

A New Model Explaining the Mass Difference Between Electron and Positron

Stefan Israelsson

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Abstract

We will in this document assume that a charged particle (electron) is built up by (similar to super string theory in a sense) of constellation of loops that has a very peculiar form of interaction that is as simple as one can possible think of. That this model has a chance of explaining the normal analytical treatment of charges in our macroscopic world is a bit of a challenge to explain. We will assume that there is a limit for how much energy density we can have and they will differ slightly between positive and negative charge meaning in the end a difference between particle mass and anti particle mass. Especially we reproduce the result that the electron and positron differs and the resulting mass of the positron is correct within measurement errors. We will also show that a stable system consists of two almost similar loops or helical paths that have opposite sign. We will show that the positive and negative charge is constant and the same. We will show how mass can be calculated and how we can calculate angular momentum which makes it possible to deduce information on this model. We will also be able to conclude why $\alpha \approx 1/137$ and why this is so and why not exactly $1/137$ and why the specific value is 137. We will show why \hbar is a fundamental constant. We also indicate how to calculate Planck's constant from knowing that α is approximately $1/137$.

1 The main model assumptions

We will base our analysis of a basic object that is a stream of charge that has no mass and move at the speed of light. It will also have the property that it only interacts if two infinitesimal line segments are parallel and directed in the same direction and if we draw the tangent lines these elements are located at the smallest distance to each other. We will assume for the specific case the basic Coulomb's law apply for these special segments. We will implicitly assume that each particle is composed of objects that is a basic object that is overlaid in all possible directions and that for two particles, there is always a a matched pair that can express the normal electrostatic law so that we can reproduce the usual macroscopic interaction. We will assume that each loop has a fixed amount of charge so that as we enlarge the loop, it will be less dense. In a sense it is a closed system That can scale. We will assume that there is a energy density limit, one for each sign of the charge that is almost the same.

2 Lorentz invariance

for two segments to be interacting they must be parallel. And in their reference frame. The energy is,

$$q^2/r$$

As the speed of reference frame (defined through a limiting procedure) is the same independent of if we move the system or not defining the interaction in this frame is hence Lorentz invariant.

The observant reader would realize that there are some issues with these objects. We will consider the streams as a limit of a sequence of objects of the kind,

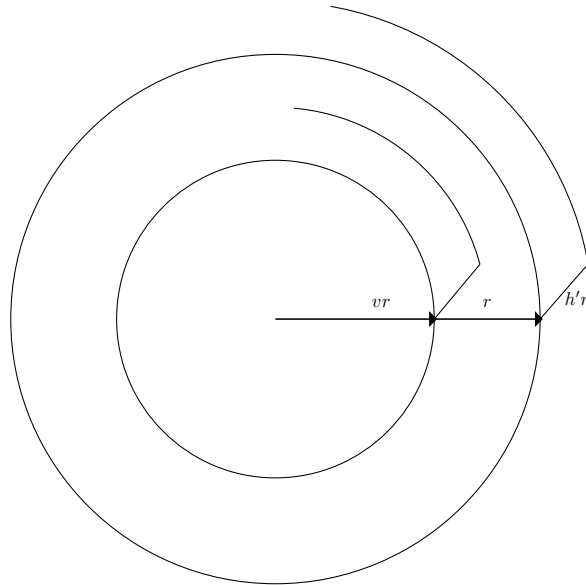
$$\lim_{n \rightarrow \infty, v \rightarrow c} \sum_{i=0}^n e_a \frac{\delta_{r_i^a(t)=(i/n+vt)\hat{x}}}{\sqrt{n}}$$

Where we have n equally spaced normal mass less electron's evenly distributed on $[0,1]$ at $t = 0$ and they move with the speed of light along the x-axis. We will assume here that nearby electrons are not interaction, but in stead if we take a parallel stream,

$$\lim_{n \rightarrow \infty, v \rightarrow c} \sum_{i=0}^n e_a \frac{\delta_{r_i^b(t)=(i/n+vt)\hat{x}+h\hat{y}}}{\sqrt{n}}$$

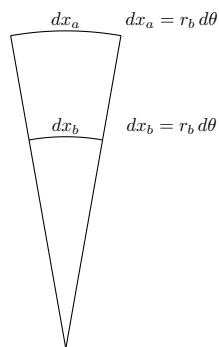
As we will close them into loops, We will assume that each current loop has the same number of charges and we will assume that this is an invariant of the world. Note that as we increase the velocity they will contract and in order to get a nonzero charge density we need to spread the charges out more and more in their reference frame. Hence when we define the interaction in that frame, any two parallel segments need to be perfectly matched. Also in that frame the next charge is infinitely large distance away so there is no self interaction and if they are offset and not at the closest distance they will be infinitely far away. Also if the streams interact the probability of two streams hitting each other is zero in a sense so we could hand wave away that part as well (e.g. they sync so that they do not interact). So will assume that only r_i^a, r_i^b are interacting as normal electric charges and the rest does not. And we will demand the streams to be parallel and likewise directed and also located so that it is in a sense closes as possible if we consider the tangent lines of the streams. Hence in any geometrical constellation one need to search for parallel tangents that are not offsets e.g. if you draw the tangents they need to be parallel and the pairing need to be at the closest distance. As the interaction is done in the rest reference system, one can always consider only the electrostatic interaction when exploring a certain geometrical setup, which is similar to quantum electrodynamics that does also not have internal magnetic terms. As we defined the energy in the rest frame we are free to also put a limit there of the energy density. One for each charge.

3 The Loop



Consider 2 concentric loops stacked above each other forming a double cylindrical entity. the charge in the inner loop is positive and the charge in the outer loop is negative (we will consider a reversed version later). We will attache a constant charge density of the streams will be constant, e_a for negative and a_b for positive. We will be a bit sloppy in the mathematical rigor and consider that all interaction terms is a limit in L_2 of their combination.

Loops of different charge sign will attract if they are concentric and have very little attraction if not centered so we expect this selection of geometry to be stable. let $r_b = vr$ be the inner radius and $r_a = r$ be the outer radius, then we will assume a scaling so that the effective charge density in the outer cylinder to have the radian contribution constraint assuming e_a to be the charge density at the outer radius and e_b the inner radius charge density, hence the constraint is,



$$er_a d\theta = (e_a r_a - e_b r_b) d\theta = (e_a - e_b v) r_a d\theta = (e_a - ve_b) r d\theta.$$

Or,

$$e = e_a - ve_b$$

We shall consider scaling properties and hence it is natural to consider $e = ue_a$. The dual setup is considering,

$$-e = be_a - e_b.$$

For this case we will considering $e = ue_b$ as a scaling. When we multiply two of these streams we will consider the “square root” of a delta measure that is made stringent by the limiting argument above. and hence will we use

$$e_a e_b r_a r_b d\theta, \quad e_a^2 r_a^2 d\theta, \quad e_b^2 r_b^2 d\theta \quad (1)$$

For the energy relation below. Also when we want to study the “energy density” we will mean then that we need to study this effect on the paired r_i^a, r_i^b . Then summing the effect on the unit length will lead to taking these values.

$$e_a e_a, \quad e_a^2, \quad e_b^2. \quad (2)$$

To see that we will assume a normalized condition on the energy density at the singly scaled pairing

You may say, this does not cut it. this makes the integrated e vary when you vary the radius, and we will see that this changes. Now this is a correct observation and in a sense it works out that way. But we will stack multiple such loops and form a torus with radius R . If you examine two of these torus-es they only interact significantly if they is located in 2 parallel planes. And there they interact only on a circle of the radius, say R and along this axis we will scale the charge when we consider the torus as a surface else we will consider it as stacked circles the effective charge on all those concentric loops will be e . This is a bit tricky to understand but it is as it is in this model and the task is not to dismiss it, but see if there is any explainable power in this model. Not to make a water tight theory as we first need to pass the first floor of the theoretical castle.

Anyway, the charge condition is a scale invariant condition in order for the final charge to be correct as argues. The attractive energy per radian of the loops are (where we use the special Coulomb’s law and the observation 1)

$$V_l d\theta = k \frac{e_a e_b r_a r_b}{r_a - r_b} d\theta = k \frac{e_a e_b v r}{1 - v} d\theta$$

Note here that $|1 - v|$ will be a term that has a scaling w.r.t. $2\pi R$ included, but this is hidden in the analysis below. Similarly as we stack loops (or helix) right on top of each other with a pitch $\langle = hr$ we will see that the forces on one segments in one direction is

$$F = k \frac{e_*^2 r_*}{(hr)^2} (1 + 2^{-2} + 3^{-2} + \dots) = \zeta(2) \frac{ke_*^2}{(h'r)^2}$$

Now this is a simplification e.g. if we connect it and turn it into a torus or helix, so we will just assume that this part transforms as ζ_h/h' , where we punt for now what ζ_h is. Hence if we consider the force on both sides we get the energy by integrating $h'r$ to,

$$V_{h,*} d\theta = 2\zeta_h \frac{ke_*^2 r_*^2}{h'r} d\theta.$$

So the total energy for one loop is,

$$E = (V_{h,1} + V_{h,2} + 2V_l)2\pi = 2\pi kr \left(\frac{2\zeta_h}{h'} (e_a^2 + e_b^2 v^2) - 2 \frac{e_a e_b v}{|1 - v|} \right).$$

Using $e = e_a - ve_b$ in this expression for the energy, we note that we can search to find the stationary point varying $x = ve_b$ and keeping the rest constant, while also introducing the obvious A, B to this leads to,

$$A(2e + 2x + 2x) - B(e + 2x) = 0.$$

Or,

$$(2A - B)(2x + e) = 0.$$

So,

$$x = ve_b = -e/2$$

This means that ve_v tend to go to zero unless,

$$2A - B = 0 \Leftrightarrow \frac{2\zeta_h}{h'} = \frac{1}{|1-v|}. \quad (3)$$

For which energy wise it can vary freely! To simplify the expression for the energy, let first $e_b = ue_a$. Let $w = uv$ and note $e = e_a(1-w)$. Then,

$$E = 2\pi kr e_a^2 \left(\frac{2\zeta_h}{h'} (1+w^2) - 2 \frac{w}{|1-v|} \right).$$

Or using 3,

$$E = 2\pi kr e_a^2 \frac{2\zeta_h}{h'} (1+w^2 - 2w).$$

Complete the square and we get,

$$E = 2\pi kr \frac{2\zeta(2)}{h'} (e_a(1-w))^2 = 4\pi kr \frac{\zeta(2)}{h'} e^2. \quad (4)$$

Note that this condition is invariant of how we combine the charges. To evaluate the energy density and apply limits on them as the system want to scale down in order to minimize energy. Assume the condition 2 for evaluating this limit. The charge densities at the loop a is,

$$\rho_a = ke_a^2 \left(\frac{2\zeta_h}{h'} - \frac{u}{1-v} \right) \frac{1}{r}.$$

Or using 3,

$$\rho_a = ke_a^2 \frac{2\zeta_h}{h'} (1-u) \frac{1}{r}. \quad (5)$$

The density at loop b is,

$$\rho_b = ke_a^2 \left(\frac{2\zeta_h}{h'} u^2 - \frac{u}{1-v} \right) \frac{1}{r}.$$

Or again using 3,

$$\rho_a = ke_a^2 \frac{2\zeta_h}{h'} (u^2 - u) \frac{1}{r}. \quad (6)$$

Hence if these two densities are at a positive and negative limit, we need to have (using 5 and 6)

$$\rho_a = c_a, \quad \rho_b = -c_b$$

To simplify the analysis of this, use 3 and take,

$$C_* = c_* * C = c_* \frac{h'}{2\zeta_h ke_a^2} = c_* \frac{|1-v|}{ke_a^2}.$$

Then,

$$\frac{|1-u|}{r} = C_a = c_a C, \quad (7)$$

$$u \frac{|1-u|}{r} = C_b = c_b C. \quad (8)$$

Note that this result is independent how we combined the charges to e . Hence dividing 8 with 7,

$$e_b/e_a = u = C_b/C_a = c_b/c_a. \quad (9)$$

The constraint 7 is,

$$\frac{|1-u|}{r} = c_a C = c_a \frac{|1-v|}{k e_a^2}.$$

Or,

$$\frac{e_a^2(|1-u|)}{r} = c_a \frac{|1-v|}{k}.$$

Hence,

$$|1-v| = \frac{k e_a^2 |1-u|}{r c_a}. \quad (10)$$

in the dual we will consider here is simple taking the anti particle of the system. We will mark this with $\mathcal{D}(\cdot)$ and we get $\mathcal{D}(u) = c_a/c_b$ and $\mathcal{D}(e_a) = e_b$ and for this case,

$$|1 - \mathcal{D}(v)| = \frac{k e_b^2 |1 - \mathcal{D}(u)|}{\mathcal{D}(r) c_b} = \frac{k e_b e_a |1 - u|}{\mathcal{D}(r) c_b} = \frac{k e_a e_a |1 - u|}{\mathcal{D}(r) c_a} = |1 - v| \frac{r}{\mathcal{D}(r)}$$

We can also reformulate the condition for e , using 10 as,

$$e = e_a(1 - uv) = e_a(1 - u) + e_a u(1 - v) = e_a - e_b + \frac{k u e_a^2}{r c_a} (e_a - e_b) = \Delta \left(1 + \frac{k e_b e_a}{r c_a} \right) \quad (11)$$

where we used $\Delta = e_a - e_b$. The dual expression is then,

$$\mathcal{D}(e) = -\Delta \left(1 + \frac{k e_b e_a}{\mathcal{D}(r) c_b} \right). \quad (12)$$

So in order for $e = -e'$ we need,

$$\mathcal{D}(r) = r/u. \quad (13)$$

Hence

$$(1 - \mathcal{D}(v)) = -(1 - v)u$$

And also $\mathcal{D}(h') = h'u$. We can solve for r in 11,

$$r = \frac{k e_b e_a}{c_a \left(\frac{e}{\Delta} - 1 \right)}. \quad (14)$$

But not only this, we also note that starting with,

$$e = e_a |1 - u| + e_a u |1 - v|$$

And using 7,

$$e = D \frac{r c_a}{e_a} + e_a u |1 - v|,$$

with,

$$D = \frac{h'}{2\zeta_h k}.$$

Assuming h', v, u constant we can minimize the energy by minimizing e to get,

$$e_a = \sqrt{\frac{Drc_a}{u|1-v|}} = \sqrt{\frac{\frac{h'}{2\zeta_h}rc_a}{ku|1-v|}} = \sqrt{\frac{rc_a}{ku}}. \quad (15)$$

Hence from 7,

$$\frac{rc_a}{ku}|1-u| = rc_a \frac{|1-v|}{k}.$$

Or,

$$|1-u| = u|1-v| \quad (16)$$

Hence,

$$e = 2e_a|1-u| = 2\sqrt{\frac{rc_a}{ku}}|1-u|. \quad (17)$$

Also,

$$e = 2\frac{e_a}{c_a}|c_a - c_b|. \quad (18)$$

The constraint 3 implies,

$$h' = |1-v|2\zeta_h = \frac{|1-u|2\zeta_h}{u} \quad (19)$$

And $\mathcal{D}(h') = h'u$. Now the actual pitch is hr is then invariant. So the argument for equal charge would that the most energetically favorable action when a negative and positive charge form is an alignment and hence equal pitch, hence the negative and positive charge is constrained to be the same and as we see below this also imply that the \hbar must be the same. Anyway 19 and squaring 17,

$$e^2 = 4\frac{rc_a}{ku}|1-u|^2. \quad (20)$$

Special relativity means that we can deduce the masses per loop from 4 as,

$$E = mc^2 = 4\pi kr \frac{\zeta_h}{h'} e^2 = 2\pi \frac{kr}{|1-v|} e^2$$

Using 16, with this, we get,

$$mc^2 = \frac{2\pi krue^2}{|1-u|}.$$

So,

$$m = \eta \frac{2\pi krue^2}{|1-u|c^2} \quad (21)$$

(we will discuss η soon). And hence the dual relationship,

$$\mathcal{D}(m) = \frac{m}{u^2}. \quad (22)$$

Note that the unit is kg/m with η currently an unknown unit. However the loop is like a delta measure and you can see it as the result of taking the limit with a scaled mass and thinner small cylindrical shell. Hence,

$$\eta = 1 \quad [m].$$

We will need that to not confuse the astute reader that checks the calculation by examining the units. Hence m will have the unit [kg].

3.1 Stacking into a torus

Previously we was working with a system where we stacked loops on top of each other to form a cylindrical structure. Now instead we connect all loops so they form a torus. When we do this we will consider the pitch defined by,

$$h'r = h2\pi Rr.$$

E.g.

$$h' = 2\pi Rh.$$

From this we get the dual condition $R' = R$. But the stacking of the loop is although possible mathematically, hard to motivate for a stable structure. However if we transform the loops to helical path's along the helix with a velocity v we have indeed produced a system that stabilizes as each path is non interacting. In the reference frame of the system, where we move with the particles along the big circle we will still make a loop and the helix will interact repulsively with a similar part one pitch away. As the number of pitches is the same, e.g. hr we realizes that we have two radius's of the torus. One in the system of the lab R and one in the rest frame R_0 . and we have,

$$R = \frac{R_0}{\gamma}$$

We will evaluate the interaction in the rest frame. So we stack n' of them and therefore,

$$nh'r = 2\pi R.$$

Or,

$$hr = \frac{1}{n}$$

As the distance between the paths are different we realize that this can't be exactly try as we have a contraction in the closest $R - r$ distance, hence we actually have,

$$nh'r = 2\pi(R - r) = \frac{2\pi R}{f},$$

with,

$$f = \frac{1}{1 - r/R}.$$

Hence,

$$hrf = \frac{1}{n}. \tag{23}$$

The attractive energy will be as before as that is independent of any movements of the loops orientation. The repulsion will however be active on only on two parts of the loops where they interact. The energy will be the mean which is the same as using the center (R) distance. However, the energy density that we use need to be analyzed at the $R - r$ distance where it is the most extreme. we will do so by doing the transformation $c_* \rightarrow c_*/f$. In this new parameterisation. The unit of h is here $[1/m]$.

3.2 Scaling

Consider scaling. As the number of loops per torus is fixed, e.g. n' . Then we know that only the loops will need to scale. hence we will get from 10, 14 and 4,

$$e \rightarrow xe \tag{24}$$

$$r \rightarrow r/x \tag{25}$$

$$v \rightarrow v, \tag{26}$$

$$u \rightarrow u, \tag{27}$$

$$h \rightarrow hx, \tag{28}$$

$$rh \rightarrow rh, \tag{29}$$

$$E \rightarrow xE, \tag{30}$$

$$m \rightarrow xm. \tag{31}$$

This means that in order to maintain the same scaling we must have,

$$R \rightarrow R/x, \tag{32}$$

$$R_0 \rightarrow R_0/x, \tag{33}$$

$$r/R \rightarrow R/r, \tag{34}$$

$$f \rightarrow f, \tag{35}$$

$$rhf \rightarrow rhf. \tag{36}$$

Now as the helix will stretch with the R we see that,

$$E_{tot} \rightarrow E_{tot}, \tag{37}$$

$$m_{tot} \rightarrow m_{tot}, \tag{38}$$

$$e_{tot} \rightarrow e_{tot}. \tag{39}$$

3.3 Angular momentum

The per loop angular momentum is,

$$l = m\gamma(v_h)v_hR = mv_hR_0.$$

The question is how v_h scales. If the length of the helix scales as R and hence the time it takes to move one turn scales as R . But as the number so turns along the helix is invariant, we find that the pitch distance also scales as R which leaves the velocity invariant. Hence v_h is invariant of the scaling and hence the total angular moment which is n' such copies is invariant of the scale.

$$v_h \rightarrow v_h, \tag{40}$$

$$l \rightarrow l, \tag{41}$$

$$l_{tot} \rightarrow l_{tot}. \tag{42}$$

If we let the length of the helix as L then v_h satisfy (in the rest frame),

$$\frac{v_h}{c} = \frac{2\pi R}{L} = \frac{2\pi R_0/\gamma}{L_0\gamma} = \frac{2\pi R_0}{L_0}.$$

Anyhow if we factor in the need to remove from the outer loop the same quantity from the inner loop we get,

$$l = mv_h R_0 |1 - v|$$

$$l_{tot} = n'l = n'mv_h R_0 |1 - v| = n'\eta \frac{2\pi k r u e^2}{|1 - u|c^2} \frac{c2\pi R_0}{L_0} R_0 |1 - v| = \eta \frac{A_0 R_0}{L_0} \frac{kue^2}{|1 - u|c} |1 - v|,$$

with A_0 being the torus area e.g,

$$A_0 = 2\pi r 2\pi R_0.$$

Using 16 we find,

$$l_{tot} = \eta \frac{A_0 R_0}{L_0} \frac{ke^2}{c}.$$

In order for the charge to be properly (hopefully) managed we need,

$$\frac{A_0 2\pi R_0}{L_0} = n$$

E.g. we need to scale down the area in order to compensate for the extra space the helical path takes. As we have $n = \frac{1}{hr}$ (forgetting about f) identical pitches and hence we conclude that (using 16),

$$\hbar = \int l_{tot} d\theta = \eta \frac{A_0 2\pi R_0}{L_0} \frac{kue^2}{|1 - u|c} |1 - v| = \eta n' \frac{ke^2}{c} = \eta \frac{ke^2}{hrc}. \quad (43)$$

Note that we here consider one helix turn per pitch, but, as discussed above, this can also be any integer number of pitches hence we actually have the Bohr condition of angular momentum.

$$l_{tot} = n\hbar$$

We can solve for hr

$$hr = \eta \frac{ke^2}{\hbar c} = \alpha \approx \frac{1}{137} = \frac{1}{n}$$

Which indicate why Wolfgang Pauli's quest to search for why $1/\alpha$ was almost a natural number (137) may have a partial answer.

3.4 Defining the zeta factor

Consider N charges evenly distributed on a unit circle. Let's study the forces on one single charge. then they are locates on $e^{2\pi k/N}$, $k = 0, \dots, N - 1$. The force at $k = 0$ is then. Now we would not like to cancel any of their contributions to action at hence we get

$$V(N) = \sum_{k=1}^N \frac{h'r}{R} \frac{1}{|e^{2\pi ik/N} - 1|}.$$

Now,

$$|e^{2\pi ik/N} - 1|^2 = (e^{2\pi ik/N} - 1)(e^{-2\pi ik/N} - 1) = 2 - 2\cos(2\pi k/N).$$

Hence we are left with,

$$V(N) = \frac{1}{\sqrt{2}} \sum_{k=1}^N \frac{h'r}{R} \frac{1}{\sqrt{1 - \cos(2\pi k/N)}}.$$

Using the trigonometric identity for the double cosine's,

$$1 - \cos(2\pi k/N) = 1 - \cos^2(\pi k/N) + \sin^2(\pi k/N) = 2 \sin^2(\pi k/N).$$

Hence

$$V(N) = \frac{1}{2} \sum_{k=1}^N \frac{h'r}{R} \frac{1}{\sin(\pi k/N)}.$$

So we will have,

$$\zeta_h(N) = N \sum_{k=1}^N \frac{\pi \alpha}{\sin(\pi k/N)} \quad (44)$$

Now as

$$\sin(\pi k/N) < \pi k/N$$

then, including that we have N charges, we get

$$\zeta_h(N) > N \sum_{k=1}^N \frac{\pi \alpha}{\pi k/N} > N^2 \alpha (\ln(N)) \quad (45)$$

hence

$$\zeta_h(N) > N^2 \alpha \ln(N) \quad (46)$$

A direct calculation with $N = 137$ gives,

$$\zeta_h(N) \approx N1382.5\alpha. \quad (47)$$

3.5 Numerology

The following expression is a quite good equation for the fine structure constant,

$$\frac{\alpha}{1 + \frac{\alpha}{1 - (2\pi - 1)^2}} = \frac{1}{137}$$

We can explore this further and find another expression,

$$\frac{\alpha}{1 + \frac{\alpha}{1 - \left(1 - \frac{2\pi}{1 - \frac{4\pi\alpha}{1 + \frac{2\pi}{1 - \frac{4\pi\alpha}{1 - 2\pi/(1+2\alpha)}}}}\right)^2}} = \frac{1}{137}$$

We could postulate from this,

$$\frac{\alpha}{1 - \frac{\alpha}{x^2 - 1}} = \frac{1}{137}.$$

With x satisfying,

$$x = 1 - \frac{2\pi}{1 - \frac{4\pi\alpha}{1 + \frac{2\pi}{1 - \frac{4\pi\alpha}{x}}}}$$

Of cause this is very numerological and are simply fined tuned with the help of trial and error. Can we motivate this? Well, as the density is higher closer to the center it is there we find the limiting energy density and this motivates,

$$hrf = \alpha f = \frac{1}{137},$$

with

$$f = \frac{R}{R-r} = \frac{1}{1-\frac{r}{R}}.$$

So,

$$\frac{\alpha}{1-\frac{r}{R}} = \frac{1}{137}.$$

Using the found expression we could match this with the found expression,

$$\frac{r}{R} \approx \frac{\alpha}{(2\pi-1)^2-1}. \quad (48)$$

But this is the same as,

$$\frac{r}{R} = \frac{rh}{hR} = \frac{\alpha}{hR}.$$

Hence we can identify,

$$x^2 = hR + 1.$$

Now $x^2 = 28.7778$ means

$$hR = 27.7778$$

Or

$$\alpha R = 27.7778r.$$

Or

$$R = 3807r.$$

Consider $L_h = rh2\pi R_0$ the pitch distance and $L_r = 2\pi r$ the distance of on helix turn in in the loop direction. Then

$$v_h/c = \frac{L_h}{L} = \frac{L_h}{\sqrt{L_h^2 + L_r^2}} = \frac{rh2\pi R_0}{\sqrt{(2\pi r)^2 + (rh2\pi R_0)^2}}.$$

Rearenging we find

$$v_r/c = \frac{1}{\sqrt{1 + (1/(hR_0))^2}}.$$

Solving for hR_0 we get,

$$hR_0 = \sqrt{\gamma^2 - 1}.$$

Or using $hr = \alpha$,

$$\alpha \frac{r}{R_0} = \sqrt{\gamma^2 - 1}.$$

Now as $R = R_0\gamma$ we have,

$$hR = \gamma\sqrt{\gamma^2 - 1}.$$

And by using $hR \approx 27.7778$,

$$\gamma \approx 5.3 \quad (49)$$

As α is invariant of the duality, we conclude that as $\mathcal{D}(r) = r/u$, we have

$$\mathcal{D}(R) = R/u.$$

As $\mathcal{D}(R_0) = R_0$ we get $\mathcal{D}(\gamma) = \gamma u$ and hence,

$$\mathcal{D}(\gamma m) = \frac{\gamma m}{u}. \quad (50)$$

3.6 On n'

Form 3 we see that h' satisfies,

$$h'r = hr2\pi R = \alpha 2\pi R = \frac{|1-u|2\zeta_h}{u} \approx \frac{|1-u|}{u} n\alpha \ln(n).$$

Or

$$R = Cn \ln(n),$$

Hence,

$$R \approx C137 \ln(137).$$

Which means,

$$r = \frac{r}{R} Cn \ln(n).$$

But we also have from 15,

$$r = Ce_a^2/n^2$$

Equating to,

$$e_a^2 = C' \frac{r}{R} n \ln(n) = n \ln(n)$$

But as we have an approximate postulated expression from 48,

$$\frac{r}{R} \approx \frac{\alpha}{1 - (2\pi - 1)^2}.$$

So,

$$e_a^2 \approx C'' \alpha n \ln(n) \approx C'' \ln(n)$$

This indicate why we have $n = 137$.

3.7 On mass

If we consider the total mass scale invariant we get (using n copies and 21),

$$m_e = \gamma nm = \eta n \gamma \frac{2\pi k r u e^2}{|1-u|c^2} = \eta n \gamma \frac{\alpha 2\pi k u e^2}{h|1-u|c^2}. \quad (51)$$

Using the electron mass equation 51 and the condition for h , 19,

$$m_e = \eta n \gamma \alpha \frac{2\pi k u e^2}{\frac{|1-u|2\zeta_h}{u} |1-u|c^2} = \eta n \alpha \gamma \frac{\pi k e^2}{h \zeta_h c^2} \left(\frac{u}{|1-u|} \right)^2 = \eta n \gamma \alpha^2 \frac{\pi \hbar}{\zeta_h c} \left(\frac{u}{|1-u|} \right)^2, \quad (52)$$

plugging in 47, we find,

$$m_e \approx \eta \gamma \frac{\alpha}{1382.5} \frac{\pi \hbar}{c} \left(\frac{u}{|1-u|} \right)^2.$$

Taking (49), $\gamma = 5.3$ we find,

$$|1-u| \approx \epsilon = 5.83 \cdot 10^{-9}$$

Also note that we know that $u = 1 \pm \epsilon$. Hence from 50,

$$m_{positron} = \mathcal{D}(m) = m/u = m/(1 \pm \epsilon) \approx m(1 \pm \epsilon)$$

Taking the lower value of this we get the positron mass,

$$m_{positron} = 0.5109989530 \text{ [Mev}/c^2].$$

Meassured is approximately, (hard to find a good reference for this value)

$$m_{positron}^* = 0.5109989519 \text{ [Mev}/c^2].$$

3.7.1 A note on the experimental value used

These measurements have not been reproduced and originates from, the most precise measurement of the positron mass to date that was conducted by the researchers at the Harvard-MIT Center for Ultracold Atoms in collaboration with other institutions. In 2018, they used a technique called Penning trap mass spectrometry to measure the positron mass with high precision. The experiment involved trapping individual positrons and antiprotons in a magnetic field and measuring their oscillation frequencies. By comparing the oscillation frequencies of the trapped particles, the researchers were able to determine the mass ratio of the positron to the antiproton with exceptional accuracy. The result yielded a positron mass value of $(1.000\,000\,038\,06 \pm 0.000\,000\,000\,38)$ times the mass of an electron. Currently as we need to wait for more confirmatins of the experiments from other groups, this information is not considered acknowledged. But it is interesting that our model are able to conform to the result of that study.

3.8 An addition theorem of charge streams and a fundamental scaling property

On the other hand if we overlay many of these geometrical structure and span a spherical symmetric object, the only interacting will be done with parallel torus structures if they are separated (far away) and there will be one such pair for every direction. And hence the symmetric usual Coulomb law naturally applies. Also as the hole constructions is defined as a limit between of proper EM theoretical objects, we will understand that the magnetic field will properly appear when we change reference frame. So in all we have managed to reproduced our macroscopic understanding from these small building blocks.

Consider what will happen when we overlay two loops at a certain point. To maintain the overall limit balance we need $c_a \rightarrow xc_a$, $c_b \rightarrow xc_b$. To leave charge invariant we then need rc_a, rc_b to be invariant as seen by 7 and 8 to be constant. Thus mean $r \rightarrow r/x$. This imply $h \rightarrow hx$ and $v \rightarrow v$ are invariant as R and the pitch is the same. Also,

$$E \rightarrow Ex, \quad \text{or,} \quad m \rightarrow mx$$

Hence in the end, $l \rightarrow lx$ for the individual systems. This means that we can average naturally the loops in the sphere and if add only loops pointing towards the upper half uniformly we realize that by vector addition, the overall angular momentum becomes the famous,

$$l_z = \hbar/2.$$