A rational setup is Algebraic Quantum Groups, for example Q [30].

Now that all starts from Q, and via Absolute Galois Group leads to periods that are values of Feynman integrals (amplitudes), is well known.

Also known: h is a central extension (infinitesimal deformation), c a central extension of $(R^3, \times)$ as a Lie algebra and Virasoro algebra (central extension of Witt algebra, the vector fields of the circle / string, or Riemann sphere / conformal transformations / Lorentz transformations) are also facts in this circle of ideas.

B.8 Primes and Frequencies

So, Riemann zeros should be some periods visible in the p-adic (adelic) vertion around Weyl conjectures ...

At the Physics level, QHE should relate with these via a 2D-lattice structure $L \to C$, where quantized magnetic flux (fluxons h/e) should be some periods, with rational relations between them ($SL_2(Z)$?).

B.9 RZF and Gauss periods / DFT

Riemann Zeta Function $\zeta = DS(1)$ as a Dirichlet serries (discrete Mellin transform / multiplicative FT) is inverse to Dirichlet transform of the *Mobius function* $DS(\mu)$, as a counting multiplicative function of prime powers as "fermions/bosons" (Values ± 1). MF is also the sum of primitive roots (generators) of roots of unity (corresponds to Lie algebra of Q; it is zero if n is not square free).

So R-zeros are rather poles of the Dirichlet transform of this multiplicative character, which is associated to primes ...

Now Gauss sums are DFT of Dirichlet characters of Z/n, hence a lot more structure to study (including Jacobi sums, as Hochschild cocycle of Gauss sums $J(c, c') = (d_H g)(c, c')$).

B.10 Electric vs. Magnetic Periods

$e/4\pi\epsilon$ suggests the Gauss integral $\oint_S^2 (1/r)$ as a related period, and as a grading.

2π is the 1D-period for fluxons $\oint Adr$ and appears when computing quantum phase monodromy $e/h \oint Adr$.

Hodge duality relates the two, in $C \times C$ or $R^{3,1}$; together with a Poincare duality yields the CY Hodge diamond symmetries.

B.10.1 Electric-Magnetic Duality

The Olive-Montonen duality is related to Laglands dual applied to the gauge group.

The electron and fluxon (charge and current sources) are dual to a "magnetic monopole" of quark fields origin (electric aspect) [51] and electric loop current (spin related). The electric aspect (monopole field) is responsible for Gravity, with $\pm$ charges, allowing for control of the probe's response to an electric field.

More specifically, the $U(1)$-EM, quantum phase related (emergent time), with electron and photon pair of source / gauge boson, "controls" / is related to the representation theory of $SU(2)$-quark fields of EM type (Cartan frame for baryon; RBB color theory) responsible for a magnetic monopole component due to spin "defects", associated with Gravity and macroscopic inertial mass.

These aspects are prominent in conditions of superconductibility at low temperature (solid state physics: quantum Hall effect, spin ice etc.).

B.11 Duality and Projective ratio

Now $R_H = \frac{e}{h/e}$ is the ratio between the E-charge (period) and M-charge (period). Note that there are 3-spatial quark field connections of EM type, and one "true" EM-connection for the $T - color$ we associate with the electron ...

So, why this ratio is 1/137 (energy level related) from the "speed of light", the "speed" at which "time stopes", i.e. quantum phase is constant?

$$\frac{1}{137}c = \frac{g_E}{g_M}.$$

We previously interpreted e/c is electric charge, but it seems that this is not a SI vs. Gauss units issue, and that c needs to be separated conceptually from this "quantum ratio" of periods.

B.12 ... and running constant feature

The "running constant" aspect is an indication that alpha is an "average" of some sort, similar to $\pi(x)/x$, the density of primes, which "runs" with the increase of x, leading asymptotically to $1/ln(x)$, which for primes is "exactly" the inverse of the Lie algebra "generator" $\log p$ (weight of a prime cycle F_p).

Then $\partial\beta(E)/\partial E$ should also be understood (see [31]).

B.13 More speculations

If indeed α is related to RH (duality between primes and zeros), solving one problem, will clarify the other ... and make both problems much harder too! Although some Math-Physicists made very good use of analogies, to "import" Physics to solve Math problems ...

Thus a certain "program" emerges:

1) Understand Low Energy Physics (superconductibility, Meissner effect and abelian Higgs mechanism, QHE etc.);

2) Relate with HEP, e.g. using Bohr model to understand QHE ($Z/n \to U(1)$ and Chinese R. Th.: refine Bohr's model);

3) From Lie Theory to NT: from $SU(2)$-Spin theory, by including finite subgroups and modular functions/ curves (Belyi functions, Galois groups and quantized B-field as monodromy of A / connection);

4) Relate "Above as bellow": $P \to M$ (Quantum / Classical), using "infotronics-spintronix" / electronics analogy;

Since the "Universe is Mathematical", Riemann zeros should be quantum periods, and alpha a (LIe algebra?) syzygy between c and e and h/e (Invariant Theory applied to Intrinsic String Theory / Belyi morphisms etc.).

When expanded as a Fourier series in α (FD) could be an analog of the case with Klein invariants and coefficients and j-invariant (finite groups characters).

B.14 Primes, Rooted Trees and QED/ST

The correspondence between Feynman Diagrams and Dessins d'Enfants, with their Belyi functions, establish a bridge between EPP (including QLD etc.) and Alg. geometry / String Theory, with a model for baryon which unifies EWT (QFD) and QCD.

These hierarchy of these structures is described by rooted trees, which have the structure of a Hopf algebra(Connes-Kreimer approach to renormalization). The fact that the set of primes have a POSet structure and a correspondence with HA of Rooted Trees is no coincidence [29].

The role of alpha for QED and FD, should reflect in a more transparent way at the level of String Theory and Belyi morphisms between Belyi maps, which represent decay processes.

B.15 Conclusions

The fine structure constant reflects the correspondence between quantum description (internal space) of fundamental quark-electron fields interaction (principal bundle $P \to R^{3,1}$) and classical theories on the base, continuum Space-Time (external space), where measurements are performed and physical units defined.

B.15.1 Is Alpha a Period?

The elementary charge density $e/4\pi$ is perhaps a better parameter when comparing with the magnetic charge $\bar{h}/e$:

$$\frac{g_E}{g_M} = \frac{e/4\pi}{h/(e/4\pi)} = \frac{(e/4\pi)^2}{h} \sim c \quad (4\pi)^2 \sim 158 \leftrightarrow 137; \qquad 1/c^2 = \epsilon \cdot \mu \quad (\textit{Hodge str.})$$

A factor of 3 should also be considered, coming from the three quark fields of EM type vs. one for the electron. The presence of ϵ and μ accounts for permitivity and magnetization, which "hide" quantum effects as effective corrections to Maxwell's equations, in terms of E, D, H, B and P, M (see Wiki: $D = \epsilon E + P$ etc.).

Note that a pointwise treatment of electric (div / work) and magnetic (curl / curvature, monodromy) fields cannot account for the non-isotropic effects of the baryon field, even if multi-pole corrections are included (needs a "non-abelian Fourier expantion", in terms of representations of $SU(2)$). The "fractional charges" aspect is not equivalent with three pointwise charges of $+2/3$ and $-1/3$ (Coulomb / $SO(3)$-symmetric).

B.15.2 Dirac Quantization and Hopf Fibration

It is an essential structure in the SM: qubit space, Klein geometry model for Cartan Moving Frame approach, QC etc.

Dirac quantization of electric charge $eg = \bar{h}$ [33] seems just a cup product relation between spheres involved: S^1 for magnetic charge (Fluxon and Ampere's Law), S^2 for electric charge (Gauss Law) and quantum of action as a triple integral, in the following vein:

$$eq = h \quad \leftrightarrow \quad \oiint 1/r \cdot \oint Adr = \oiiint_{S^3} dV ...??^{6}.$$

We now know magnetic charge (fluxon) is due to the "current of vector potential on a loop (Wilson loop, monodromy etc.), and it is Hodge dual to electric charge (related to cup product).

Their ratio should be related to the T vs. RGB quark connections, with a simple $1/3 \cdot 1/3$ factor involved. Then $4\pi\alpha \sim 1/12.5$ should be comparable to $1/9$.

The precise correspondence should come from representation theory of finite groups (modular curves and $PSL_2(F_p)$) and $SU(2)$ with its $SU(3)$ symmetry group of the quark fields frame (Cartan Geometry). Including more representations could account for the dependence on energy level.

B.16 Monopoles, Spin ice and Gravity

While fluxons are magnetic currents (monodromy), magnetic monopole fields were reported as being associated to spin-spin correlations in spin ice, yielding a *Magnetic Coulomb Law* [51]. The author claims that these are the superposition of radial / electric component of the three quark fields of nuclei and responsible for Gravity of Quantum origin, as formulated in [10]. Macro Gravity is an average, solving the Hierarchy Problem [46]. The thermodynamic properties (spin orientation statistics and spin currents) are consistent with Alzofon's Theory and Verlinde's approach involving entropy).

A report on a *Modified Coulomb Law*, quite similar with the above mentioned Magnetic Coulomb Law, is presented in [45].

[6] Tentativelly from the Hopf fibration $S^1 \to S^3 \to S^2$ ...

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