Dimensional Regularization as Mass Generating Mechanism

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Abstract

Relativistic Quantum Field Theory (QFT) develops divergences caused by perturbative corrections to Feynman diagrams. Dimensional Regularization (DR) is a technique that isolates divergences using analytic continuation to *non-integer dimensions*. In this introductory tutorial we argue that DR provides an alternative mechanism for mass generation in particle physics. This mechanism reconciles the Higgs model of electroweak symmetry breaking with the minimal fractal topology of spacetime above the Fermi scale. Mass predictions agree reasonably well with experimental data.

Key words: Dimensional Regularization, fractal spacetime, particle masses, Standard Model.

In relativistic QFT, the *S-matrix* (or scattering matrix) relates the initial and final states of a physical system undergoing a scattering process. S-matrix is associated with an unitary operator S that determines the evolution between two asymptotic states at $t=-\infty$ and $t=+\infty$, respectively, [1-2]

$$\left|\psi_b(+\infty)\right\rangle = S\left|\psi_a(-\infty)\right\rangle \tag{1}$$

The basic quantities of interest are the S – matrix elements, defined through

$$S_{ba} = {}_{out}\left\langle \psi_b(t) \middle| \psi_a(t) \right\rangle_{in} \tag{2}$$

On account of (2), the S-matrix represents a set of complex amplitudes that are used to compute the probabilities of various scattering processes. In the Standard Model (SM), the scattering matrix may include arbitrary combinations of elementary stable particles such as neutrinos, electrons, and photons, as well as combinations of other stable composite particles.

The analysis of scattering processes employs the concept of *n point function* $\Gamma^{(n)}(p_1, p_2, ..., p_n)$, which corresponds to n external particles, any of each can be either *incoming* or *outgoing*. Another key concept is the particle *self-energy*

($\Sigma(p)$), which denotes the overall contribution of virtual processes that start and end with that particle.

To fix ideas, consider the textbook φ^4 model of a real scalar field described by the Lagrangian

$$L = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{g}{4!}\varphi^4 \tag{3}$$

where m is a mass parameter and g the self-interaction coupling of the field. The continuous emission and absorption of *virtual particles* generates corrections to the parameters of (3). Fig. 1 illustrates an elementary self-energy correction to the free propagator resulting from the emission and absorption of a virtual particle with arbitrary momentum *k*.

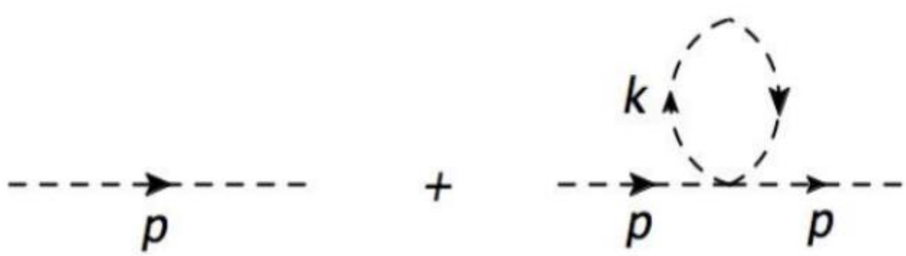

Fig. 1: Elementary self-energy correction to the free propagator.

The self-energy contribution to the free propagator takes the form [3]

$$-i\Sigma(p^2) = \frac{-ig}{2}\int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2 - m^2 + i\eta} \tag{4}$$

Like several other momentum integrals of QFT, it is apparent that (4) diverges as $k \to \infty$. The goal of the Regularization program is to bring divergences under control and enable observables of the underlying theory to stay finite.

To this end, one proceeds by choosing a *regulator,* that can be either be an *Euclidean momentum cutoff* (Λ_{UV}) or a *continuous deviation* from four spacetime dimensions ($\varepsilon = 4 - d$). In either case, the expectation is that the regulated observables of the theory consist of,

a) a regulator-dependent term containing the divergence and,

b) a sum of finite terms that is asymptotically independent of the regulator as $\Lambda_{UV} \to \infty$ or $\varepsilon \to 0$.

Regardless of the choice, the regularization method must comply with the symmetries of the theory. In a relativistic QFT, these include Lorentz symmetry, as well as local gauge and global symmetries.

We briefly review below two traditional regularization methods, namely *Pauli-Villars* (PV) and *Dimensional Regularization* (DR).

- **The PV method**

With reference to Fig. 2, consider the one-loop contribution to the 4-point function $\Gamma(s,t,u)$ in scalar field theory (3). Here, s,t,u are the Mandelstam variables defined as

$$s=(p_1+p_2)^2 \tag{5}$$

$$t=(p_1-p_3)^2 \tag{6}$$

$$u=(p_1-p_4)^2 \tag{7}$$

The $s-$channel correction to Γ is given by [3]

$$\Gamma(p^2) = \frac{(-ig)^2}{2} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\eta} \frac{i}{(p-k)^2 - m^2 + i\eta} \tag{8}$$

where momentum conservation requires

$$p = p_1 + p_2 = p_3 + p_4 \tag{9}$$

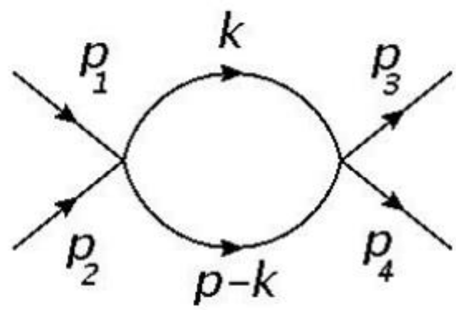

Fig. 2: One-loop contribution to the 4-point function.

The above integral contains the contribution from the two segments of the loop, the k segment, and the $p-k$ segment. After tedious manipulations, the divergent piece of the 4-point function is found to be,

$$\Gamma(0) \approx \frac{ig^2}{32\pi^2} \ln\left(\frac{\Lambda_{UV}^2}{m^2}\right) \tag{10}$$

Using similar arguments, the divergent part of the 2-point function amounts to [3],

$$\Sigma(0) = \frac{g}{32\pi^2}\Lambda_{UV}^2 \tag{11}$$

- **The DR method**

The DR method makes use of the fact that momentum integrals can be formulated as analytic functions in *continuous dimensions* defined as

$$d = 4 - \varepsilon\,; \quad \varepsilon << 1 \tag{12}$$

The d - dimensional integral corresponding to (8) can be written as,

$$I_d = \mu^{\varepsilon}\int \frac{d^d k}{(2\pi)^d}\frac{i}{k^2 - m^2 + i\eta}\frac{i}{(p-k)^2 - m^2 + i\eta} \tag{13}$$

where μ is an arbitrary mass scale whose role is to maintain dimensional consistency of (13). Calculations show that the divergent part of the 4- and 2-point functions in DR are given by, respectively,

$$\Gamma(0) = \frac{ig^2}{32\pi^2}(\frac{2}{\varepsilon}) \tag{14}$$

$$\Sigma(0) = \frac{g}{32\pi^2}(-\frac{2m^2}{\varepsilon}) \tag{15}$$

The table below summarizes the divergent parts of the 4-point and 2-point functions following the PV and DR methods, respectively.

	Pauli-Villars Regularization	Dimensional Regularization
$\Gamma(0)$	$\frac{ig^2}{32\pi^2}\ln(\Lambda_{UV}^2/m^2)$	$\frac{ig^2}{32\pi^2}(2/\varepsilon)$
$\Sigma(0)$	$\frac{g}{32\pi^2}\Lambda_{UV}^2$	$\frac{g}{32\pi^2}(-2m^2/\varepsilon)$

Tab. 1: Pauli-Villars versus Dimensional Regularization of φ^4 theory

Since the computation of any observable must give answers independent of the regularization method, observables obtained via PV and DR methods must coincide. It follows that, in the limit $m << \Lambda_{UV}$,

$$\boxed{\varepsilon(\mu) = 4 - d(\mu) = O(m^2/\Lambda_{UV}^2) << 1} \tag{16}$$

Although deceptively simple, this relationship has unexpected implications for particle physics near or beyond the Fermi scale ($v = 246\, GeV$), as well as for the non-integrable sector of gravitational dynamics and relativistic cosmology. For example, (16) leads - either directly or indirectly - to the following findings:

1) Particle masses derive from dimensional deviations $\varepsilon(\mu)$, which represent *topological polarizations* of classical spacetime above the Fermi scale [4].

2) Minimal fractality of spacetime encoded in $\varepsilon(\mu)$ implies that there is *mixing* between the ultraviolet and infrared sectors of field theory. This, in turn, means that ultraviolet dynamics makes the transition from the ordinary calculus to *fractional differential and integral operators*, reflecting the non-local, dissipative, and structure-forming nature of interactions above the Fermi scale [5].

3) The Standard Model of particle physics represents a *self-contained multifractal set*, whose flavor and mass composition follows the bifurcation

scenario of transition to chaos [4]. The following table highlights the repetitive distribution of particle masses and coupling strengths, based on the universal transition to chaos of the Renormalization Group flow [6].

Table 2: Actual versus predicted SM parameters

Parameter ratio	**Behavior**	**Actual**	**Predicted**
m_u/m_c	δ^{-4}	3.365×10^{-3}	2.104×10^{-3}
m_c/m_t	δ^{-4}	3.689×10^{-3}	2.104×10^{-3}
m_d/m_s	δ^{-2}	0.052	0.046
m_s/m_b	δ^{-2}	0.028	0.046
m_e/m_μ	δ^{-4}	4.745×10^{-3}	2.104×10^{-3}
m_μ/m_τ	δ^{-2}	0.061	0.046
$(\alpha_{EM}/\alpha_W)^2$	δ^{-2}	0.045	0.046
$(\alpha_{EM}/\alpha_s)^2$	δ^{-4}	2.368×10^{-3}	2.104×10^{-3}
$1-(M_W/M_Z)^2$	δ^{-1}	0.2215	0.2142

4) The Fermi scale, cosmological constant (Λ_{cc}) and the Planck scale (M_{Pl}) satisfy the following relationship [7]

$$\varepsilon_{\min} \propto \frac{\Lambda_{cc}^{1/4}}{v} \approx \frac{v}{M_{Pl}} \tag{17}$$

where $\varepsilon_{\min}$ denotes the minimal dimensional deviation.

5) Particle masses obey the sum-of-squares constraint

$$m_W^2 + m_Z^2 + m_H^2 + \sum_f m_f^2 = v^2 \tag{18}$$

where the last term of the right-hand side extends to the full set of fermion masses [8]. The boson and fermion contributions are divided into nearly equal shares, that is,

$$\sum_b m_b^2 \approx \sum_f m_f^2 \approx \frac{v^2}{2} \tag{19}$$

6) Taking complex-scalar field theory as baseline model points out that the SM group unfolds sequentially from bifurcations driven by the Renormalization Group scale μ [9-10]. Specifically, the following relationships are shown to hold:

$$m_H^2 = 2\lambda v^2 \tag{20}$$

$$v \approx 2m_H \tag{21}$$

$$v \approx 2m_W + m_Z \tag{22}$$

$$m_t = 173.95\ GeV \tag{23}$$

where $\lambda = \lambda(\varepsilon(\mu))$ stands for the scale-dependent Higgs self-interaction coupling and m_t is the mass of the top quark.

7) Other features of the SM (fermion chirality, mixing angles, the existence of three generations, the strong CP problem, the g-2 anomaly, the Cabibbo angle) appear to be linked to (16).

8) Baryon asymmetry is a direct consequence of (16) [11].

8) Dark Matter consists of Cantor Dust, a *cosmic web* structure formed by topological condensation (clumping) of continuous dimensions in the early Universe [12-17]. An attractive feature of this finding is that it reconciles the gravitational and particle interpretations of Dark Matter.

Additional relevant references can be located at [18].

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