

# Dimensionless theory of everything

Stergios Pellis

sterpellis@gmail.com

ORCID iD: 0000-0002-7363-8254

Greece

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## Abstract

In this paper we will present the Dimensionless theory of everything. In an elegant way we will present the Dimensionless unification of the fundamental interactions. We will calculate the unity formulas that connect the coupling constants of the fundamental forces. We will find the formulas for the Gravitational constant. It will be presented that the gravitational fine-structure constant is a simple analogy between atomic physics and cosmology. We will find the expression that connects the gravitational fine-structure constant with the four coupling constants. Perhaps the gravitational fine-structure constant is the coupling constant for the fifth force. It will present the law of the gravitational fine-structure constant followed by ratios of maximum and minimum theoretical values for natural quantities. Also we will present the Dimensionless unification of atomic physics and cosmology. We find the formulas for the cosmological constant. We will prove that the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions will propose a possible solution for the density parameter of baryonic matter, dark matter and dark energy. The sum of the contributions to the total density parameter at the current time is  $\Omega_0=1.0139$ . It is surprising that Plato used a dodecahedron as the quintessence to describe the cosmos.

## Keywords

Theory of everything , Fine-structure constant , Proton to electron mass ratio , Dimensionless physical constants , Coupling constant , Gravitational constant , Avogadro's number , Fundamental Interactions , Gravitational fine-structure constant , Cosmological parameters , Cosmological constant , Unification of the microcosm and the macrocosm , Poincaré dodecahedral space

## 1. Introduction

A theory of everything is a hypothetical, singular, all-encompassing, coherent theoretical framework of physics that fully explains and links together all aspects of the universe. Finding a theory of everything is one of the major unsolved problems in physics. String theory and M-theory have been proposed as theories of everything. Among ancient Greek and Hellenistic philosophers, notable proponents of the microcosm–macrocosm analogy included Anaximander, Plato, the Hippocratic authors and the Stoics. In later periods, the analogy was especially prominent in the works of those philosophers who were heavily influenced by Platonic and Stoic thought, such as Philo of Alexandria the authors of the early Greek Hermetica and the Neoplatonists. Since time immemorial philosophers, poets, and scientists have pondered the relationship between the microcosm and the macrocosm. This theory was started by Pythagoras who saw the universe and the body as a harmonious unity. The microcosm and the macrocosm. The relevant scale, what counts as the micro- and the mega-, has always been determined by the scientific knowledge of the time. Since Newton, the scales of the largest and the smallest have extended by ten orders of magnitude in both directions. Equally strikingly, the meanings of ‘micro’ and ‘mega’ have changed in the historical development from the unification of celestial and terrestrial mechanics, to the physical study of stars by means of spectral analysis, to the micro-physical explanation of the baryon asymmetry of the universe. It was only in the late 1910s, however, that the first physical fact was discovered that could provide a quantitative clue to the interconnection between the micro- and mega-worlds. It was a mathematician, Hermann Weyl, who made this discovery. His discovery later gave rise to such different ideas as the hypothetical variation of the gravitational constant and the anthropic principle. More cautiously, it was referred to as “an unexplained empirical connection between meta-galactic

parameters and micro-physical constants". Although this link between the micro-and mega-worlds is regarded as an empirical fact, its recognition was intertwined with developments in advanced theoretical physics. Before turning to the circumstances of the discovery of this fact, let us look at its contemporary status, which clearly points to its empirical nature. The laws of physics have a set of fundamental constants, and it is generally admitted that only dimensionless combinations of constants have physical significance. The most fully developed version of the idea in antiquity was made by Plato, but fragmentary evidence indicates that philosophers before him articulated some version of it. The idea may have begun as an archetypal theme of mythology that the pre-Socratic philosophers reworked into a more systematic form. Unfortunately, it is impossible to reconstruct their thinking in much detail, and clear references attributing the doctrine to Democritus and Pythagoras are quite late, dating to the fifth and ninth centuries C.E., respectively. Some form of the idea seems to have been common among most ancient cultures. Since comparisons of human beings and the universe were made in India and China, the concept may ultimately be of Asian origin but the available sources do not indicate that the theory in Greece was the result of cultural diffusion. Among extant Greek texts, the term first appears in the *Physics* of Aristotle, where it occurs in an incidental remark. Plato did not use the terminology when he developed the idea. The laws of physics have a set of fundamental constants, and it is generally admitted that only dimensionless combinations of constants have physical significance. These combinations include the electromagnetic and gravitational fine structure, along with the ratios of elementary particles masses. Cosmological measurements clearly depend on the values of these constants in the past and can therefore give information on their time dependence if the effects of time-varying constants can be separated from the effects of cosmological parameters. Of the fifth Platonic solid, the dodecahedron, Plato obscurely remarked, "...the god used for arranging the constellations on the whole heaven". Aristotle added a fifth element, *aithēr* (aether in Latin, "ether" in English) and postulated that the heavens were made of this element, but he had no interest in matching it with Plato's fifth solid. Euclid completely mathematically described the Platonic solids in the *Elements*, the last book (Book XIII) of which is devoted to their properties. Propositions 13–17 in Book XIII describe the construction of the tetrahedron, octahedron, cube, icosahedron, and dodecahedron in that order. For each solid Euclid finds the ratio of the diameter of the circumscribed sphere to the edge length. In Proposition 18 he argues that there are no further convex regular polyhedra. Andreas Speiser has advocated the view that the construction of the five regular solids is the chief goal of the deductive system canonized in the *Elements*. Much of the information in Book XIII is probably derived from the work of Theaetetus.

According to a recent theory the Universe could be a dodecahedron. It is surprising that Plato used a dodecahedron as the quintessence to describe the cosmos! Plato also stated that time had a beginning; it came together with the universe in one instant of creation. A polyhedron bounded by a number of congruent polygonal faces, so that the same number of faces meet at each vertex, and in each face all the sides and angles are equal (i.e. faces are regular polygons) is called a regular polyhedron. One morning the young Werner Heisenberg discovered reading Plato's *Timaeus* a description of the world with regular polyhedra. Heisenberg could not understand why Plato, being so rational, started to use speculative ideas. But finally he was fascinated by the idea that it could be possible to describe the Universe mathematically. He could not understand why Plato used the Polyhedra as the basic units in his model, but Heisenberg considered that in order to understand the world it is necessary to understand the *Physics* of the atoms. Theaetetus was a member of Plato's Academy. He was a son of Euphronius of Sounion, student of Theodore of Cyrene. Theaetetus died on his return to Athens after he was wounded at the Battle of Corinth. His friend Plato dedicated one of his dialogues to him. Euclid's *elements* chapter X and XIII are based on the work of Theaetetus. Hippasus, from Metapontum in Magna Graecia (south Italy), who wrote around 465 BC about a "sphere of 12 pentagons" refers to the dodecahedron. Hippasus performed acoustics Experiments with vessels filled with different amounts of water and with copper discs of different thicknesses.

In ancient Greek philosophy, the pre-Socratic philosophers speculated that the apparent diversity of observed phenomena was due to a single type of interaction, namely the motions and collisions of atoms. The concept of "atom" proposed by Democritus was an early philosophical attempt to unify phenomena observed in nature. Archimedes was possibly the first philosopher to have described nature with axioms and then deduce new results from them. In the late 17th century, Isaac Newton's description of the long-distance force of gravity implied that not all forces in nature result from things coming into contact. Newton's work in his *Mathematical Principles of Natural Philosophy* dealt with this in a further example of unification, in this case unifying Galileo's work on terrestrial gravity, Kepler's laws of planetary motion and the phenomenon of tides by explaining these apparent actions at a distance under one single law: the law of universal gravitation. In 1814, building on these results, Laplace famously suggested that a sufficiently powerful intellect could, if it knew the position and velocity of every particle at a given time, along with the laws of nature, calculate the position of any particle at any other time. Laplace thus envisaged a combination of gravitation and mechanics as a theory of everything. In 1820, Hans Christian Ørsted discovered a connection between electricity and magnetism, triggering decades of work that culminated in 1865, in James Clerk Maxwell's theory of electromagnetism. In his experiments of 1849–50, Michael Faraday was the first to search for a

unification of gravity with electricity and magnetism. However, he found no connection. In 1900, David Hilbert published a famous list of mathematical problems. In Hilbert's sixth problem, he challenged researchers to find an axiomatic basis to all of physics. In this problem he thus asked for what today would be called a theory of everything. After 1915, when Albert Einstein published the theory of gravity (general relativity), the search for a unified field theory combining gravity with electromagnetism began with a renewed interest. In Einstein's day, the strong and the weak forces had not yet been discovered, yet he found the potential existence of two other distinct forces, gravity and electromagnetism, far more alluring. coupling), is a number that determines the strength of the force exerted in an interaction. In attributing a relative strength to the four fundamental forces, it has proved useful to quote the strength in terms of a coupling constant. The coupling constant for each force is a dimensionless constant.

In physics, a coupling constant or gauge coupling parameter is a number that determines the strength of the force exerted in an interaction. In attributing a relative strength to the four fundamental forces, it has proved useful to quote the strength in terms of a coupling constant. The coupling constant for each force is a dimensionless constant. A coupling constant is a parameter in field theory, which determines the relative strength of interaction between particles or fields. In the quantum field theory the coupling constants are associated with the vertices of the corresponding Feynman diagrams. Dimensionless parameters are used as coupling constants, as well as the quantities associated with them that characterize the interaction and have dimensions. In physics, the fundamental interactions, also known as fundamental forces, are the interactions that do not appear to be reducible to more basic interactions. There are four fundamental interactions known to exist: the gravitational and electromagnetic interactions, which produce significant long-range forces whose effects can be seen directly in everyday life, and the strong and weak interactions, which produce forces at minuscule, subatomic distances and govern nuclear interactions. Some scientists hypothesize that a fifth force might exist, but these hypotheses remain speculative. Each of the known fundamental interactions can be described mathematically as a field. The gravitational force is attributed to the curvature of spacetime, described by Einstein's general theory of relativity. The other three are discrete quantum fields, and their interactions are mediated by elementary particles described by the Standard Model of particle physics.

## 2. Maths constants

Archimedes constant  $\pi$  is a mathematical constant that expresses the ratio of the circumference  $C$  to the diameter  $\delta$  of a circle. It is symbolized by the Greek letter  $\pi$  from the middle of the 18th century. It is referred to as Archimedes' constant because the method of determining  $\pi$  is attributed to Greek Mathematician Archimedes from the perimeter  $P_n$  of a regular polygon with  $n$  sides that describe a circle with diameter  $d$ . The number  $\pi$  is an implicit and transcendental number. Number is celebrated on the "day of  $\pi$ " and the record of counting the digits of  $\pi$  is often mentioned in news headlines. Several people tried to memorize the value of  $\pi$  as accurately as possible, leading to a record of over 67000 digits. Archimedes constant  $\pi$  appears in many types in all fields of mathematics and physics. It is found in many types of trigonometry and geometry, especially in terms of circles, ellipses or spheres. It is also found in various types from other disciplines, such as Cosmology, numbers, Statistics, fractals, thermodynamics, engineering, and electromagnetism. Also Archimedes constant  $\pi$  appears in the cosmological constant, Heisenberg's uncertainty principle, Einstein's field equation of general relativity, Coulomb's law for the electric force in vacuum, Magnetic permeability of free space, Period of a simple pendulum with small amplitude, Kepler's third law of planetary motion, the buckling formula, etc.

In mathematics, two quantities have a golden ratio  $\phi$  if the ratio of their sum to the largest quantity is equal to the ratio of the largest quantity to the smallest. A number that is often encountered when taking distance ratios in simple geometric shapes such as the pentagon, the pentagram, the hexagon and the dodecahedron. The golden ratio  $\phi$  is a transcendental number. The ancient Greek mathematicians studied for the first time the golden ratio  $\phi$ , due to its frequent appearance in geometry. He discovered that the golden ratio was neither an integer nor a fraction. The golden ratio is important in the geometry of regular pentagrams and pentagons. The golden word was known to the Pythagoreans. In their secret symbol, the pentacle, the golden word appears on the sides of a star as well as on the quotient of the area of the regular pentagon with vertices at the edges of the pentagon to the area of the regular pentagon. Pheidias (500 BC-432 BC), the Greek sculptor and mathematician applied the golden ratio  $\phi$  to the design of sculptures for the Parthenon. Plato (Circa 428 BC-347 BC), in his views on natural science and cosmology presented in "Timaeus" considered the golden section as the most binding of all mathematical relations and the key to physics. Euclid's Elements provide the first written definition of what we now call the golden ratio. Euclid cites one for dividing the line into end and middle ratio. In the 20th century, the golden ratio is represented by the Greek letter  $\phi$ , from the initial letter of the sculptor Pheidias who is said to have been one of the first to use it in his works. The golden ratio was studied regionally during the next millennium (850-930) and used in the geometric calculations of pentagons and decagons. 18th century mathematicians Abraham de Moivre, Daniel Bernoulli and Leonhard Euler used a formula based on the golden ratio that finds the value of a Fibonacci number based on its placement in the

sequence. Golden ratio  $\phi$  is an omnipresent number in nature, found in the architecture of living creatures as well as human buildings, music, finance, medicine, philosophy, and of course in physics and mathematics including quantum computation. It is the most irrational number known and a number-theoretical chameleon with a self-similarity property. The golden ratio can be found in nearly all domains of Science, appearing when self-organization processes are at play and/or expressing minimum energy configurations. Several non-exhaustive examples are given in biology (natural and artificial phyllotaxis, genetic code and DNA), physics (hydrogen bonds, chaos, superconductivity), astrophysics (pulsating stars, black holes), chemistry (quasicrystals, protein AB models), and technology (tribology, resistors, quantum computing, quantum phase transitions, photonics). The fifth power of the golden mean appears in Phase transition of the hard hexagon lattice gas model, Phase transition of the hard square lattice gas model, One-dimensional hard-boson model, Baryonic matter relation according to the E-infinity theory, Maximum quantum probability of two particles, Maximum of matter energy density, Reciprocity relation between matter and dark matter, Superconductivity phase transition, etc.

Euler's number  $e$  is an important mathematical constant, which is the base of the natural logarithm. It is the limit of the sequence  $(1+1/n)^n$  as  $n$  approaches infinity, an expression derived from the study of compound interest. Euler's number frequently appears in problems related to growth or decay, where the rate of change is determined by the present value of the number being measured. Euler's number has many practical uses, especially in higher level mathematics such as calculus, differential equations, trigonometry, complex analysis, statistics, etc. One example is in biology, where bacterial populations are expected to double at reliable intervals. Another case is radiometric dating, where the number of radioactive atoms is expected to decline over the fixed half-life of the element being measured. The number  $e$  is sometimes called Euler's number, after the Swiss mathematician Leonhard Euler or Napier's constant. However, Euler's choice of the symbol  $e$  is said to have been retained in his honor. The constant was discovered by the Swiss mathematician Jacob Bernoulli while studying compound interest. Like the constant  $\pi$ ,  $e$  is irrational (that is, it cannot be represented as a ratio of integers) and transcendental (that is, it is not a root of any non-zero polynomial with rational coefficients). The number  $e$  is prominent in mathematics, along with  $0$ ,  $1$ ,  $\pi$  and  $i$ . All five of these numbers play important and repetitive roles in mathematics and these five constants appear in a formulation of Euler's identity. Euler's number has many practical uses, especially in higher level mathematics such as calculus, differential equations, trigonometry, complex analysis, statistics, etc. From Euler's identity the following relation of the mathematical constant  $e$  can emerge  $e = i^{-2i/\pi}$ . The history of the mathematical constant  $e$  begins with John Napier (1550-1617) who defined logarithms through a process called dynamic ratio. However, this did not contain the constant itself, but simply a list of logarithms calculated from the constant. That the table was written by William Oughtred. The first known use of the constant, represented by the letter  $b$ , was the correspondence from Gottfried Leibniz to Christiaan Huygens in 1690 and 1691. However the discovery of the constant itself is credited to Jacob Bernoulli in 1683. Leonhard Euler introduced the letter  $e$  as the basis for natural logarithms, writing in a letter to Christian Goldbach on November 25, 1731. Leonhard Euler (1707-1783) named his letter after the constant  $e$  and discovered many of these remarkable properties.

The imaginary unit  $i$  is a solution to the quadratic equation  $x^2+1=0$ . Although there is no real number with this property, it can be used to extend the real numbers to what are called complex numbers, using addition and multiplication. Despite their misleading name, imaginary numbers are not only real but also very useful, with application in electricity, signal processing and many other applications. They are widely used in electronics, for the representation of alternating currents and in waves. Although the Greek mathematician and engineer Hero of Alexandria is noted as the first to have conceived imaginary numbers, it was Rafael Bombelli who first set down the rules for multiplication of complex numbers in 1572. The concept had appeared in print earlier, such as in work by Gerolamo Cardano. At the time, imaginary numbers and negative numbers were poorly understood and were regarded by some as fictitious or useless much as zero once was. Many other mathematicians were slow to adopt the use of imaginary numbers, including René Descartes, who wrote about them in his *La Géométrie* in which the term imaginary was used and meant to be derogatory. The use of imaginary numbers was not widely accepted until the work of Leonhard Euler (1707-1783) and Carl Friedrich Gauss (1777-1855). The geometric significance of complex numbers as points in a plane was first described by Caspar Wessel (1745-1818).

Gelfond's constant, in mathematics, is the number  $e^\pi$ ,  $e$  raised to the power  $\pi$ . Like  $e$  and  $\pi$ , this constant is a transcendental number. It was named after the Soviet mathematician Aleksandr Gelfond. Gelfond's constant were singled out in Hilbert's 7th problem as an example of numbers whose excess was an open problem. This was first established by Gelfond and may now be considered as an application of the Gelfond-Schneider theorem, noting that  $e^\pi = (e^{i\pi})^{-i} = (-1)^{-i} = i^{-2} \approx 23.14069263277926900572\dots$

Euler's constant is a mathematical constant usually denoted by the lowercase Greek letter gamma ( $\gamma$ ). The number  $\gamma$  has not been proved algebraic or transcendental. In fact, it is not even known whether  $\gamma$  is irrational. The numerical value of Euler's constant is  $\gamma = 0.57721566490153286\dots$

Euler's identity is considered to be an exemplar of mathematical beauty as it shows a profound connection between

the most fundamental numbers in mathematics:

$$e^{i\pi} + 1 = 0$$

All five of the numbers play important and repetitive roles in mathematics. The expression who connects the six basic mathematical constants, the number 0, the number 1, the golden ratio  $\phi$ , the Archimedes constant  $\pi$ , the Euler's number  $e$  and the imaginary unit  $i$  is:

$$e^{\frac{i\pi}{1+\phi}} + e^{\frac{-i\pi}{1+\phi}} + e^{\frac{i\pi}{\phi}} + e^{\frac{-i\pi}{\phi}} = 0$$

In [1] we presented exact and approximate expressions between the Archimedes constant  $\pi$ , the golden ratio  $\phi$ , the Euler's number  $e$  and the imaginary number  $i$ .

### 3. Fine-structure constant

The fine-structure constant is one of the most fundamental constants of physics. It describes the strength of the force of electromagnetism between elementary particles in what is known as the standard model of particle physics. In particular, the fine-structure constant sets the strength of electromagnetic interaction between light (photons) and charged elementary particles such as electrons and muons. The quantity  $\alpha$  was introduced into physics by A. Sommerfeld in 1916 and in the past has often been referred to as the Sommerfeld fine-structure constant. In order to explain the observed splitting or fine structure of the energy levels of the hydrogen atom, Sommerfeld extended the Bohr theory to include elliptical orbits and the relativistic dependence of mass on velocity. One of the most important numbers in physics is the fine-structure constant  $\alpha$  which defines the strength of the electro-magnetic field. It is a dimensionless number independent of how we define our units of mass, length, time or electric charge. A change in these units of measurement leaves the dimensionless constant unchanged. The number can be seen as the chance that an electron emits or absorbs a photon. It's a pure number that shapes the universe to an astonishing degree. Paul Dirac considered the origin of the number «the most fundamental unsolved problem of physics». The constant is everywhere because it characterizes the strength of the electromagnetic force affecting charged particles such as electrons and protons. Many eminent physicists and philosophers of science have pondered why  $\alpha$  itself has the value that it does, because the value shows up in so many important scenarios and aspects of physics. Nobody has come up with any ideas that are even remotely convincing. A similar situation occurs with the proton-electron mass ratio  $\mu$ , not because of its ubiquity, but rather how chemistry can be based on two key electrically charged particles of opposite electric charge that are opposite but of seemingly identical magnitude while their masses have a ratio that is quite large yet finite. These two questions have a huge bearing on why physics and chemistry behave the way they do. The product of the two quantities appears, at least at first glance, not to be so important. The fine-structure constant  $\alpha$  is defined as:

$$\alpha = \frac{q_e^2}{4\pi\epsilon_0\hbar c}$$

Based on the precise measurement of the hydrogen atom spectrum by Michelson and Morley in 1887, Arnold Sommerfeld extended the Bohr model to include elliptical orbits and relativistic dependence of mass on velocity. He introduced a term for the fine-structure constant in 1916. The first physical interpretation of the fine-structure constant was as the ratio of the velocity of the electron in the first circular orbit of the relativistic Bohr atom to the speed of light in the vacuum. The 2018 CODATA recommended value of  $\alpha$  is

$$\alpha = 0.0072973525693(11).$$

With standard uncertainty  $0.000000011 \times 10^{-3}$  and relative standard uncertainty  $1.5 \times 10^{-10}$ . For reasons of convenience, historically the value of the reciprocal of the fine-structure constant is often specified. The 2018 CODATA recommended value of  $\alpha^{-1}$  is given by:

$$\alpha^{-1} = 137.035999084(21)$$

With standard uncertainty  $0.000000021 \times 10^{-3}$  and relative standard uncertainty  $1.5 \times 10^{-10}$ . There is general

agreement for the value of  $\alpha$ , as measured by these different methods. The preferred methods in 2019 are measurements of electron anomalous magnetic moments and of photon recoil in atom interferometry. The most precise value of  $\alpha$  obtained experimentally (as of 2012) is based on a measurement of  $g$  using a one-electron so-called "quantum cyclotron" apparatus, together with a calculation via the theory of QED that involved 12672 tenth-order Feynman diagrams:

$$\alpha^{-1} = 137.035999174(35)$$

This measurement of  $\alpha$  has a relative standard uncertainty of  $2.5 \times 10^{-10}$ . This value and uncertainty are about the same as the latest experimental results. Further refinement of this work were published by the end of 2020, giving the value:

$$\alpha^{-1} = 137.035999206(11)$$

with a relative accuracy of 81 parts per trillion. Also the fine-structure constant is universal scaling factor:

$$\alpha = \frac{2\pi r_e}{\lambda_e} = \frac{\lambda_e}{2\pi \alpha_0} = \frac{r_e}{l_{pl}} \frac{m_e}{m_{pl}} = \sqrt{\frac{r_e}{\alpha_0}}$$

There is a dream, which, albeit more often not confessed, occupies the most secret aspirations of theoreticians and is that of reducing the various constants of Physics to simple formula involving integers and transcendent numbers. The fine-structure constant plays an important role in modern physics. Yet it continues to be a mystery as to exactly what it represents and why it has the mystical value it has.

The mystery about the fine-structure constant is actually a double mystery. The first mystery – the origin of its numerical value – has been recognized and discussed for decades. The second mystery – the range of its domain – is generally unrecognized.

— M. H. MacGregor (2007). *The Power of Alpha*.

When I die my first question to the Devil will be: What is the meaning of the fine structure constant?

— Wolfgang Pauli

“God is a pure mathematician!” declared British astronomer Sir James Jeans. The physical Universe does seem to be organized around elegant mathematical relationships. And one number above all others has exercised an enduring fascination for physicists: 137.0359991.... It is known as the fine-structure constant and is denoted by the Greek letter alpha ( $\alpha$ ).”

— Paul Davies

“While twentieth-century physicists were not able to identify any convincing mathematical constants underlying the fine structure, partly because such thinking has normally not been encouraged, a revolutionary suggestion was recently made by the Czech physicist Raji Heyrovská, who deduced that the fine structure constant, ...really is defined by the [golden] ratio ....”

— Carl Johan Calleman, *The Purposeful Universe: How Quantum Theory and Mayan Cosmology Explain the Origin and Evolution of Life*

The fine-structure constant plays an important role in modern physics. Yet it continues to be a mystery as to exactly what it represents and why it has the mystical value it has. The elementary charge of electron  $e$  was proposed by Stoney in 1894 and discovered by Thomson in 1896, then Planck introduced the energy quanta  $h \cdot \nu$  in 1901 and explained it as photon  $E = h \cdot \nu$  by Einstein in 1905. Planck first noticed in 1905 that  $e^2/c$  and  $h$  have the same dimension. In 1909, Einstein found that there are two fundamental velocities in physics:  $c$  and  $e^2/h$  requiring explanation. He said, “It seems to me that we can conclude from  $h = e^2/c$  that the same modification of theory that contains the elementary quantum  $e$  as a consequence, will also contain as a consequence the quantum structure of radiation.” Albert Einstein was the first to use a mathematical formula for the fine-structure constant  $\alpha$  in 1909. This expression is:

$$\alpha = \frac{7\pi}{3000}$$

with numerical value  $\alpha=0.00733038286$  with an error accuracy of 0.45%. Later many scientists used other mathematical formulas for fine-structure constant but they are not at all accurate. These are Jeans 1913, Lewis Adams 1914, Lunn in 1922, Peirles in 1928 and others. Arthur Eddington was the first to focus on its inverse value and suggested that it should be an integer, that the theoretical value is  $\alpha^{-1}=136$ . In his original document 1929 he applied the value:

$$\alpha^{-1}=16+1/2 \times 16 \times (16-1)=136$$

However, the experiments themselves consistently showed that  $\alpha^{-1}=137$ . This forced him to look for an error in his original theory. He soon came to the conclusion that:

$$\alpha^{-1}=137$$

He thus argued that the extra unit was a consequence of the initial exclusion of every elementary particle pair in the universe. In the document of 1929, Eddington considered that fine-structure constant relates in a simple way to the cosmological constants, as given by the expression:

$$\alpha = \frac{2\pi mcR_E}{h\sqrt{N}}$$

where N the cosmic number, the number of electrons and protons in the closed universe. Eddington always kept the name and the symbol  $\alpha$ :

$$\alpha = \frac{hc}{2\pi q_e^2}$$

The first to find an exact formula for the fine-structure constant  $\alpha$  was the Swiss mathematician Armand Wyler in 1969. Based on the arguments concerning the congruent group, the group consists of simple Lorentz transformations such as the space-time dimensions that leave the Maxwell equations unchanged. The first form of the Wyler constant type is:

$$\alpha_w = \left( \frac{9}{16\pi^3} \right) \left( \frac{\pi}{5!} \right)^{\frac{1}{4}}$$

With numerical value  $\alpha_w=0.00729735252\dots$ . At the time it was proposed, they agreed with the experiment to be within 1.5 ppm for the value  $\alpha^{-1}$ .

#### **4. Fine-structure constant from the golden angle, the relativity factor and the fifth power of the golden mean**

Dr. Rajalakshmi Heyrovská in [2] has found that the golden ratio  $\phi$  provides a quantitative link between various known quantities in atomic physics. While searching for the exact values of ionic radii and for the significance of the ionization potential of hydrogen, Dr. Heyrovská has found that the Bohr radius can be divided into two Golden sections pertaining to the electron and proton. More generally, it was found that  $\phi$  is also the ratio of anionic to cationic radii of any atom, their sum being the covalent bond length. After that she showed, among other facts, that many bond lengths in organic and inorganic molecules behave additively, and are the sum of the covalent and the ionic radii, whether partially or fully ionic or covalent. An interpretation and a value of the fine-structure constant  $\alpha^{-1}$  has been discovered in terms of the golden angle. He proposed another interpretation of  $\alpha$  based on the observation that it is close to the golden angle. Fine-structure constant can also be formulated in [3], [4] and [5] exclusively in terms of the golden angle, the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360 \cdot \phi^{-2} - 2 \cdot \phi^{-3} + (3 \cdot \phi)^{-5} \quad (1)$$

with numerical values:

$$\alpha^{-1}=137.03599916476564\dots\dots$$

$$\alpha=0.00729735256498292\dots\dots$$

The numerical value is the average of all the measurements. The formula is the exact formula for the fine-structure constant  $\alpha$ . Another beautiful forms of the equations are:

$$\frac{1}{\alpha} = \frac{360}{\phi^2} - \frac{2}{\phi^3} + \frac{1}{3^5\phi^5} \quad (2)$$

$$\frac{1}{\alpha} = \frac{360}{\phi^2} - \frac{2}{\phi^3} + \frac{3^{-5}}{\phi^5} \quad (3)$$

Other equivalent expressions for the fine-structure constant are:

$$\alpha^{-1}=(362-3^{-4})\cdot\phi^{-2}-(1-3^{-5})\cdot\phi^{-1} \quad (4)$$

$$\alpha^{-1}=(362-3^{-4})+(3^{-4}+2\cdot 3^{-5}-364)\cdot\phi^{-1} \quad (5)$$

$$\alpha^{-1}=1-2\cdot\phi^{-1}+360\cdot\phi^{-2}-\phi^{-3}+(3\cdot\phi)^{-5} \quad (6)$$

$$\alpha^{-1}=\phi^0-2\cdot\phi^{-1}+360\cdot\phi^{-2}-\phi^{-3}+(3\cdot\phi)^{-5} \quad (7)$$

$$\alpha^{-1}=\phi^0-2\cdot\phi^{-1}+360\cdot\phi^{-2}-\phi^{-3}-241\cdot 3^{-5}\cdot\phi^{-4}-(3\cdot\phi)^{-5} \quad (8)$$

$$\alpha^{-1}=(174474\cdot\phi+86995)\cdot(243\cdot\phi^5)^{-1} \quad (9)$$

$$\alpha^{-1}=(87480\cdot\phi^3-486\cdot\phi^2+1)\cdot(243\cdot\phi^5)^{-1} \quad (10)$$

$$\frac{360}{\phi^2} - \frac{2}{\phi^3} + \frac{1}{(3\phi)^5} = -\frac{2}{(2\cos(\frac{\pi}{5}))^3} + \frac{360}{(2\cos(\frac{\pi}{5}))^2} + \frac{1}{(6\cos(\frac{\pi}{5}))^5} \quad (11)$$

$$\frac{360}{\phi^2} - \frac{2}{\phi^3} + \frac{1}{(3\phi)^5} = -\frac{2}{(1+2\sin(\frac{\pi}{10}))^3} + \frac{360}{(1+2\sin(\frac{\pi}{10}))^2} + \frac{1}{(3(1+2\sin(\frac{\pi}{10})))^5} \quad (12)$$

$$\frac{360}{\phi^2} - \frac{2}{\phi^3} + \frac{1}{(3\phi)^5} = -\frac{2}{(2\sin(54^\circ))^3} + \frac{360}{(2\sin(54^\circ))^2} + \frac{1}{(6\sin(54^\circ))^5} \quad (13)$$

$$\frac{360}{\phi^2} - \frac{2}{\phi^3} + \frac{1}{(3\phi)^5} = \frac{1}{(-6\sin(666^\circ))^5} - \frac{2}{(-2\sin(666^\circ))^3} + \frac{360}{(-2\sin(666^\circ))^2} \quad (14)$$





$$\frac{1}{243 \phi^5} + \sum_{k=1}^3 \frac{\begin{cases} -\frac{2}{\phi^3} & k=1 \\ 180 \phi & k=2 \\ -180 \phi & k=3 \end{cases}}{1}$$

## 5. Fine-structure constant from the Archimedes constant

We proposed in [5], [6] and [7] the simple and absolutely accurate expression for the fine-structure constant in terms of the Archimedes constant  $\pi$ :

$$\alpha^{-1} = \frac{2706}{43} \pi \ln 2 \quad (18)$$

with absolutely accurate numerical values:

$$\alpha^{-1} = 137.035999078\dots$$

$$\alpha = 0.00729735256959\dots$$

The equivalent expression for the fine-structure constant is:

$$\alpha^{-1} = 2 \cdot 3 \cdot 11 \cdot 41 \cdot 43^{-1} \cdot \pi \cdot \ln 2 \quad (19)$$

Other equivalent expression for the fine-structure constant are:

$$\alpha^{-1} = [(14^3 - 38) \cdot 43^{-1}] \cdot \ln 2 \cdot \pi \quad (20)$$

$$\frac{2}{43} \times 3 \times 11 \times 41 \pi \log(2) = \frac{2706}{43} \pi \log(a) \log_a(2) \quad (21)$$

$$\frac{2}{43} \times 3 \times 11 \times 41 \pi \log(2) = \frac{5412}{43} \pi \coth^{-1}(3) \quad (22)$$

$$\frac{2}{43} \times 3 \times 11 \times 41 \pi \log(2) = \frac{5412}{43} \pi \tanh^{-1}\left(\frac{1}{3}\right) \quad (23)$$

$$\frac{2}{43} \times 3 \times 11 \times 41 \pi \log(2) = \frac{5412}{43} i \pi \cot^{-1}(3i) \quad (24)$$

$$\frac{2}{43} \times 3 \times 11 \times 41 \pi \log(2) = -\frac{2706}{43} \pi S_{0,1}(-1) \quad (25)$$

$$\frac{2}{43} \times 3 \times 11 \times 41 \pi \log(2) = \frac{5412}{43} i \pi \tan^{-1}\left(-\frac{i}{3}\right) \quad (26)$$

$$\frac{2}{43} \times 3 \times 11 \times 41 \pi \log(2) = \frac{2706 \pi}{43} \int_1^2 \frac{1}{t} dt \quad (27)$$

$$\frac{2}{43} \times 3 \times 11 \times 41 \pi \log(2) = -\frac{1353 i}{43} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0 \quad (28)$$

This accurate expression is the most impressive since it is simple and contains just a few prime numbers and the Archimedes constant. These prime numbers can be possibly connected to finite groups (Group of Lie type). The series representations for the fine-structure constant are:

$$\begin{aligned} \frac{2}{43} \times 3 \times 11 \times 41 \log(2) \pi &= \frac{5412}{43} i \pi^2 \left[ \frac{\arg(2-x)}{2\pi} \right] + \\ &\frac{2706}{43} \pi \log(x) - \frac{2706}{43} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \quad \text{for } x < 0 \end{aligned} \quad (29)$$

$$\begin{aligned} \frac{2}{43} \times 3 \times 11 \times 41 \log(2) \pi &= \frac{2706}{43} \pi \left[ \frac{\arg(2-z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \frac{2706}{43} \pi \log(z_0) + \\ &\frac{2706}{43} \pi \left[ \frac{\arg(2-z_0)}{2\pi} \right] \log(z_0) - \frac{2706}{43} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \end{aligned} \quad (30)$$

$$\begin{aligned} \frac{2}{43} \times 3 \times 11 \times 41 \log(2) \pi &= \frac{5412}{43} i \pi^2 \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \\ &\frac{2706}{43} \pi \log(z_0) - \frac{2706}{43} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \end{aligned} \quad (31)$$

We pose  $a_b = 2706/43 = 2 \cdot 3 \cdot 11 \cdot 41 \cdot 43^{-1}$ . This combination of prime numbers we will call the coefficient for the fine-structure constant. So the expression for the fine-structure constant can be written as:

$$\alpha^{-1} = -a_b \cdot \pi \cdot \ln 2 \quad (32)$$

The repeating decimal of the coefficient for the fine-structure constant  $a_b$  is:

$$\overline{62.930232558139534883720} \quad (\text{period } 21)$$

The continued fraction for the  $a_b$  is:

$$62 + \frac{1}{1 + \frac{1}{13 + \frac{1}{3}}}$$

The Egyptian fraction expansion for the  $a_b$  is:

$$62 + \frac{1}{2} + \frac{1}{3} + \frac{1}{11} + \frac{1}{167} + \frac{1}{473946}$$

The repeating decimal of the  $ab^{-1}$  is:

$$\overline{0.01589061345} \text{ (period 10)}$$

So the expression for the fine-structure constant with the Archimedes constant is:

$$a^{-1} = -ab \cdot n \cdot \ln 2$$

$$a \cdot n \cdot \ln 2 = -\overline{0.01589061345} \tag{33}$$

The pattern of the continued fraction for the fine-structure constant is:

[137; 27, 1, 3, 1, 1, 16, 1, 4, 45, 12, 2, 4, 1, 6, 4, 2, 1, 155, 2, 7, 26, 1, 2, 1, 2, 10, 2, 3, 11, 2, 1, 2, 1, 4, 2, 1, 2, 7, 7, 1, 1, 1, 1, 2, 1, 5, 1, 1, 1, 1, 5, 1, 3, 1, 1, 1, 1, 1, 1, 5, 2, 9, 1, 1, 3, 2, 1, 27, 1, 11, 1, 1, 1, 11, 2, 5, 1, 3, 41, 1, 4, 1, 13, 1, 1, 38, 1, 4, 2, 1, 2, 6, 1, 2, 18, 33, 1, 1, 4, 1, 9611, 1, 6, 5, 5, 1, 5, 1, 1, 2, 2, 1, 2, 1, 2, 2, 1, 2, 12, 2, 1, 3, 8, 1, 1, 5, 2, 8, 2, 8, 111, 12, 3, 3, 2, 1, 1, 18, 1, 1, 1, 1, 15, 1, 6, 3, 7, 1, 1, 1, 1, 2, 9, 1, 2, 1, 2, 2, 1, 1, 1, 1, 1, 3, 1, 9, 2, 2, 1, 12, 16, 1, 4, 1, 1, 3, 3, 54, 1, 1, 1, 2, 1, 2, 2, 3, 1, 9, 1, 2, 4, 1, 1, 2, 1, 6, 4, 8, 42, 1, 1, 9, 12, 1, 7, 9, 1, 1, 1, 3, 3, 3, 16, 2, 1, 2, 12, 1, 2, 2, 1, 3, 1, 5, 7, 2, 1, 3, 3, 1, 2, 1, 3, 2, 2, 6, 1, 6, 1, 2, 1, 1, 9, 1, 2, 2, 106, 1, 1, 1, 1, 8, 1, 1, 1, 27, 2, 55, 2, 1, 6, 2, 1, 3, 1, 9, 1, 3, 2, 1, 6, 4, 1, 13, 2, 2, 1, 2, 6, 1, 2, 1, 1, 3, 6, 1, 2, 189, 3, 8, 3, 4, 4, 1, 3, 5, 1, 1, 6, 565, 1, 1, 2, 7, 2, 1, 1, 4, 1, 2, 2, 3, 1, 3, 96, 9, 2, 1, 2, 67, 1, 1, 1, 1, 8, 3, 1, 4, 1, 2, 26, 1, 2, 1, 3, 5, 13, 11, 3, 1, 1, 10, 1, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 1, 1, 1, 2, 1, 1, 1, 1, 11, 5, 9, 14, 1, 1, 4, 2, 1, 5, 1, 9, 2, 77, 5, 1, 1, 1, 5, 4, 1, 1, 3, 1, 10, 2, 1, 5, 10, 4, 1, 6, 1, 2, 1, 2, 1, 2, 16, 708, 1, 4, 3, 1, 1, 14, 1, 3, 1, 1, 2, 38, 46, 1, 1, 4, 1, 1, 2, 3, 2, 2, 15, 1, 3, 1, 1, 20, 1, 14, 6, 2, 7, 2, 1, 2, 14, 4, 3, 1, 1, ...]

The continued fraction for the fine-structure constant is:

$$137 + \cfrac{1}{27 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{16 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{45 + \cfrac{1}{12 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{1 + \dots}}}}}}}}}}}}}}}}}}}}}}$$

### 6. Fine-structure constant from the Madelung constant

The Madelung constant is used in determining the electrostatic potential of a single ion in a crystal by approximating the ions by point charges. It is named after Erwin Madelung, a German physicist. Because the anions and cations in an ionic solid attract each other by virtue of their opposing charges, separating the ions requires a certain amount of energy. This energy must be given to the system in order to break the anion-cation bonds. The energy required to

break these bonds for one mole of an ionic solid under standard conditions is the lattice energy. The Madelung constant is also a useful quantity in describing the lattice energy of organic salts. Izgorodina and coworkers have described a generalized method (called the EUGEN method) of calculating the Madelung constant for any crystal structure. The quantities obtained [8] from cubic, hexagonal, etc. lattice sums, evaluated at  $s=1$ , are called Madelung constants. For cubic lattice sums:

$$b_n(2s) \equiv \sum_{k_1, \dots, k_n = -\infty}^{\infty} \frac{(-1)^{k_1 + \dots + k_n}}{(k_1^2 + \dots + k_n^2)^s},$$

The Madelung constants are expressible in closed form for even indices  $n$ , a few examples of which are summarized in the following table, where  $\beta(n)$  is the Dirichlet beta function and  $\eta(n)$  is the Dirichlet eta function. To obtain the closed form for  $b_2(s)$ , break up the double sum into pieces that do not include  $i=j=0$ :

$$b_2(2s) = 4 \left[ \sum_{i,j=1}^{\infty} \frac{(-1)^{i+j}}{(i^2 + j^2)^s} + \sum_{i=1}^{\infty} \frac{(-1)^i}{i^{2s}} \right].$$

The second of these sums can be done analytically as:

$$\sum_{i=1}^{\infty} \frac{(-1)^i}{i^{2s}} = -4^s (4^s - 2) \zeta(2s),$$

which in the case  $s=1$  reduces to:

$$\sum_{i=1}^{\infty} \frac{(-1)^i}{i^2} = -\frac{1}{12} \pi^2.$$

The first sum is more difficult, but in the case  $s=1$  can be written:

$$\sum_{i,j=1}^{\infty} \frac{(-1)^{i+j}}{i^2 + j^2} = \frac{1}{12} \pi (\pi - 3 \ln 2).$$

Combining these then gives the original sum as:

$$b_2(2) = -\pi \ln 2.$$

So the equivalent expressions for the fine-structure constant with the Madelung constant  $b_2(2)$  are:

$$\alpha^{-1} = -\frac{2706}{43} b_2(2) \quad (34)$$

$$\alpha^{-1} = -2 \cdot 3 \cdot 11 \cdot 41 \cdot 43^{-1} \cdot b_2(2) \quad (35)$$

$$\alpha^{-1} = -[(14^3 - 38) \cdot 43^{-1}] \cdot b_2(2) \quad (36)$$

Also the expression for the fine-structure constant can be written as:

$$\alpha^{-1} = -ab \cdot b_2(2)$$

So from the expression for the fine-structure constant with the madelung constant  $b_2(2)$  is:

$$\alpha \cdot b_2(2) = -0.01589061345 \quad (37)$$

## 7. Proton to electron mass ratio

In Physics, the ratio of the mass of a proton to an electron is simply the remainder of the mass of the proton divided by that of the electron, from the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious ratio of mass between a proton and an electron. The values of  $m_e$  and  $m_p$ , and the equilibrium between them, govern nuclear reactions such as the decay of protons and the nuclear synthesis of stars, leading to the formation of basic biochemical elements, including carbon. The space where stars and planets form and support life and molecular structures can appear. The mass ratio of protons to electrons, two constant particles that make up about 95% of the visible Universe, may be related to the total computational value of the Universe. Thus, as pure numbers they are supposed to be associated with prime numbers, entropy, binary and complexity. The proton to electron mass ratio  $\mu$  is a ratio of like-dimensioned physical quantities, it is a dimensionless quantity, a function of the dimensionless physical constants, and has numerical value independent of the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious mass ratio between the proton and electron. The numerical challenge of the mass ratio of proton to electron in the field of elementary particle physics began with the discovery of the electron by JJ Thomson in 1897, and with the identification of the point nature of the proton by E. Rutherford in 1911. These two particles have electric charges that are identical in size but opposite charges. The 2018 CODATA recommended value of the proton to electron mass ratio  $\mu$  is:

$$\mu = 1836.15267343$$

with standard uncertainty 0.00000011 and relative standard uncertainty  $6.0 \times 10^{-11}$ . The value of  $\mu$  is a solution of the equation:

$$3 \cdot \mu^4 - 5508 \cdot \mu^3 - 841 \cdot \mu^2 + 10 \cdot \mu - 2111 = 0$$

The search for mathematical expression for this dimensionless number motivated many serious scientists. First Peirls in 1928 proposed the mathematical expression:

$$\mu = \frac{2(\pi - 1)\pi}{\alpha}$$

A year later Reinhold Furth in 1929 assumed that proton to electron mass ratio  $\mu$  could be derived from the quadratic equation containing the fine-structure constant  $\alpha$ :

$$\mu = \frac{64\pi}{15\alpha}$$

Later in 1935, A. Eddington, who accepted some of Furth's ideas, presented in his book «New Pathways in Science» the equation for the the proton to electron mass ratio  $\mu$ :

$$10 \cdot \mu^2 - 136 \cdot \mu + 1 = 0$$

However both approaches can not be used nowadays as they give very high deviation from the currently known experimental value of  $\mu$ . Haas in 1938 presented the expression:

$$\mu = \frac{3\sqrt{2}\pi}{\alpha}$$

Later in 1951 Lenz noted that  $\mu$  can be approximated with the formula:

$$\mu = 6 \cdot \pi^5$$

In 1990, I.J. Good, a British mathematician assembled eight conjectures of numerology for the ratio of the rest masses of the proton and the electron. Recently the professional approach to mathematically decode  $\mu$  was done by Simon Plouffe. He used a large database of mathematical constants and specialized programs to directly find an expression. Along with his main remarkable result for the expression for  $\mu$  via Fibonacci and Lucas numbers and golden ratio he also noted that expression for  $\mu$  using  $\pi$  can be improved as:

$$\mu = \frac{6\pi^5 + 328}{\pi^8}$$

We propose in [9] the exact mathematical expression for the proton to electron mass ratio using Fibonacci and Lucas numbers:

$$\mu = 11^{47/32} \cdot 5^{5/2} \cdot 9349^{5/76} \cdot \phi^{-21/16} \quad (38)$$

However:

$$(2 \cdot \phi - 1)^2 = 5$$

$$\phi^5 - \phi^{-5} = 11$$

$$\phi^{19} - \phi^{-19} = 9349$$

So the exact mathematical expression for the proton to electron mass ratio is:

$$\mu^{32} = (\phi^5 - \phi^{-5})^{47} \cdot (2 \cdot \phi - 1)^{160} \cdot (\phi^{19} - \phi^{-19})^{40/19} \cdot \phi^{-42} \quad (39)$$

$$\mu^{32} = \phi^{-42} \cdot F_5^{160} \cdot L_5^{47} \cdot L_{19}^{40/19} \quad (40)$$

with numerical value:

$$\mu = 1836.15267343\dots$$

Also we propose the exact mathematical expression for the proton to electron mass ratio:

$$\mu = 165 \sqrt[3]{\frac{\ln^{11} 10}{7}} \quad (41)$$

with numerical value:

$$\mu = 1836.15267392\dots$$

Other equivalent expressions for the proton to electron mass ratio are:

$$\mu^3 = 7^{-1} \cdot 165^3 \cdot \ln^{11} 10 \quad (42)$$

$$7 \cdot \mu^3 = (3 \cdot 5 \cdot 11)^3 \cdot \ln^{11}(2 \cdot 5) \quad (43)$$

Other exact mathematical expression for the proton to electron mass ratio is:

$$\mu = 6 \cdot \pi^5 + \pi^{-3} + 2 \cdot \pi^{-6} + 2 \cdot \pi^{-8} + 2 \cdot \pi^{-10} + 2 \cdot \pi^{-13} + \pi^{-15} \quad (44)$$

with numerical value:

$$\mu = 1836.15267343\dots$$

## 8. Unity formula that connect the fine-structure constant and the proton to electron mass ratio

Also in [9] was presented the exact mathematical expressions that connects the proton to electron mass ratio  $\mu$  and the fine-structure constant  $\alpha$ :

$$9 \cdot \mu - 119 \cdot \alpha^{-1} = 5 \cdot (\varphi + 42) \quad (45)$$

$$\mu - 6 \cdot \alpha^{-1} = 360 \cdot \varphi - 165 \cdot \pi + 345 \cdot e + 12 \quad (46)$$

$$\mu - 182 \cdot \alpha = 141 \cdot \varphi + 495 \cdot \pi - 66 \cdot e + 231 \quad (47)$$

$$\mu - 807 \cdot \alpha = 1205 \cdot \pi - 518 \cdot \varphi - 411 \cdot e \quad (48)$$

In [10] was presented the unity formula that connects the fine-structure constant and the proton to electron mass ratio. It was explained that  $\mu \cdot \alpha^{-1}$  is one of the roots of the following trigonometric equation:

$$2 \cdot 10^2 \cdot \cos(\mu \cdot \alpha^{-1}) + 13^2 = 0 \quad (49)$$

The exponential form of this equation is:

$$10^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) + 13^2 = 0 \quad (50)$$

This exponential form can also be written with the beautiful form:

$$10^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) = 13^2 \cdot e^{i\pi} \quad (51)$$

Also this unity formula can also be written in the form:

$$10 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha})^{1/2} = 13 \cdot i \quad (52)$$

So other beautiful formula that connects the fine-structure constant, the proton to electron mass ratio and the fifth power of the golden mean is:

$$5^2 \cdot (5 \cdot \varphi^{-2} + \varphi^{-5})^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) + (5 \cdot \varphi^2 - \varphi^{-5})^2 = 0 \quad (53)$$

The formula that connects the fine-structure constant, the proton to electron mass ratio and the mathematical constants  $\pi, \varphi, e$  and  $i$  is:

$$10^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) = (5 \cdot \varphi^2 - \varphi^{-5})^2 \cdot e^{i\pi} \quad (54)$$

All these equations are simple, elegant and symmetrical in a great physical meaning.

## 9. Gravitational coupling constant for the electron

In physics, the gravitational coupling constant  $\alpha_G$  is a constant that characterizes the gravitational pull between a given pair of elementary particles. For the electron pair this constant is denoted by  $\alpha_G$ . The choice of units of measurement, but only with the choice of particles. The gravitational coupling constant  $\alpha_G$  is a scaling ratio that can be used to compare similar unit values from different scaling systems (Planck scale, atomic scale, and cosmological scale). The gravitational coupling constant can be used for comparison of length, range and force values. The gravitational coupling constant  $\alpha_G$  is defined as:

$$\alpha_G = \frac{Gm_e^2}{\hbar c}$$

There is so far no known way to measure  $\alpha_G$  directly. The value of the constant gravitational coupling  $\alpha_G$  is only known in four significant digits. The approximate value of the constant gravitational coupling  $\alpha_G$  is:

$$\alpha_G = 1.7518099 \times 10^{-45}$$

Also the gravitational coupling constant is universal scaling factor:



$$\alpha_G = \frac{m_e^2}{m_{pl}^2} = \frac{\alpha_{G(p)}}{\mu^2} = \frac{\alpha}{\mu N_1} = \frac{\alpha^2}{N_1^2 \alpha_{G(p)}} = \left( \frac{2\pi l_{pl}}{\lambda_e} \right)^2 = \left( \alpha \frac{l_{pl}}{r_e} \right)^2 = \left( \frac{l_{pl}}{\alpha \alpha_0} \right)^2$$

The gravitational coupling constant  $\alpha_{G(p)}$  for the proton is produced similar to the electron, but replaces the mass of electrons with the mass of the protons. The gravitational coupling constant of the proton  $\alpha_{G(p)}$  is defined as:

$$\alpha_{G(p)} = \frac{G m_p^2}{\hbar c}$$

The approximate value of the constant gravitational coupling of the proton is:

$$\alpha_{G(p)} = 5.9061512 \times 10^{-39}$$

Also other expression for the gravitational coupling constant is:

$$\alpha_{G(p)} = \frac{m_p^2}{m_{pl}^2} = \mu^2 \alpha_G = \frac{\alpha \mu}{N_1} = \frac{\alpha^2}{N_1^2 \alpha_G}$$

## 10. Ratio of electric force to gravitational force between electron and proton

The enormous value of the ratio of electric force to gravitational force was first pointed out by Bergen Davis in 1904. But Weyl and Eddington suggested that the number was about  $10^{40}$  and was related to cosmological quantities. The electric force  $F_c$  between electron and proton is defined as:

$$F_c = \frac{q_e^2}{4\pi\epsilon_0 r^2}$$

The gravitational force  $F_g$  between electron and proton is defined as:

$$F_g = \frac{G m_e m_p}{r^2}$$

So from these expressions we have:

$$N_1 = \frac{F_c}{F_g}$$

$$N_1 = \frac{q_e^2}{4\pi\epsilon_0 G m_e m_p}$$

$$N_1 = \frac{k_e q_e^2}{G m_p m_e}$$

$$N_1 = \frac{\alpha \hbar c}{G m_e m_p}$$

So the ratio  $N_1$  of electric force to gravitational force between electron and proton is defined as:

$$N_1 = \frac{\alpha}{\mu \alpha_G} = \frac{\alpha \mu}{\alpha_{G(p)}} = \frac{\alpha}{\sqrt{\alpha_G \alpha_{G(p)}}} = \frac{k_e q_e^2}{G m_e m_p} = \frac{\alpha \hbar c}{G m_e m_p}$$

The approximate value of the ratio of electric force to gravitational force between electron and proton is  $N_1=2.26866072 \times 10^{39}$ . The ratio  $N_1$  of electric force to gravitational force between electron and proton can also be written in expression:

$$N_1 = \frac{5}{3} 2^{130} = 2,26854911 \times 10^{39}$$

According to current theories  $N_1$  should be constant. The ratio  $N_2$  of electric force to gravitational force between two electrons is defined as:

$$N_2 = \mu N_1 = \frac{\alpha}{\alpha_G} = \frac{N_1^2 \alpha_{G(p)}}{\alpha} = \frac{k_e q_e^2}{G m_e^2} = \frac{\alpha \hbar c}{G m_e^2}$$

The approximate value of  $N_2$  is  $N_2=4.16560745 \times 10^{42}$ . According to current theories  $N_2$  should grow with the expansion of the universe.

## 11. Avogadro's number

Avogadro's number  $N_A$  is defined as the number of carbon-12 atoms in twelve grams of elemental carbon-12 in its standard state. Avogadro's number  $N_A$  is the fundamental physical constant that links the macroscopic physical world of objects that we can see and feel with the submicroscopic, invisible world of atoms. The name honors the Italian mathematical physicist Amedeo Avogadro, who proposed that equal volumes of all gases at the same temperature and pressure contain the same number of molecules. The most accurate definition of the Avogadro's number value involves the change in molecular quantities and, in particular, the change in the value of an elementary charge. The exact value of the Avogadro's number is:

$$N_A = 6.02214076 \times 10^{23}$$

The value of the Avogadro's number  $N_A$  can also be written in expressions:

$$N_A = 84446885^3 = 6.02214076 \times 10^{23}$$

$$N_A = 2^{79} = 6.04462909 \times 10^{23} \quad (55)$$

## 12. Exact mathematical formula that connects 6 dimensionless physical constants

In [11] was presented the exact mathematical formula that connects 6 dimensionless physical constants. In physics, a dimensionless physical constant is a physical constant that is dimensionless, a pure number having no units attached and having a numerical value that is independent of whatever system of units may be used. The term fundamental physical constant is used to refer to some universal dimensionless constants. A long-sought goal of theoretical physics is to find first principles from which all of the fundamental dimensionless constants can be calculated and compared to the measured values. In the 1920s and 1930s, Arthur Eddington embarked upon extensive mathematical investigation into the relations between the fundamental quantities in basic physical theories, later used as part of his effort to construct an overarching theory unifying quantum mechanics and cosmological physics. The mathematician Simon Plouffe has made an extensive search of computer databases of mathematical formulas, seeking formulas for the mass ratios of the fundamental particles. An empirical relation between the masses of the electron, muon and tau has been discovered by physicist Yoshio Koide, but this formula remains unexplained. Dimensionless physical constants cannot be derived and have to be measured. Developments in physics may lead to either a reduction or an extension of their number: discovery of new particles, or new relationships between physical phenomena, would introduce new constants, while the development of a more fundamental theory might allow the derivation of several constants from a more fundamental constant. A long-sought goal of theoretical physics is to find first principles from which all of the fundamental dimensionless constants can be calculated and compared to the measured values. The laws of physics have a set of fundamental constants, and it is generally admitted that only dimensionless combinations of constants have physical significance. These combinations include the electromagnetic and gravitational fine structure, along with the ratios of elementary particles masses. Cosmological measurements clearly depend on the

values of these constants in the past and can therefore give information on their time dependence if the effects of time-varying constants can be separated from the effects of cosmological parameters.

A Planck length  $l_{pl}$  is about  $10^{-20}$  times the diameter of a proton, meaning it is so small that immediate observation at this scale would be impossible in the near future. The length Planck  $l_{pl}$  has dimension [L]. The length Planck  $l_{pl}$  can be defined by three fundamental natural constants, the speed of light at vacuum  $c$ , the reduced Planck constant and the gravity constant  $G$  as:

$$l_{pl} = \sqrt{\frac{\hbar G}{c^3}} = \frac{\hbar}{m_{pl}c} = \frac{h}{2\pi m_{pl}c} = \frac{m_p r_p}{4m_{pl}}$$

The 2018 CODATA recommended value of the Planck length is  $l_{pl}=1.616255 \times 10^{-35}$  m with standard uncertainty  $0.000018 \times 10^{-35}$  m and relative standard uncertainty  $1.1 \times 10^{-5}$ .

The Bohr radius  $a_0$  is a physical constant, approximately equal to the most probable distance between the nucleus and the electron in a hydrogen atom in its ground state. The Bohr radius  $a_0$  is defined as:

$$a_0 = \frac{\hbar}{\alpha m_e c} = \frac{r_e}{\alpha^2} = \frac{\lambda_c}{2\pi\alpha}$$

The 2018 CODATA recommended value of the Bohr radius is  $a_0=5.29177210903 \times 10^{-11}$  m with standard uncertainty  $0.00000000080 \times 10^{-11}$  m and relative standard uncertainty  $1.5 \times 10^{-10}$ .

The Planck constant, or Planck's constant, is a fundamental physical constant of foundational importance in quantum mechanics. The constant gives the relationship between the energy of a photon and its frequency, and by the mass-energy equivalence, the relationship between mass and frequency. Specifically, a photon's energy is equal to its frequency multiplied by the Planck constant. The constant is generally denoted by  $h$ . The reduced Planck constant, equal to the constant divided by  $2\pi$ , is denoted by  $\hbar$ . For the reduced Planck constant  $\hbar$  apply:

$$\hbar = a \cdot m_e \cdot a_0 \cdot c$$

So from these expressions we have:

$$\hbar^2 = a^2 \cdot m_e^2 \cdot a_0^2 \cdot c^2$$

$$(\hbar \cdot G / c^3) = a^2 \cdot m_e^2 \cdot a_0^2 \cdot (G / \hbar \cdot c)$$

$$(\hbar \cdot G / c^3) = a^2 \cdot a_0^2 \cdot (G \cdot m_e^2 / \hbar \cdot c)$$

$$l_{pl}^2 = a^2 \cdot a_0 \cdot G$$

So the new formula for the Planck length  $l_{pl}$  is:

$$l_{pl} = a \sqrt{a_0 G} \quad (56)$$

Jeff Yee proposed in [12] that the mole and charge are related by deriving Avogadro's number from three constants, the Bohr radius, the Planck length and Euler's number. The fundamental unit of length in this unit system is the Planck length  $l_{pl}$ . Spacetime is proposed to be a lattice structure, in which its unit cells have sides of length  $a$ , marked below in the next figure. The lattice contains repeating cells with this structure, so it can be simplified to model a single unit cell of this repeating structure. These types of structures are commonly found in molecules. The center point of wave convergence is referred to here as a wave center. The separation length between granules in the unit cell is the diameter of a granule ( $2 \cdot l_{pl}$ ) multiplied by Euler's number ( $e$ ), which is the base of the natural logarithm. There are exactly Avogadro's number of unit cells in the radius of hydrogen. The Avogadro's number  $N_A$  can be calculated from the Planck length  $l_{pl}$ , the Bohr radius  $a_0$  and Euler's number  $e$ :

$$N_A = \frac{a_0}{2e l_{pl}}$$

We will use this expression and the new formula for the Planck length  $l_p$  to resulting the unity formula that connects the fine-structure constant  $\alpha$  and the gravitational coupling constant  $\alpha_G$ :

$$\alpha_0 = 2 \cdot e \cdot N_A \cdot l_p$$

$$\alpha_0 = 2eN_A\alpha\sqrt{\alpha_G\alpha_0}$$

$$2eN_A\alpha\sqrt{\alpha_G} = 1$$

Therefore the unity formula that connect the fine-structure constant  $\alpha$ , the gravitational coupling constant  $\alpha_G$  and the Avogadro's number  $N_A$  is:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (57)$$

The unity formula is equally valid:

$$\alpha^2 \cdot \alpha_G = (2 \cdot e \cdot N_A)^{-2} \quad (58)$$

This formula is the simple unification of the electromagnetic and the gravitational interactions. So from this expression the new formula for the Avogadro number  $N_A$  is:

$$N_A = \left( 2e\alpha\sqrt{\alpha_G} \right)^{-1} \quad (59)$$

The exact mathematical formula that connect the mass ratio of proton to electron, the fine-structure constant  $\alpha$  and the proton-proton gravitational coupling constant  $\alpha_{G(pp)}$  is:

$$\alpha^7 = \mu^7 \cdot [\alpha_{G(pp)} \cdot \log_2(2 \cdot \pi)] \quad (60)$$

The exact mathematical formula that connect the mass ratio of proton to electron, the fine-structure constant  $\alpha$  and the proton-electron gravitational coupling constant  $\alpha_{G(pe)}$  is:

$$\alpha^7 = \mu^8 \cdot [\alpha_{G(pp)} \cdot \log_2(2 \cdot \pi)] \quad (61)$$

The exact mathematical formula that connect the mass ratio of proton to electron, the fine-structure constant  $\alpha$  and the gravitational coupling constant of electrons-electrons  $\alpha_{G(ee)}$  is:

$$\alpha^7 = \mu^9 \cdot [\alpha_{G(pp)} \cdot \log_2(2 \cdot \pi)] \quad (62)$$

The exact mathematical formula that connect the proton to electron mass ratio  $\mu$ , the gravitational coupling constant  $\alpha_G$  of the electron and the gravitational coupling constant of the proton  $\alpha_{G(p)}$  is:

$$\alpha_{G(p)} = \mu^2 \cdot \alpha_G \quad (63)$$

The exact mathematical formula that connect the fine-structure constant  $\alpha$ , the gravitational coupling constant  $\alpha_G$  of the electron and the Avogadro number  $N_A$  is:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (64)$$

The exact mathematical formula that connect the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton and the gravitational coupling constant  $\alpha_G$  of the electron is:

$$\alpha = \mu \cdot N_1 \cdot \alpha_G \quad (65)$$

The exact mathematical formula that connect the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N_1$  of the electric force to the gravitational force between electron and proton and the gravitational coupling

constant of the proton  $a_{G(p)}$  is:

$$\alpha \cdot \mu = N_1 \cdot a_{G(p)} \quad (66)$$

The exact mathematical formula that connect the fine-structure constant  $\alpha$ , the ratio  $N_1$  of the electric force to the gravitational force between electron and proton, the gravitational coupling constant  $a_G$  of the electron and the gravitational coupling constant of the proton  $a_{G(p)}$  is:

$$\alpha^2 = N_1^2 \cdot a_G \cdot a_{G(p)} \quad (67)$$

The exact mathematical formula that connect the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the Avogadro's number  $N_A$  and the gravitational coupling constant of the proton  $a_{G(p)}$  is:

$$\mu^2 = 4 \cdot e^2 \cdot \alpha^2 \cdot a_{G(p)} \cdot N_A^2 \quad (68)$$

The exact mathematical formula that connect the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N_1$  of the electric force to the gravitational force between electron and proton and the Avogadro's number  $N_A$  is:

$$\mu \cdot N_1 = 4 \cdot e^2 \cdot \alpha^3 \cdot N_A^2 \quad (69)$$

The exact mathematical formula that connect the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N_1$  of the electric force to the gravitational force between electron and proton, the Avogadro's number  $N_A$  and the gravitational coupling constant  $a_G$  of the electron is:

$$4 \cdot e^2 \cdot \alpha \cdot \mu \cdot a_G^2 \cdot N_A^2 \cdot N_1 = 1 \quad (70)$$

The exact mathematical formula that connect the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N_1$  of the electric force to the gravitational force between electron and proton, the Avogadro's number  $N_A$  and the gravitational coupling constant of the the proton  $a_{G(p)}$  is:

$$\mu^3 = 4 \cdot e^2 \cdot \alpha \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1 \quad (71)$$

The exact mathematical formula that connect the proton to electron mass ratio  $\mu$ , the ratio  $N_1$  of the electric force to the gravitational force between electron and proton, the Avogadro's number  $N_A$ , the gravitational coupling constant  $a_G$  of the electron and the gravitational coupling constant of the proton  $a_{G(p)}$  is:

$$\mu^2 = 4 \cdot e^2 \cdot a_G \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1^2 \quad (72)$$

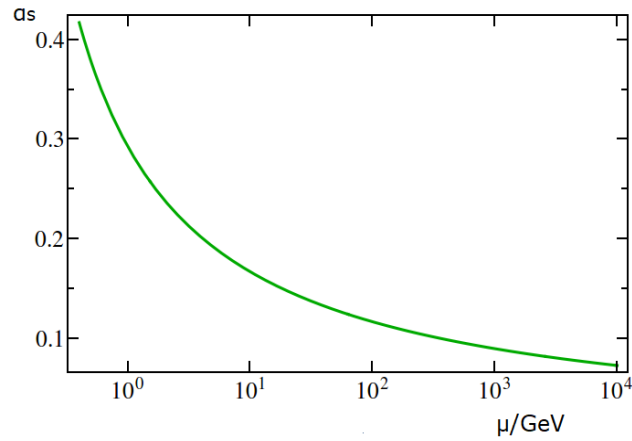
The exact mathematical formula that connect the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton, the Avogadro's number  $N_A$ , the gravitational coupling constant  $a_G$  of the electron and the gravitational coupling constant of the proton  $a_{G(p)}$  is:

$$\mu = 4 \cdot e^2 \cdot \alpha \cdot a_G \cdot a_{G(p)} \cdot N_A^2 \cdot N_1 \quad (73)$$

## 12. Dimensionless unification of the strong nuclear and the weak nuclear interactions

In nuclear physics and particle physics, the strong interaction is one of the four known fundamental interactions, with the others being electromagnetism, the weak interaction, and gravitation. Strong force involves the exchange of huge particles and therefore has a very small range. It is clear that strong force is much stronger simply than the fact that the nuclear magnitude (dominant strong force) is about  $10^{-15}$  m while the atom (dominant electromagnetic force) has a size of about  $10^{-10}$  m. At the range of  $10^{-15}$  m, the strong force is approximately 137 times as strong as electromagnetism,  $10^6$  times as strong as the weak interaction, and  $10^{38}$  times as strong as gravitation. The strong coupling constant  $\alpha_s$  is one of the fundamental parameters of the typical model of particle physics. The strong coupling constant  $\alpha_s$  is one of the fundamental parameters of the typical model of particle physics. The strong nuclear force confines quarks into hadron particles such as the proton and neutron. In addition, the strong force binds these neutrons and protons to create atomic nuclei, where it is called the nuclear force. Most of the mass of a common proton or neutron is the result of the strong force field energy; the individual quarks provide only about 1% of the

mass of a proton. The electromagnetic force is infinite in range and obeys the inverse square law, while the strong force involves the exchange of massive particles and it therefore has a very short range. The value of the strong coupling constant, like other coupling constants, depends on the energy scale. As the energy increases, this constant decreases as shown in Figure 1.



**Figure 1.** Strong coupling constant as a function of the energy.

The last measurement [13] in 23 November 2021 of European organization for nuclear research (CERN) is used in a comprehensive QCD analysis at next-to next-to-leading order, which results in significant improvement in the accuracy of the parton distributions in the proton. Simultaneously, the value of the strong coupling constant at the Z boson mass is extracted as:

$$\alpha_s(m_Z) = 0.1170 \pm 0.0019$$

A measurement of the inclusive jet production in proton-proton collisions at the LHC at  $\sqrt{s} = 13$  TeV is presented. The double-differential cross sections are measured as a function of the jet transverse momentum  $p_T$  and the absolute jet rapidity  $|\eta|$ . The anti-kT clustering algorithm is used with distance parameter of 0.4 (0.7) in a phase space region with jet  $p_T$  from 97 GeV up to 3.1 TeV and  $|\eta| < 2.0$ . Data collected with the CMS detector are used, corresponding to an integrated luminosity of 36.3 fb<sup>-1</sup> (33.5 fb<sup>-1</sup>). The measurement is used in a comprehensive QCD analysis at next-to next-to-leading order, which results in significant improvement in the accuracy of the parton distributions in the proton. Simultaneously, the value of the strong coupling constant at the Z boson mass is extracted as  $\alpha_s(m_Z) = 0.1170 \pm 0.0019$ . For the first time, these data are used in a standard model effective field theory analysis at next to-leading order, where parton distributions and the QCD parameters are extracted simultaneously with imposed constraints on the Wilson coefficient  $c_1$  of 4-quark contact interactions.

Interaction phenomena in field theory are often defined using perturbation theory, in which the functions in the equations are extended to forces of constant interaction. Usually, for all interactions except the strong one, the coupling constant is much smaller than the unit. This makes the application of perturbation theory effective, as the contribution from the main terms of the extensions decreases rapidly and their calculation becomes redundant. In the case of strong interactions, perturbation theory becomes useless and other calculation methods are required. One of the predictions of quantum field theory is the so-called "floating constants" phenomenon, according to which interaction constants change slowly with the increase of energy transferred during the interaction of particles. Thus, the constant of the electromagnetic interaction increases, and the constant of the strong interaction decreases with increasing energy. For quarks in quantum chromodynamics, a strong interaction constant is introduced:

$$\alpha_s = \frac{q_{gg}^2}{4\pi\hbar c} = \frac{q_{gg}^2 \epsilon_0 \alpha}{q_e^2} = \frac{\epsilon_0 q_{gg}^2}{q_{pl}^2}$$

where  $q_{gg}$  is the active color charge of a quark that emits virtual gluons to interact with another quark. By reducing the distance between the quarks, which is achieved in high-energy particle collisions, a logarithmic reduction of  $\alpha_s$  and a weakening of the strong interaction (the effect of the asymptotic freedom of the quarks) is expected. In [14] we presented the recommended value for the strong coupling constant:

$$\alpha_s = \frac{\text{Eulers' number}}{\text{Gerford's constant}}$$

$$\alpha_s = \frac{e}{e^\pi}$$

$$\alpha_s = e^{1-\pi}$$

(74)

with numerical value:

$$\alpha_s = 0.11746\dots$$

This value is the current world average value for the coupling evaluated at the Z-boson mass scale. It fits perfectly in the measurement of the strong coupling constant of the European organization for nuclear research (CERN). Also for the value of the strong coupling constant we have the equivalent expressions:

$$\alpha_s = e \cdot e^{-\pi} = e \cdot i^{2i} = i^{-2i/n} \cdot i^{2i} = i^{2i-(2i/n)} = i^{2i(n-1)/n}$$

The series representations for the strong coupling constant is:

$$e^{1-\pi} = \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{1-\pi}$$

$$e^{1-\pi} = e^{1-4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$e^{1-\pi} = \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{1-\pi}$$

$$e^{1-\pi} = \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{1-4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$e^{1-\pi} = \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{1-4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

The pattern of the continued fraction for the strong coupling constant is:





$e, +1 e, 0 e,$  and  $-1 e$ . The  $\Delta$  baryons have a mass of about  $1232 \text{ MeV}/c^2$ , a spin of  $3/2$ , and an isospin of  $3/2$ . Ordinary protons and neutrons, by contrast, have a mass of about  $939 \text{ MeV}/c^2$ , a spin of  $1/2$ , and an isospin of  $1/2$ . The  $\Delta^+$  (uud) and  $\Delta^0$  (udd) particles are higher-mass excitations of the proton ( $N^+, \text{uud}$ ) and neutron ( $N^0, \text{udd}$ ), respectively. However, the  $\Delta^{++}$  and  $\Delta^-$  have no direct nucleon analogues. The decays of the delta baryons is:

$$\Delta^+ \rightarrow p + \pi^0$$

The lifetime of the delta baryons is:

$$\tau_{\Delta} = 6 \times 10^{-24} \text{ s}$$

The sigma baryons are a family of subatomic hadron particles which have two quarks from the first flavor generation (up or down quarks), and a third quark from a higher flavor generation, in a combination where the wavefunction sign remains constant when any two quark flavors are swapped. They are thus baryons, with total isospin of 1, and can either be neutral or have an elementary charge of  $+2, +1, 0,$  or  $-1$ . They are closely related to the Lambda baryons, which differ only in the wavefunction's behavior upon flavor exchange. The decays of the sigma baryons is:

$$\Sigma^+ \rightarrow p + \pi^0$$

The lifetime of the delta baryons is:

$$\tau_{\Sigma} = 8 \times 10^{-11} \text{ s}$$

The extraordinary difference of 13 orders of magnitude in the lifetimes comes from the fact that the sigma decay does not conserve strangeness and therefore can proceed only by the weak interaction. The lifetime of a decay is proportional to the inverse square of the coupling constant between the initial and final products, and since the final products are identical, the difference in lifetime must come from the difference in coupling constants. The coupling constant ratio can then be estimated for this situation:

$$\frac{\alpha_w}{\alpha_s} = \sqrt{\frac{\tau_{\Delta}}{\tau_{\Sigma}}} = 10^{-7} e$$

$$\frac{\alpha_w}{\alpha_s} = 10^{-7} e \tag{75}$$

From this expression we can result the world average value of the weak coupling constant  $\alpha_w$ :

$$\alpha_w = e \cdot \alpha_s \cdot 10^{-7}$$

$$\alpha_w = e^{2 \cdot n} \cdot 10^{-7}$$

$$\alpha_w = e \cdot e \cdot i^{2i} \cdot 10^{-7}$$

$$\alpha_w = e^2 \cdot i^{2i} \cdot 10^{-7}$$

So the recommended theoretical current world average value for the weak coupling constant is:

$$\alpha_w = (e \cdot i^i)^2 \cdot 10^{-7} \tag{76}$$

with numerical value:

$$\alpha_w = 3.19310 \cdot 10^{-8}$$

From expression can result other equivalent expressions:

$$\alpha_w \cdot \alpha_s^{-1} = e \cdot 10^{-7}$$

$$\begin{aligned} \alpha_s \cdot \alpha_w^{-1} &= e^{-1} \cdot 10^7 \\ e \cdot \alpha_s &= 10^7 \cdot \alpha_w \end{aligned} \quad (77)$$

From this expression apply:

$$\begin{aligned} e^n \cdot \alpha_s \cdot \alpha_s &= 10^7 \cdot \alpha_w \\ e^n \cdot \alpha_s^2 &= 10^7 \cdot \alpha_w \\ \alpha_s^2 &= 10^7 \cdot e^{-n} \cdot \alpha_w \\ \alpha_s^2 &= i^{2i} \cdot 10^7 \cdot \alpha_w \end{aligned} \quad (78)$$

From this expression and Euler's identity resulting the beautiful formulas:

$$\begin{aligned} e^{in} + 1 &= 0 \\ (10^7 \cdot \alpha_s^{-2} \cdot \alpha_w)^i + 1 &= 0 \\ (10^{-7} \cdot \alpha_s^2 \cdot \alpha_w^{-1})^i + 1 &= 0 \\ 10^{-7i} \cdot \alpha_s^{2i} \cdot \alpha_w^{-i} + 1 &= 0 \\ 10^{-7i} \cdot \alpha_s^{2i} \cdot \alpha_w^{-i} &= i^2 \\ \alpha_s^{2i} &= i^2 \cdot 10^{7i} \cdot \alpha_w^i \\ \frac{\alpha_s^{2i}}{\alpha_w^i} &= i^2 10^{7i} \end{aligned} \quad (79)$$

We reached the conclusion of the dimensionless unification of the strong nuclear and the weak nuclear forces:

$$\begin{aligned} e \cdot \alpha_s &= 10^7 \cdot \alpha_w \\ \alpha_s^2 &= i^{2i} \cdot 10^7 \cdot \alpha_w \end{aligned}$$

(Dimensionless unification of the strong nuclear and the weak nuclear force interactions)

### 13. Dimensionless unification of the strong nuclear and the electromagnetic interactions

Based on Einstein's light quantum hypothesis, the duality of the photon was confirmed through quantum-mechanical experiments and examination. The photon is now regarded as a particle in fields related to the interaction of material with light that is absorbed and emitted; and regarded as a wave in regions relating to light propagation. It is known that among the four forces constituting the universe, the photon serves to convey electromagnetic force. The other three forces are gravitational force, strong force, and weak force. The photon plays an important role in the structure of the world where we live and is deeply involved with sources of matter and life. Through the work of Max Planck, Albert Einstein, Louis de Broglie, Arthur Compton, Niels Bohr, Erwin Schrödinger and many others, current scientific theory holds that all particles exhibit a wave nature and vice versa. This phenomenon has been verified not only for elementary particles, but also for compound particles like atoms and even molecules. For macroscopic particles, because of their extremely short wavelengths, wave properties usually cannot be detected. Bohr regarded the "duality paradox" as a fundamental or metaphysical fact of nature. He regarded renunciation of the cause-effect relation, or complementarity, of the space-time picture, as essential to the quantum mechanical account. Werner Heisenberg considered the question further. He saw the duality as present for all quantum entities, but not quite in the usual quantum mechanical account considered by Bohr. He saw it in what is called second quantization, which generates an entirely new concept of fields that exist in ordinary space-time, causality still being visualizable. Jesús Sánchez in [20] explained that the fine-structure constant is one of the roots of the following trigonometric equation:

$$\cos a^{-1} = e^{-1} \quad (80)$$

Another elegant expression is the following exponential form equations:

$$\begin{aligned} e^{i/a} - e^{-1} &= -e^{-i/a} + e^{-1} \\ e^{i/a} + e^{-i/a} &= 2 \cdot e^{-1} \end{aligned} \quad (81)$$

These expressions show the wave nature of light. The modern theory of quantum mechanics came to picture light as both a particle and a wave and ,as a phenomenon which is neither a particle or a wave. Instead,modern physics sees light as something that can be described sometimes with mathematics appropriate to one type of macroscopic metaphor (particles) and sometimes another macroscopic metaphor (water waves),but is actually something that cannot be fully imagined. Also the fine-structure constant is one of the roots of the following trigonometric equation:

$$\begin{aligned} \cos(10^3 \cdot a^{-1}) &= \phi^2 \cdot e^{-1} \\ e \cdot \cos(10^3 \cdot a^{-1}) &= \phi^2 \end{aligned} \quad (82)$$

Another elegant expression is the following exponential form equation:

$$e^{1000i/a} + e^{-1000i/a} = 2 \cdot \phi^2 \cdot e^{-1} \quad (83)$$

From these expressions resulting the following equations:

$$\begin{aligned} \cos^{-1} a^{-1} \cdot \cos(10^3 \cdot a^{-1}) &= \phi^2 \\ \cos(10^3 \cdot a^{-1}) &= \phi^2 \cdot \cos a^{-1} \end{aligned} \quad (84)$$

We will use the expressions to resulting the unity formulas that connects the strong coupling constant  $a_s$  and the fine-structure constant  $\alpha$ :

$$\begin{aligned} \cos a^{-1} &= e^{-1} \\ a_s &= e^{1-n} \\ \cos a^{-1} &= (e^n \cdot a_s)^{-1} \\ \cos a^{-1} &= e^{-n} \cdot a_s^{-1} \\ e^n \cdot a_s \cdot \cos a^{-1} &= 1 \end{aligned} \quad (85)$$

Other forms of the equations are:

$$\begin{aligned} \cos a^{-1} &= (i^{-2i} \cdot a_s)^{-1} \\ i^{-2i} \cdot a_s \cdot \cos a^{-1} &= 1 \\ \cos a^{-1} &= i^{2i} \cdot a_s^{-1} \\ a_s \cdot \cos a^{-1} &= i^{2i} \end{aligned} \quad (86)$$

So the beautiful formulas for the strong coupling constant  $a_s$  are:

$$\begin{aligned} a_s &= e^{-n} \cdot \cos^{-1} a^{-1} \\ a_s &= i^{2i} \cdot \cos^{-1} a^{-1} \end{aligned}$$

Now we need to study the following equivalent equations:

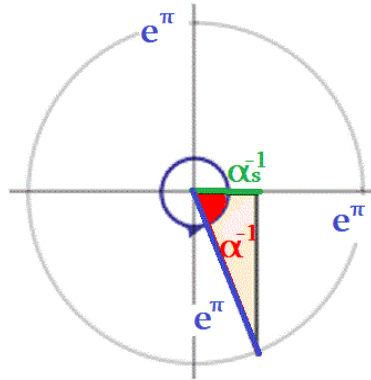
$$\cos \alpha^{-1} = \frac{e^{-\pi}}{\alpha_s}$$

$$\cos \alpha^{-1} = \frac{i^{2i}}{\alpha_s}$$

$$\cos \alpha^{-1} = \frac{\alpha_s^{-1}}{e^{\pi}}$$

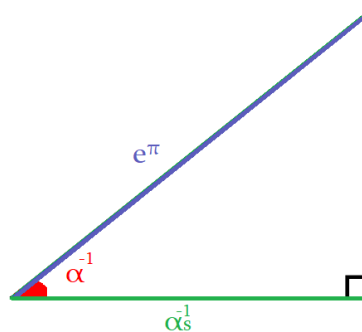
$$\cos \alpha^{-1} = \frac{\alpha_s^{-1}}{i^{-2i}}$$

The figure below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $e^{\pi}$ . The strong coupling constant  $\alpha_s$  and the fine-structure constant  $\alpha$  are in a right triangle with the variable acute angle  $\alpha^{-1}$  radians. The adjacent side is the variable side  $\alpha_s^{-1}$  while the hypotenuse is constant  $e^{\pi}$ . The fine structure constant is the ratio of the speed of the electron compared to the speed of light, in the first level of an atom. It is also related to the ratio of the Bohr radius of an atom to the Compton wavelength of an electron. We could try to relate it to the electromagnetic interactions in the atom. The figure 2 below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $e^{\pi}$ .



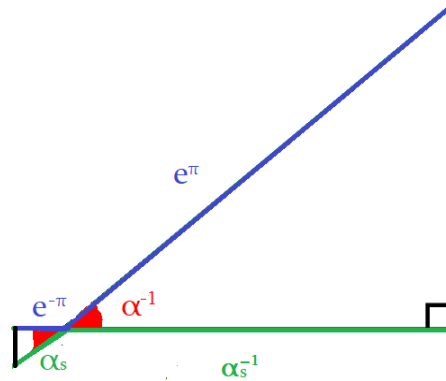
**Figure 2.** The angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $e^{\pi}$ .

The nucleus emits a photon that has an intrinsic property associated with the electromagnetic interaction represented by a vector. This vector can rotate as the photon moves along. The electron has an additional property related to the electromagnetic interaction that is also represented by a vector. The figure 3 and 4 below shows the geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.



**Figure 3.** The strong coupling constant  $\alpha_s$  and the fine-structure constant  $\alpha$  are in a right triangle with the variable acute angle  $\alpha^{-1}$  radians. The adjacent side is the variable side  $\alpha_s^{-1}$  while the hypotenuse is constant  $e^{\pi}$ .

Thus, when the photon reaches the electron, the electromagnetic interaction between them is related to the relative position of these vectors at the time of interaction. When we talk about relative placement between vectors, we can talk about a projection of one of them onto the other. This means that  $\cos\alpha^{-1}$  will be related to the interaction of these two properties of the photon and the electron. It would be related to their relative vector position at the time of interaction.



**Figure 4.** Geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.

The angle in  $\alpha^{-1}$  radians is not only the final interaction angle, but also includes, for example, the number of rotations the photon or electron vector has made before the interactions. From expressions resulting the formulas that connects the strong coupling constant  $\alpha_s$  and the fine-structure constant  $\alpha$ :

$$\begin{aligned}
 e^{i/\alpha} + e^{-i/\alpha} &= 2 \cdot e^{-1} \\
 e^{i/\alpha} + e^{-i/\alpha} &= 2 \cdot (e^n \cdot \alpha_s)^{-1} \\
 e^{i/\alpha} - (e^n \cdot \alpha_s)^{-1} &= -e^{-i/\alpha} + (e^n \cdot \alpha_s)^{-1} \\
 e^{i/\alpha} + e^{-i/\alpha} &= 2 \cdot e^{-n} \cdot \alpha_s^{-1} \\
 e^n \cdot \alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) &= 2
 \end{aligned} \tag{87}$$

Other forms of the equations are:

$$\begin{aligned}
 e^{i/\alpha} + e^{-i/\alpha} &= 2 \cdot (i^{-2i} \cdot \alpha_s)^{-1} \\
 e^{i/\alpha} + e^{-i/\alpha} &= 2 \cdot i^{2i} \cdot \alpha_s^{-1} \\
 e^{i/\alpha} - i^{2i} \cdot \alpha_s^{-1} &= -e^{-i/\alpha} + i^{2i} \cdot \alpha_s^{-1} \\
 \alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) &= 2 \cdot i^{2i}
 \end{aligned} \tag{88}$$

These equations are applicable for all energy scales. So the beautiful formulas for the strong coupling constant  $\alpha_s$  are:

$$\begin{aligned}
 \alpha_s &= 2 \cdot e^{-n} \cdot (e^{i/\alpha} + e^{-i/\alpha})^{-1} \\
 \alpha_s &= 2 \cdot i^{2i} \cdot (e^{i/\alpha} + e^{-i/\alpha})^{-1}
 \end{aligned}$$

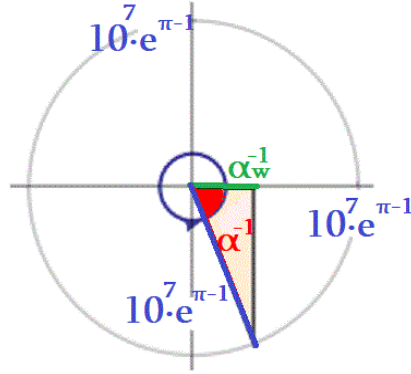
We reached the conclusion of the dimensionless unification of the strong nuclear and the electromagnetic forces:

$$\alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot i^{2i}$$

(Dimensionless unification of the strong nuclear and the electromagnetic interactions)

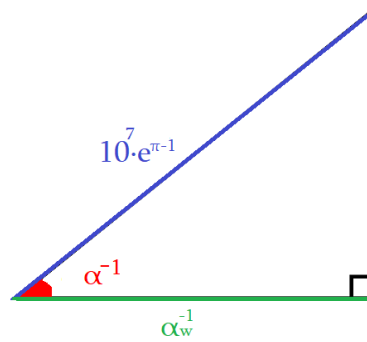
## 14. Dimensionless unification of the weak nuclear and electromagnetic interactions

In particle physics, the electroweak interaction or electroweak force is the unified description of two of the four known fundamental interactions of nature: electromagnetism and the weak interaction. The electroweak theory, in physics, is the theory that describes both the electromagnetic force and the weak force. Although these two forces appear very different at everyday low energies, the theory models them as two different aspects of the same force. The figure 5 below shows the angle in  $\alpha^{-1}$  radians.



**Figure 5.** The angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7 \cdot e^{\pi-1}$ .

The weak force acts only across distances smaller than the atomic nucleus, while the electromagnetic force can extend for great distances (as observed in the light of stars reaching across entire galaxies), weakening only with the square of the distance. Moreover, comparison of the strength of these two fundamental interactions between two protons, for instance, reveals that the weak force is some 10 million times weaker than the electromagnetic force. Yet one of the major discoveries of the 20th century has been that these two forces are different facets of a single, more-fundamental electroweak force. The figure 6 below shows the geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions.



**Figure 6.** Geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions

We will use the expressions to resulting the unity formula that connect the weak coupling constant  $a_w$  and the fine-structure constant  $\alpha$ :

$$e \cdot a_s = 10^7 \cdot a_w$$

$$e^n \cdot a_s \cdot \cos \alpha^{-1} = 1$$

$$e^n \cdot 10^7 \cdot a_w \cdot \cos \alpha^{-1} = e$$

$$e^{n-1} \cdot 10^7 \cdot a_w \cdot \cos \alpha^{-1} = 1$$

$$10^7 \cdot a_w \cdot \cos \alpha^{-1} = e^{1-n} \quad (89)$$

Other forms of the equations are:

$$a_w \cdot \cos \alpha^{-1} = e \cdot i^{2i} \cdot 10^{-7}$$

$$10^7 \cdot a_w \cdot \cos \alpha^{-1} = e \cdot i^{2i} \quad (90)$$

The figure below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7 \cdot e^{n-1}$ . The strong coupling constant  $a_s$  and the fine-structure constant  $\alpha$  are in a right triangle with the variable acute angle  $\alpha^{-1}$  radians. The adjacent side is the variable side  $a_w^{-1}$  while the hypotenuse is constant  $10^7 \cdot e^{n-1}$ . So the formulas for the weak coupling constant  $a_w$  are:

$$a_w = (e^{n-1} \cdot 10^7 \cdot \cos \alpha^{-1})^{-1}$$

$$a_w = e^{1-n} \cdot 10^{-7} \cdot \cos^{-1} \alpha^{-1}$$

$$a_w = e \cdot i^{2i} \cdot (10^7 \cdot \cos \alpha^{-1})^{-1}$$

$$a_w = e \cdot i^{2i} \cdot 10^{-7} \cdot \cos^{-1} \alpha^{-1}$$

Resulting the unity formulas that connects weak coupling constant  $a_w$  and the fine-structure constant  $\alpha$ :

$$e \cdot a_s = 10^7 \cdot a_w$$

$$e^n \cdot a_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2$$

$$e^n \cdot 10^7 \cdot a_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot e$$

$$e^{i/\alpha} + e^{-i/\alpha} = 2 \cdot (e^{n-1} \cdot 10^7 \cdot a_w)^{-1}$$

$$e^{i/\alpha} - (e^{n-1} \cdot 10^7 \cdot a_w)^{-1} = -e^{-i/\alpha} + (e^{n-1} \cdot 10^7 \cdot a_w)^{-1}$$

$$10^7 \cdot a_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot e^{1-n} \quad (91)$$

Other form of the equations is:

$$10^7 \cdot a_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot e \cdot i^{2i} \quad (92)$$

So the formulas for the weak coupling constant  $a_w$  are:

$$a_w = 2 \cdot [e^{n-1} \cdot 10^7 \cdot (e^{i/\alpha} + e^{-i/\alpha})]^{-1}$$

$$a_w = 2 \cdot e^{1-n} \cdot 10^{-7} \cdot (e^{i/\alpha} + e^{-i/\alpha})^{-1}$$

$$a_w = 2 \cdot e \cdot i^{2i} \cdot [10^7 \cdot (e^{i/\alpha} + e^{-i/\alpha})]^{-1}$$

$$a_w = 2 \cdot e \cdot i^{2i} \cdot 10^{-7} \cdot (e^{i/\alpha} + e^{-i/\alpha})^{-1}$$

These equations are applicable for all energy scales. We reached the conclusion of the dimensionless unification of the weak nuclear and the electromagnetic forces:

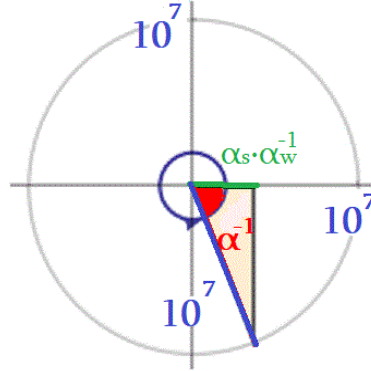
$$10^7 \cdot a_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot e \cdot i^{2i}$$

(Dimensionless unification of the weak nuclear and the electromagnetic interactions)

## 15. Dimensionless unification of the strong nuclear, the weak nuclear and electromagnetic interactions

Quantum mechanics is a theoretical framework that only focuses on the three non-gravitational forces for understanding the universe in regions of both very small scale and low mass: subatomic particles, atoms, molecules, etc. Quantum mechanics successfully implemented the Standard Model that describes the three non-gravitational forces: strong nuclear, weak nuclear, and electromagnetic force—as well as all observed elementary particles. A Grand Unified Theory (GUT) is a model in particle physics in which, at high energies, the three gauge interactions of the Standard

Model comprising the electromagnetic, weak, and strong forces are merged into a single force. Although this unified force has not been directly observed, many GUT models theorize its existence. If unification of these three interactions is possible, it raises the possibility that there was a grand unification epoch in the very early universe in which these three fundamental interactions were not yet distinct. Experiments have confirmed that at high energy the electromagnetic interaction and weak interaction unify into a single electroweak interaction. GUT models predict that at even higher energy, the strong interaction and the electroweak interaction will unify into a single electron nuclear interaction. The figure 7 below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7$ .



**Figure 7.** The angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7$ .

We will use the expressions to find the expression that connects the strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$  and the fine-structure constant  $\alpha$ :

$$e \cdot \alpha_s = 10^7 \cdot \alpha_w$$

$$\cos \alpha^{-1} = e^{-1}$$

$$\cos \alpha^{-1} = \alpha_s \cdot \alpha_w^{-1} \cdot 10^{-7}$$

So the unity formula that connects the strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$  and the fine-structure constant  $\alpha$  is:

$$10^7 \cdot \alpha_w \cdot \cos \alpha^{-1} = \alpha_s \tag{93}$$

Now we need to study the following equivalent equations:

$$\cos \alpha^{-1} = \frac{10^{-7} \alpha_w^{-1}}{\alpha_s^{-1}}$$

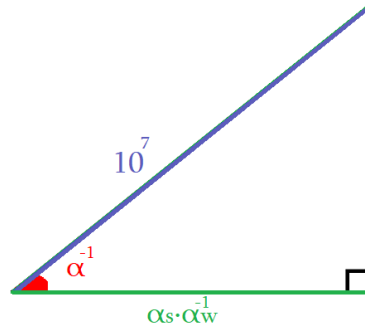
$$\cos \alpha^{-1} = \frac{\alpha_s}{10^7 \alpha_w}$$

$$10^7 \cos \alpha^{-1} = \frac{\alpha_s}{\alpha_w}$$

$$\cos \alpha^{-1} = \frac{\alpha_s \alpha_w^{-1}}{10^7}$$

The figure below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7$ . The strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$  and the fine-structure constant  $\alpha$  are in a right triangle with the variable acute angle  $\alpha^{-1}$  radians. The adjacent side is the variable side  $\alpha_s \cdot \alpha_w^{-1}$  while the hypotenuse is constant  $10^7$ . The figure 8 below shows the dimensionless unification of the strong nuclear, the weak nuclear and the electromagnetic interactions.





**Figure 8.** Geometric representation of the dimensionless unification of the strong nuclear, the weak nuclear and the electromagnetic interactions.

Resulting the beautiful formulas that connects the strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$  and the fine-structure constant  $\alpha$ :

$$\begin{aligned}
 e \cdot \alpha_s &= 10^7 \cdot \alpha_w \\
 e^{i/\alpha} + e^{-i/\alpha} &= 2 \cdot e^{-1} \\
 e^{i/\alpha} + e^{-i/\alpha} &= 2 \cdot 10^{-7} \cdot \alpha_s \cdot \alpha_w^{-1} \\
 \alpha_w \cdot \alpha_s^{-1} \cdot (e^{i/\alpha} + e^{-i/\alpha}) &= 2 \cdot 10^{-7} \\
 10^7 \cdot \alpha_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) &= 2 \cdot \alpha_s \tag{94}
 \end{aligned}$$

These equations are applicable for all energy scales. We reached the conclusion of the dimensionless unification of the strong nuclear, the weak nuclear and the electromagnetic forces:

$$10^7 \cdot \alpha_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot \alpha_s$$

(Dimensionless unification of the strong nuclear, the weak nuclear and the electromagnetic interactions)

## 16. Dimensionless unification of the gravitational and the electromagnetic interactions

It was presented in [11] the mathematical formulas that connects the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton, the Avogadro's number  $N_A$ , the gravitational coupling constant  $\alpha_G$  of the electron and the gravitational coupling constant of the proton  $\alpha_G(p)$ :

$$\alpha_G(p) = \mu^2 \cdot \alpha_G \tag{95}$$

$$\alpha = \mu \cdot N_1 \cdot \alpha_G \tag{96}$$

$$\alpha \cdot \mu = N_1 \cdot \alpha_G(p) \tag{97}$$

$$\alpha^2 = N_1^2 \cdot \alpha_G \cdot \alpha_G(p) \tag{98}$$

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \tag{99}$$

$$\mu^2 = 4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G(p) \cdot N_A^2 \tag{100}$$

$$\mu \cdot N_1 = 4 \cdot e^2 \cdot \alpha^3 \cdot N_A^2 \tag{101}$$

$$4 \cdot e^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = 1 \tag{102}$$

$$\mu^3 = 4 \cdot e^2 \cdot a \cdot a_G(p)^2 \cdot NA^2 \cdot N1 \quad (103)$$

$$\mu^2 = 4 \cdot e^2 \cdot a_G \cdot a_G(p)^2 \cdot NA^2 \cdot N1^2 \quad (104)$$

$$\mu = 4 \cdot e^2 \cdot a \cdot a_G \cdot a_G(p) \cdot NA^2 \cdot N1 \quad (105)$$

Also resulting the expressions:

$$\cos(\alpha^{-1}) = e^{-1}$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot NA^2 = 1$$

$$4 \cdot a^2 \cdot a_G \cdot NA^2 = e^{-2}$$

$$\cos^2 \alpha^{-1} = 4 \cdot a^2 \cdot a_G \cdot NA^2$$

$$\alpha^{-2} \cdot \cos^2 \alpha^{-1} = 4 \cdot a_G \cdot NA^2 \quad (106)$$

This unity formula is equally valid:

$$\alpha^{-1} \cos \alpha^{-1} = 2N_A \sqrt{\alpha_G} \quad (107)$$

Also resulting another elegant exponential form equations:

$$e^{i/a} + e^{-i/a} = 2 \cdot e^{-1}$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot NA^2 = 1$$

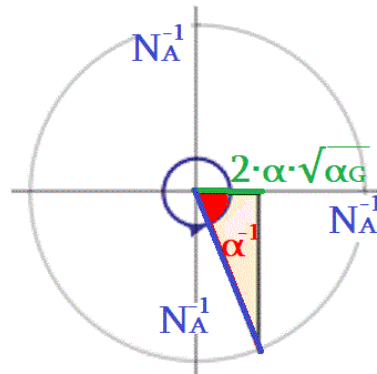
$$4 \cdot a^2 \cdot a_G \cdot NA^2 = e^{-2}$$

$$16 \cdot a^2 \cdot a_G \cdot NA^2 = (e^{i/a} + e^{-i/a})^2 \quad (108)$$

This unity formula is equally valid:

$$\alpha^{-1} \left( e^{\frac{i}{\alpha}} + e^{\frac{-i}{\alpha}} \right) = 4N_A \sqrt{\alpha_G} \quad (109)$$

The concept of power of two supports an idea of holographic concepts of the Universe or some of the fractal theories. Also it is used in wave mechanics, and it could be viewed in accordance with wave properties of the elementary particles in quantum physics. The figure 9 below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $NA^{-1}$ .



**Figure 9.** The angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $NA^{-1}$ .

Also resulting the expression with power of two:

$$2^{160} \cdot e^2 \cdot a^2 \cdot a_G = 1 \quad (110)$$

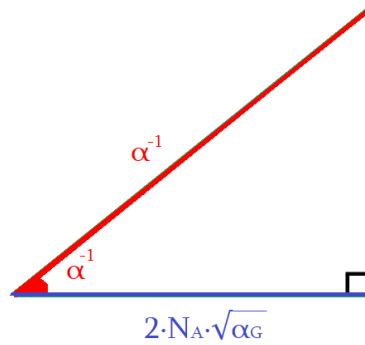
$$a^{-2} \cdot \cos^2 a^{-1} = 2^{160} \cdot a_G \quad (111)$$

$$2^{162} \cdot a^2 \cdot a_G = (e^{i/a} + e^{-i/a})^2 \quad (112)$$

Other form of the equations is:

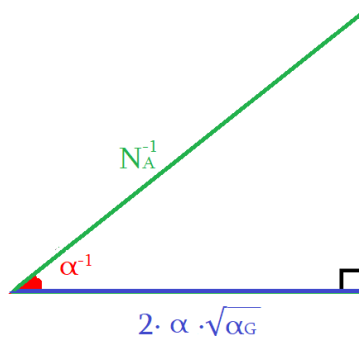
$$\alpha^{-1} \cos \alpha^{-1} = 2^{34} \sqrt{\alpha_G} \quad (113)$$

In his experiments of 1849–50, Michael Faraday was the first to search for a unification of gravity with electricity and magnetism. However, he found no connection. In 1900, David Hilbert published a famous list of mathematical problems. In Hilbert's sixth problem, he challenged researchers to find an axiomatic basis for all of physics. The figures 10 and 11 below show the geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions.



**Figure 10.** First geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions

Gravity and electromagnetism are able to coexist as entries in a list of classical forces, but for many years it seemed that gravity could not be incorporated into the quantum framework, let alone unified with the other fundamental forces. For this reason, work on unification, for much of the twentieth century, focused on understanding the three forces described by quantum mechanics: electromagnetism and the weak and strong forces.



**Figure 11.** Second geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions

We reached the conclusion of the dimensionless unification of the gravitational and the electromagnetic forces:

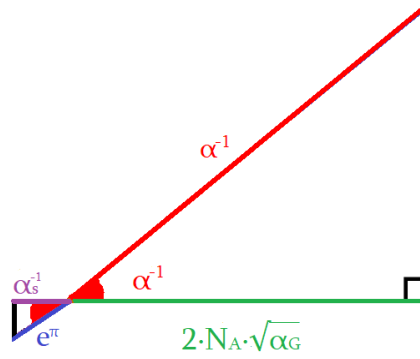
$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1$$

$$16 \cdot a^2 \cdot a_G \cdot N_A^2 = (e^{i/a} + e^{-i/a})^2$$

(Dimensionless unification of the gravitational and the electromagnetic interactions)

## 17. Dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions

Although the Standard Model is believed to be theoretically self-consistent and has demonstrated huge successes in providing experimental predictions,it leaves some phenomena unexplained. It falls short of being a complete theory of fundamental interactions. For example,it does not fully explain baryon asymmetry,incorporate the full theory of gravitation as described by general relativity,or account for the universe's accelerating expansion as possibly described by dark energy. The model does not contain any viable dark matter particle that possesses all of the required properties deduced from observational cosmology. It also does not incorporate neutrino oscillations and their non-zero masses. Now we will find the equation that connect the coupling constants of the strong nuclear,the gravitational and the electromagnetic interactions. The figure 12 below shows the geometric representation of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions.



**Figure 12.** Geometric representation of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions

We will use the previous expressions to resulting the unity formulas that connect the strong coupling constant  $\alpha_s$ ,the fine-structure constant  $\alpha$  and the gravitational coupling constant  $\alpha_G$ :

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1$$

$$4 \cdot e^2 \cdot (e^n \cdot \alpha_s)^2 \cdot \alpha_G \cdot N_A^2 = 1$$

$$4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (114)$$

Other form of the equation is:

$$4 \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i} \quad (115)$$

Also resulting the mathematical formulas that connects the strong coupling constant  $\alpha_s$ ,the proton to electron mass ratio  $\mu$ ,the fine-structure constant  $\alpha$ ,the ratio  $N_1$  of electric force to gravitational force between electron and proton,the Avogadro number  $N_A$ ,the gravitational coupling constant  $\alpha_G$  of the electron and the gravitational coupling constant of the proton  $\alpha_{G(p)}$ :

$$4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (116)$$

$$\mu^2 = 4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_{G(p)} \cdot N_A^2 \quad (117)$$

$$\mu \cdot N_1 = 4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha^3 \cdot N_A^2 \quad (118)$$

$$4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = 1 \quad (119)$$

$$\mu^3 = 4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1 \quad (120)$$

$$\mu^2 = 4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha_G \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1^2 \quad (121)$$

$$\mu = 4 \cdot e^{2n} \cdot \alpha_s^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_{G(p)} \cdot N_A^2 \cdot N_1 \quad (122)$$

Other equivalent forms of the equations are:

$$4 \cdot a_s^2 \cdot a^2 \cdot a_G \cdot NA^2 = i^{4i} \quad (123)$$

$$i^{4i} \cdot \mu = a_s^2 \cdot a^2 \cdot a_G(p) \cdot NA^2 \quad (124)$$

$$i^{4i} \cdot \mu \cdot N_1 = 4 \cdot a_s^2 \cdot a^3 \cdot NA^2 \quad (125)$$

$$4 \cdot a_s^2 \cdot a \cdot \mu \cdot a_G^2 \cdot NA^2 \cdot N_1 = i^{4i} \quad (126)$$

$$i^{4i} \cdot \mu^3 = 4 \cdot a_s^2 \cdot a \cdot a_G(p)^2 \cdot NA^2 \cdot N_1 \quad (127)$$

$$i^{4i} \cdot \mu^2 = 4 \cdot e^{2n} \cdot a_s^2 \cdot a_G \cdot a_G(p)^2 \cdot NA^2 \cdot N_1^2 \quad (128)$$

$$i^{4i} \cdot \mu = 4 \cdot a_s^2 \cdot a \cdot a_G \cdot a_G(p) \cdot NA^2 \cdot N_1 \quad (129)$$

Also apply:

$$a_s \cdot \cos a^{-1} = i^{2i}$$

$$2 \cdot NA \cdot a_G^{1/2} = a^{-1} \cdot \cos a^{-1}$$

$$2 \cdot a_s \cdot a \cdot a_G^{1/2} \cdot NA = i^{2i}$$

$$2 \cdot a_s \cdot a \cdot NA \cdot a_G^{1/2} \cdot a_s \cdot \cos a^{-1} = i^{2i} \cdot i^{2i}$$

$$2 \cdot a \cdot \cos a^{-1} \cdot a_s^2 \cdot a_G^{1/2} \cdot NA = i^{4i}$$

$$4 \cdot a^2 \cdot \cos^2 a^{-1} \cdot a_s^4 \cdot a_G \cdot NA^2 = i^{8i} \quad (130)$$

$$a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot i^{2i}$$

$$2 \cdot NA \cdot a_G^{1/2} = a^{-1} \cdot (e^{i/a} + e^{-i/a})$$

$$2 \cdot a_s \cdot a \cdot NA \cdot a_G^{1/2} = i^{2i}$$

$$a_s \cdot (e^{i/a} + e^{-i/a}) \cdot 2 \cdot a_s \cdot a \cdot NA \cdot a_G^{1/2} = 2 \cdot i^{2i} \cdot i^{2i}$$

$$a \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^2 \cdot a_G^{1/2} \cdot NA = i^{4i}$$

$$a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^4 \cdot a_G \cdot NA^2 = i^{8i} \quad (131)$$

Also resulting the expressions with power of two:

$$2^{80} \cdot a_s \cdot a \cdot a_G^{1/2} = i^{2i}$$

$$2^{160} \cdot a_s^2 \cdot a^2 \cdot a_G = i^{4i} \quad (132)$$

$$2^{80} \cdot a \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^2 \cdot a_G^{1/2} = i^{4i}$$

$$2^{160} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_s^4 \cdot a_G = i^{8i} \quad (133)$$

We reached the conclusion of the dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions:

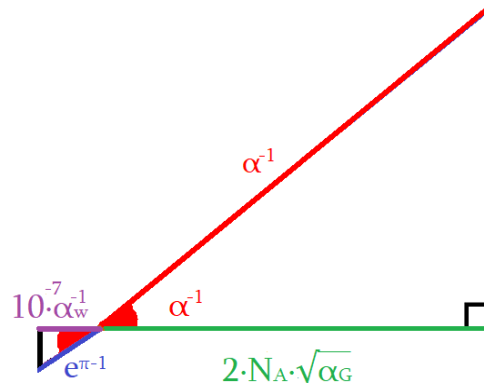
$$4 \cdot a_s^2 \cdot a^2 \cdot a_G \cdot NA^2 = i^{4i}$$

$$a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^4 \cdot a_G \cdot NA^2 = i^{8i}$$

(Dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions)

## 18. Dimensionless unification of the weak nuclear, the gravitational and the electromagnetic interactions

Now we will find the equation that connects the coupling constants of the weak nuclear, the gravitational and the electromagnetic interactions. The figure 13 below shows the geometric representation of the dimensionless unification of the weak nuclear, the gravitational and the electromagnetic interactions.



**Figure 13.** Geometric representation of the dimensionless unification of the weak nuclear, the gravitational and the electromagnetic interactions

We will use the previous expressions to resulting the unity formulas that connects the weak coupling constant  $\alpha_w$ , the fine-structure constant  $\alpha$  and the gravitational coupling constant  $\alpha_G$ :

$$e \cdot \alpha_s = 10^7 \cdot \alpha_w$$

$$2 \cdot e^\pi \cdot \alpha_s \cdot \alpha \cdot \alpha_G^{1/2} \cdot N_A = 1$$

$$2 \cdot \alpha_s \cdot \alpha \cdot \alpha_G^{1/2} \cdot N_A = i^{2i}$$

$$2 \cdot e^\pi \cdot 10^7 \cdot \alpha_w \cdot \alpha \cdot \alpha_G^{1/2} \cdot N_A = e$$

$$4 \cdot 10^{14} \cdot e^{2\pi} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = e^2 \quad (134)$$

$$2 \cdot 10^7 \cdot \alpha_w \cdot \alpha \cdot \alpha_G^{1/2} \cdot N_A = i^{2i} \cdot e$$

$$4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i} \cdot e^2 \quad (135)$$

Also resulting the mathematical formulas that connects the weak coupling constant  $\alpha_w$ , the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton, the Avogadro's number  $N_A$ , the gravitational coupling constant  $\alpha_G$  of the electron and the gravitational coupling constant of the proton  $\alpha_G(p)$ :

$$4 \cdot 10^{14} \cdot e^{2\pi} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = e^2 \quad (136)$$

$$e^2 \cdot \mu^2 = 4 \cdot 10^{14} \cdot e^{2\pi} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G(p) \cdot N_A^2 \quad (137)$$

$$e^2 \cdot \mu \cdot N_1 = 4 \cdot 10^{14} \cdot e^{2\pi} \cdot \alpha_w^2 \cdot \alpha^3 \cdot N_A^2 \quad (138)$$

$$4 \cdot 10^{14} \cdot e^{2\pi} \cdot \alpha_w^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = e^2 \quad (139)$$

$$e^2 \cdot \mu^3 = 4 \cdot 10^{14} \cdot e^{2\pi} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1 \quad (140)$$

$$e^2 \cdot \mu^2 = 4 \cdot 10^{14} \cdot e^{2\pi} \cdot \alpha_w^2 \cdot \alpha_G \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1^2 \quad (141)$$

$$e^2 \cdot \mu = 4 \cdot e^\pi \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_G(p) \cdot N_A^2 \cdot N_1 \quad (142)$$

Other equivalent forms of the equations are:

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2 = i^{4i} \cdot e^2 \quad (143)$$

$$i^{4i} \cdot e^2 \cdot \mu^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G(p) \cdot NA^2 \quad (144)$$

$$i^{4i} \cdot e^2 \cdot \mu \cdot N_1 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^3 \cdot NA^2 \quad (145)$$

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot \mu \cdot a_G^2 \cdot NA^2 \cdot N_1 = i^{4i} \cdot e^2 \quad (146)$$

$$i^{4i} \cdot e^2 \cdot \mu^3 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot a_G(p)^2 \cdot NA^2 \cdot N_1 \quad (147)$$

$$i^{4i} \cdot e^2 \cdot \mu^2 = 4 \cdot 10^{14} \cdot e^{2n} \cdot a_w^2 \cdot a_G \cdot a_G(p)^2 \cdot NA^2 \cdot N_1^2 \quad (148)$$

$$i^{4i} \cdot e^2 \cdot \mu = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot a_G \cdot a_G(p) \cdot NA^2 \cdot N_1 \quad (149)$$

$$a_w^{-1} \cdot a_s^2 = i^{2i} \cdot 10^7$$

$$a_s^2 = i^{2i} \cdot 10^7 \cdot a_w$$

$$2 \cdot a \cdot \cos a^{-1} \cdot a_s^2 \cdot a_G^{1/2} \cdot NA = i^{4i}$$

$$2 \cdot a \cdot \cos a^{-1} \cdot i^{2i} \cdot 10^7 \cdot a_w \cdot a_G^{1/2} \cdot NA = i^{4i}$$

$$2 \cdot 10^7 \cdot a \cdot \cos a^{-1} \cdot a_w \cdot a_G^{1/2} \cdot NA = i^{2i}$$

$$4 \cdot 10^{14} \cdot a^2 \cdot \cos^2 a^{-1} \cdot a_w^2 \cdot a_G \cdot NA^2 = i^{4i} \quad (150)$$

$$a \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^2 \cdot a_G^{1/2} \cdot NA = i^{4i}$$

$$a \cdot (e^{i/a} + e^{-i/a}) \cdot i^{2i} \cdot 10^7 \cdot a_w \cdot a_G^{1/2} \cdot NA = i^{4i}$$

$$10^7 \cdot a \cdot (e^{i/a} + e^{-i/a}) \cdot a_w \cdot a_G^{1/2} \cdot NA = i^{2i}$$

$$10^{14} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G \cdot NA^2 = i^{4i} \quad (151)$$

Also resulting the expression with power of two:

$$2^{80} \cdot 10^7 \cdot a_w \cdot a \cdot a_G^{1/2} = i^{2i} \cdot e$$

$$2^{160} \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G = i^{4i} \cdot e^2 \quad (152)$$

$$2^{80} \cdot 10^7 \cdot a \cdot (e^{i/a} + e^{-i/a}) \cdot a_w \cdot a_G^{1/2} = i^{2i}$$

$$2^{160} \cdot 10^{14} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G = i^{4i} \quad (153)$$

We reached the conclusion of the dimensionless unification of the weak nuclear, the gravitational and electromagnetic forces:

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2 = i^{4i} \cdot e^2$$

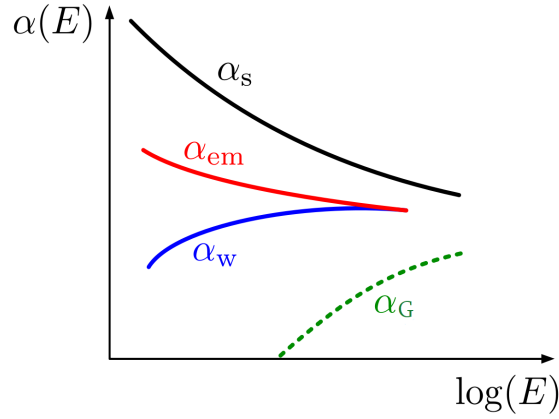
$$10^{14} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G \cdot NA^2 = i^{8i}$$

(Dimensionless unification of the weak nuclear, the gravitational and the electromagnetic interactions)

## 19. Dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions

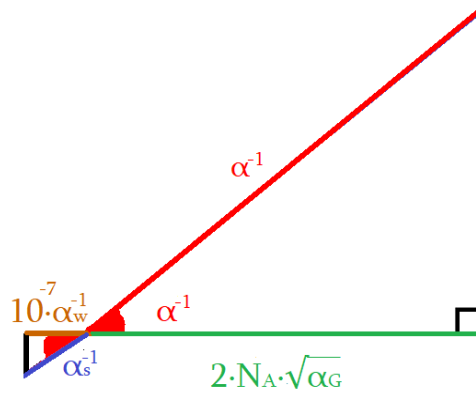
A theory of everything is a hypothetical, singular, all-encompassing, coherent theoretical framework of physics that fully explains and links together all aspects of the universe. Finding a theory of everything is one of the major

unsolved problems in physics. String theory and M-theory have been proposed as theories of everything. The figure 14 below shows the variation of the coupling constants of the four fundamental interactions of physics as a function of energy.



**Figure 14.** Variation of the coupling constants of the four fundamental interactions of physics as a function of energy.

Over the past few centuries, two theoretical frameworks have been developed that, together, most closely resemble a theory of everything. These two theories upon which all modern physics rests are general relativity and quantum mechanics. The figure 15 below shows the geometric representation of the dimensionless unification of the strong nuclear, the weak nuclear, the



**Figure 15.** Geometric representation of the dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions

General relativity is a theoretical framework that only focuses on gravity for understanding the universe in regions of both large scale and high mass: planets, stars, galaxies, clusters of galaxies etc. Now we will find the equation that connects the four coupling constants. We will use the previous expressions to resulting the unity formulas that connects the strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$ , the fine-structure constant  $\alpha$  and the gravitational coupling constant  $\alpha_G$ :

$$\alpha_w^{-1} \cdot \alpha_s^2 = i^{2i} \cdot 10^7$$

$$2 \cdot \alpha_s \cdot \alpha \cdot NA \cdot \alpha_G^{1/2} = i^{2i}$$

$$\alpha_w^{-1} \cdot \alpha_s^2 = 2 \cdot 10^7 \cdot \alpha_s \cdot \alpha \cdot NA \cdot \alpha_G^{1/2}$$

$$\alpha_w^{-1} \cdot \alpha_s = 2 \cdot 10^7 \cdot \alpha \cdot NA \cdot \alpha_G^{1/2}$$

$$2 \cdot 10^7 \cdot NA \cdot \alpha_w \cdot \alpha \cdot \alpha_G^{1/2} \cdot \alpha_s^{-1} = 1$$

$$\alpha_w \cdot \alpha \cdot \alpha_G^{1/2} \cdot \alpha_s^{-1} = (2 \cdot 10^7 \cdot NA)^{-1} \quad (154)$$

$$2 \cdot 10^7 \cdot NA \cdot \alpha_w \cdot \alpha \cdot \alpha_G^{1/2} = \alpha_s$$

$$\alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot \alpha_s^{-2} = (2 \cdot 10^7 \cdot NA)^{-2} \quad (155)$$



So the beautiful unity formula that connects the strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$ , the fine-structure constant  $\alpha$  and the gravitational coupling constant  $\alpha_G$  is:

$$\begin{aligned}(2 \cdot 10^7 \cdot N_A \cdot \alpha_w \cdot \alpha)^2 \cdot \alpha_G &= \alpha_s^2 \\ 4 \cdot 10^{14} \cdot N_A^2 \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G &= \alpha_s^2\end{aligned}\quad (156)$$

Sometimes the gravitational coupling constant for the proton  $\alpha_{G(p)}$  is used instead of the gravitational coupling constant  $\alpha_G$  for the electron:

$$\begin{aligned}\alpha_{G(p)} &= \mu^2 \cdot \alpha_G \\ \alpha_G^{1/2} &= \alpha_{G(p)}^{1/2} \cdot \mu^{-1} \\ \alpha_s \cdot \mu \cdot (\alpha_w \cdot \alpha \cdot \alpha_{G(p)}^{1/2})^{-1} &= 2 \cdot 10^7 \cdot N_A \\ \alpha_s \cdot \mu &= 2 \cdot 10^7 \cdot N_A \cdot \alpha_w \cdot \alpha \cdot \alpha_{G(p)}^{1/2} \\ 2 \cdot 10^7 \cdot N_A \cdot \alpha_w \cdot \alpha \cdot \alpha_{G(p)}^{1/2} \cdot \alpha_s^{-1} \cdot \mu^{-1} &= 1 \\ 2 \cdot 10^7 \cdot N_A \cdot \alpha_w \cdot \alpha \cdot \alpha_{G(p)}^{1/2} \cdot \alpha_s^{-1} &= \mu \cdot \alpha_s \\ \alpha_w \cdot \alpha \cdot \alpha_{G(p)}^{1/2} \cdot \alpha_s^{-1} &= (2 \cdot 10^7 \cdot N_A)^{-1} \cdot \mu\end{aligned}\quad (157)$$

So the beautiful unity formula that connect the strong coupling constant  $\alpha_s$ , weak coupling constant  $\alpha_w$ , the fine-structure constant  $\alpha$  and the gravitational coupling constant  $\alpha_{G(p)}$  for the proton is:

$$\begin{aligned}(2 \cdot 10^7 \cdot N_A \cdot \alpha_w \cdot \alpha)^2 \cdot \alpha_{G(p)} &= \mu^2 \cdot \alpha_s^2 \\ 4 \cdot 10^{14} \cdot N_A^2 \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_{G(p)} &= \mu^2 \cdot \alpha_s^2\end{aligned}\quad (158)$$

$$\begin{aligned}\cos \alpha^{-1} &= 2 \cdot \alpha \cdot \alpha_G^{1/2} \cdot N_A \\ 2 \cdot 10^7 \cdot N_A \cdot \alpha_w \cdot \alpha \cdot \alpha_G^{1/2} &= \alpha_s \\ 2 \cdot \alpha \cdot \alpha_G^{1/2} \cdot N_A \cdot 2 \cdot 10^7 \cdot N_A \cdot \alpha_w \cdot \alpha \cdot \alpha_G^{1/2} &= \alpha_s \cos \alpha^{-1} \\ 4 \cdot 10^7 \cdot \alpha^2 \cdot \alpha_G \cdot \alpha_w \cdot N_A^2 &= \alpha_s \cos \alpha^{-1} \\ \alpha^{-1} \cdot (e^{i/\alpha} + e^{-i/\alpha}) &= 4 \cdot N_A \cdot \alpha_G^{1/2} \\ 2 \cdot 10^7 \cdot N_A \cdot \alpha_w \cdot \alpha \cdot \alpha_G^{1/2} &= \alpha_s \\ 2 \cdot 10^7 \cdot N_A \cdot \alpha_w \cdot \alpha \cdot \alpha_G^{1/2} \cdot 4 \cdot N_A \cdot \alpha_G^{1/2} &= \alpha_s \cdot \alpha^{-1} \cdot (e^{i/\alpha} + e^{-i/\alpha}) \\ 8 \cdot 10^7 \cdot N_A^2 \cdot \alpha_w \cdot \alpha^2 \cdot \alpha_G &= \alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha})\end{aligned}$$

Resulting the mathematical formulas that connects the strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$ , the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton, the Avogadro's number  $N_A$ , the gravitational coupling constant  $\alpha_G$  of the electron and the gravitational coupling constant of the proton  $\alpha_{G(p)}$ :

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 \quad (159)$$

$$\mu^2 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_{G(p)} \cdot N_A^2 \quad (160)$$

$$\mu \cdot N_1 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^3 \cdot N_A^2 \quad (161)$$

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 \quad (162)$$

$$\mu^3 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1 \quad (163)$$

$$\mu \cdot a_s = 4 \cdot 10^{14} \cdot a_w^2 \cdot a_G \cdot G(p)^2 \cdot NA^2 \cdot N_1^2 \quad (164)$$

$$\mu \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot a_G \cdot a_G(p) \cdot NA^2 \cdot N_1 \quad (165)$$

Also resulting the expressions with power of two:

$$2^{80} \cdot 10^7 \cdot a_w \cdot a \cdot a_G^{1/2} \cdot a_s^{-1} = 1$$

$$2^{160} \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} = 1 \quad (166)$$

$$a_s = 2^{80} \cdot 10^7 \cdot a_w \cdot a \cdot a_G^{1/2}$$

$$a_s^2 = 2^{160} \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \quad (167)$$

We reached the conclusion of the dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions:

$$a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2$$

$$8 \cdot 10^7 \cdot NA^2 \cdot a_w \cdot a^2 \cdot a_G = a_s \cdot (e^{i/a} + e^{-i/a})$$

(Dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions)

## 11. Gravitational constant

In [21] and [22] it presented the theoretical value of the Gravitational constant  $G = 6.67448 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$ . This value is very close to the CODATA recommended value of gravitational constant and two experimental measurements from a research group announced new measurements based on torsion balances. Gravity is a natural phenomenon by which all things with mass or energy, including planets, stars, galaxies, and even light, are brought toward one another. On Earth, gravity gives weight to physical objects, and the Moon's gravity causes the ocean tides. The gravitational attraction of the original gaseous matter present in the Universe caused it to begin coalescing and forming stars and caused the stars to group together into galaxies, so gravity is responsible for many of the large-scale structures in the Universe. Gravity has an infinite range, although its effects become increasingly weaker as objects get further away. Gravity is most accurately described by the general theory of relativity, which describes gravity not as a force, but as a consequence of masses moving along geodesic lines in a curved spacetime caused by the uneven distribution of mass. However, for most applications, gravity is well approximated by Newton's law of universal gravitation, which describes gravity as a force causing any two bodies to be attracted toward each other, with magnitude proportional to the product of their masses and inversely proportional to the square of the distance between them. Gravity is the weakest of the four fundamental interactions of physics. As a consequence, it has no significant influence at the level of subatomic particles. In contrast, it is the dominant interaction at the macroscopic scale, and is the cause of the formation, shape and trajectory of astronomical bodies. Attempts to develop a theory of gravity consistent with quantum mechanics, a quantum gravity theory, which would allow gravity to be united in a common mathematical framework with the other three fundamental interactions of physics, are a current area of research.

In the late 17th century, Isaac Newton's description of the long-distance force of gravity implied that not all forces in nature result from things coming into contact. Newton's work in his *Mathematical Principles of Natural Philosophy* dealt with this in a further example of unification, in this case unifying Galileo's work on terrestrial gravity, Kepler's laws of planetary motion and the phenomenon of tides by explaining these apparent actions at a distance under one single law: the law of universal gravitation. In 1814, building on these results, Laplace famously suggested that a sufficiently powerful intellect could, if it knew the position and velocity of every particle at a given time, along with the laws of nature, calculate the position of any particle at any other time. In 1900, David Hilbert published a famous list of mathematical problems. In Hilbert's sixth problem, he challenged researchers to find an axiomatic basis to all of physics. In this problem he thus asked for what today would be called a theory of everything. After 1915, when Albert Einstein published the theory of gravity (general relativity), the search for a unified field theory combining gravity with electromagnetism began with a renewed interest. In Einstein's day, the strong and the weak forces had not yet been discovered, yet he found the potential existence of two other distinct forces, gravity and electromagnetism, far more alluring. coupling), is a number that determines the strength of the force exerted in an interaction. In attributing a relative strength to the four fundamental forces, it has proved useful to quote the strength in terms of a coupling

constant. The coupling constant for each force is a dimensionless constant. The gravitational constant is an empirical physical constant that participates in the calculation of gravitational force between two bodies. It usually appears in Isaac Newton's law of universal gravitation and Albert Einstein's general theory of relativity. In Newton's law, it is the proportionality constant connecting the gravitational force between two bodies with the product of their masses and the inverse square of their distance. In the Einstein field equations, it quantifies the relation between the geometry of spacetime and the energy–momentum tensor. The modern notation of Newton's law involving  $G$  was introduced in the 1890s by C.V. Boys. The first implicit measurement with an accuracy within about 1% is attributed to Henry Cavendish in a 1798 experiment.

The physicist Sir Isaac Newton in 1687 published his book "Philosophiæ Naturalis Principia Mathematica" where he presented the law of universal gravity to describe and calculate the mutual attraction of particles and huge objects in the universe. In this paper, Isaac Newton concluded that the attraction between two bodies is proportional to the product of their masses and inversely proportional to the square of the distance separating them. However, these must be adjusted by introducing the gravity constant  $G$ . The gravitational constant  $G$  occupies an anomalous position among the other constants of physics. The mass  $M$  of any celestial object cannot be determined independently of the gravitational attraction that it exerts. Thus, the combination  $G \cdot M$ , not the separate value of  $M$ , is the only meaningful property of a star, planet, or galaxy. According to general relativity and the principle of equivalence,  $G$  does not depend on material properties but is in a sense a geometric factor.

The concept of a different cosmology  $G$  first appears in the work of Edward Arthur Milne a few years before Dirac formulated LNH. Milne was inspired not by a large number of coincidences but by a contradiction of Einstein's general theory of relativity. For Milne, the space was not a structured object but merely a frame of reference in which relations such as this could accommodate Einstein's conclusions:

$$G = \frac{c^3}{M_U} T_U$$

According to this relationship,  $G$  increases with time. Dirac hypothesized that the constant of universal attraction  $G$  varies with time. Dirac's hypothesis went so far as to claim that such coincidences could be explained if the very physical constants changed with  $T_U$ , especially the gravitational constant  $G$ , which must decrease with time:

$$G \approx \frac{1}{t}$$

However, according to general relativity,  $G$  must also be constant over time. Although George Gamow noted that such a time variation is not necessarily due to Dirac's assumptions, no corresponding change of  $G$  has been found. According to general relativity,  $G$  is constant, otherwise the law of conservation of energy is violated. Dirac dealt with this difficulty by introducing into the Einstein field equations a gauge function  $\beta$  that describes the structure of spacetime in terms of a ratio of gravitational and electromagnetic units. The gravitational constant is defined as:

$$G = \alpha_G \frac{\hbar c}{m_e^2}$$

The expressions for the gravitational constant  $G$  in terms of Planck units are:

$$G = \frac{c^3 l_{pl}^2}{\hbar} = \frac{\hbar c}{m_{pl}^2} = \frac{l_{pl} c^2}{m_{pl}} = \frac{c^5 t_{pl}^2}{\hbar}$$

A surprisingly close relationship between gravity and the electrostatic interaction. The gravitational constant  $G$  and the Coulomb constant  $k_e$  are expressed in terms of Planck units as:

$$G = \frac{K_e q_e^2}{a m_{pl}^2}$$

Also another beautiful expression that proves the close relationship between gravity and electrostatic interaction is:

$$G = \frac{\alpha c^4 l_{pl}^2}{K_e q_e^2}$$

The 2018 CODATA recommended value of gravitational constant is  $G=6.67430 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$  with standard uncertainty  $0.00015 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$  and relative standard uncertainty  $2.2 \times 10^{-5}$ . The gravitational constant  $G$  is quite difficult to measure because gravity is much weaker than other fundamental forces, and an experimental apparatus cannot be separated from the gravitational influence of other bodies. The first direct measurement of gravitational attraction between two bodies in the laboratory was performed in 1798, seventy-one years after Newton's death, by Henry Cavendish. He determined a value for  $G$  implicitly, using a torsion balance invented by the geologist Rev. John Michell. His result corresponds to the value of  $G=6.74(4) \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$ . It is surprisingly accurate, about 1% above the modern value. The accuracy of the measured value of  $G$  has increased only modestly since the original Cavendish experiment. In the January 2007 issue of Science, Fixler et al. described a measurement of the gravitational constant by a new technique, atom interferometry, reporting a value of  $G=6.693(34) \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$ , 0.28% (2800 ppm) higher than the 2006 CODATA value. An improved cold atom measurement by Rosi et al. was published in 2014 of  $G=6.67191(99) \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$ . Although much closer to the accepted value (suggesting that the Fixler et al. measurement was erroneous), this result was 325 ppm below the recommended 2014 CODATA value, with non-overlapping standard uncertainty intervals. As of 2018, efforts to re-evaluate the conflicting results of measurements are underway, coordinated by NIST, notably a repetition of the experiments reported by Quinn et al.

In August 2018, a Chinese research group announced new measurements based on torsion balances,  $6.674184(78) \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$  and  $6.674484(78) \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$  based on two different methods [23]. These are claimed as the most accurate measurements ever made, with standard uncertainties cited as low as 12 ppm. The difference of  $2.7\sigma$  between the two results suggests there could be sources of error unaccounted for. The study is an example of excellent craftsmanship in precision measurements. However, the true value of  $G$  remains unclear. Various determinations of  $G$  that have been made over the past 40 years have a wide spread of values. Although some of the individual relative uncertainties are of the order of 10 parts per million, the difference between the smallest and largest values is about 500 parts per million. Now we will find the formulas for the gravitational constant  $G$  using the unity formulas for the coupling constants that we calculated. The gravitational coupling constant  $\alpha_G$  can be written in the form:

$$\begin{aligned} 4 \cdot e^2 \cdot N_A^2 \cdot a^2 \cdot \alpha_G &= 1 \\ \alpha_G &= (2 \cdot e \cdot a \cdot N_A)^{-2} \end{aligned} \quad (168)$$

Therefore from this expression the formula for the gravitational constant is:

$$G = (2e\alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (169)$$

From equivalent expressions (39) the gravitational coupling constant  $\alpha_G$  can be written in the forms:

$$\begin{aligned} 4 \cdot e^{2n} \cdot a_s^2 \cdot a^2 \cdot \alpha_G \cdot N_A^2 &= 1 \\ \alpha_G &= (2 \cdot e^n \cdot a_s \cdot a \cdot N_A)^{-2} \end{aligned} \quad (170)$$

$$\begin{aligned} 4 \cdot a_s^2 \cdot a^2 \cdot \alpha_G \cdot N_A^2 &= i^{4i} \\ \alpha_G &= i^{4i} \cdot (2 \cdot a_s \cdot a \cdot N_A)^{-2} \end{aligned} \quad (171)$$

Therefore from these expressions the equivalent formulas for the gravitational constant are:

$$G = (2e^\pi \alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (172)$$

$$G = i^{4i} (2\alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (173)$$

The gravitational coupling constant  $\alpha_G$  can be written in the form:

$$4 \cdot 10^{14} \cdot e^{2n} \cdot a_w^2 \cdot a^2 \cdot \alpha_G \cdot N_A^2 = e^2$$

$$\alpha_G = (2 \cdot e^{n-1} \cdot 10^7 \cdot a_w \cdot a \cdot N_A)^{-2} \quad (174)$$

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot \alpha_G \cdot N_A^2 = i^{4i} \cdot e^2$$

$$\alpha_G = i^{4i} \cdot e^2 \cdot (2 \cdot 10^7 \cdot a_w \cdot a \cdot N_A)^{-2} \quad (175)$$

Therefore from these expressions the equivalent formulas for the gravitational constant are:

$$G = (2e^{\pi-1} 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (176)$$

$$G = i^{4i} e^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (177)$$

The gravitational coupling constant  $\alpha_G$  can be written in the form:

$$4 \cdot 10^{14} \cdot N_A^2 \cdot a_w^2 \cdot a^2 \cdot \alpha_G = a_s^2$$

$$\alpha_G = a_s^2 \cdot (2 \cdot 10^7 \cdot a_w \cdot a \cdot N_A)^{-2} \quad (178)$$

Therefore from this expression the formula for the gravitational constant is:

$$G = \alpha_s^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (179)$$

Now we will find the theoretical value of the Gravitational constant  $G$  using the unity formulas for the coupling constants that we calculated. The gravitational coupling constant  $\alpha_G$  can be written in the form:

$$a^{-2} \cdot \cos^2 a^{-1} = 4 \cdot \alpha_G \cdot N_A^2$$

$$\alpha_G = (2 \cdot a \cdot N_A)^{-2} \cdot \cos^2 a^{-1} \quad (180)$$

Therefore from this expression the formula for the gravitational constant is:

$$G = (2 \alpha N_A)^{-2} \cos^2 a^{-1} \frac{\hbar c}{m_e^2} \quad (181)$$

Using the 2018 CODATA recommended value of the the fundamental constants resulting the theoretical value of the Gravitational constant  $G$ :

$$G = 6.67448 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2 \quad (182)$$

## 12. Gravitational fine-structure constant

In [24] and [25] resulting in the dimensionless unification of atomic physics and cosmology. The relevant constant in atomic physics is the fine-structure constant  $\alpha$ , which plays a fundamental role in atomic physics and quantum electrodynamics. The analogous constant in cosmology is the gravitational fine-structure constant  $\alpha_g$ . It plays a fundamental role in cosmology. The mysterious value of the gravitational fine-structure constant  $\alpha_g$  is an equivalent way to express the biggest issue in theoretical physics. The mysterious value of the gravitational fine-structure constant  $\alpha_g$  is an equivalent way to express the biggest issue in theoretical physics. The new formula for the Planck length  $l_p$  is:

$$l_{pl} = a\sqrt{\alpha_G}\alpha_0$$

The fine-structure constant equals:

$$\alpha^2 = \frac{r_e}{a_0}$$

From these expressions we have:

$$l_{pl} = \frac{\alpha\sqrt{\alpha_G}r_e}{\alpha^2}$$

$$l_{pl} = \frac{\sqrt{\alpha_G}}{\alpha}r_e$$

$$\frac{l_{pl}^3}{r_e^3} = \frac{\sqrt{\alpha_G^3}}{\alpha^3}$$

The gravitational fine structure constant  $\alpha_g$  is defined as:

$$\alpha_g = \frac{l_{pl}^3}{r_e^3}$$

$$\alpha_g = \frac{\sqrt{\alpha_G^3}}{\alpha^3}$$

$$\alpha_g = \sqrt{\frac{\alpha_G^3}{\alpha^6}} \quad (183)$$

with numerical value:

$$\alpha_g = 1.886837 \times 10^{-61}$$

Also equals:

$$\alpha_g^2 \cdot \alpha^6 = \alpha_G^3$$

$$\alpha_g^2 = \alpha_G^3 \cdot \alpha^{-6}$$

$$\alpha_g^2 = \left( \frac{\alpha_G}{\alpha^2} \right)^3$$

Now we will try to find the best mathematical expression of the gravitational fine structure constant  $\alpha_g$  with the mathematical constants. In trying to do this we found surprising coincidences and various approaches for the math constants. Now we will try to find the best mathematical expression of the gravitational fine structure constant  $\alpha_g$  with the mathematical constants. In trying to do this we found surprising coincidences and various approaches for the math constants. Two approaches for Archimedes constant  $\pi$  are:

$$6 \cdot 7^{103} \cdot \pi^5 \approx 2^{300} \quad (184)$$

$$6 \cdot \pi^5 \approx 150^{3/2} - 1 \quad (185)$$

A approach for the Gelfond's constant  $e^\pi$  is:

$$e^\pi \simeq \frac{55}{\pi} \sqrt{\frac{2}{\ln \pi}} \quad (186)$$

A approximation expression that connects the golden ratio  $\phi$ , the Archimedes constant  $\pi$  and the Euler's number  $e$  is:

$$2^2 11^2 e \simeq 3^4 \phi^5 \sqrt[3]{\pi} \quad (187)$$

Two approximations expressions that connects the golden ratio  $\phi$ , the Archimedes constant  $\pi$ , the Euler's number  $e$  and the Euler's constant  $\gamma$  are:

$$4e^2 \gamma \ln^2(2\pi) \simeq \sqrt{3^3} \phi^5 \quad (188)$$

$$\sqrt{3^5} e \gamma \ln(2\pi) \sqrt[3]{\pi} \simeq 11^2 \quad (189)$$

The expression that connects the gravitational fine-structure constant  $\alpha_g$  with the Archimedes constant  $\pi$ , the Euler's number  $e$  and the Euler's constant  $\gamma$  is:

$$\alpha_g = [e \cdot \gamma \cdot \ln^2(2 \cdot \pi)]^{-1} \times 10^{-60} = 1.886837 \times 10^{-61} \quad (190)$$

The expression that connects the gravitational fine-structure constant  $\alpha_g$  with the golden ratio  $\phi$  and the Euler's number  $e$  is:

$$\alpha_g = \frac{4e}{3\sqrt{3}\phi^5} \times 10^{-60} = 1,886837 \times 10^{-61} \quad (191)$$

The expression that connects the gravitational fine-structure constant  $\alpha_g$  with the Archimedes constant  $\pi$  is:

$$\alpha_g = \frac{\sqrt{3^5} \sqrt[3]{\pi}}{11^2} \times 10^{-60} = 1,886837 \times 10^{-61} \quad (192)$$

The expression that connects the gravitational fine-structure constant  $\alpha_g$  with the golden ratio  $\phi$  and the Euler's constant  $\gamma$  is:

$$\alpha_g = \frac{7\phi\gamma^2}{2} \times 10^{-60} = 1,886826 \times 10^{-61} \quad (193)$$

The expression that connects the gravitational fine-structure constant  $\alpha_g$  with the Archimedes constant and the golden ratio  $\phi$  is:

$$\alpha_g = \frac{2\pi}{3\phi^5} \times 10^{-60} = 1,888514 \times 10^{-61} \quad (194)$$

Resulting the unity formula for the gravitational fine-structure constant  $\alpha_g$ :

$$\alpha_g = (2 \cdot e \cdot a^2 \cdot NA)^{-3} \quad (195)$$

Also apply the expressions:

$$(2 \cdot e \cdot a^2 \cdot NA)^3 \cdot a_g = 1$$

$$8 \cdot e^3 \cdot a^6 \cdot a_g \cdot NA^3 = 1$$

Resulting the unity formula for the gravitational fine-structure constant  $a_g$ :

$$a_g = i^{6i} \cdot (2 \cdot a_s \cdot a^2 \cdot NA)^{-3} \quad (196)$$

Also apply the expression:

$$(2 \cdot a_s \cdot a^2 \cdot NA)^3 \cdot a_g = i^{6i}$$

$$8 \cdot a_s^3 \cdot a^6 \cdot a_g \cdot NA^3 = i^{6i}$$

Resulting the unity formula for the gravitational fine-structure constant  $a_g$ :

$$a_g = i^{6i} \cdot e^3 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot NA)^{-3} \quad (197)$$

Also apply the expression:

$$(2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot NA)^3 \cdot a_g = i^{6i} \cdot e^3$$

$$8 \cdot 10^{21} \cdot a_w^3 \cdot a^9 \cdot a_g \cdot NA^3 = i^{6i} \cdot e^3$$

Resulting the unity formulas for the gravitational fine-structure constant  $a_g$ :

$$a_g = (10^7 \cdot a_w \cdot a_G^{1/2} \cdot e^{-1} \cdot a_s^{-1} \cdot a^{-1})^3 \quad (198)$$

Also apply the expressions:

$$a_g = 10^{21} \cdot a_w^3 \cdot a_G^{3/2} \cdot a_s^{-3} \cdot a^{-3} \cdot e^{-3}$$

$$a_g \cdot a_s^3 \cdot a^3 \cdot e^3 = 10^{21} \cdot a_w^3 \cdot a_G^{3/2}$$

So the unity formula for the gravitational fine-structure constant  $a_g$  is:

$$a_g^2 = (10^{14} \cdot a_w^2 \cdot a_G \cdot e^{-2} \cdot a_s^{-2} \cdot a^{-2})^3 \quad (199)$$

Also apply the expressions:

$$a_g^2 = 10^{42} \cdot a_w^6 \cdot a_G^3 \cdot e^{-6} \cdot a_s^{-6} \cdot a^{-6}$$

$$e^6 \cdot a_s^6 \cdot a^6 \cdot a_g^2 = 10^{42} \cdot a_w^6 \cdot a_G^3$$

$$a_g^2 \cdot (e \cdot a_s \cdot a)^6 = (10^{14} \cdot a_w^2 \cdot a_G)^3$$

Resulting the unity formula for the gravitational fine-structure constant  $a_g$ :

$$a_g = i^{6i} \cdot (10^7 \cdot a_w \cdot a_G^{1/2} \cdot a_s^{-2} \cdot a^{-1})^3$$

$$a_g = 10^{21} \cdot i^{6i} \cdot (a_w \cdot a_G^{1/2} \cdot a_s^{-2} \cdot a^{-1})^3$$

$$a_g = 10^{21} \cdot i^{6i} \cdot a_w^3 \cdot a_G^{3/2} \cdot a_s^{-6} \cdot a^{-3} \quad (200)$$

Also apply the expressions:

$$a_g^{1/3} \cdot a_s^2 \cdot a \cdot a_w^{-1} \cdot a_G^{-1/2} = i^{2i} \cdot 10^7$$



$$\alpha_g \cdot \alpha_s^6 \cdot a^3 = 10^{21} \cdot i^{6i} \cdot \alpha_w^3 \cdot \alpha_G^{3/2}$$

So the unity formulas for the gravitational fine-structure constant  $\alpha_g$  are:

$$\alpha_g^2 = i^{6i} \cdot (10^{14} \cdot \alpha_w^2 \cdot \alpha_G \cdot \alpha_s^{-4} \cdot a^{-2})^3$$

$$\alpha_g^2 = 10^{42} \cdot i^{12i} \cdot (\alpha_w^2 \cdot \alpha_G \cdot \alpha_s^{-4} \cdot a^{-2})^3$$

$$\alpha_g^2 = 10^{42} \cdot i^{12i} \cdot \alpha_w^6 \cdot \alpha_G^3 \cdot \alpha_s^{-12} \cdot a^{-6} \quad (201)$$

Also apply the expressions:

$$\alpha_g^2 \cdot \alpha_s^{12} \cdot a^6 \cdot \alpha_w^{-6} \cdot \alpha_G^{-3} = i^{12i} \cdot 10^{42}$$

$$(\alpha_s^6 \cdot a^3 \cdot \alpha_g)^2 = (10^{14} \cdot i^{4i} \cdot \alpha_w^2 \cdot \alpha_G)^3$$

$$\alpha_s^{12} \cdot a^6 \cdot \alpha_g^2 = 10^{42} \cdot i^{12i} \cdot \alpha_w^6 \cdot \alpha_G^3$$

So the unity formulas for the gravitational fine-structure constant  $\alpha_g$  are:

$$\alpha_g = \left( \frac{10^7 \alpha_w \sqrt{\alpha_G}}{e \alpha_s a} \right)^3 \quad (202)$$

$$\alpha_g^2 = 10^{42} \left( \frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 a^2} \right)^3 \quad (203)$$

$$\alpha_g = 10^{21} i^{6i} \left( \frac{\alpha_w \sqrt{\alpha_G}}{\alpha_s^2 a} \right)^3 \quad (204)$$

$$\alpha_g^2 = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha_s^2 a^4} \right)^3 \quad (205)$$

This expression connects the gravitational fine-structure constant  $\alpha_g$  with the four coupling constants. Perhaps the gravitational fine structure constant  $\alpha_g$  is the coupling constant for the fifth force. Some speculative theories have proposed a fifth force to explain various anomalous observations that do not fit existing theories. The characteristics of this fifth force depend on the hypothesis being advanced. Many postulate a force roughly the strength of gravity with a range of anywhere from less than a millimeter to cosmological scales. Another proposal is a new weak force mediated by W and Z bosons. The search for a fifth force has increased in recent decades due to two discoveries in cosmology which are not explained by current theories. It has been discovered that most of the mass of the universe is accounted for by an unknown form of matter called dark matter. Most physicists believe that dark matter consists of new, undiscovered subatomic particles, but some believe that it could be related to an unknown fundamental force. Second, it has also recently been discovered that the expansion of the universe is accelerating, which has been attributed to a form of energy called dark energy. Some physicists speculate that a form of dark energy called quintessence could be a fifth force.

### 13. Dimensionless unification of atomic physics and cosmology

In the context of cosmology the cosmological constant is a homogeneous energy density that causes the expansion of the universe to accelerate. Originally proposed early in the development of general relativity in order to allow a static universe solution it was subsequently abandoned when the universe was found to be expanding. Now the

cosmological constant is invoked to explain the observed acceleration of the expansion of the universe. The cosmological constant is the simplest realization of dark energy, which is the more generic name given to the unknown cause of the acceleration of the universe. Its existence is also predicted by quantum physics, where it enters as a form of vacuum energy, although the magnitude predicted by quantum theory does not match that observed in cosmology.

The cosmological constant  $\Lambda$  is presumably an enigmatic form of matter or energy that acts in opposition to gravity and is considered by many physicists to be equivalent to dark energy. Nobody really knows what the cosmological constant is exactly, but it is required in cosmological equations in order to reconcile theory with our observations of the universe. One potential explanation for the cosmological constant lies in the realm of modern particle physics. Experiments have verified that empty space is permeated by countless virtual particles constantly popping in and out of existence. It is commonly believed that the cosmological constant problem can only be solved ultimately in a unified theory of quantum gravity and the standard model of electroweak and strong interactions, which is still absent so far. But connecting vacuum energy to the cosmological constant is not straightforward. Based on their observations of supernovas, astronomers estimate that dark energy should have a small and sedate value, just enough to push everything in the universe apart over billions of years. Yet when scientists try to calculate the amount of energy that should arise from virtual particle motion, they come up with a result that's 120 orders of magnitude greater than what the supernova data suggest. The cosmological constant has the same effect as an intrinsic energy density of the vacuum,  $\rho_{vac}$  and an associated pressure. In this context, it is commonly moved onto the right-hand side of the equation, and defined with a proportionality factor of  $\Lambda = 8 \cdot \pi \cdot \rho_{vac}$  where unit conventions of general relativity are used (otherwise factors of  $G$  and  $c$  would also appear, i.e.:

$$\Lambda = 8\pi\rho_{vac} \frac{G}{c^4} = \kappa\rho_{vac}$$

where  $\kappa$  is Einstein's rescaled version of the gravitational constant  $G$ . The cosmological constant has been introduced in gravitational field equations by Einstein in 1917 in order to satisfy Mach's principle of the relativity of inertia. Then it was demonstrated by Cartan in 1922 that the Einstein field tensor including a cosmological constant  $\Lambda$ :

$$E_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu}$$

is the most general tensor in Riemannian geometry having null divergence like the energy momentum tensor  $T_{\mu\nu}$ . This theorem has set the general form of Einstein's gravitational field equations as  $E_{\mu\nu} = \kappa \cdot T_{\mu\nu}$  and established from first principles the existence of  $\Lambda$  as an unvarying true constant. The cosmological constant problem dates back to the realization that it is equivalent to a vacuum energy density. One of the main consequences in cosmology of a positive cosmological constant is an acceleration of the expansion of the universe. Such an acceleration has been first detected in 1981 in the Hubble diagram of infrared elliptical galaxies, yielding a positive value close to the presently measured one, but with still large uncertainties. Accurate measurements of the acceleration of the expansion since 20 years have reinforced the problem. The cosmological constant  $\Lambda$ , as it appears in Einstein's equations, is a curvature. As such, besides being an energy density, it is also the inverse of the square of an invariant cosmic length  $L$ .

In the early-mid 20th century Dirac and Zel'dovich were among the first scientists to suggest an intimate connection between cosmology and atomic physics. Though a revolutionary proposal for its time, Dirac's Large Number Hypothesis (1937) adopted a standard assumption of the non-existence of the cosmological constant term  $\Lambda=0$ . Zel'dovich insight (1968) was to realize that a small but nonzero cosmological term  $\Lambda>0$  allowed the present day radius of the Universe to be identified with the de Sitter radius which removed the need for time dependence in the fundamental couplings. Thus, he obtained the formula:

$$\Lambda = \frac{m_p^6 G^2}{\hbar^6}$$

where  $m$  is a mass scale characterizing the relative strengths of the gravitational and electromagnetic interactions, which he identified with the proton mass  $m_p$ . Laurent Nottale in [26] which, instead, suggests the identification  $m = m_e/\alpha$ . He assumed that the cosmological constant  $\Lambda$  is the sum of a general-relativistic term and of the quantum, scale-varying, gravitational self-energy of virtual pairs. A renormalization group approach is used to describe its scale-dependence. We argue that the large scale value of  $\Lambda$  is reached at the classical electron scale. This reasoning provides with a large-number relation:

$$\alpha \frac{m_{pl}}{m_e} = \left( \frac{L}{l_{pl}} \right)^{\frac{1}{3}}$$

The cosmological constant  $\Lambda$  has the dimension of an inverse length squared. The cosmological constant is the inverse of the square of a length  $L$ :

$$L = \sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d = \sqrt{3}L$$

So the de Sitter radius and the cosmological constant are related through a simple equation:

$$R_d = \sqrt{\frac{3}{\Lambda}}$$

From this equation resulting the expressions for the gravitational fine structure constant  $\alpha_g$ :

$$\alpha \frac{m_{pl}}{m_e} = \left( l_{pl} \sqrt{\Lambda} \right)^{-\frac{1}{3}}$$

$$\alpha_g = l_{pl} \sqrt{\Lambda}$$

$$\alpha_g = \sqrt{\frac{G \hbar \Lambda}{c^3}}$$

So the cosmological constant  $\Lambda$  equals:

$$\Lambda = \alpha_g^2 l_{pl}^{-2}$$

$$\Lambda = \frac{l_{pl}^4}{r_e^6}$$

$$\Lambda = \alpha_g^2 \frac{c^3}{G \hbar}$$

$$\Lambda = \frac{G}{\hbar^4} \left( \frac{m_e}{a} \right)^6$$

Resulting the dimensionless unification of the atomic physics and the cosmology:

$$\alpha_g = (2 \cdot e \cdot a^2 \cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = (2 \cdot e \cdot a^2 \cdot N_A)^{-6} \quad (206)$$

$$(2 \cdot e \cdot a^2 \cdot N_A)^6 \cdot l_{pl}^2 \cdot \Lambda = 1 \quad (207)$$

Now we will use the unity formulas of the dimensionless unification of atomic physics and cosmology to find the equations of the cosmological constant. For the cosmological constant equals:

$$\Lambda = \left(2e\alpha^2 N_A\right)^{-6} \frac{c^3}{G\hbar} \quad (208)$$

Resulting the dimensionless unification of atomic physics and cosmology:

$$a_g = i^{6i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-3}$$

$$|p|^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-6} \quad (209)$$

$$(2 \cdot a_s \cdot a^2 \cdot N_A)^6 \cdot |p|^2 \cdot \Lambda = i^{12i} \quad (210)$$

For the cosmological constant equals:

$$\Lambda = i^{12i} (2\alpha_s a^2 N_A)^{-6} \frac{c^3}{G\hbar} \quad (211)$$

Resulting the dimensionless unification of atomic physics and cosmology:

$$a_g = i^{6i} \cdot e^3 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^{-3}$$

$$|p|^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^{-6} \quad (212)$$

$$(2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^6 \cdot |p|^2 \cdot \Lambda = i^{12i} \cdot e^6 \quad (213)$$

For the cosmological constant equals:

$$\Lambda = i^{12i} e^6 (2 \cdot 10^7 \alpha_w a^3 N_A)^{-6} \frac{c^3}{G\hbar} \quad (214)$$

Resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g^2 = 10^{42} \left( \frac{\alpha_G a_w^2}{e^2 \alpha_s^2 a^2} \right)^3$$

$$|p|^2 \Lambda = 10^{42} \left( \frac{\alpha_G a_w^2}{e^2 \alpha_s^2 a^2} \right)^3 \quad (215)$$

$$e^6 \cdot a_s^6 \cdot a^6 \cdot |p|^2 \cdot \Lambda = 10^{42} \cdot \alpha_G^3 \cdot a_w^6 \quad (216)$$

For the cosmological constant equals:

$$\Lambda = 10^{42} \left( \frac{\alpha_G a_w^2}{e^2 \alpha_s^2 a^2} \right)^3 \frac{c^3}{G\hbar} \quad (217)$$

Resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g^2 = 10^{42} i^{12i} \left( \frac{\alpha_G a_w^2}{a^2 \alpha_s^4} \right)^3$$

$$l_{pl}^2 \Lambda = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \quad (218)$$

$$\alpha_s^{12} \cdot \alpha^6 \cdot |pl|^2 \cdot \Lambda = 10^{42} \cdot i^{12i} \cdot \alpha_G^3 \cdot \alpha_w^6 \quad (219)$$

This unity formula is a simple analogy between atomic physics and cosmology. For the cosmological constant equals:

$$\Lambda = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \frac{c^3}{G \hbar} \quad (220)$$

In [27] we presented the Equation of the Universe:

$$\frac{\Lambda G \hbar}{c^3} = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \quad (221)$$

$$e^{6\pi} \frac{\Lambda G \hbar}{c^3} = 10^{42} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \quad (222)$$

#### 14. Maximum and minimum values for natural quantities

There have been theories of the shortest and largest natural quantities. Physics can be summed up in a few limiting statements. They imply that in nature every physical observable is bounded by a value close to the Planck value. The speed limit is equivalent to special relativity, the force limit to general relativity, and the action limit to quantum theory. The newly discovered maximum force principle makes it possible to summarize special relativity, quantum theory and general relativity into a fundamental limiting principle each. In [28], [29] and [30] we presented the law of the gravitational fine-structure constant  $\alpha_g$  followed by ratios of maximum and minimum theoretical values for natural quantities. This theory uses quantum mechanics, cosmology, thermodynamics, and special and general relativity. Length  $l$ , time  $t$ , speed  $v$  and temperature  $T$  have the same min/max ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{max}} = \frac{t_{min}}{t_{max}} = \frac{v_{min}}{v_{max}} = \frac{T_{min}}{T_{max}} \quad (223)$$

Energy  $E$ , mass  $M$ , action  $A$ , momentum  $P$  and entropy  $S$  have another min/max ratio, which is the square of  $\alpha_g$ :

$$\alpha_g^2 = \frac{E_{min}}{E_{max}} = \frac{M_{min}}{M_{max}} = \frac{A_{min}}{A_{max}} = \frac{P_{min}}{P_{max}} = \frac{S_{min}}{S_{max}} \quad (224)$$

Force  $F$  has min/max ratio which is  $\alpha_g^4$ :

$$\alpha_g^4 = \frac{F_{min}}{F_{max}} \quad (225)$$

Mass density has min/max ratio which is  $\alpha_g^5$ :

$$\alpha_g^5 = \frac{\rho_{min}}{\rho_{max}} \quad (226)$$

## 15. Length scales

In [31] we presented the Unification of the Microcosm and the Macrocosm. A Planck length  $l_{pl}$  is about  $10^{-20}$  times the diameter of a proton, meaning it is so small that immediate observation at this scale would be impossible in the near future. The length Planck  $l_{pl}$  defined as:

$$l_{pl} = \sqrt{\frac{\hbar G}{c^3}} = \frac{\hbar}{m_{pl}c} = \frac{h}{2\pi m_{pl}c} = \frac{m_p r_p}{4m_{pl}}$$

The classical electron radius is a combination of fundamental physical quantities that define a length scale for problems involving an electron interacting with electromagnetic radiation. The classical electron radius is given as:

$$r_e = \alpha^2 \alpha_0 = \frac{\hbar \alpha}{m_e c} = \frac{\lambda_c \alpha}{m_e c^2} = \frac{\mu_0 q_e^2}{4\pi m_e} = \frac{k_e q_e^2}{m_e c^2} = \frac{\alpha^3}{4\pi R_\infty}$$

The Bohr radius  $\alpha_0$  is a physical constant, approximately equal to the most probable distance between the nucleus and the electron in a hydrogen atom in its ground state. The Bohr radius  $\alpha_0$  is defined as:

$$\alpha_0 = \frac{\hbar}{\alpha m_e c} = \frac{r_e}{\alpha^2} = \frac{\lambda_c}{2\pi \alpha}$$

The proton radius  $r_p$  is the distance from the center of the proton to the tip of the proton. The proton radius  $r_p$  is an unanswered physics problem related to the size of the proton. In atomic physics, there are two common and "natural" scales of length. The first scale of length is given by Compton's wavelength of electrons. Using the de Broglie equation, we get that Compton's wavelength is the wavelength of a photon whose energy is the same as the rest mass of the particle, or mathematically speaking: The Compton wavelength of a particle is equal to the wavelength of a photon whose energy is the same as the mass of that particle. It was introduced by Arthur Compton in his explanation of the scattering of photons by electrons. The standard Compton wavelength  $\lambda_c$  of a particle is given by  $\lambda_c = h/m \cdot c$ . Thus respectively the Compton wavelength  $\lambda_c$  of the electron with mass  $m_e$  is given by the formula:

$$\lambda_c = \frac{2\pi r_e}{\alpha} = \frac{h}{m_e c}$$

Sometimes the Compton wavelength is expressed by the reduced Compton  $\tilde{\lambda}_c$  wavelength. When the Compton  $\lambda_c$  wavelength is divided by  $2 \cdot \pi$ , we obtain the reduced Compton  $\tilde{\lambda}_c$  wavelength, i.e. the Compton wavelength for 1 radius instead of  $2 \cdot \pi$  rad  $\tilde{\lambda}_c = \lambda_c / 2 \cdot \pi$ . The fine-structure constant is universal scaling factor:

$$\alpha = \frac{2\pi r_e}{\lambda_e} = \frac{\lambda_e}{2\pi \alpha_0} = \frac{r_e}{l_{pl}} \frac{m_e}{m_{pl}} = \sqrt{\frac{r_e}{\alpha_0}}$$

Also the gravitational coupling constant is universal scaling factor:

$$\alpha_G = \frac{m_e^2}{m_{pl}^2} = \frac{\alpha_{G(p)}}{\mu^2} = \frac{\alpha}{\mu N_1} = \frac{\alpha^2}{N_1^2 \alpha_{G(p)}} = \left( \frac{2\pi l_{pl}}{\lambda_e} \right)^2 = \left( \alpha \frac{l_{pl}}{r_e} \right)^2 = \left( \frac{l_{pl}}{\alpha \alpha_0} \right)^2$$

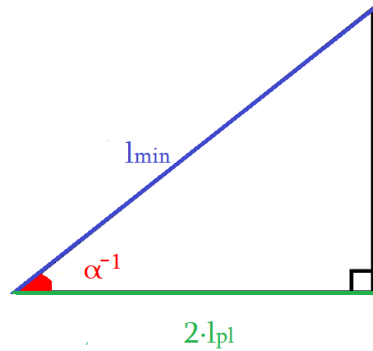
A smallest length in nature thus implies that there is no way to define exact boundaries of objects or elementary particles. Max Planck proposed natural units that indirectly discovered the lowest-level properties of free space, all born from equations that simplified the mathematics of physics equations. The fundamental unit of length in this unit system is the Planck length  $l_{pl}$ . The smallest components of spacetime will never be seen with the human eye as it is orders of magnitudes smaller than an atom. Thus, it will never be directly observed but it can be deduced by mathematics. We proposed to be a lattice structure, in which its unit cells have sides of length  $2 \cdot e \cdot l_{pl}$ . Perhaps for the minimum distance  $l_{min}$  apply:

$$l_{min} = 2 \cdot e \cdot l_{pl} \quad (227)$$

From expressions apply:

$$\begin{aligned} \cos \alpha^{-1} &= e^{-1} \\ \cos \alpha^{-1} \cdot l_{\min} &= 2 \cdot |p| \\ \cos \alpha^{-1} &= \frac{2l_{pl}}{l_{\min}} \end{aligned} \quad (228)$$

The figures 16 below show the geometric representation of the fundamental unit of length.

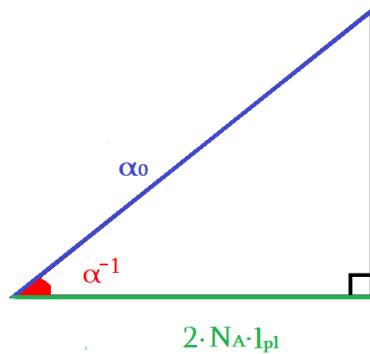


**Figure 16.** Geometric representation of the fundamental unit of length.

For the Bohr radius  $\alpha_0$  apply:

$$\begin{aligned} \alpha_0 &= N_A \cdot l_{\min} \\ \alpha_0 &= 2 \cdot e \cdot N_A \cdot |p| \end{aligned} \quad (229)$$

The figures 17 below show the geometric representation of the relationship between the Bohr radius and the Planck length.



**Figure 17.** Geometric representation of the relationship between the Bohr radius and the Planck length.

The cosmological constant  $\Lambda$  has the dimension of an inverse length squared. The cosmological constant is the inverse of the square of a length  $L$ :

$$L = \sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d = \sqrt{3}L$$

So the de Sitter radius and the cosmological constant are related through a simple equation:

$$R_d = \sqrt{\frac{3}{\Lambda}}$$

The Hubble length or Hubble distance is a unit of distance in cosmology, defined as:

$$L_H = c \cdot H_0^{-1}$$

It is equivalent to 4.420 million parsecs or 14.4 billion light years. (The numerical value of the Hubble length in light years is, by definition, equal to that of the Hubble time in years.) The Hubble distance would be the distance between the Earth and the galaxies which are currently receding from us at the speed of light, as can be seen by substituting  $D = c \cdot H_0^{-1}$  into the equation for Hubble's law,  $v = H_0^{-1} \cdot D$ . For the density parameter for dark energy apply:

$$\Omega_\Lambda = \left( \frac{L_H}{R_d} \right)^2 = \frac{L_H^2}{R_d^2}$$

From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda = 2 \cdot e^{-1} = 0.73576 = 73.57\%$$

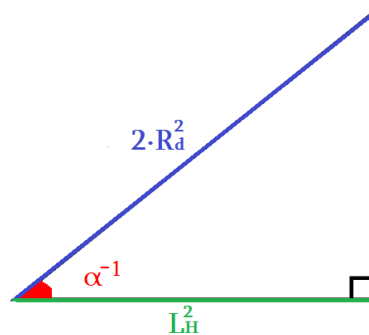
So from this expression apply:

$$2 \cdot R_d^2 = e \cdot L_H^2 \quad (230)$$

So apply the expression:

$$\cos \alpha^{-1} = \frac{L_H^2}{2R_d^2} \quad (231)$$

The figure 18 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.



**Figure 18.** Geometric representation of the relationship between the de Sitter radius and the Hubble length.

The maximum distance  $l_{\max}$  corresponds to the distance of the universe  $l_U = c \cdot H_0^{-1}$ . Therefore:

$$l_{\max} = l_U = c \cdot H_0^{-1}$$

This distance is equivalent to 4.550 million parsecs, or 14.4 billion light-years, 13.8 billion years. Hubble's distance would be the distance between the Earth and the galaxies currently falling away from us at the speed of light, as shown by the substitution  $r = c \cdot H_0^{-1}$  in the equation for Hubble's law,  $v = H_0 \cdot r$ . The Schwarzschild radius or gravitational radius is a physical parameter that appears in the Schwarzschild solution in Einstein's field equations, which corresponds to the radius defining the event horizon of a Schwarzschild black hole. It is a



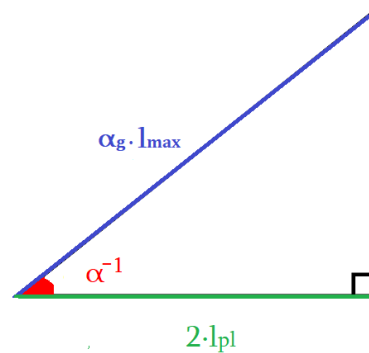
characteristic radius associated with any quantity. (from its center) which must have a celestial body in order to reach a black hole. That is, if a celestial body has a radius less than its Schwarzschild radius then it is a black hole. The term is used especially in physics and astronomy, with emphasis. The Schwarzschild ray was named after the German astronomer Karl Schwarzschild, who calculated this exact solution for the theory of general relativity in 1916. Length  $l$  has the min/max ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{max}} \quad (232)$$

The maximum distance  $l_{max}$  corresponds to the distance of the universe:

$$l_{max} = LH = c \cdot H_0^{-1} = \alpha_g^{-1} \cdot l_{min} \quad (233)$$

The figure 19 shows the geometric representation of the relationship between the maximum distance and the Planck length.



**Figure 19.** Geometric representation of the relationship between the maximum distance and the Planck length.

The value of the maximum distance  $l_{max}$  is  $l_{max} = 4.656933 \times 10^{26}$  m. In [32] we presented the New Large Number Hypothesis of the universe. By the early 1930s, the unsuccessful program of unified theories had withered under the impact of quantum theory. No one was more decisive in proclaiming the end of the era than Weyl himself. Einstein paid attention to Friedman's achievements. It is very more surprising that, in 1931, Weyl failed to notice a new stage in the development of relativistic cosmology. As a mathematician, Weyl could ignore developments in astrophysics. However, there was a deeper reason for his neglect. Weyl tried, in 1919, to connect the unified theory with physical reality. He was the first to pay attention to the empirical fact of the "coincidence of large numbers", which he saw as a connection between cosmology and microphysics. He abandoned the project, but the result he obtained survived and took on a life of its own. The grand idea of possible connections between cosmology and microphysics fascinated Eddington and drew Dirac's interest. The material presented can be seen as a logical continuation and development of Dirac and Eddington's (LNH) large number hypothesis. From 1929 until his death in 1944, Arthur Eddington worked to develop an ambitious theory of fundamental physics for the theory of everything in the physical world. His incomplete theory included abstract mathematics but was difficult for other scientists to understand. The constants of nature were of particular importance to his work. Although highly original, Arthur Eddington's effort provided ideas for other British physicists, including P. Dirac and E. A. Milne. However, Eddington's work was a major failure and was rejected by the great majority of physicists. An important reason was the unorthodox view of quantum mechanics. Arthur Eddington developed a standard model of the internal structure of stars and was the first to propose nuclear reactions as the main source of stellar energy. In 1919 he rose to fame when, together with Frank Dyson, they confirmed Einstein's prediction of the bending star around the Sun. Six years later he proposed the general theory of relativity in white dwarfs, and in 1930 he developed one of the first relativistic models of the expanding universe known as the Lemaître-Eddington model. Dirac's cosmological theory based on the  $G(t)$  hypothesis was directly inspired by the ideas of Milne and Eddington. His more general view of fundamental physics involved the assertion of a predetermined harmony between mathematics and physics. By the late 1930s Dirac had concluded that physics and mathematics would eventually merge into a single branch of higher knowledge. Although Eddington's work had elements in common with the ideas of Milne and other physicists of the period, it was unique in the way he interpreted it philosophically. Eddington was convinced that the laws of nature were subjective and not objective. Laws, he argued, were not abstract expressions of an external world, but essentially constructs of physics. This also applies to the fundamental constants of nature. Eddington's numerological and philosophical approach to

fundamental physics attracted much attention among British scientists and philosophers. The general attitude was critical and sometimes dismissive.

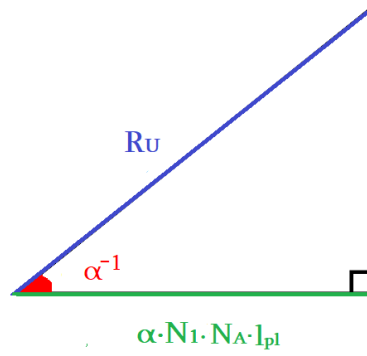
Paul Dirac (1902–1984) was one of the greatest physicists of the twentieth century. Dirac's contribution to the early stages of quantum theory was enormous. The equation that bears his name describes the behavior of particles with half-integer spin, like electrons, and predicts the existence of antimatter. This equation is compatible with the special theory of relativity, in contrast to the corresponding Schrödinger equation that applies to particles moving at non-relativistic speeds. He was awarded the Nobel Prize in Physics in 1933 (together with Erwin Schrödinger). Dirac was the first to succeed in putting the formalism of quantum physics, which was formulated by physicists such as Heisenberg and Schrödinger, on a mathematically transparent and therefore easy-to-use basis. His book "The Principles of Quantum Mechanics" was so groundbreaking that almost all of his colleagues had nothing but praise for this publication. However, Dirac was often questioned. Many of his assumptions later proved untenable. Perhaps it takes imaginative thinking to come up with ideas that no one else has. It may look like success: it is built on trial and error, successes and failures, but in the end, the outside world perceives only success. The rocky road to success is usually hidden. Dirac's successes overshadowed his failures, but maybe it was just the strategy he needed to show success. The Dirac Large Number Hypothesis (LNH) is an observation that he made by Paul Dirac in 1937 relating the ratios of size scales in the Universe to those of force scales. The ratios are very large dimensionless numbers, about 40 orders of magnitude at the present cosmological epoch. The Dirac Large Number Hypothesis (LNH) is a coincidence attributed to an explanation then applied as an axiom. His LNH was highly questionable. There were few colleagues who took it seriously. Dirac's theory has inspired and continues to inspire significant scientific research in various disciplines. Within geophysics, for example, Edward Teller appeared to raise a serious objection to LNH in 1948 when he argued that fluctuations in the force of gravity were inconsistent with the paleontological data. However, George Gamow demonstrated in 1962 how a simple revision of the parameters (in this case, the age of the solar system) can invalidate Teller's conclusions. The debate is further complicated by the choice of LNH cosmologies. In 1978, G. Blake argued that paleontological data are consistent with the "multiplicative" scenario but not with the "additive" scenario. Arguments both for and against the LNH are also made from astrophysical considerations. For example, D. Falik argued that the LNH is not consistent with experimental results for the microwave background radiation, whereas Canuto and Hsieh argued that it is. An argument that generated considerable controversy was put forward by Robert Dicke in 1961. Known as the human coincidence or the beautiful, simply states that the large numbers in the LNH are a necessary coincidence for intelligent beings, since they parameterize the fusion of hydrogen in stars, and thus carbon-based life would not have arisen otherwise.

The anthropocentric approach to the "coincidence of large numbers" originated in two papers published by Robert Dicke in 1957 and 1961. Dicke's theoretical ideas were far from consistent. He expressed doubts that the theory had a reliable experimental basis. Dicke suggested, as a consequence of his approach, that the constants of these interactions depended on time and space. He tried to formulate a new theory of gravity which he saw as a manifestation of electromagnetism with variable permeability of vacuum. Dicke believed that the "coincidence of large numbers" was certain proof that the gravitational constant changed with time. Discussing the variability of physical constants, Dicke insisted that "the age of the Universe is not random. Various authors have introduced new sets of numbers into the original 'coincidence' considered by Dirac and his contemporaries thus expanding or distancing from Dirac's own conclusions. Jordan (1947) noted that the mass ratio for a typical star (specifically, a star of Chandrasekhar mass, itself a constant of nature, about 1.44 solar masses) and an electron approaches  $10^{60}$ , an interesting variation on the  $10^{40}$  and  $10^{80}$  usually associated with Dirac and Eddington respectively. Several authors have recently identified and speculated on the importance of an even larger number, about 120 orders of magnitude. This is for example the reason for the theoretical and observational estimates of the vacuum energy density that Nottale (1993) and Matthews (1997) linked to an LNH frame with a scaling law for the cosmological constant.

The observable universe is a ball-shaped region of the universe comprising all matter that can be observed from Earth or its space-based telescopes and exploratory probes at the present time, as the electromagnetic radiation from these objects has had time to reach the Solar System and Earth since the beginning of the cosmological expansion. Initially it was estimated that there may be 2 trillion galaxies in the observable universe, although that number was reduced in 2021 to only several hundred billion based on data from New Horizons. Assuming the universe is isotropic, the distance to the edge of the observable universe is roughly the same in every direction. That is, the observable universe is a spherical region centered on the observer. Every location in the universe has its own observable universe, which may or may not overlap with the one centered on Earth.

According to calculations, the current comoving distance proper distance, which takes into account that the universe has expanded since the light was emitted to particles from which the cosmic microwave background radiation (CMBR) was emitted, which represents the radius of the visible universe, is about 14.0 billion parsecs (about 45.7 billion light-years), while the comoving distance to the edge of the observable universe is about 14.3 billion parsecs (about 46.6 billion light-years), about 2% larger. The radius of the observable universe is therefore estimated to be

about 46.5 billion light-years and its diameter about 28.5 gigaparsecs (93 billion light-years, or  $8.8 \times 10^{26}$  meters or  $2.89 \times 10^{27}$  feet), which equals 880 yottameters. The figure 20 shows the geometric representation of the relationship between the radius of the universe with the Planck length.



**Figure 20.** Geometric representation of the relationship between the radius of the universe with the Planck length

The diameter of the observable universe will be calculated to be equal to the product of the ratio of electric force to gravitational force between electron and proton on the reduced Compton wavelength of the electron:

$$2 \cdot R_U = N_1 \cdot \lambda_c \quad (234)$$

Diameter of the universe = ratio of electric force to gravitational force  $\times$  reduced Compton wavelength of the electron

So the expressions for the radius of the observable universe are:

$$\frac{2R_U}{r_e} = \frac{N_1}{\alpha} \quad (235)$$

$$\frac{2R_U}{r_e} = \frac{1}{\mu \alpha_G} \quad (236)$$

$$\frac{2R_U}{\alpha_0} = \alpha N_1 \quad (237)$$

$$\frac{2R_U}{l_{\min}} = \alpha N_1 N_A \quad (238)$$

The expression between the radius of the observable universe  $R_U$  with the Planck length  $l_{pl}$  is:

$$R_U = e \cdot \alpha \cdot N_1 \cdot N_A \cdot l_{pl} \quad (239)$$

The expression between the radius of the observable universe  $R_U$  with the minimum distance  $l_{\min}$  is:

$$2 \cdot R_U = \alpha \cdot N_1 \cdot N_A \cdot l_{\min} \quad (240)$$

So apply the expressions for the radius of the observable universe:

$$R_U = \frac{\alpha N_1}{2} \alpha_0 \quad (241)$$

$$R_U = \frac{N_1}{2\alpha} r_e \quad (242)$$

$$R_U = \frac{1}{2\mu\alpha_G} r_e \quad (243)$$

$$R_U = \frac{m_{pl}^2 r_e}{2m_e m_p} \quad (244)$$

$$R_U = \frac{\hbar c r_e}{2G m_e m_p} \quad (245)$$

$$R_U = \frac{\alpha \hbar}{2G m_e^2 m_p} \quad (246)$$

For the value of the radius of the universe apply  $R_U = 4.38 \times 10^{26}$  m. The expressions for the gravitational constant are:

$$G = \frac{\hbar c r_e}{2m_e m_p} \frac{1}{R_U} \quad (247)$$

$$G = \frac{\alpha \hbar}{2m_e^2 m_p} \frac{1}{R_U} \quad (248)$$

## 16. Mass scales

The Planck mass  $m_{pl}$  appears everywhere in astrophysics, cosmology, quantum gravity, string theory, etc. Its mass is enormous compared to any subatomic particle and even the mass of heavier atoms. The mass Planck  $m_{pl}$  can be defined by three fundamental natural constants, the speed of light in vacuum  $c$ , the reduced Planck constant  $\hbar$  and the gravity constant  $G$  as:

$$m_{pl} = \sqrt{\frac{\hbar c}{G}} = \frac{\hbar}{l_{pl} c} = \frac{\mu_0 q_{pl}^2}{4\pi l_{pl}}$$

In [33] J. Forsythe and T. Valev found an extended mass relation for seven fundamental masses. Six of these masses are successfully identified as mass of the observable universe, Eddington mass limit of the most massive stars, mass of hypothetical quantum "Gravity Atom" whose gravitational potential is equal to electrostatic potential, Planck mass, Hubble mass and mass dimension constant relating masses of stable particles with coupling constants of fundamental interactions. The seventh mass is unidentified and could be considered as a prediction of the suggested mass relation for an unknown fundamental mass, potentially a yet unobserved light particle. First triad of these masses describes macro objects, the other three masses belong to particle physics masses, and the Planck mass appears intermediate in relation to these two groups. We found a similar mass relation for seven fundamental masses:

$$M_n = a^{-1} \cdot a_g^{(2-n)/3} \cdot m_e \quad (249)$$

$$n = 0, 1, 2, 3, 4, 5, 6$$

For  $n=0$   $M_0$  is the minimum mass  $M_{min}$ :

$$M_0 = M_{min} = a^{-1} \cdot a_g^{(2-0)/3} \cdot m_e$$

$$M_0 = M_{\min} = a^{-1} \cdot a_g^{2/3} \cdot m_e \quad (250)$$

For  $n=1$   $M_1$  is unidentified and could be regarded as a prediction by the suggested mass relation for unknown fundamental mass  $M_{Un}$ , most likely a yet unobserved light particle:

$$M_1 = M_{Un} = a^{-1} \cdot a_g^{(2-1)/3} \cdot m_e$$

$$M_1 = M_{Un} = a^{-1} \cdot a_g^{1/3} \cdot m_e \quad (251)$$

For  $n=2$   $M_2$  is a mass dimension constant in a basic mass equation relating masses of stable particles and coupling constants of the four interactions approximately a half charged pion mass  $M_{pi}$ :

$$M_2 = M_{pi} = a^{-1} \cdot a_g^{(2-2)/3} \cdot m_e$$

$$M_2 = M_{pi} = a^{-1} \cdot m_e \quad (252)$$

For  $n=3$   $M_3$  is the Planck mass  $m_{pl}$ :

$$M_3 = m_{pl} = a^{-1} \cdot a_g^{(2-3)/3} \cdot m_e$$

$$M_3 = m_{pl} = a^{-1} \cdot a_g^{-1/3} \cdot m_e \quad (253)$$

For  $n=4$  is the central mass of a hypothetical quantum "Gravity Atom"  $M_{GA}$ .

$$M_4 = M_{GA} = a^{-1} \cdot a_g^{(2-4)/3} \cdot m_e$$

$$M_4 = M_{GA} = a^{-1} \cdot a_g^{-2/3} \cdot m_e \quad (254)$$

For  $n=5$  is of the order of the Eddington mass  $M_{Edd}$  limit of the most massive stars:

$$M_5 = M_{Edd} = a^{-1} \cdot a_g^{(2-5)/3} \cdot m_e$$

$$M_5 = M_{Edd} = a^{-1} \cdot a_g^{-1} \cdot m_e \quad (255)$$

For  $n=6$  is the mass of the Hubble sphere and the mass of the observable universe  $M_U$ :

$$M_6 = M_U = a^{-1} \cdot a_g^{(2-6)/3} \cdot m_e$$

$$M_6 = M_U = a^{-1} \cdot a_g^{-4/3} \cdot m_e \quad (256)$$

The similar mass relation for seven fundamental masses is:

$$M_n = a_g^{-n/3} \cdot M_{\min} \quad (257)$$

$$n = 0, 1, 2, 3, 4, 5, 6$$

For  $n=0$   $M_0$  is the minimum mass  $M_{\min}$ :

$$M_0 = M_{\min} \quad (258)$$

For  $n=1$   $M_1$  is unidentified and could be regarded as a prediction by the suggested mass relation for unknown fundamental mass  $M_{Un}$ , most likely a yet unobserved light particle:

$$M_1 = M_{Un} = a_g^{-1/3} \cdot M_{\min} \quad (259)$$

For  $n=2$   $M_2$  is a mass dimension constant in a basic mass equation relating masses of stable particles and coupling constants of the four interactions approximately a half charged pion mass  $M_{pi}$ :

$$M_2 = M_{pi} = a_g^{-2/3} \cdot M_{\min} \quad (260)$$

For  $n=3$   $M_3$  is the Planck mass  $m_{pl}$ :

$$M_3 = m_{pl} = \alpha_g^{-1} \cdot M_{min} \quad (261)$$

For n=4 is the central mass of a hypothetical quantum ‘‘Gravity Atom’’ MGA:

$$M_4 = M_{GA} = \alpha_g^{-4/3} \cdot M_{min} \quad (262)$$

For n=5 is of the order of the Eddington mass  $M_{Edd}$  limit of the most massive stars:

$$M_5 = M_{Edd} = \alpha_g^{-5/3} \cdot M_{min} \quad (263)$$

For n=6 is the mass of the Hubble sphere and the mass of the observable universe  $M_U$ :

$$M_6 = M_U = \alpha_g^{-2} \cdot M_{min} \quad (264)$$

Gravitons do indeed have mass, and their motions generate kinetic energy. Thus, they have both energy and mass, and they obey the law of conservation of energy and matter. If gravitons did not have mass there would be no physics that we could understand. Other particles have mass, but they are much larger, much less numerous, and cannot substitute for the gravitational effects which generate space curvature. The great mystery of so-called force at a distance is explained by the mass of gravitons. What escapes our logical eyes is the incredibly small dimensionality of gravitons, expressed on a logarithmic scale. Where things get really interesting is in the smallest dimensions. Even an incredibly small and nearly massless particle can have great adjacent gravitational powers, as long as the centers of two attracted particles are sufficiently close. The reality of dark energy is the reality of gravity flows, seen from different perspectives. Dark energy and dark matter are thus aspects of a phenomenon. That phenomenon is the flow of gravitons on a Planck scale, expressed as spacetime foam. Gravitons flow on a massive scale among universe bubbles and the matter between. Given enough flowing gravitons in the spacetime foam, on a scale the human mind can hardly comprehend, there is apparent force at a distance, expressed as the bending of space. Where gravitons gravitate, there is an accelerating flow of gravitons to their collective core, along with a release of energy to help avoid quickly forming singularities. The following applies to the minimum mass  $M_{min}$ :

$$M_{min} c^2 = \frac{\hbar}{t_{max}}$$

$$M_{min} c^2 = \hbar H_0$$

$$M_{min} = \frac{\hbar H_0}{c^2} \quad (265)$$

$$M_{min} = \frac{\hbar}{c l_{max}} \quad (266)$$

So apply the expressions:

$$M_{min} = \frac{\hbar}{c} \sqrt{\Lambda} \quad (267)$$

$$M_{min} = \frac{m_{pl}^2}{M_{max}} \quad (268)$$

$$M_{min} = \frac{m_{pl}^2}{M_{max}} \quad (269)$$

Therefore for the minimum mass  $M_{min}$  apply:

$$M_{\min} = \alpha_g m_{pl} \quad (270)$$

$$M_{\min} = \frac{\alpha_G}{\alpha^3} m_e \quad (271)$$

$$M_{\min} = \frac{\sqrt[3]{\alpha_g^2}}{\alpha} m_e \quad (272)$$

$$M_{\min} = (2 \cdot e \cdot N_A)^{-2} \cdot \alpha^{-1} \cdot m_e \quad (273)$$

For the value of the minimum mass  $M_{\min}$  apply  $M_{\min} = 4.06578 \times 10^{-69}$  kg. Three independent calculations calculate the mass of the universe  $1.46 \times 10^{53}$  kg,  $1.7 \times 10^{53}$  kg and  $1.20 \times 10^{53}$  kg. In this context, mass refers to ordinary matter and includes the interstellar medium (ISM) and the intergalactic medium (IGM). However, it excludes dark matter and dark energy. This reported value for the mass of ordinary matter in the universe can be estimated on the basis of critical density infinite. So to estimate the mass value of the universe we will calculate the average of the three independent calculations that produce relatively close results. So the  $M_U$  mass of the observable universe is approximately  $M_U = 1.45 \times 10^{53}$  kg. Mass  $M$  have max/min ratio, which is the square of  $\alpha_g$ :

$$\alpha_g^2 = \frac{M_{\min}}{M_{\max}} \quad (274)$$

For the maximum mass  $M_{\max}$  applies:

$$M_{\max} = \frac{F_{\max} l_{\max}}{c^2} \quad (275)$$

$$M_{\max} = \frac{m_{pl}^2}{M_{\min}} \quad (276)$$

The expressions for the mass of the observable universe are:

$$M_U = \alpha^{-1} \cdot \alpha_g^{-4/3} \cdot m_e \quad (277)$$

$$M_U = \alpha^3 \cdot \alpha_G^{-2} \cdot m_e \quad (278)$$

$$M_U = (2 \cdot e \cdot \alpha^2 \cdot N_A)^2 \cdot N_1 \cdot m_p \quad (279)$$

$$M_U = \mu \cdot \alpha \cdot N_1^2 \cdot m_p \quad (280)$$

For the value of the mass of the observable universe  $M_U$  apply  $M_U = 1.153482 \times 10^{53}$  kg. In astrophysics, the Eddington number,  $N_{\text{Edd}}$ , is the number of protons in the observable universe. Eddington originally calculated it as about  $1.57 \times 10^{79}$ , current estimates make it approximately  $10^{80}$ . The term is named for British astrophysicist Arthur Eddington, who in 1940 was the first to propose a value of  $N_{\text{Edd}}$  and to explain why this number might be important for physical cosmology and the foundations of physics. The expressions who calculate the number of protons in the observable universe are:

$$N_{\text{Edd}} = \frac{M_U}{m_p} = \mu \alpha N_1^2 = 6.9 \times 10^{79} \quad (281)$$

$$\frac{M_U}{m_p} = (2e\alpha^2 N_A)^2 N_1 \quad (282)$$

$$\frac{M_U}{m_p} = \frac{N_1}{\alpha_g^{\frac{2}{3}}} \quad (283)$$

$$\frac{M_U}{m_p} = \left( \frac{r_e}{l_{pl}} \right)^2 N_1 \quad (284)$$

Also apply the expressions:

$$m_{pl} \cdot l_{max} = M_U \cdot l_{pl} \quad (285)$$

$$l_{max}^2 \cdot M_{min} = l_{min}^2 \cdot M_U \quad (286)$$

The expressions for the relationship between the mass of the observable universe  $M_U$  with the radius of the universe  $R_U$  are:

$$\frac{M_U}{R_U^2} = 4\alpha\mu^2 \frac{m_e}{r_e^2} \quad (287)$$

$$\frac{M_U}{R_U^2} = 16 \frac{m_p}{r_p r_e} \quad (288)$$

$$\frac{M_U}{R_U^2} = \frac{64}{\alpha} \frac{m_e}{r_p^2} \quad (289)$$

$$\frac{M_U}{m_p} = \alpha\mu \left( \frac{2R_U}{r_e} \right)^2 \quad (290)$$

$$\frac{\frac{M_U}{m_p}}{\left( \frac{2R_U}{r_e} \right)^2} = \alpha\mu \quad (291)$$

## 17. Energy scales

R. Adler in [34] calculated the energy ratio in cosmology, the ratio of the dark energy density to the Planck energy density. Atomic physics has two characteristic energies, the rest energy of the electron  $E_e$ , and the binding energy of the hydrogen atom  $E_H$ . The rest energy of the electron  $E_e$  is defined as:

$$E_e = m_e c^2$$

The binding energy of the hydrogen atom  $E_H$  is defined as:

$$E_H = \frac{m_e e^4}{2\hbar^2}$$

Their ratio is equal to half the square of the fine-structure constant:



$$\frac{E_H}{E_e} = \frac{\alpha^2}{2}$$

Cosmology also has two characteristic energy scales, the Planck energy density  $\rho_{pl}$ , and the density of the dark energy  $\rho_\Lambda$ . The Planck energy density is defined as:

$$\rho_{pl} = \frac{E_{pl}}{l_{pl}} = \frac{c^7}{\hbar G^2}$$

To obtain an expression for the dark energy density in terms of the cosmological constant we recall that the cosmological term in the general relativity field equations may be interpreted as a fluid energy momentum tensor of the dark energy according to so the dark energy density  $\rho_\Lambda$  is given by:

$$\rho_\Lambda = \frac{\Lambda c^4}{8\pi G}$$

The ratio of the energy densities is thus the extremely small quantity:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{\alpha_g^2}{8\pi}$$

So for the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{2e^2\varphi^{-5}}{3^3\pi\varphi^5} \times 10^{-120} \quad (292)$$

Some authors consider the small value of the ratio to be arguably one of the most mysterious problems in present day physics. The understanding of atomic structure required the discovery of the fundamental dynamical constant  $\hbar$ . Viewed in this way the cosmological analog of  $\hbar$  is  $\Lambda$ , but any dynamical role it may play is not yet apparent. It is amusing to note that in the presence of two length scales, and their dimensionless ratio, dimensional analysis becomes problematic, a dimensional estimate can contain an arbitrary function of the ratio, for example a power or a logarithm. In the case of cosmology it is clear that dimensional estimates, with two disparate length scales, may be much worse than useless.

## 18. Time scales

The Planck time  $t_{pl}$  is the time required for light to travel a distance of 1 Planck length in vacuum. No current physical theory can describe timescales shorter than the Planck time, such as the earliest events after the Big Bang. Some conjecture that the structure of time need not remain smooth on intervals comparable to the Planck time. Today it plays a tantalizing role in our understanding of the Big Bang and the search for a theory of quantum gravity. All scientific experiments and human experiences occur on time scales that are many orders of magnitude larger than the Planck time  $t_{pl}$ , making events on the Planck time  $t_{pl}$  undetectable with current scientific technology. The Planck time  $t_{pl}$  is defined as:

$$t_{pl} = \frac{l_{pl}}{c} = \sqrt{\frac{\hbar G}{c^5}} = \frac{\hbar}{m_{pl}c^2}$$

The 2018 CODATA recommended value of the Planck time is  $t_{pl}=5.391247 \times 10^{-44}$  s with standard uncertainty  $0.000060 \times 10^{-44}$  s and relative standard uncertainty  $1.1 \times 10^{-5}$ . For the minimum distance  $l_{min}$  apply:

$$l_{min}=2 \cdot e \cdot l_{pl}$$

So for the minimum time  $t_{min}$  apply:

$$t_{min} = \frac{l_{min}}{c}$$

$$t_{min} = \frac{2el_{pl}}{c}$$

$$t_{min} = 2 \cdot e \cdot t_{pl} \quad (293)$$

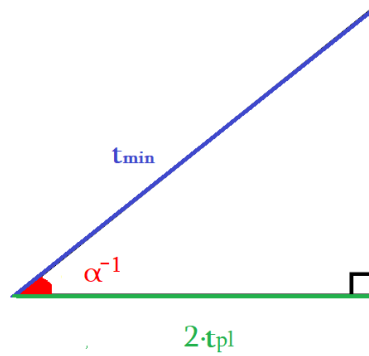
From expressions apply:

$$\cos \alpha^{-1} = e^{-1}$$

$$\cos \alpha^{-1} \cdot t_{min} = 2 \cdot t_{pl}$$

$$\cos \alpha^{-1} = \frac{2t_{pl}}{t_{min}} \quad (294)$$

The figures 21 below show the geometric representation of the fundamental unit of time.



**Figure 21.** Geometric representation of the fundamental unit of time.

The Hubble constant  $H_0$  is one of the most important numbers in cosmology because it is required to estimate the size and age of the universe. This number indicates the rate at which the universe is expanding. The Hubble constant can be used to determine the inherent brightness and masses of stars in nearby galaxies, examine the same properties in more distant galaxies and galaxy clusters, infer the amount of dark matter in the universe, and obtain the scale size of distant clusters as far as clusters test for theoretical cosmological models. In 1929, American astronomer Edwin Hubble announced his discovery that galaxies, in all directions, seemed to be moving away from us and have greater displacement for attenuated galaxies. The unit of the Hubble constant is 1 km/s/Mpc. The 2018 CODATA recommended value of the Hubble constant is  $H_0 = 67.66 \pm 0.42$  (km/s)/Mpc =  $(2.1927664 \pm 0.0136) \times 10^{-18} \text{ s}^{-1}$ . Hubble length or Hubble distance is a unit of distance in cosmology, defined as the speed of light multiplied by Hubble time  $L_H = c \cdot H_0^{-1}$ . This distance is equivalent to 4.550 million parsecs, or 14.4 billion light-years, 13.8 billion years. Hubble's distance would be the distance between the Earth and the galaxies currently falling away from us at the speed of light, as shown by the substitution  $r = c \cdot H_0^{-1}$  in the equation for Hubble's law,  $v = H_0 \cdot r$ .

The maximum time period  $t_{max}$  is the time from the time of Bing Bang to the present day. This time period corresponds to the time of the universe  $t_U = H_0^{-1}$ . Therefore:

$$t_{max} = t_U = H_0^{-1}$$

Time  $t$  has the min/max ratio which is:

$$\alpha_g = \frac{t_{min}}{t_{max}}$$

$$\alpha_g = t_{\min} \cdot H_0 \quad (295)$$

$$\alpha_g = 2 \cdot e \cdot t_{pl} \cdot H_0 \quad (296)$$

In physical cosmology, the age of the universe is the time elapsed since the Big Bang. Astronomers have derived two different measurements of the age of the universe: a measurement based on direct observations of an early state of the universe, which indicate an age of  $13.787 \pm 0.020$  billion years as interpreted with the Lambda-CDM concordance model as of 2021.

In 1961, Dicke observed that a dimensionless number must necessarily be large to make the lifetime of stars long enough to produce heavy chemical elements such as carbon. Knowing that carbon is the most essential element for biological materials, this is the first claim called "Human Coincidence", which infers that the connection between physical constants is necessary for the existence of life in the universe. Thermonuclear combustion is necessary for the production of elements heavier than hydrogen. Again it takes several billion years for this to occur. type of conversion inside a star. According to the general theory of relativity no universe can provide several billion years of time unless it is several light-years in extent. Serious criticisms and interpretations have been made on the issue of the large number hypothesis and the existence of intelligent beings or life. One of the most difficult issues in understanding consciousness is understanding how information is synthesized to form our subjective experience. The widely accepted hypothesis is that gamma currents ranging from many places in the brain combine to create a unified subjective experience. In this way, neurons performing different tasks in separate areas of the brain are divided into a single instantaneous activity. From [35] the gamma rhythm is a pattern of neuronal oscillations whose frequency ranges from 25 Hz to 100 Hz although 40 Hz is typical. Gamma frequency oscillations are present during wakefulness and REM sleep. Changes in electrical membrane potential generate neuronal action potentials. Oscillatory activity of neurons is connected to these spikes. The oscillation of the single neuron can be observed in fluctuations at the threshold of the membrane potential. The time quantum in the brain  $t_B$ , the smallest unit of time that related to the 40 Hz oscillation of the gamma rate:

$$\frac{t_B}{t_{pl}} = \sqrt[3]{\alpha_g^2} \quad (297)$$

The Planck time depends on the fundamental constants such as the Planck constant  $h$ , the gravitational constant  $G$  and the speed of light  $c$ , while it is not clear whether the shorter time scale in the brain also depends on these fundamental constants. Thus, the observer who can observe a universe tuned to the various fundamental constants must have synchronous activity of gamma oscillations of about 40 Hz in his nervous system. This is what we find from the experimental results in modern neuroscience.

In the 18th century, the concept that the age of Earth was millions, if not billions, of years began to appear. Nonetheless, most scientists throughout the 19th century and into the first decades of the 20th century presumed that the universe itself was Steady State and eternal, possibly with stars coming and going but no changes occurring at the largest scale known at the time. The first scientific theories indicating that the age of the universe might be finite were the studies of thermodynamics, formalized in the mid-19th century. The concept of entropy dictates that if the universe (or any other closed system) were infinitely old, then everything inside would be at the same temperature, and thus there would be no stars and no life. No scientific explanation for this contradiction was put forth at the time.

In 1915 Albert Einstein published the theory of general relativity and in 1917 constructed the first cosmological model based on his theory. In order to remain consistent with a steady-state universe, Einstein added what was later called a cosmological constant to his equations. Einstein's model of a static universe was proved unstable by Arthur Eddington. The first direct observational hint that the universe was not static but expanding came from the observations of 'recession velocities', mostly by Vesto Slipher, combined with distances to the 'nebulae' (galaxies) by Edwin Hubble in a work published in 1929. Earlier in the 20th century, Hubble and others resolve individual stars within certain nebulae, thus determining that they were galaxies, similar to, but external to, our Milky Way Galaxy. In addition, these galaxies were very large and very far away. Spectra taken of these distant galaxies showed a red shift in their spectral lines presumably caused by the Doppler effect, thus indicating that these galaxies were moving away from the Earth. In addition, the farther away these galaxies seemed to be (the dimmer they appeared to us) the greater was their redshift, and thus the faster they seemed to be moving away. This was the first direct evidence that the universe is not static but expanding. The first estimate of the age of the universe came from the calculation of when all of the objects must have started speeding out from the same point. Hubble's initial value for the universe's age was very low, as the galaxies were assumed to be much closer than later observations found them to be. The first reasonably accurate measurement of the rate of expansion of the universe, a numerical value now known as the

Hubble constant, was made in 1958 by astronomer Allan Sandage. His measured value for the Hubble constant came very close to the value range generally accepted today. Sandage, like Einstein, did not believe his own results at the time of discovery. Sandage proposed new theories of cosmogony to explain this discrepancy. This issue was more or less resolved by improvements in the theoretical models used for estimating the ages of stars. As of 2013, using the latest models for stellar evolution, the estimated age of the oldest known star is  $14.46 \pm 0.8$  billion years. The discovery of microwave cosmic background radiation announced in 1965 finally brought an effective end to the remaining scientific uncertainty over the expanding universe. It was a chance result from work by two teams less than 60 miles apart. In 1964, Arno Penzias and Robert Wilson were trying to detect radio wave echoes with a supersensitive antenna. The antenna persistently detected a low, steady, mysterious noise in the microwave region that was evenly spread over the sky, and was present day and night. After testing, they became certain that the signal did not come from the Earth, the Sun, or our galaxy, but from outside our own galaxy, but could not explain it. At the same time another team, Robert H. Dicke, Jim Peebles, and David Wilkinson, were attempting to detect low level noise that might be left over from the Big Bang and could prove whether the Big Bang theory was correct. The two teams realized that the detected noise was in fact radiation left over from the Big Bang, and that this was strong evidence that the theory was correct. Since then, a great deal of other evidence has strengthened and confirmed this conclusion, and refined the estimated age of the universe to its current figure. The space probes WMAP, launched in 2001, and Planck, launched in 2009, produced data that determines the Hubble constant and the age of the universe independent of galaxy distances, removing the largest source of error.

In physical cosmology, the age of the universe is the time elapsed since the Big Bang. Astronomers have derived two different measurements of the age of the universe: a measurement based on direct observations of an early state of the universe, which indicate an age of  $13.787 \pm 0.020$  billion years as interpreted with the Lambda-CDM concordance model as of 2021. This distance is equivalent to 4.550 million parsecs, or 14.4 billion light-years, 13.8 billion years. Hubble's distance would be the distance between the Earth and the galaxies currently falling away from us at the speed of light, as shown by the substitution  $r = c \cdot H_0^{-1}$  in the equation for Hubble's law,  $u = H_0 \cdot r$ . The Schwarzschild radius or gravitational radius is a physical parameter that appears in the Schwarzschild solution in Einstein's field equations, which corresponds to the radius defining the event horizon of a Schwarzschild black hole. It is a characteristic radius associated with any quantity. (from its center) which must have a celestial body in order to reach a black hole. That is, if a celestial body has a radius less than its Schwarzschild radius then it is a black hole. The term is used especially in physics and astronomy, with emphasis The Schwarzschild ray was named after the German astronomer Karl Schwarzschild, who calculated this exact solution for the theory of general relativity in 1916. The age of the universe is the time elapsed since the Big Bang. Astronomers have derived two different measurements of the age of the universe: a measurement based on direct observations of an early state of the universe, which indicate an age of  $13.787 \pm 0.020$  billion years as interpreted with the Lambda-CDM concordance model as of 2021 and a measurement based on the observations of the local, modern universe, which suggest a younger age. The uncertainty of the first kind of measurement has been narrowed down to 20 million years, based on a number of studies that all show similar figures for the age and that includes studies of the microwave background radiation by the Planck spacecraft, the Wilkinson Microwave Anisotropy Probe and other space probes. Measurements of the cosmic background radiation give the cooling time of the universe since the Big Bang, and measurements of the expansion rate of the universe can be used to calculate its approximate age by extrapolating backwards in time. The range of the estimate is also within the range of the estimate for the oldest observed star in the universe. For the age of the universe apply:

$$T_U = \frac{R_U}{c} \quad (298)$$

$$T_U = \frac{N_1 r_e}{2ac} \quad (299)$$

$$T_U = \frac{r_e}{2\mu\alpha_G c} \quad (300)$$

$$T_U = \frac{\alpha N_1 \alpha_0}{2c} \quad (301)$$

$$T_U = \frac{\alpha \hbar}{2c G m_e^2 m_p} \quad (302)$$

$$T_U = \frac{\hbar r_e}{2G m_e m_p} \quad (303)$$

For the value of the age of the universe apply  $T_U = 1.46 \times 10^{18}$  s. The expressions for the gravitational constant are:

$$G = \frac{\alpha \hbar}{2c m_e^2 m_p} \frac{1}{T_U} \quad (304)$$

$$G = \frac{\hbar r_e}{2m_e m_p} \frac{1}{T_U} \quad (305)$$

## 19. The Shape of The Universe

In [36] and [37] we proved that the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions will propose a possible solution for the density parameter of baryonic matter, dark matter and dark energy. The Platonic solids have been known since antiquity. It has been suggested that certain carved stone balls created by the late Neolithic people of Scotland represent these shapes; however, these balls have rounded knobs rather than being polyhedral, the numbers of knobs frequently differed from the numbers of vertices of the Platonic solids, there is no ball whose knobs match the 20 vertices of the dodecahedron, and the arrangement of the knobs was not always symmetric. The ancient Greeks studied the Platonic solids extensively. Some sources credit Pythagoras with their discovery. Other evidence suggests that he may have only been familiar with the tetrahedron, cube, and dodecahedron and that the discovery of the octahedron and icosahedron belong to Theaetetus, a contemporary of Plato. In any case, Theaetetus gave a mathematical description of all five and may have been responsible for the first known proof that no other convex regular polyhedra exists. The Platonic solids are prominent in the philosophy of Plato, their namesake. Plato wrote about them in the dialogue *Timaeus* c. 360 B.C. in which he associated each of the four classical elements (earth, air, water, and fire) with a regular solid. Earth was associated with the cube, air with the octahedron, water with the icosahedron, and fire with the tetrahedron. There was intuitive justification for these associations: the heat of fire feels sharp and stabbing (like little tetrahedra). Air is made of the octahedron; its minuscule components are so smooth that one can barely feel it. Water, the icosahedron, flows out of one's hand when picked up, as if it is made of tiny little balls. By contrast, a highly nonspherical solid, the hexahedron (cube) represents "earth". These clumsy little solids cause dirt to crumble and break when picked up in stark difference to the smooth flow of water. Moreover, the cube being the only regular solid that tessellates Euclidean space was believed to cause the solidity of the Earth. Of the fifth Platonic solid, the dodecahedron, Plato obscurely remarked, "...the god used for arranging the constellations on the whole heaven". Aristotle added a fifth element, *aithēr* (aether in Latin, "ether" in English) and postulated that the heavens were made of this element, but he had no interest in matching it with Plato's fifth solid. Euclid completely mathematically described the Platonic solids in the *Elements*, the last book (Book XIII) of which is devoted to their properties. Propositions 13–17 in Book XIII describe the construction of the tetrahedron, octahedron, cube, icosahedron, and dodecahedron in that order. For each solid Euclid finds the ratio of the diameter of the circumscribed sphere to the edge length. In Proposition 18 he argues that there are no further convex regular polyhedra. Andreas Speiser has advocated the view that the construction of the five regular solids is the chief goal of the deductive system canonized in the *Elements*. Much of the information in Book XIII is probably derived from the work of Theaetetus.

According to a recent theory the Universe could be a dodecahedron. It is surprising that Plato used a dodecahedron as the quintessence to describe the cosmos! Plato (c. 427 BC – c. 347 BC) also stated that time had a beginning; it

came together with the universe in one instant of creation. A polyhedron bounded by a number of congruent polygonal faces, so that the same number of faces meet at each vertex, and in each face all the sides and angles are equal (i.e. faces are regular polygons) is called a regular polyhedron. One morning the young Werner Heisenberg discovered reading Plato's *Timaeus* a description of the world with regular polyhedra. Heisenberg could not understand why Plato, being so rational, started to use speculative ideas. But finally he was fascinated by the idea that it could be possible to describe the Universe mathematically. He could not understand why Plato used the Polyhedra as the basic units in his model, but Heisenberg considered that in order to understand the world it is necessary to understand the Physics of the atoms. Theaetetus (c. 414-367 BC), was a member of Plato's Academy. He was a son of Euphronius of Sounion, student of Theodore of Cyrene. Theaetetus died on his return to Athens after he was wounded at the Battle of Corinth. His friend Plato dedicated one of his dialogues to him. Euclid's elements chapter X and XIII are based on the work of Theaetetus. Hippasus, from Metapontum in Magna Graecia (south Italy), who wrote around 465 BC about a "sphere of 12 pentagons" refers to the dodecahedron. Hippasus performed acoustics Experiments with vessels filled with different amounts of water and with copper discs of different thicknesses.

Finite well-proportioned spaces, especially the Poincaré dodecahedron, open something of a Pandora's box for the physics of the early universe. The standard model of cosmology relies in the main on the hypothesis that the early universe underwent a phase of exponential expansion called inflation, which produced density fluctuations on all scales. In the simplest inflationary models, space is supposed to have become immensely larger than the observable universe. Therefore, a positive curvature (i.e.  $\Omega > 1$ ), even if weak, implies a finite space and sets strong constraints on inflationary models. It is possible to build "low scale" inflationary universes in which the inflation phase ends more quickly than it does in general inflationary modes, leading to a detectable space curvature. In other words, even if space is not flat, a multi connected topology does not contradict the general idea of inflation. However, no convincing physical scenario for this has yet been proposed. Perhaps the most fundamental challenge is to link the present-day topology of space to a quantum origin, since general relativity does not allow for topological changes during the course of cosmic evolution. A quantum theory of gravity could allow us to address this problem, but there is currently no indication about how such a unified theory might actually describe the emergence of multiple connected spaces. Data from the European Planck Surveyor, which is scheduled for launch in 2007 will be able to determine  $\Omega$  with a precision of 1%. A value lower than 1.01 will rule out the Poincaré dodecahedron model, since the size of the corresponding dodecahedron would become greater than the observable universe and would not leave any observable imprint on the microwave background. A value greater than 1.01, on the other hand, would strengthen the models' cosmological pertinence. Whether or not some multiply connected model of space such as the Poincaré dodecahedron is refuted by future astronomical data, cosmic topology will continue to remain at the heart of our understanding about the ultimate structure of our universe.

Surprisingly, not all small-volume universes suppress the large-scale fluctuations. In 2003 J.-P. Luminet in [38] proved that the long-wavelength modes tend to be relatively lowered only in a special family of finite, multi connected spaces that are called "well-proportioned spaces" because they have a similar extent in all three dimensions. More specifically, we discovered that the best candidate to fit the observed power spectrum is a well-proportioned space called the Poincaré dodecahedral space. This space may be represented by a polyhedron with 12 pentagonal faces, with opposite faces being "glued" together after a twist of  $36^\circ$ . This is the only consistent way to obtain a spherical (i.e. positively curved) space from a dodecahedron: if the twist was  $108^\circ$ , for example, we would end up with a radically different hyperbolic space. The Poincaré dodecahedral space is essentially a multiply connected variant of a simply connected hypersphere, although its volume is 120 times smaller. A rocket leaving the dodecahedron through a given face immediately re-enters through the opposite face, and light propagates such that any observer whose line-of-sight intercepts one face has the illusion of seeing a slightly rotated copy of their own dodecahedron. This means that some photons from the cosmic microwave background, for example, would appear twice in the sky. The power spectrum associated with the Poincaré dodecahedral space is different from that of a flat space because the fluctuations in the cosmic microwave background will change as a function of their wavelengths. In other words, due to a cut-off in space corresponding to the size of the dodecahedron, one expects fewer fluctuations at large angular scales than in an infinite flat space, but at small angular scales one must recover the same pattern as in the flat infinite space. In order to calculate the power spectrum we varied the mass-energy density of the dodecahedral universe and computed the quadrupole and the octopole modes relative to the WMAP data. To our delight, we found a small interval of values over which both these modes matched the observations perfectly. Moreover, the best fit occurred in the range  $1.01 < \Omega < 1.02$ , which sits comfortably with the observed value.

The Poincaré dodecahedral space therefore accounts for the lack of large-scale fluctuations in the microwave background and also for the slight positive curvature of space inferred from WMAP and other observations. Moreover, given the observed values of the mass-energy densities and of the expansion rate of the universe, the size of the dodecahedral universe can be calculated. We found that the smallest dimension of the Poincaré dodecahedron space is 43 billion light-years, compared with 53 billion light-years for the "horizon radius" of the observable

universe. Moreover, the volume of this universe is about 20% smaller than the volume of the observable universe. (There is a common misconception that the horizon radius of a flat universe is 13.7 billion light-years, since that is the age of the universe multiplied by the speed of light. However, the horizon radius is actually much larger because photons from the horizon that are reaching us now have had to cross a much larger distance due to the expansion of the universe.) If physical space is indeed smaller than the observable universe, some points on the map of the cosmic microwave background will have several copies. As first shown by Neil Cornish of Montana State University and co-workers in 1998, these ghost images would appear as pairs of so-called matched circles in the cosmic microwave background where the temperature fluctuations should be the same. This “lensing” effect, which can be precisely calculated, is thus purely attributable to the topology of the universe. Due to its 12-sided regular shape, the Poincaré dodecahedral model actually predicts six pairs of diametrically opposite matched circles with an angular radius of 10-50°, depending on the precise values of cosmological parameters such as the mass-energy density.

## 20. Solution for the density parameter of baryonic matter

The Hubble constant  $H_0$  is one of the most important numbers in cosmology because it is required to estimate the size and age of the universe. This number indicates the rate at which the universe is expanding. The Hubble constant can be used to determine the inherent brightness and masses of stars in nearby galaxies, examine the same properties in more distant galaxies and galaxy clusters, infer the amount of dark matter in the universe, and obtain the scale size of distant clusters as far as clusters test for theoretical cosmological models. In 1929, American astronomer Edwin Hubble announced his discovery that galaxies, in all directions, seemed to be moving away from us and have greater displacement for attenuated galaxies. However, the true value for  $H_0$  is very complicated. Astronomers need two measurements:

- a) First, spectroscopic observations reveal the redshift of the galaxy, showing its radial velocity.
- b) The second measurement, the most difficult value, is the exact distance of the galaxy from Earth.

The unit of the Hubble constant is 1 km/s/Mpc. The 2018 CODATA recommended value of the Hubble constant is  $H_0 = 67.66 \pm 0.42$  (km/s)/Mpc =  $(2.1927664 \pm 0.0136) \times 10^{-18} \text{ s}^{-1}$ . Hubble length or Hubble distance is a unit of distance in cosmology, defined as the speed of light multiplied by Hubble time  $L_H = c \cdot H_0^{-1}$ . This distance is equivalent to 4.550 million parsecs, or 14.4 billion light-years, 13.8 billion years. Hubble's distance would be the distance between the Earth and the galaxies currently falling away from us at the speed of light, as shown by the substitution  $r = c \cdot H_0^{-1}$  in the equation for Hubble's law,  $u = H_0 \cdot r$ .

The critical density is the average density of matter required for the Universe to just halt its expansion, but only after an infinite time. A Universe with a critical density is said to be flat. In his theory of general relativity, Einstein demonstrated that the gravitational effect of matter is to curve the surrounding space. In a Universe full of matter, both its overall geometry and its fate are controlled by the density of the matter within it. If the density of matter in the Universe is high (a closed Universe), self-gravity slows the expansion until it halts, and ultimately re-collapses. In a closed Universe, locally parallel light rays converge at some extremely distant point. This is referred to as spherical geometry. If the density of matter in the Universe is low (an open Universe), self-gravity is insufficient to stop the expansion, and the Universe continues to expand forever (albeit at an ever decreasing rate). In an open Universe, locally parallel light rays ultimately diverge. This is referred to as hyperbolic geometry. Balanced on a knife edge between Universes with high and low densities of matter, there exists a Universe where parallel light rays remain parallel. This is referred to as a flat geometry, and the density is called the critical density. In a critical density Universe, the expansion is halted only after an infinite time. To date, the critical density is estimated to be approximately five atoms per cubic meter, whereas the average density of ordinary matter in the Universe is believed to be 0.2–0.25 atoms per cubic meter. A much greater density comes from the unidentified dark matter; both ordinary and dark matter contribute in favor of contraction of the universe. However, the largest part comes from so-called dark energy, which accounts for the cosmological constant term. Although the total density is equal to the critical density the dark energy does not lead to contraction of the universe but rather may accelerate its expansion. Therefore, the universe will likely expand forever. An expression for the critical density is found by assuming  $\Lambda$  to be zero and setting the normalized spatial curvature,  $k$ , equal to zero. When the substitutions are applied to the first of the Friedmann equations we find:

$$\rho_c = \frac{3H_0^3}{8\pi G}$$

It should be noted that this value changes over time. The critical density changes with cosmological time, but the energy density due to the cosmological constant remains unchanged throughout the history of the universe. The amount of dark energy increases as the universe grows, while the amount of matter does not. The density parameter  $\Omega$

is defined as the ratio of the actual density  $\rho$  to the critical density  $\rho_c$  of the Friedmann universe. The relation between the actual density and the critical density determines the overall geometry of the universe, when they are equal, the geometry of the universe is flat (Euclidean). The galaxies we see in all directions are moving away from the Earth, as evidenced by their red shifts. Hubble's law describes this expansion. Remarkably, study of the expansion rate has shown that the universe is very close to the critical density that would cause it to expand forever. The density parameter  $\Omega$  is defined as the ratio of the average density of matter and energy in the Universe  $\rho$  to the critical density  $\rho_c$  of the Friedmann universe. The relation between the actual density and the critical density determines the overall geometry of the universe; when they are equal, the geometry of the universe is flat (Euclidean). In earlier models, which did not include a cosmological constant term, critical density was initially defined as the watershed point between an expanding and a contracting Universe. The density parameter is given by:

$$\Omega_0 = \frac{\rho}{\rho_c}$$

where  $\rho$  is the actual density of the Universe and  $\rho_c$  the critical density. Although current research suggests that  $\Omega_0$  is very close to 1, it is still of great importance to know whether  $\Omega_0$  is slightly greater than 1, less than 1, or exactly equal to 1, as this reveals the ultimate fate of the Universe. If  $\Omega_0$  is less than 1, the Universe is open and will continue to expand forever. If  $\Omega_0$  is greater than 1, the Universe is closed and this will eventually halt its expansion and recollapse. If  $\Omega_0$  is exactly equal to 1 then the Universe is flat and contains enough matter to halt the expansion but not enough to recollapse it. It is important to note that the  $\rho$  used in the calculation of  $\Omega_0$  is the total mass/energy density of the Universe. In other words, it is the sum of a number of different components including both normal and dark matter as well as the dark energy suggested by recent observations. We can therefore write:

$$\Omega_0 = \Omega_B + \Omega_D + \Omega_\Lambda$$

$$\Omega_0 = \Omega_m + \Omega_\Lambda$$

$$\Omega_0 = \Omega_B + \Omega_{D+\Lambda}$$

where:

$\Omega_B$  is the density parameter for normal baryonic matter,

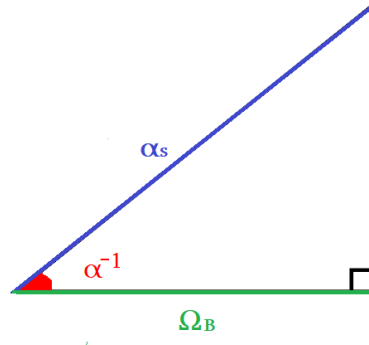
$\Omega_D$  is the density parameter for dark matter,

$\Omega_\Lambda$  is the density parameter for dark energy,

$\Omega_m$  is the sum of the density parameter for normal baryonic matter and the density parameter for dark matter,

$\Omega_{D+\Lambda}$  is the sum of the density parameter for the density parameter for dark matter and the density parameter for dark energy. The sum of the contributions to the total density parameter  $\Omega_0$  at the current time is  $\Omega_0 = 1.02 \pm 0.02$ . Current observations suggest that we live in a dark energy dominated Universe with  $\Omega_\Lambda = 0.73$ ,  $\Omega_D = 0.23$  and  $\Omega_B = 0.04$ . To the accuracy of current cosmological observations, this means that we live in a flat,  $\Omega_0 = 1$  Universe. Instead of the cosmological constant  $\Lambda$  itself, cosmologists often refer to the ratio between the energy density due to the cosmological constant and the critical density of the universe, the peak point of a density sufficient to prevent the universe from expanding forever, at one level of the universe is the ratio between the energy of the universe due to the cosmological constant  $\Lambda$  and the critical density of the universe, that is what we would call the fraction of the universe consisting of dark energy [39]. By definition, baryonic matter should only include matter composed of baryons. In other words, it should include protons, neutrons and all the objects composed of them (i.e. atomic nuclei), but exclude things such as electrons and neutrinos which are actually leptons. In astronomy, however, the term 'baryonic matter' is used more loosely, since on astronomical scales, protons and neutrons are always accompanied by electrons. Astronomers therefore use the term 'baryonic' to refer to all objects made of normal atomic matter, essentially ignoring the presence of electrons which, after all, represent only  $\sim 0.0005$  of the mass. Neutrinos, on the other hand, are considered non-baryonic by astronomers. Another slight oddity in the usage of the term baryonic matter in astronomy is that black holes are included as baryonic matter. While the matter from which black holes form is mainly baryonic matter, once swallowed by the black hole, this distinction is lost. For example, a theoretical black hole constructed purely out of photons is indistinguishable from one made from normal baryonic matter. This is often referred to as the 'black holes have no hair' theorem which simply states that black holes do not have properties such as baryonic or non-baryonic. Objects in the Universe composed of baryonic matter include Clouds of cold gas, Planets, Comets and asteroids, Stars, Neutron stars and Black holes. The figure 22 shows the Geometric representation of the density parameter for the baryonic matter .





**Figure 22.** Geometric representation of the the density parameter for the baryonic matter

The assessment of baryonic matter at the current time was assessed by WMAP to be  $\Omega_B=0.044\pm 0.004$ . From the dimensionless unification of the fundamental interactions the density parameter for the normal baryonic matter is:

$$\Omega_B=e^{-\pi} \quad (306)$$

$$\Omega_B=i^{2i} \quad (307)$$

$$\Omega_B=0.0432 \quad (308)$$

$$\Omega_B=4.32\% \quad (309)$$

Series representations for the density parameter for the normal baryonic matter  $\Omega_B$  are:

$$e^{-\pi} = \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{-\pi} \quad (310)$$

$$e^{-\pi} = e^{-4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \quad (311)$$

$$e^{-\pi} = \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{-\pi} \quad (312)$$

$$e^{-\pi} = \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{-4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \quad (313)$$

$$e^{-\pi} = \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{-4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \quad (314)$$

The pattern of the continued fraction for the density parameter for the normal baryonic matter is:



$$\Omega_B = 2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) \quad (322)$$

$$\Omega_B = 2 \cdot 10^7 \cdot N_A \cdot e^{-1} \cdot a_w \cdot a \cdot a_G^{1/2} \quad (323)$$

$$\Omega_B = 10^{-7} \cdot a_g^{1/3} \cdot a_s^2 \cdot a \cdot a_w^{-1} \cdot a_G^{-1/2} \quad (324)$$

## 21. Solution for the density parameter of dark energy

In [40] we presented the solution for the Density Parameter of Dark Energy. In physical cosmology and astronomy, dark energy is an unknown form of energy that affects the universe on the largest scales. The first observational evidence for its existence came from measurements of supernovas, which showed that the universe does not expand at a constant rate; rather, the universe's expansion is accelerating. Understanding the universe's evolution requires knowledge of its starting conditions and composition. Before these observations, scientists thought that all forms of matter and energy in the universe would only cause the expansion to slow down over time. Measurements of the cosmic microwave background (CMB) suggest the universe began in a hot Big Bang, from which general relativity explains its evolution and the subsequent large-scale motion. Without introducing a new form of energy, there was no way to explain how scientists could measure an accelerating universe. Since the 1990s, dark energy has been the most accepted premise to account for the accelerated expansion. As of 2021, there are active areas of cosmology research to understand the fundamental nature of dark energy.

The fraction of the effective mass of the universe attributed to dark energy or the cosmological constant is  $\Omega_\Lambda = 0.73 \pm 0.04$ . With 73% of the influence on the expansion of the universe in this era, dark energy is viewed as the dominant influence on that expansion. The previous history of the big bang is viewed as being at first radiation dominated, then matter dominated, and now having passed into the era where dark energy is the dominant influence. The density parameter for dark energy is defined as:

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}$$

The cosmological constant is the inverse of the square of a length L:

$$L = \sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d = \sqrt{3}L$$

So for the density parameter for dark energy apply:

$$\Omega_\Lambda = \frac{c^2}{R_d^2 H_0^2}$$

The Hubble length or Hubble distance is a unit of distance in cosmology, defined as:

$$L_H = c \cdot H_0^{-1}$$

the speed of light multiplied by the Hubble time. It is equivalent to 4.420 million parsecs or 14.4 billion light years. (The numerical value of the Hubble length in light years is, by definition, equal to that of the Hubble time in years.) The Hubble distance would be the distance between the Earth and the galaxies which are currently receding from us at the speed of light, as can be seen by substituting  $D = c \cdot H_0^{-1}$  into the equation for Hubble's law,  $v = H_0^{-1} \cdot D$ . So for the density parameter for dark energy apply:

$$\Omega_\Lambda = \left( \frac{L_H}{R_d} \right)^2 = \frac{L_H^2}{R_d^2}$$

From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_{\Lambda} = 2 \cdot e^{-1} \quad (325)$$

$$\Omega_{\Lambda} = 0.73576 \quad (326)$$

$$\Omega_{\Lambda} = 73.57\% \quad (327)$$

Series representations for the density parameter for dark energy are:

$$\frac{2}{e} = 2 \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \quad (328)$$

$$\frac{2}{e} = \frac{2}{\sum_{k=0}^{\infty} \frac{1}{k!}} \quad (329)$$

$$\frac{2}{e} = \frac{4}{\sum_{k=0}^{\infty} \frac{1+k}{k!}} \quad (330)$$

$$\frac{2}{e} = \frac{2z}{\sum_{k=0}^{\infty} \frac{-1+k+z}{k!}} \quad (331)$$

$$\frac{2}{e} = \frac{2}{\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}} \quad (332)$$

$$\frac{2}{e} = \frac{2}{\sum_{k=0}^{\infty} \frac{1+2k}{(2k)!}} \quad (333)$$

$$\frac{2}{e} = -\frac{2}{-3 + \sum_{k=0}^{\infty} \frac{1+k}{(3+k)!}} \quad (334)$$

$$\frac{2}{e} = \frac{1}{\sum_{k=0}^{\infty} \frac{1+k}{(1+2k)!}} \quad (335)$$

The pattern of the continued fraction for the density parameter for dark energy is:

[0; 1, 2, 1, 3, 1, 1, 1, 3, 3, 3, 1, 3, 1, 3, 5, 3, 1, 5, 1, 3, 7, 3, 1, 7, 1, 3, 9, 3, 1,  
 9, 1, 3, 11, 3, 1, 11, 1, 3, 13, 3, 1, 13, 1, 3, 15, 3, 1, 15, 1, 3, 17, 3, 1, 17, 1,  
 3, 19, 3, 1, 19, 1, 3, 21, 3, 1, 21, 1, 3, 23, 3, 1, 23, 1, 3, 25, 3, 1, 25, 1, 3, 27,  
 3, 1, 27, 1, 3, 29, 3, 1, 29, 1, 3, 31, 3, 1, 31, 1, 3, 33, 3, 1, 33, 1, 3, 35, 3, 1,  
 35, 1, 3, 37, 3, 1, 37, 1, 3, 39, 3, 1, 39, 1, 3, 41, 3, 1, 41, 1, 3, 43, 3, 1, 43, 1,  
 3, 45, 3, 1, 45, 1, 3, 47, 3, 1, 47, 1, 3, 49, 3, 1, 49, 1, 3, 51, 3, 1, 51, 1, 3, 53,  
 3, 1, 53, 1, 3, 55, 3, 1, 55, 1, 3, 57, 3, 1, 57, 1, 3, 59, 3, 1, 59, 1, 3, 61, 3, 1,  
 61, 1, 3, 63, 3, 1, 63, 1, 3, 65, 3, 1, 65, 1, 3, 67, 3, 1, 67, 1, 3, 69, 3, 1, 69, 1,  
 3, 71, 3, 1, 71, 1, 3, 73, 3, 1, 73, 1, 3, 75, 3, 1, 75, 1, 3, 77, 3, 1, 77, 1, 3, 79,  
 3, 1, 79, 1, 3, 81, 3, 1, 81, 1, 3, 83, 3, 1, 83, 1, 3, 85, 3, 1, 85, 1, 3, 87, 3, 1,  
 87, 1, 3, 89, 3, 1, 89, 1, 3, 91, 3, 1, 91, 1, 3, 93, 3, 1, 93, 1, 3, 95, 3, 1, 95, 1,  
 3, 97, 3, 1, 97, 1, 3, 99, 3, 1, 99, 1, 3, 101, 3, 1, 101, 1, 3, 103, 3, 1, 103, 1,  
 3, 105, 3, 1, 105, 1, 3, 107, 3, 1, 107, 1, 3, 109, 3, 1, 109, 1, 3, 111, 3, 1, 111,  
 1, 3, 113, 3, 1, 113, 1, 3, 115, 3, 1, 115, 1, 3, 117, 3, 1, 117, 1, 3, 119, 3,  
 1, 119, 1, 3, 121, 3, 1, 121, 1, 3, 123, 3, 1, 123, 1, 3, 125, 3, 1, 125, 1, 3, 127,  
 3, 1, 127, 1, 3, 129, 3, 1, 129, 1, 3, 131, 3, 1, 131, 1, 3, 133, 3, 1, 13, ...]

The continued fraction for the density parameter for dark energy is:

$$\begin{array}{r}
 1 \\
 \hline
 1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{3 + \frac{1}{5 + \frac{1}{3 + \frac{1}{1 + \frac{1}{5 + \frac{1}{1 + \frac{1}{3 + \frac{1}{7 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}} \\
 \end{array}$$

So apply:

$$2 \cdot R d^2 = e \cdot L H^2 \tag{336}$$

Also from the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda = 2 \cdot \cos \alpha^{-1} \tag{337}$$

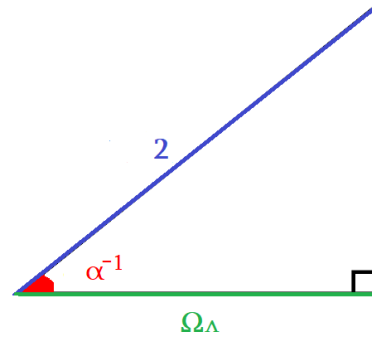
So apply the expression:

$$\cos \alpha^{-1} = \frac{\Omega_\Lambda}{2} \tag{338}$$

So the beautiful equation for the density parameter for dark energy is:

$$\Omega_\Lambda = e^{i/a} + e^{-i/a} \quad (339)$$

The figure 23 shows the geometric representation of the density parameter for dark energy.

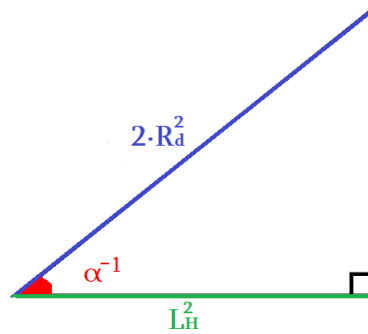


**Figure 23.** Geometric representation of the the density parameter for the dark energy

So apply the expression:

$$\cos \alpha^{-1} = \frac{L_H^2}{2R_d^2} \quad (340)$$

The figure 24 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.



**Figure 24.** Geometric representation of the relationship between the de Sitter radius and the Hubble length

From the dimensionless unification of the fundamental interactions for the density parameter of dark energy apply:

$$\Omega_\Lambda = 2 \cdot 10^{-7} \cdot a_s \cdot a_w^{-1} \quad (341)$$

$$\Omega_\Lambda = 2 \cdot i^{2i} \cdot a_s^{-1} \quad (342)$$

$$\Omega_\Lambda = 2 \cdot e \cdot 10^{-7} \cdot i^{2i} \cdot a_w^{-1} \quad (343)$$

$$\Omega_\Lambda = 2 \cdot 10^{-7} \cdot a_s \cdot a_w^{-1} \quad (344)$$

$$\Omega_\Lambda = 4 \cdot a \cdot a_G^{-1/2} \cdot N_A \quad (345)$$

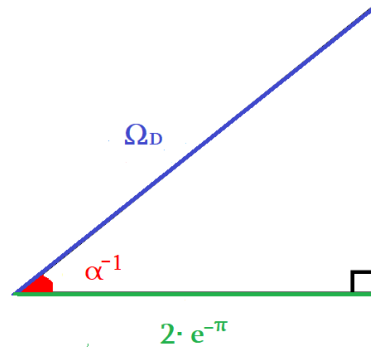
$$\Omega_\Lambda = i^{8i} \cdot a^{-2} \cdot a_s^{-4} \cdot a_G^{-1} \cdot N_A^{-2} \quad (346)$$

$$\Omega_\Lambda = 10^7 \cdot i^{4i} \cdot a^{-1} \cdot a_w^{-1} \cdot a_G^{-1/2} \cdot N_A^{-1} \quad (347)$$

$$\Omega_\Lambda = 8 \cdot 10^7 \cdot N_A^2 \cdot a_w \cdot a^2 \cdot a_G \cdot a_s^{-1} \quad (348)$$

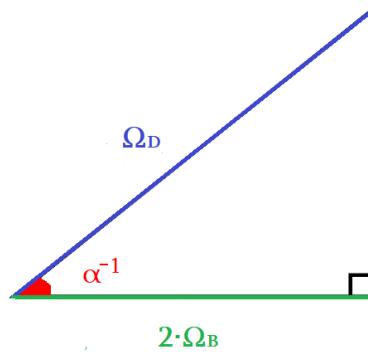
## 22. Solution for the density parameter of dark matter

Dark matter is a hypothetical form of matter thought to account for approximately 85% of the matter in the universe. Dark matter is called "dark" because it does not appear to interact with the electromagnetic field, which means it does not absorb, reflect, or emit electromagnetic radiation and is, therefore, difficult to detect. Various astrophysical observations – including gravitational effects which cannot be explained by currently accepted theories of gravity unless more matter is present than can be seen – imply dark matter's presence. For this reason, most experts think that dark matter is abundant in the universe and has had a strong influence on its structure and evolution. The figure 25 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.



**Figure 25.** Geometric representation of the density parameter of dark matter.

The primary evidence for dark matter comes from calculations showing that many galaxies would behave quite differently if they did not contain a large amount of unseen matter. Some galaxies would not have formed at all and others would not move as they currently do. Other lines of evidence include observations in gravitational lensing and the cosmic microwave background, along with astronomical observations of the observable universe's current structure, the formation and evolution of galaxies, mass location during galactic collisions, and the motion of galaxies within galaxy clusters. The figure 26 shows the geometric representation of the relationship between the density parameter of dark and baryonic matter.



**Figure 26.** Geometric representation of the relationship between the density parameter of dark and baryonic matter.

Current observations suggest that we live in a dark energy dominated Universe with density parameters for dark matter  $\Omega_D=0.23$ . From the dimensionless unification of the fundamental interactions the density parameter for dark matter is:

$$\Omega_D=2 \cdot e^{1-n} \quad (349)$$

$$\Omega_D=2 \cdot e \cdot i^{2i} \quad (350)$$

$$\Omega_D=0.2349 \quad (351)$$

$$\Omega_D=23.49\% \quad (352)$$

Series representations for the density parameter of dark matter are:

$$2e^{1-\pi} = 2 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{1-\pi} \quad (353)$$

$$2e^{1-\pi} = 2e^{1-4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \quad (354)$$

$$2e^{1-\pi} = 2 \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{1-\pi} \quad (355)$$

$$2e^{1-\pi} = 2 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{1-4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \quad (356)$$

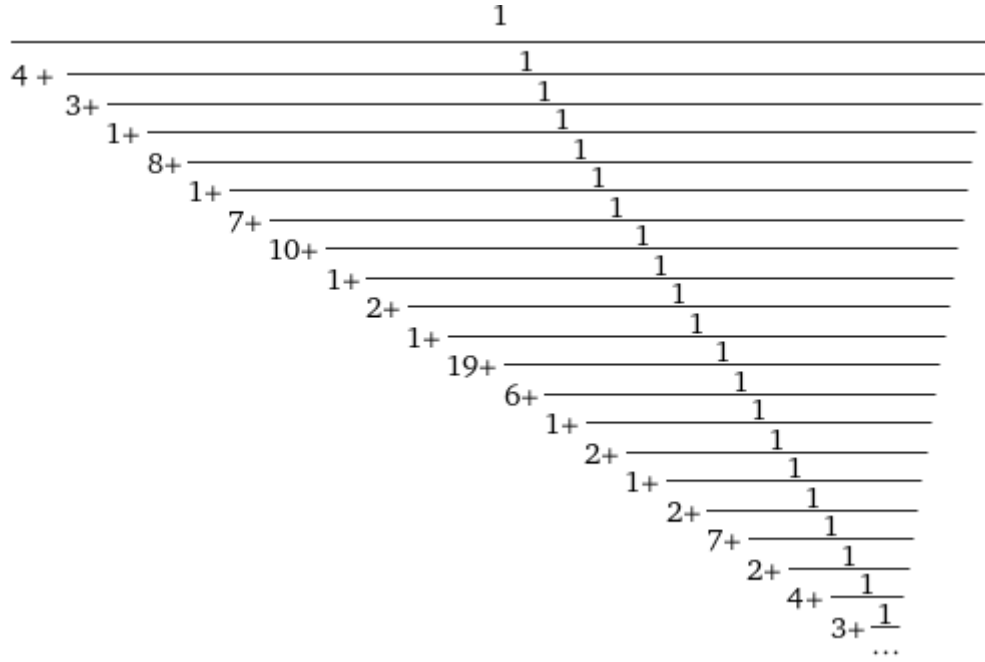
$$2e^{1-\pi} = 2 \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{1-4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \quad (357)$$

The pattern of the continued fraction for the density parameter for the normal baryonic matter is:

[0; 4, 3, 1, 8, 1, 7, 10, 1, 2, 1, 19, 6, 1, 2, 1, 2, 7, 2, 4, 3, 1, 50, 3, 15, 1, 1, 2, 2, 8, 9, 1, 3, 11, 8, 4, 1, 6, 34, 1, 5, 2, 1, 9, 4, 1, 1, 2, 1, 4, 3, 4, 1, 1, 4, 1, 1, 1, 8, 2, 1, 8, 9, 2, 1, 2, 5, 11, 1, 3, 1, 4, 1, 4, 71, 7, 4, 1, 1, 1, 3, 18, 1, 1, 4, 1, 2, 2, 1, 1, 9, 3, 4, 5, 1, 57, 2, 2, 2, 2, 23, 1, 1, 2, 2, 1, 3, 2, 1, 3, 2, 4, 1, 1, 3, 14, 4, 3, 1, 11, 3, 1, 2, 2, 8, 1, 6, 1, 3, 2, 2, 11, 8, 3, 6, 2, 1, 2, 2, 11, 1, 1, 6, 1, 4, 7, 18, 4, 1, 1, 6, 1, 1, 8, 1, 1, 4, 1, 2, 15, 1, 1, 3, 1, 1, 1, 1, 2, 8, 1, 3, 7, 1, 4, 4, 15, 1, 1, 15, 1, 9500, 2, 1, 10, 4, 1, 5, 1, 3, 2, 8, 2, 2, 1, 1, 1, 4, 4, 4, 2, 1, 5, 1, 1, 2, 1, 1, 1, 1, 9, 2, 26, 1, 5, 4, 5, 1, 1, 1, 1, 1, 1, 1, 11, 4, 4, 1, 3, 1, 1, 3, 2, 1, 9, 2, 1, 7, 17, 1, 161, 2, 4, 1, 1, 1, 1, 1, 2, 1, 2, 2, 11, 2, 47, 1, 1, 10, 7, 19, 3, 1, 1, 3, 3, 6, 3, 1, 4, 1, 1, 3, 9, 1, 1, 32, 2, 19, 18, 15, 42, 1, 6, 1, 3, 2, 1, 1, 1, 5, 2, 6, 1, 1, 1, 1, 1, 2, 63, 2, 4, 1381, 2, 6, 1, 5, 2, 1, 2, 2, 1, 8, 1, 8, 2, 1, 4, 2, 3, 1, 8, 9, 1, 2, 26, 14, 2, 1, 2, 5, 2, 292, 3, 1, 2, 36, 1, 2, 1, 1, 5, 11, 1, 1, 5, 6, 19, 67, 597, 4, 2, 1, 50, 53, 1, 1, 1, 2, 1, 1, 1, 22, 1, 4, 5, 236, 1, 2, 28, 4, 2, 1, 27, 9, 3, 1, 1, 2, 1, 2, 1, 4, 2, 2, 3, 1, 7, 1, 1, 2, 1, 3, 1, 1, 1, 1, 2, 2, 3, 1, 1, 3, 4, 2, 3, 1, 3, 1, 4, 1, 1, 1, 3, 1, 2, 1, 1, 27, 9, 16, 10, 3, 1, 1, 4, 1, 2, 2, 1, 25, 1, 2, 8, 3, 14, 2, 1, 2, 1, 2, 3, 2, 1, 3, 1, ...]

The continued fraction for the density parameter for dark energy is:





From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_D = 2 \cdot a_s \tag{358}$$

$$\Omega_D = 2 \cdot 10^7 \cdot e^{-1} \cdot a_w \tag{359}$$

$$\Omega_D = 2 \cdot (i^{2i} \cdot 10^7 \cdot a_w)^{1/2} \tag{360}$$

$$\Omega_D = 4 \cdot i^{2i} \cdot (e^{i/a} + e^{-i/a})^{-1} \tag{361}$$

$$\Omega_D = 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) \tag{362}$$

$$\Omega_D = 4 \cdot 10^7 \cdot a_w \cdot a \cdot a_G^{1/2} \cdot N_A \tag{363}$$

$$\Omega_D = 16 \cdot 10^7 \cdot N_A^2 \cdot a_w \cdot a^2 \cdot a_G \cdot (e^{i/a} + e^{-i/a})^{-1} \tag{364}$$

The relationship between the density parameter of dark matter and baryonic matter is:

$$\Omega_D = 2 \cdot e \cdot \Omega_B \tag{365}$$

The relationship between the density parameter of dark energy, dark matter and baryonic matter is:

$$\Omega_D \cdot \Omega_\Lambda = 4 \cdot \Omega_B \tag{366}$$

### 23. Poincaré dodecahedral space

The shape of the universe, in physical cosmology, is the local and global geometry of the universe. The local features of the geometry of the universe are primarily described by its curvature, whereas the topology of the universe describes general global properties of its shape as a continuous object. The spatial curvature is defined by general relativity, which describes how spacetime is curved due to the effect of gravity. The spatial topology cannot be determined from its curvature, due to the fact that there exist locally indistinguishable spaces that may be endowed with different topological invariants. Cosmologists distinguish between the observable universe and the entire universe, the former being a ball-shaped portion of the latter that can, in principle, be accessible by astronomical observations. Assuming the cosmological principle, the observable universe is similar from all contemporary vantage points, which allows cosmologists to discuss properties of the entire universe with only information from studying

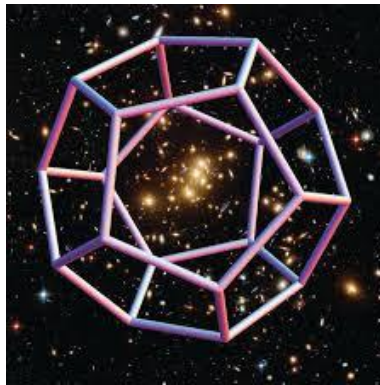
their observable universe. The main discussion in this context is whether the universe is finite, like the observable universe, or infinite. Several potential topological and geometric properties of the universe need to be identified. Its topological characterization remains an open problem. Some of these properties are Boundedness (whether the universe is finite or infinite), Flatness (zero curvature), hyperbolic (negative curvature), or spherical (positive curvature) and Connectivity: how the universe is put together as a manifold, i.e., a simply connected space or a multiply connected space. There are certain logical connections among these properties. For example, a universe with positive curvature is necessarily finite. Although it is usually assumed in the literature that a flat or negatively curved universe is infinite, this need not be the case if the topology is not the trivial one. For example, a multiply connected space may be flat and finite, as illustrated by the three-torus. Yet, in the case of simply connected spaces, flatness implies infinitude. From the dimensionless unification of the fundamental interactions the sum of the contributions to the total density parameter  $\Omega_0$  at the current time is:

$$\Omega_0 = \Omega_B + \Omega_D + \Omega_\Lambda$$

$$\Omega_0 = e^{-n} + 2 \cdot e^{1-n} + 2 \cdot e^{-1} \quad (367)$$

$$\Omega_0 = 1.0139 \quad (368)$$

A positively curved universe is described by elliptic geometry, and can be thought of as a three-dimensional hypersphere, or some other spherical 3-manifold (such as the Poincaré dodecahedral space), all of which are quotients of the 3-sphere. Poincaré dodecahedral space is a positively curved space, colloquially described as "soccer ball-shaped", as it is the quotient of the 3-sphere by the binary icosahedral group, which is very close to icosahedral symmetry, the symmetry of a soccer ball. This was proposed by Jean-Pierre Luminet and colleagues in 2003 and an optimal orientation on the sky for the model was estimated in 2008. When the universe expands sufficiently, the cosmological constant  $\Lambda$  becomes more important than the energy density of matter in determining the fate of the universe. If  $\Lambda > 0$  there will be an approximately exponential expansion. This seems to be happening now in our universe. The figure 27 shows that the shape of the universe is Poincaré dodecahedral space.



**Figure 27.** *The shape of the universe is Poincaré dodecahedral space.*

In [41] J.-P. Luminet, J. Weeks, A. Riazuelo, R. Lehoucq and J.-P. Uzan presents a simple geometrical model of a finite, positively curved space, the Poincaré dodecahedral space – which accounts for WMAP's observations with no fine-tuning required. Circle searching (Cornish, Spergel and Starkman, 1998) may confirm the model's topological predictions, while upcoming Planck Surveyor data may confirm its predicted density of:

$$\Omega_0 = 1.013 > 1$$

If confirmed, the model will answer the ancient question of whether space is finite or infinite, while retaining the standard Friedmann-Lemaître foundation for local physics. The Poincaré dodecahedral space is a dodecahedral block of space with opposite faces abstractly glued together, so objects passing out of the dodecahedron across any face return from the opposite face. Light travels across the faces in the same way, so if we sit inside the dodecahedron and look outward across a face, our line of sight re-enters the dodecahedron from the opposite face. We have the illusion of looking into an adjacent copy of the dodecahedron. If we take the original dodecahedral block of space not as a Euclidean dodecahedron (with edge angles =  $117^\circ$ ) but as a spherical dodecahedron (with edge angles exactly  $120^\circ$ ), then adjacent images of the dodecahedron fit together snugly to tile the hypersphere, analogously to the way adjacent images of spherical pentagons (with perfect  $120^\circ$  angles) fit snugly to tile an ordinary sphere. Thus the Poincaré space is a positively curved space, with a multiply connected topology whose volume is 120 times smaller

than that of the simply connected hypersphere (for a given curvature radius).

The Poincaré dodecahedral space's power spectrum depends strongly on the assumed mass-energy density parameter  $\Omega_0$ . The octopole term ( $\ell=3$ ) matches WMAP's octupole best when  $1.010 < \Omega_0 < 1.014$ . Encouragingly, in the subinterval  $1.012 < \Omega_0 < 1.014$  the quadrupole ( $\ell=2$ ) also matches the WMAP value. More encouragingly still, this subinterval agrees well with observations, falling comfortably within WMAP's best fit range of  $\Omega_0 = 1.02 \pm 0.02$ . The excellent agreement with WMAP's results is all the more striking because the Poincaré dodecahedral space offers no free parameters in its construction. The Poincaré space is rigid, meaning that geometrical considerations require a completely regular dodecahedron. By contrast, a 3-torus, which is nominally made by gluing opposite faces of a cube but may be freely deformed to any parallelepiped, has six degrees of freedom in its geometrical construction. Furthermore, the Poincaré space is globally homogeneous, meaning that its geometry—and therefore its power spectrum - looks statistically the same to all observers within it. By contrast a typical finite space looks different to observers sitting at different locations. Confirmation of a positively curved universe ( $\Omega_0 > 1$ ) would require revisions to current theories of inflation, but the jury is still out on how severe those changes would be. Some researchers argue that positive curvature would not disrupt the overall mechanism and effects of inflation, but only limit the factor by which space expands during the inflationary epoch to about a factor of ten. Others claim that such models require fine-tuning and are less natural than the infinite flat space model. Having accounted for the weak observed quadrupole, the Poincaré dodecahedral space will face two more experimental tests in the next few years:

The Cornish-Spergel-Starkman circles-in-the-sky method predicts temperature correlations along matching circles in small multi connected spaces such as this one. When  $\Omega_0 = 1.013$  the horizon radius is about 0,38 in units of the curvature radius, while the dodecahedron's inradius and outradius are 0.31 and 0.39, respectively, in the same units; as a result, the volume of the physical space is only 83% the volume of the horizon sphere. In this case the horizon sphere self intersects in six pairs of circles of angular radius about  $35^\circ$ , making the dodecahedral space a good candidate for circle detection if technical problems (galactic foreground removal, integrated Sachs-Wolfe effect, Doppler effect of plasma motion) can be overcome. Indeed the Poincaré dodecahedral space makes circle searching easier than in the general case, because the six pairs of matching circles must a priori lie in a symmetrical pattern like the faces of a dodecahedron, thus allowing the searcher to slightly relax the noise tolerances without increasing the danger of a false positive. The Poincaré dodecahedral space predicts  $\Omega_0 = 1.013 > 1$ . The upcoming Planck surveyor data (or possibly even the existing WMAP data in conjunction with other data sets) should determine  $\Omega_0$  to within 1%. Finding  $\Omega_0 < 1.01$  would refute the Poincaré space as a cosmological model, while  $\Omega_0 > 1.01$  would provide strong evidence in its favor. In [42] we proposed a possible solution for the cosmological parameters. The density parameter for normal baryonic matter is:

$$\Omega_B = e^{-n} = i^{2i} = 0.04321 = 4.32\% \quad (369)$$

The density parameter for dark matter is:

$$\Omega_D = 6 \cdot e^{-n} = 6 \cdot i^{2i} = 0.25928 = 25.92\% \quad (370)$$

The density parameter for the dark energy is:

$$\Omega_\Lambda = 17 \cdot e^{-n} = 17 \cdot i^{2i} = 0.73463 = 73.46\% \quad (371)$$

The sum of the density parameter for normal baryonic matter and the density parameter for the dark energy is:

$$\Omega_0 = 24 \cdot e^{-n} = 24 \cdot i^{2i} = 1.03713 \quad (372)$$

In [43] we proposed a possible solution for the Equation of state in cosmology. In cosmology, the equation of state of a perfect fluid is characterized by a dimensionless number  $w$ , equal to the ratio of its pressure  $p$  to its energy density  $\rho$ :

$$w = \frac{p}{\rho}$$

Stable  $w$  of the state equation is the ratio of the pressure exerted by dark energy on the universe to the energy per unit volume. This ratio is  $w = -1$  for a real cosmological constant and is generally different for alternating time changes of vacuum energy forms quintessence. This ratio is often used by scientists. The state equation  $w$  has value  $w = -1.028 \pm 0.032$ . This number means how quickly the dark energy density changes as the universe expands. If  $w = -1$ , the density is strictly constant, if  $w > -1$ , the density decreases, and if  $w < -1$ , the density actually increases with

time. Einstein's cosmological constant is just the idea that there is a fixed minimum energy density everywhere in the universe; this vacuum energy would correspond to  $w=-1$ . It's easy enough to get an energy density that slowly diminishes, with  $w>-1$ , all you need to do is invent some scalar field slowly rolling down a very gentle potential, so that the energy is nearly constant but in fact gradually diminishes. If  $w<-1$ , corresponding to a gradually increasing energy density. It's not what you would typically expect; the expansion of the universe tends to dilute energy, not increase it. So for some time cosmologists who put observational limits on the value of  $w$  would exclude  $w<-1$  by hand. The energy density thus tends to increase, implying  $w<-1$  called "phantom energy" because the Phantom Menace had just come out and also because negative-kinetic-energy fields also appear in the context of quantized gauge theories, where they are called "ghost" fields. If  $w$  is less than  $-1$  and constant, the energy density grows without bound and everything in the universe is ripped to shreds at some finite point in the future. From the dimensionless unification of the fundamental interactions the state equation  $w$  has value:

$$w = -24 \cdot e^{-\pi} = -24 \cdot i^{2i} = -1.037134 \quad (373)$$

For as much as  $w<-1$ , the density actually increases with time. The famous formula  $E=m \cdot c^2$  of Einstein is better replaced by  $E=K \cdot m \cdot c^2$ . In this  $E$  becomes the sum of two types of energy, the measured normal energy density of the universe  $E(O)$  and the sum of the dark energy and the dark matter density of the universe  $E(D)$ . This reveals hitherto unsuspected quantum roots for the equation  $E=m \cdot c^2$ . Einstein's equation  $E=m \cdot c^2$  is actually the sum of two parts of quantum relativity  $E(O)$  from the quantum particle and  $E(D)$  from the quantum wave. From the dimensionless unification of the fundamental interactions for the measurable ordinary energy  $E(O)$  apply:

$$E(O) = i^{2i} \cdot m \cdot c^2$$

Also from the dimensionless unification of the fundamental interactions for the sum of the dark energy with the dark matter density of the universe  $E(D)$  apply:

$$E(D) = 23 \cdot i^{2i} \cdot m \cdot c^2$$

So for the total energy  $E$  apply:

$$E = K \cdot m \cdot c^2$$

$$E = E(O) + E(D)$$

$$E = i^{2i} \cdot m \cdot c^2 + 23 \cdot i^{2i} \cdot m \cdot c^2$$

$$E = (i^{2i} + 23 \cdot i^{2i}) \cdot m \cdot c^2$$

$$E = 24 \cdot i^{2i} \cdot m \cdot c^2 \quad (374)$$

Other forms of the equation are:

$$E = 12 \cdot i^{2i} \cdot m \cdot c^2 + 12 \cdot i^{2i} \cdot m \cdot c^2$$

$$E = 12 \cdot i^{2i} \cdot m \cdot c^2 - i^2 \cdot 12 \cdot i^{2i} \cdot m \cdot c^2$$

$$E = 12 \cdot i^{2i} \cdot m \cdot c^2 - 12 \cdot i^{2i} \cdot m \cdot (i \cdot c)^2$$

$$12 \cdot i^{2i} \cdot m \cdot (i \cdot c)^2 + E = 12 \cdot i^{2i} \cdot m \cdot c^2 \quad (375)$$

## 24. Conclusions

We presented new exact formula for the fine-structure constant  $\alpha$  in terms of the golden angle, the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1} = 360 \cdot \phi^{-2} - 2 \cdot \phi^{-3} + (3 \cdot \phi)^{-5}$$

We propose in a simple and accurate expression for the fine-structure constant  $\alpha$  in terms of the Archimedes constant  $\pi$ :

$$\alpha^{-1} = 2 \cdot 3 \cdot 11 \cdot 41 \cdot 43^{-1} \cdot \pi \cdot \ln 2$$

We propose the exact equivalent mathematical expression for the proton to electron mass ratio using Fibonacci and Lucas numbers:

$$\mu^{32} = \varphi^{-42} \cdot F_5^{160} \cdot L_5^{47} \cdot L_{19}^{40/19}$$

We propose the exact mathematical expressions for the proton to electron mass ratio:

$$\mu^3 = 7^{-1} \cdot 165^3 \cdot \ln^{11} 10$$

$$\mu = 6 \cdot \pi^5 + \pi^{-3} + 2 \cdot \pi^{-6} + 2 \cdot \pi^{-8} + 2 \cdot \pi^{-10} + 2 \cdot \pi^{-13} + \pi^{-15}$$

We present the exact mathematical expressions that connect the proton to electron mass ratio and the fine-structure constant:

$$9 \cdot \mu - 119 \cdot \alpha^{-1} = 5 \cdot (\varphi + 42)$$

$$\mu - 6 \cdot \alpha^{-1} = 360 \cdot \varphi - 165 \cdot \pi + 345 \cdot e + 12$$

$$\mu - 182 \cdot \alpha = 141 \cdot \varphi + 495 \cdot \pi - 66 \cdot e + 231$$

$$\mu - 807 \cdot \alpha = 1205 \cdot \pi - 518 \cdot \varphi - 411 \cdot e$$

The new formula for the Planck length  $l_{pl}$  is:

$$l_{pl} = a \sqrt{a_G} \alpha_0$$

The new formula for the Avogadro's number  $N_A$  is:

$$N_A = \left( 2e\alpha\sqrt{\alpha_G} \right)^{-1}$$

The mathematical formulas that connect dimensionless physical constants are:

$$\alpha_G(p) = \mu^2 \cdot \alpha_G$$

$$\alpha = \mu \cdot N_1 \cdot \alpha_G$$

$$\alpha \cdot \mu = N_1 \cdot \alpha_G(p)$$

$$\alpha^2 = N_1^2 \cdot \alpha_G \cdot \alpha_G(p)$$

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1$$

$$\mu^2 = 4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G(p) \cdot N_A^2$$

$$\mu \cdot N_1 = 4 \cdot e^2 \cdot \alpha^3 \cdot N_A^2$$

$$4 \cdot e^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = 1$$

$$\mu^3 = 4 \cdot e^2 \cdot \alpha \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1$$

$$\mu^2 = 4 \cdot e^2 \cdot \alpha_G \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1^2$$

$$\mu = 4 \cdot e^2 \cdot a \cdot a_G \cdot a_{G(p)} \cdot NA^2 \cdot N_1$$

We reached the conclusion of the simple unification of the nuclear and the atomic physics:

$$10 \cdot (e^{i\mu/a} + e^{-i\mu/a})^{1/2} = 13 \cdot i$$

We presented the recommended value for the strong coupling constant:

$$\alpha_s = \frac{\text{Euler's number}}{\text{Gerford's constant}} = \frac{e}{e^\pi} = e^{1-\pi} = 0,11748..$$

It presented the dimensionless unification of the fundamental interactions. We calculated the unity formulas that connect the coupling constants of the fundamental forces. The dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$e \cdot \alpha_s = 10^7 \cdot \alpha_w$$

$$\alpha_s^2 = i^{2i} \cdot 10^7 \cdot \alpha_w$$

The dimensionless dimensionless unification of the strong nuclear and electromagnetic interactions:

$$\alpha_s \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot i^{2i}$$

The dimensionless dimensionless unification of the weak nuclear and electromagnetic interactions:

$$10^7 \cdot \alpha_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot e \cdot i^{2i}$$

The dimensionless unification of the strong nuclear, the weak nuclear and electromagnetic interactions:

$$10^7 \cdot \alpha_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot \alpha_s$$

The dimensionless unification of the gravitational and the electromagnetic interactions:

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot NA^2 = 1$$

$$16 \cdot a^2 \cdot a_G \cdot NA^2 = (e^{i/a} + e^{-i/a})^2$$

The dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions:

$$4 \cdot \alpha_s^2 \cdot a^2 \cdot a_G \cdot NA^2 = i^{4i}$$

$$a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot \alpha_s^4 \cdot a_G \cdot NA^2 = i^{8i}$$

The dimensionless unification of of the weak nuclear, the gravitational and the electromagnetic interactions:

$$4 \cdot 10^{14} \cdot \alpha_w^2 \cdot a^2 \cdot a_G \cdot NA^2 = i^{4i} \cdot e^2$$

$$10^{14} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot \alpha_w^2 \cdot a_G \cdot NA^2 = i^{8i}$$

The dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions:

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot a^2 \cdot a_G \cdot NA^2$$

$$8 \cdot 10^7 \cdot NA^2 \cdot \alpha_w \cdot a^2 \cdot a_G = \alpha_s \cdot (e^{i/a} + e^{-i/a})$$

From these expressions resulting the unity formulas that connects the strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$ , the proton to electron mass ratio  $\mu$ , the fine-structure constant  $a$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton, the Avogadro's number  $NA$ , the gravitational coupling constant  $a_G$  of

the electron, the gravitational coupling constant of the proton  $\alpha_{G(p)}$ , the strong coupling constant  $\alpha_s$  and the weak coupling constant  $\alpha_w$ :

$$\begin{aligned}\alpha_s^2 &= 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot a^2 \cdot \alpha_G \cdot N_A^2 \\ \mu^2 \cdot \alpha_s^2 &= 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot a^2 \cdot \alpha_{G(p)} \cdot N_A^2 \\ \mu \cdot N_1 \cdot \alpha_s^2 &= 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot a^3 \cdot N_A^2 \\ \alpha_s^2 &= 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot a \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 \\ \mu^3 \cdot \alpha_s^2 &= 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot a \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1 \\ \mu \cdot \alpha_s &= 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha_G \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1^2 \\ \mu \cdot \alpha_s^2 &= 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot a \cdot \alpha_G \cdot \alpha_{G(p)} \cdot N_A^2 \cdot N_1\end{aligned}$$

We found the formula for the Gravitational constant:

$$G = (2e\alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

$$G = i^{4i} (2\alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

$$G = i^{4i} e^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

$$G = \alpha_s^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

It presented the theoretical value of the Gravitational constant  $G = 6.67448 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$ . This value is very close to the 2018 CODATA recommended value of gravitational constant and two experimental measurements from a research group announced new measurements based on torsion balances. They ended up measuring  $6.674184 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$  and  $6.674484 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$  - of swinging and angular acceleration methods, respectively.

We calculated the expression that connects the gravitational fine structure constant with the four coupling constants:

$$\alpha_g^2 = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3$$

Perhaps the gravitational fine structure constant is the coupling constant for the fifth force. It presented that the gravitational fine structure constant is a simple analogy between atomic physics and cosmology. Resulting the dimensionless unification of the atomic physics and the cosmology:

$$|pl|^2 \cdot \Lambda = (2 \cdot e \cdot a^2 \cdot N_A)^{-6}$$

$$|pl|^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot \alpha_s \cdot a^2 \cdot N_A)^{-6}$$

$$|pl|^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot a^3 \cdot N_A)^{-6}$$

$$e^6 \cdot \alpha_s^6 \cdot a^6 \cdot |pl|^2 \cdot \Lambda = 10^{42} \cdot \alpha_G^3 \cdot \alpha_w^6$$

$$\alpha_s^{12} \cdot a^6 \cdot |pl|^2 \cdot \Lambda = 10^{42} \cdot i^{12i} \cdot \alpha_G^3 \cdot \alpha_w^6$$

For the cosmological constant equals:

$$\Lambda = \left(2e\alpha^2 N_A\right)^{-6} \frac{c^3}{G\hbar}$$

$$\Lambda = i^{12i} \left(2\alpha_s a^2 N_A\right)^{-6} \frac{c^3}{G\hbar}$$

$$\Lambda = i^{12i} e^6 \left(2 \cdot 10^7 \alpha_w a^3 N_A\right)^{-6} \frac{c^3}{G\hbar}$$

$$\Lambda = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 \alpha^2}\right)^3 \frac{c^3}{G\hbar}$$

$$\Lambda = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4}\right)^3 \frac{c^3}{G\hbar}$$

The Equation of the Universe is:

$$\frac{\Lambda G\hbar}{c^3} = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4}\right)^3$$

We presented the law of the gravitational fine-structure constant  $\alpha_g$  followed by ratios of maximum and minimum theoretical values for natural quantities. Length  $l$ , time  $t$ , speed  $v$  and temperature  $T$  have the same min/max ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{max}} = \frac{t_{min}}{t_{max}} = \frac{v_{min}}{v_{max}} = \frac{T_{min}}{T_{max}}$$

Energy  $E$ , mass  $M$ , action  $A$ , momentum  $P$  and entropy  $S$  have another min/max ratio, which is the square of  $\alpha_g$ :

$$\alpha_g^2 = \frac{E_{min}}{E_{max}} = \frac{M_{min}}{M_{max}} = \frac{A_{min}}{A_{max}} = \frac{P_{min}}{P_{max}} = \frac{S_{min}}{S_{max}}$$

Force  $F$  has min/max ratio which is  $\alpha_g^4$ :

$$\alpha_g^4 = \frac{F_{min}}{F_{max}}$$

Mass density has min/max ratio which is  $\alpha_g^5$ :

$$\alpha_g^5 = \frac{\rho_{min}}{\rho_{max}}$$

Perhaps for the minimum distance  $l_{min}$  apply:

$$l_{min} = 2 \cdot e \cdot |p|$$



The maximum distance  $l_{\max}$  is:

$$l_{\max} = LH = c \cdot H_0^{-1} = \alpha_g^{-1} \cdot l_{\min}$$

For the minimum mass  $M_{\min}$  apply:

$$M_{\min} = \frac{m_{pl}^2}{M_{\max}} = \alpha_g m_{pl} = \frac{\alpha_G}{\alpha^3} m_e = \frac{\sqrt[3]{\alpha_g^2}}{\alpha} m_e$$

From the dimensionless unification of the fundamental interactions we discover a new simple Large Number Hypothesis which calculates the Mass, the Age and the Radius of the universe. The expressions for the mass of the observable universe are:

$$M_U = \alpha^{-1} \cdot \alpha_g^{-4/3} \cdot m_e = \alpha^3 \cdot \alpha_g^{-2} \cdot m_e = (2 \cdot e \cdot \alpha^2 \cdot N_A)^2 \cdot N_1 \cdot m_p = \mu \cdot \alpha \cdot N_1^2 \cdot m_p = 1.153482 \times 10^{53} \text{ kg}$$

The expressions who calculate the number of protons in the observable universe are:

$$N_{Edd} = \frac{M_U}{m_p} = \mu \alpha N_1^2 = \frac{N_1}{\alpha_g^{2/3}} = (2e\alpha^2 N_A)^2 N_1 = \left(\frac{r_e}{l_{pl}}\right)^2 N_1 = 6.9 \times 10^{79}$$

The diameter of the observable universe will be calculated to be equal to the ratio of electric force to gravitational force between electron and proton on the reduced Compton wavelength of the electron:

$$2 \cdot R_U = N_1 \cdot \lambda_c$$

The expressions for the radius of the observable universe are:

$$R_U = \frac{\alpha N_1}{2} \alpha_0 = \frac{N_1}{2\alpha} r_e = \frac{1}{2\mu\alpha_G} r_e = \frac{m_{pl}^2 r_e}{2m_e m_p} = \frac{\hbar c r_e}{2G m_e m_p} = \frac{\alpha \hbar}{2G m_e^2 m_p}$$

We Found the value of the radius of the universe  $R_U = 4.38 \times 10^{26}$  m. The expressions for the radius of the observable universe are:

$$T_U = \frac{R_U}{c} = \frac{N_1 r_e}{2\alpha c} = \frac{r_e}{2\mu\alpha_G c} = \frac{\alpha N_1 \alpha_0}{2c} = \frac{\alpha \hbar}{2c G m_e^2 m_p} = \frac{\hbar r_e}{2G m_e m_p}$$

For the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{2e^2 \varphi^{-5}}{3^3 \pi \varphi^5} \times 10^{-120}$$

Perhaps for the minimum time  $t_{\min}$  apply:

$$t_{\min} = 2 \cdot e \cdot t_{pl}$$

We proved the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions propose a possible solution for the density parameters of baryonic matter, dark matter and dark energy:

$$\Omega_B = e^{-\pi} = i^{2i} = 0.0432 = 4.32\%$$

$$\Omega_\Lambda = 2 \cdot e^{-1} = 0.7357 = 73.57\%$$

$$\Omega_D = 2 \cdot e^{1-n} = 2 \cdot e \cdot i^{2i} = 0.2349 = 23.49\%$$

The sum of the contributions to the total density parameter at the current time is  $\Omega_0 = 1.0139$ . It is surprising that Plato used a dodecahedron as the quintessence to describe the cosmos. A positively curved universe is described by elliptic geometry, and can be thought of as a three-dimensional hypersphere, or some other spherical 3-manifold, such as the Poincaré dodecahedral space, all of which are quotients of the 3-sphere. These results prove that the weather space is finite. The state equation  $w$  has value:

$$w = -24 \cdot e^{-n} = -24 \cdot i^{2i} = -1.037134$$

For as much as  $w < -1$ , the density actually increases with time [44].

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