# Part Application of Relativity Lead to Errors & Paradox in Doppler Effect and Incorrect Explanation for Muons Reaching Earth

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## Abstract:

Special Relativity envisages neither time dilation nor length contraction in isolation, independent of each other; but instead, the two occur simultaneously, in any transformation, with values that together have to mandatorily conform to the Lorentz Transformation Condition. However, if one of them is taken as zero to start with theoretically, the transformation obviously reduces to the other parameter (distance or time) with the non-zero value. Even though such assumed cases are nearly impossible in practice, their results are orchestrated as general laws of nature that exist independently, without any riders. The oft-quoted phenomena like 'time ticking slower' and 'length contraction' in moving frames are examples. These are quoted as if these happen always, without any conditions, which is incorrect. The reason would be clearer in the discussions ahead.

To render the classical Doppler Effect with Relativity, time dilation is applied to the time period of the electromagnetic waves in the receiver's frame, ignoring the inseparable length contraction of its wavelength. Further, since Relativity does not discriminate, in respect of relative motion, between the source and the receiver, one could consider the source as moving to apply the time dilation on the emitted wave. In such a case, the universally professed redshift change to blueshift, thus creating a paradox.

Similarly, in case of muons, the time dilation is applied to its mean lifespan to explain as to how these, with an extremely short lifespan of about 2.2 microseconds, are able to cover disproportionately large distances in Earth's atmosphere. Here again, no heed is paid to the inseparable length transformation occurring in almost the same proportion. Explanations are also presented from the perspective of muons experiencing drastic length contraction of Earth's atmosphere, while ignoring the inseparable contraction of all the timespans in Earth's frame. When Relativity is applied in entirety, the current explanations fail to sustain in face of the mandatory Lorentz Transformation Condition.

**Key Words**: Special Relativity, Relativistic Doppler Effect, Muons, Astrophysics, Cosmology

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## Introduction -The Mandatory Requirement:

The transformations of time and length are inseparable in Relativity, and one cannot occur without the other. This mandatory requirement is stipulated by the Lorentz Transformation Condition as follows [1][2].

$$c^2t'^2 - x'^2 = c^2t^2 - x^2 = Constant$$

where one of the frames – non-primed or primed – is stationary and the other one is moving with respect to the other with a uniform velocity along $x$-axis.

**The Condition also represents the invariance of Minkowski's spacetime hyperbola in all inertial frames.**

On rearranging, the Condition becomes

$$c^2t'^2 - c^2t^2 = x'^2 - x^2 \; . \; . \; . \; . \; . \; (1)$$

The Condition stipulates that, on transformation from one frame to the other, if time of an event changes in a particular direction, (+)ve or (-)ve, the corresponding distance of the event has also to change in the same direction. The relation between the magnitudes of their changes is also decided by the Condition.

That leads to the conclusion that if time of a frame appears increased in another frame, all the lengths of the former would necessarily have to appear longer in almost (not exactly) the same proportion in the latter. If, however, one is dealing with transformation of distance and time of an electromagnetic signal/ray, the ratio of the times, in the two frames, has to be exactly equal to that of distances, as $x = ct$ and $x' = ct'$.

The two cases are discussed below.

## I. Relativistic Doppler Effect:

Let us take the case of the source (S) of an electromagnetic wave and its receiver (R) moving away from each other with a relative velocity $v$, along the line joining them.

Let the frequencies of the wave in their frames respectively be $f_s$ And $f_r$.

The classical Doppler Effect is expressed by the following relation

$$f_r = \frac{f_s}{1+\beta}$$

where $\beta = \frac{v}{c}$

Now, the current practice to render the classical effect with Relativity is to apply time dilation to the time period of the wave in the moving frame.

If the receiver is considered moving, the time period of the received wave has to be multiplied by a factor of $\sqrt{1-\beta^2}$ and therefore, the frequency $f_r$ to be multiplied by a factor of $1/\sqrt{1-\beta^2}$. On doing so and rearranging, one gets

$$\frac{f_s}{f_r} = \sqrt{\frac{1+\beta}{1-\beta}}$$

Thus one gets the formula extensively used in astronomy, which predicts redshift observed by receivers on the Earth.

However, according to Relativity, the source could also be considered as moving, with the receiver considered as stationary. When one takes to this option, the time period of the emitted wave is to be multiplied by a factor of $\sqrt{1-\beta^2}$ and therefore, the frequency $f_s$ to be multiplied by a factor of $1/\sqrt{1-\beta^2}$. On doing so, one gets the following relation

$$\frac{f_s}{f_r} = (1+\beta)\sqrt{1-\beta^2}$$

The result predicts a redshift for $\beta < 0.6185$, no shift for $\beta = 0.6185$ and a blueshift for $\beta > 0.6185$.

This is widely different from what is achieved by taking to the previous option and thus, it leads to a paradox.

It highlights the pitfalls of motivated application of only parts of Relativity to physical phenomena, in disregard of the Condition at (1) above.

**Application in Entirety Leaves no Room for Paradox but Changes Predictions:**

Application of Relativity in its entirety, in fact, dispenses altogether with working out of the classical Doppler Effect. Further, the Lorentz transformation of a single electromagnetic wavelength and its time period in either of the frames lead to the same result, which is shown below.

Let $\lambda_s$, $t_s$ and $f_s$ be the wavelength, the time period and the frequency of the wave emitted from the source. Similarly, let the corresponding parameters of the wave received by the receiver be $\lambda_r$, $t_r$ and $f_r$ respectively.

**Imp:** Before proceeding further, it is reminded that the structure of the Lorentz transformation requires [1] that the stationary frame holds the distance and time parameters ***to-be-transformed*** (connected by expressions on RHS), and the moving frame holds the corresponding ***transformed*** parameters (appearing as sole parameters on LHS).

The two equivalent setups, based on the principle of reciprocity of velocity stipulated by Relativity, are taken below, to work out the change in frequency of emitted/received wave on account of the relative motion between the source and the observer.

## A. Source Stationary and Receiver Moving:

Let an electromagnetic wave crest incoming from the source (stationary frame) coincide with the origin of the moving frame i.e. receiver, at the instant the latter starts moving with a velocity $v$ in the direction of motion of the wave. The assumption is in accordance with the setup of increasing distance between the source and the receiver.

The origins of time and distance of the stationary frame (source) are also considered coinciding with those of the moving frame (receiver) at start.

In the stationary frame (source), after a time equal to $t_s$, the wave would have moved by a distance equal to $\lambda_s$ towards the receiver and the receiver would have moved by a distance $vt_s$ in the same direction, from the origin.

On applying the Lorentz transformation on a single wavelength and its time period in the stationary frame (source) to get their values in the moving frame (receiver), one gets following relations

$$\left.\begin{aligned} \lambda_r &= \gamma(\lambda_s - vt_s) \\ t_r &= \gamma\left(t_s - \frac{v\,\lambda_s}{c^2}\right) \end{aligned}\right\}$$

Or,

$$\left.\begin{aligned} \lambda_r &= \frac{1}{\sqrt{1-\beta^2}}\left(\lambda_s - \frac{v}{c}\lambda_s\right) \\ t_r &= \frac{1}{\sqrt{1-\beta^2}}\left(t_s - \frac{v}{c}t_s\right) \end{aligned}\right\}$$

Or,

$$\left.\begin{aligned} \lambda_r &= \sqrt{\frac{1-\beta}{1+\beta}}\lambda_s \\ t_r &= \sqrt{\frac{1-\beta}{1+\beta}}\,t_s \end{aligned}\right\}$$

The relations obtained represent blueshift in the receiver's frame, which is just the inverse of what is currently being worked out by applying only the time dilation in the receiver's frame.

It is, however, noteworthy that the soundness of the results can be verified by dividing the first relation by the second, as follows.

$$\frac{\lambda_r}{t_r} = \frac{\lambda_s}{t_s} = c$$

which is in accordance with the second postulate of Relativity i.e. constancy of light speed in all inertial frames. The speed of the electromagnetic wave is the same at the receiver as at the source.

**B. Receiver Stationary and Source Moving:**

At the origin of the moving frame (source), let a wave crest be emitted towards the receiver, at the instant the former starts moving with a velocity $v$ in the direction opposite to that of the emitted wave. The assumption is once again in accordance with the setup of increasing distance between the source and the receiver.

The origins of time and distance of the stationary frame (receiver) are also considered coinciding with those of the moving frame (source) at start.

In the stationary frame (receiver), after a time equal to $t_r$, the wave would have moved by a distance equal to $\lambda_r$ towards the receiver and the source would have moved by a distance $vt_r$ in opposite direction, from the origin.

On applying the Lorentz transformation on a single wavelength and its time period in the stationary frame (receiver) to get their values in the moving frame (source), one gets following relations, with $v$ getting replaced by $-v$ in view of the source moving opposite to the direction of the wave.

$$\left.\begin{aligned} \lambda_s &= \gamma(\lambda_r + vt_r) \\ t_s &= \gamma\left(t_r + \frac{v\,\lambda_r}{c^2}\right) \end{aligned}\right\}$$

Or,

$$\left.\begin{aligned} \lambda_s &= \frac{1}{\sqrt{1-\beta^2}}\left(\lambda_r + \frac{v}{c}\lambda_r\right) \\ t_s &= \frac{1}{\sqrt{1-\beta^2}}\left(t_r + \frac{v}{c}t_r\right) \end{aligned}\right\}$$

Or,

$$\left.\begin{aligned} \lambda_s &= \sqrt{\frac{1+\beta}{1-\beta}}\,\lambda_r \\ t_s &= \sqrt{\frac{1+\beta}{1-\beta}}\,t_r \end{aligned}\right\}$$

Or,

$$\left.\begin{aligned} \lambda_r &= \sqrt{\frac{1-\beta}{1+\beta}}\lambda_s \\ t_r &= \sqrt{\frac{1-\beta}{1+\beta}}\, t_s \end{aligned}\right\}$$

which is the same as obtained previously in sub-section 'A'.

Thus the results are the same, no matter which one of the two – the source or the receiver - is considered moving. This leaves no room for any paradox.

It is also conceivable that **there cannot be any classical Doppler Effect**, in view of the speed of light being the same with respect to the source as well as to the receiver.

The above results show that when Relativity is applied in its entirety, the Relativistic Doppler Effect predicts a blueshift, on the earth, of the electromagnetic waves received from receding stars and galaxies. This is just the opposite of what is professed. The phenomena of redshift on the Earth, therefore, requires explanations from factors other than Relativity, and calls for identification of agents leading to loss of energy of the incoming photons.

**Imp Note:**

The above results also go against the derivation of redshift, as shown by Einstein in his 1905 paper, Section 7 titled "Theory of Doppler's Principle and of Aberration". On this, it is clarified that Einstein's working out of angular frequency $\omega'$ in the moving (observer's) frame as being $\beta(1-v/c)$ times that in the stationary frame (receding light emitting body) $\omega$ is obviously incorrect, as this holds true for the time period of the wave and not for its frequency. Therefore, as the angular frequency is inversely proportional to the time period, its correct value in the moving frame is $(1/\beta(1-v/c))$ times that in the stationary frame. This correction, by way of reversal of the ratio, changes the redshift to blueshift.

Thus Einstein's working out of redshift by Relativity is also found to be incorrect, and therefore, the view that reasons for redshift should be searched from elsewhere gains ground.

## II. The Enigma of Muons Reaching Earth:

The enigma is: a muon, with a mean lifespan of 2.2 microseconds and travelling with a speed as high as $0.999c$ can travel only a distance of $0.999 \times 300000 \times 2.2 \times 10^{-6} = 0.659\ \text{km} = 659\ \text{m}$ before decaying. With this magnitude of travel capacity, even those generated at the heights of 15 km are detected on the Earth's surface.

Such phenomena are explained by invoking Relativity in part i.e. either time dilation or length contraction, in contradiction with the mandatory requirements of Relativity highlighted in the opening section titled "Introduction -The Mandatory Requirement".

The **current explanations** from both the approaches are first discussed below, followed by the exercise with complete (not part) application of Relativity.

**A. Current Explanation With Time Dilation approach:**

Muons live for 2.2 microseconds of their time, which observers on the Earth would measure $2.2/\sqrt{1-v^2/c^2}$ microseconds. With the figures assumed above, this works out to 49.2 microseconds, which translates into a travel distance of $0.999 \times 300000 \times 49.2 \times 10^{-6} = 14.75$ km. This is quite close to 15 km. Thus it makes arrival of muons possible on the Earth.

**B. Current Explanation With Length Contraction approach:**

For the muon, in its own frame, the entire length of Earth's atmosphere, say 15 km, is treated as moving upwards, and therefore, it appears to it as contracted by the factor i.e. $\sqrt{1-v^2/c^2}$. With the figures assumed above, this leads to a contracted length of only 0.671 km or 671 m, which is easily covered by the muon within its lifespan of 2.2 microseconds.

**Correction:**

The above explanations fail to sustain, when **corrected below** by applying together both the inseparable effects of time dilation and length contraction.

Invoking the Condition at (1) above, let the primed parameters be of the Earth and the non-primed parameters be of a muon.

**A1. Time Dilation Approach Supplemented with Length Transformation:**

The Condition with relation (1) may be rearranged as follows.

$$x'^2 = c^2t'^2 - c^2t^2 + x^2$$

On substituting the expression of $t'$ from sub-section 'A' above, one gets

$$x'^2 = c^2\left(\frac{1}{1-\beta^2} - 1\right)t^2 + x^2 = c^2t^2\frac{\beta^2}{1-\beta^2} + x^2$$

Therefore, the expression for length in the Earth's frame becomes as follows.

$$x' = \sqrt{\frac{\beta^2}{1-\beta^2}c^2t^2 + x^2}$$

On changing the units of $c$ in a more convenient form, we have $c = 300$ m/microsec.

On substituting $t = 2.2$ microseconds and $\beta = 0.999$, as taken in sub-section 'A' above, the expression for $x'$ becomes as follows, with unit of distances in meters.

$$x' = \sqrt{\frac{(0.999\times300\times2.2)^2}{1-(0.999)^2} + x^2} = \sqrt{217473354 + x^2} \text{ meters}$$

If $x$ is taken as the muon's proper length in meters, the term $x^2$ may be neglected in the above expression, and its length in the Earth frame, $x'$ works out to $14747\ m \approx 14.75\ km$.

It shows that a negligible length of muon should also be seen, in the Earth's frame, as lengthened to $\approx 14.75$ km, along with the lengthening of its lifespan to 49.2 microseconds.

This obviously is impossible and thus, Relativity fails to explain the phenomena.

**B1. Length Contraction Approach Supplemented with Time Transformation:**

The Condition with relation (1) may be rearranged as follows.

$$c^2t^2 = c^2t'^2 - x'^2 + x^2$$

On substituting the expression of $x$ from sub-section 'B' above, one gets

$$t = \sqrt{t'^2 + \frac{-x'^2 + x'^2(1-\beta^2)}{c^2}} = \sqrt{t'^2 - \beta^2\frac{x'^2}{c^2}}$$

On changing the units of $c$ in a more convenient form, we have $c = 0.3$ km/microsec.

On substituting $x' = 15$ km and $\beta = 0.999$, as taken in sub-section 'B' above, the expression for $t$ becomes as follows, with unit of time in microseconds.

$$t = \sqrt{t'^2 - \left(\frac{0.999\times15}{0.3}\right)^2} = \sqrt{t'^2 - 2495} \text{ microseconds}$$

From the above, it is obvious that $t = 0$ for $t'^2 = 2495$

Or, $t = 0$ for $t' = \sqrt{2495} = 49.95$ microseconds.

It means that for a time span of 49.95 microseconds in the Earth's frame, the corresponding time in the muon's frame is zero, and for all lesser timespans, the corresponding time in the muon's frame become imaginary.

Its implications may be understood as follows.

Let us first assume that numerous stationary or slow moving muons are also getting generated continuously on the Earth's surface (say in a laboratory), in addition to those in the atmosphere. The muons generated in the Earth's atmosphere at a height of 15 km are able to reach the surface because the 15 km length contracts for them to 0.671 km, as the atmosphere is treated as moving upwards with respect to the muon with the same velocity. The muons would be watching this upward motion for their proper lifespan of 2.2 microseconds.

Quite obviously, at the instant when the Earth's surface and the atmospheric muons meet, all the muons generated in the laboratory in the last 2.2 microseconds (Earth's time) should be available to meet with the atmospheric muons.

Now, think of the situation from the muon's reference frame. While the atmosphere's length of 15 km reduces to 0.671 km for them, even the lifespans of the muons generated on the Earth in the last 2.2 microseconds (Earth's time) would get transformed, in accordance with the Condition stated by (1). However, as shown above, the transformation of the lifespans would lead to imaginary values, as the lifespans of 2.2 microseconds are too less than the threshold value of 49.95 microseconds, as calculated above.

Thus, the explanation offered by length contraction of atmosphere also fails to sustain in face of the Condition.

## Conclusion:

The part applications of Relativity, which may well be termed as a motivated steps, pervade everywhere in physics. The parts like time dilation and length contraction are incorrectly treated as independent and separate phenomena under Relativity, which are mutually exclusive of each other.

The Relativity does not envisage any such separation, but physicists have been applying it only by parts to force-explain numerous phenomena of great importance.

It has been shown in the article that the part application, by way of only time dilation on Doppler Effect, has been projecting redshift from receding stars and galaxies, but the full application projects blueshift. The part application, though incorrect, gratifies us, as the projections somewhat go in line with the observations. The misplaced complacency, however, holds us from exploring other factors/theories that go behind so much of mystery still prevailing in the domain of cosmology.

The later part of the paper has also shown that the explanations of extremely short-lived muons reaching the Earth's surface after crossing a 15 km long atmosphere, by part application of Relativity (either time dilation or length contraction), fail to stand up to the mandatory requirements of the Lorentz Transformation Condition. Such incorrect explanations have been holding us, similar to the previous case, from search of other explanations from quantum theory or a new particle theory.

## References:


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