

# The Dimensionless Fractal Universe

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## Abstract

In this paper from the Dimensionless unification of the fundamental interactions we will calculate the unity formulas that connect the coupling constants of the fundamental forces. We will present the Dimensionless unification of atomic physics and cosmology. We will prove that the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions will propose a possible solution for the density parameter of baryonic matter, dark matter and dark energy.

## Keywords

Hubble constant , Dimensionless unification of the fundamental interactions , Fine-structure constant , Proton to electron mass ratio , Dimensionless physical constants , Coupling constant , Gravitational constant , Avogadro's number , Fundamental Interactions , Cosmological parameters , Cosmological constant , Poincaré dodecahedral space

## 1. Introduction

Euler's identity is considered to be an exemplar of mathematical beauty as it shows a profound connection between the most fundamental numbers in mathematics:

$$e^{i\pi} + 1 = 0$$

All five of the numbers play important and repetitive roles in mathematics. The expression who connects the six basic mathematical constants, the number 0, the number 1, the golden ratio  $\phi$ , the Archimedes constant  $\pi$ , the Euler's number  $e$  and the imaginary unit  $i$  is:

$$e^{\frac{i\pi}{1+\phi}} + e^{\frac{-i\pi}{1+\phi}} + e^{\frac{i\pi}{\phi}} + e^{\frac{-i\pi}{\phi}} = 0$$

In [1] we presented exact and approximate expressions between the Archimedes constant  $\pi$ , the golden ratio  $\phi$ , the Euler's number  $e$  and the imaginary number  $i$ . New interpretation and very accurate values of the fine-structure constant has been discovered in terms of the Archimedes constant and the golden ratio. We propose in [2] , [3] and [4] the exact formula for the fine-structure constant  $\alpha$  with the golden angle, the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1} = 360 \cdot \phi^{-2} - 2 \cdot \phi^{-3} + (3 \cdot \phi)^{-5} = 137.035999164... \quad (1)$$

Also we propose in [4] , [5] and [6] a simple and accurate expression for the fine-structure constant  $\alpha$  in terms of the Archimedes constant  $\pi$ :

$$\alpha^{-1} = 2 \cdot 3 \cdot 11 \cdot 41 \cdot 43^{-1} \cdot \pi \cdot \ln 2 = 137.035999078... \quad (2)$$

We propose in [7] the exact mathematical expressions for the proton to electron mass ratio  $\mu$ :

$$7 \cdot \mu^3 = 165^3 \cdot \ln^{11} 10 \Rightarrow \mu = 1836.15267392... \quad (3)$$

Also in [7] was presented the exact mathematical expressions that connects the proton to electron mass ratio  $\mu$  and the fine-structure constant  $\alpha$ :

$$9 \cdot \mu - 119 \cdot \alpha^{-1} = 5 \cdot (\phi + 42) \quad (4)$$

In [8] was presented the unity formula that connects the fine-structure constant and the proton to electron mass ratio. It was explained that  $\mu \cdot \alpha^{-1}$  is one of the roots of the following trigonometric equation:

$$2 \cdot 10^2 \cdot \cos(\mu \cdot \alpha^{-1}) + 13^2 = 0 \quad (5)$$

The exponential form of this equation is:

$$10^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) + 13^2 = 0 \quad (6)$$

Also this unity formula can also be written in the form:

$$10 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha})^{1/2} = 13 \cdot i \quad (7)$$

It was presented in [9] the mathematical formulas that connects the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton, the Avogadro's number  $N_A$ , the gravitational coupling constant  $\alpha_G$  of the electron and the gravitational coupling constant of the proton  $\alpha_G(p)$ :

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (8)$$

$$\mu^2 = 4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G(p) \cdot N_A^2 \quad (9)$$

$$\mu \cdot N_1 = 4 \cdot e^2 \cdot \alpha^3 \cdot N_A^2 \quad (10)$$

$$4 \cdot e^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = 1 \quad (11)$$

$$\mu^3 = 4 \cdot e^2 \cdot \alpha \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1 \quad (12)$$

$$\mu^2 = 4 \cdot e^2 \cdot \alpha_G \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1^2 \quad (13)$$

$$\mu = 4 \cdot e^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_G(p) \cdot N_A^2 \cdot N_1 \quad (14)$$

In [10] we presented the recommended value for the strong coupling constant:

$$\alpha_s = \frac{\text{Euler's number}}{\text{Gerford's constant}} = \frac{e}{e^\pi} = e^{1-\pi} = 0,11748.. \quad (15)$$

This value is the current world average value for the coupling evaluated at the Z-boson mass scale. In the papers [11], [12], [13] and [14] was presented the unification of the fundamental interactions. We found the unity formulas that connect the strong coupling constant  $\alpha_s$  and the weak coupling constant  $\alpha_w$ . We reached the conclusion of the dimensionless unification of the strong nuclear and the weak nuclear interactions:

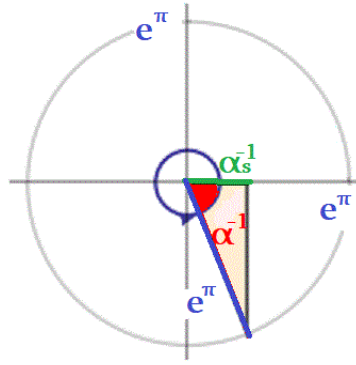
$$e \cdot \alpha_s = 10^7 \cdot \alpha_w \quad (16)$$

$$\alpha_s^2 = i^{2i} \cdot 10^7 \cdot \alpha_w \quad (17)$$

Resulting the unity formulas that connects the strong coupling constant  $\alpha_s$  and the fine-structure constant  $\alpha$ :

$$\cos \alpha^{-1} = \frac{\alpha_s^{-1}}{e^\pi} \quad (18)$$

The figure 1 below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $e^\pi$ .



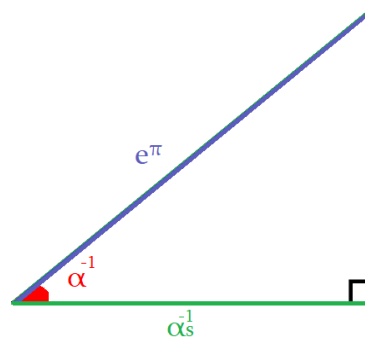
**Figure 1.** The angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $e^\pi$ .

We reached the conclusion of the dimensionless unification of the strong nuclear and the electromagnetic interactions:

$$e^\pi \cdot a_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \quad (19)$$

$$a_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot i^{2i} \quad (20)$$

The figure 2 below shows the geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.

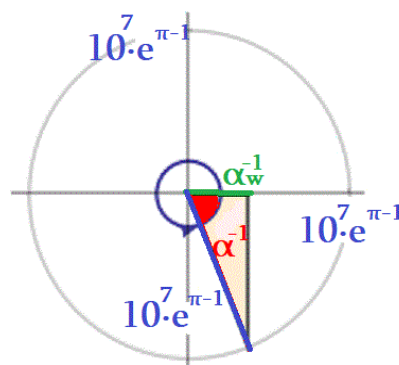


**Figure 2.** Geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.

The electroweak theory, in physics, is the theory that describes both the electromagnetic force and the weak force. We reached the conclusion of the dimensionless unification of the weak nuclear and the electromagnetic forces:

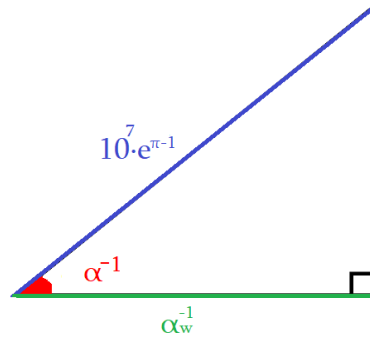
$$10^7 \cdot a_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot e \cdot i^{2i} \quad (21)$$

The figure 3 below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7 \cdot e^{\pi-1}$ .



**Figure 3.** The angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7 \cdot e^{\pi-1}$ .

The figure 4 below shows the geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions.



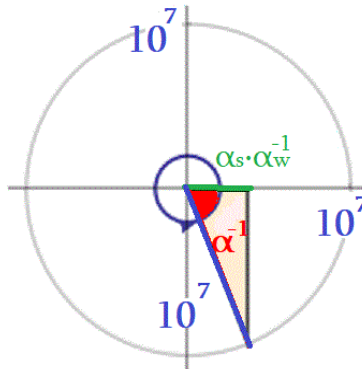
**Figure 4.** Geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions

Resulting the unity formulas that connects the strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$  and the fine-structure constant  $\alpha$ :

$$10^7 \cdot \alpha_w \cdot \cos \alpha^{-1} = \alpha_s \quad (22)$$

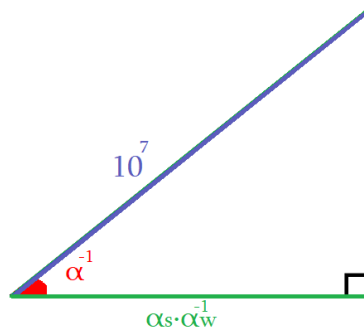
$$\cos \alpha^{-1} = \frac{\alpha_s \alpha_w^{-1}}{10^7} \quad (23)$$

The figure 5 below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7$ .



**Figure 5.** The angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $10^7$ .

The figure 6 below shows the dimensionless unification of the strong nuclear, the weak nuclear and the electromagnetic interactions.



**Figure 6.** Geometric representation of the dimensionless unification of the strong nuclear, the weak nuclear and the electromagnetic interactions.

We reached the conclusion of the dimensionless unification of the strong nuclear, the weak nuclear and the electromagnetic forces:

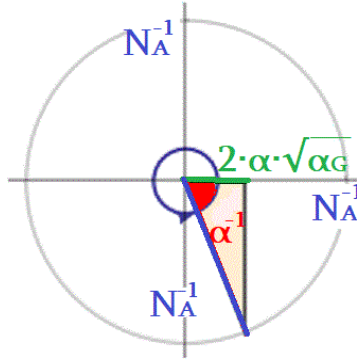
$$10^7 \cdot \alpha_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot \alpha_s \quad (24)$$

Resulting the unity formula that connects the fine-structure constant  $\alpha$ , the gravitational coupling constant  $\alpha_G$  and the Avogadro's number  $N_A$ :

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (25)$$

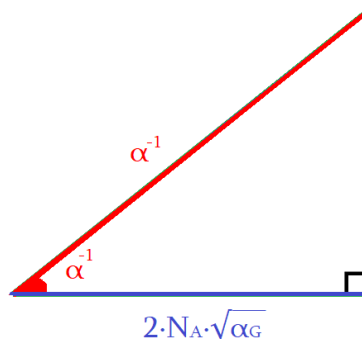
$$\alpha^{-2} \cdot \cos^2 \alpha^{-1} = 4 \cdot \alpha_G \cdot N_A^2 \quad (26)$$

The figure 7 below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $N_A^{-1}$ .

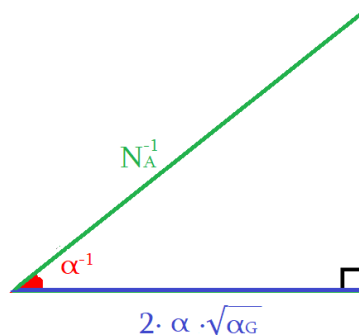


**Figure 7.** The angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $N_A^{-1}$ .

The figures 8 and 9 below show the geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions.



**Figure 8.** First geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions



**Figure 9.** Second geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions

We reached the conclusion of the dimensionless unification of the gravitational and the electromagnetic forces:

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot NA^2 = 1 \quad (27)$$

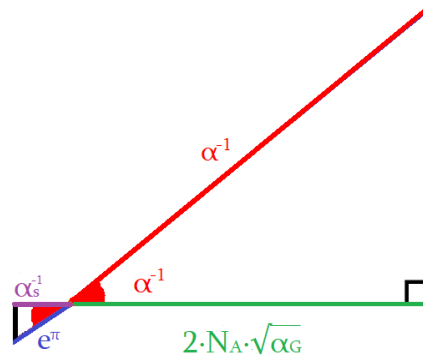
$$16 \cdot a^2 \cdot a_G \cdot NA^2 = (e^{i/a} + e^{-i/a})^2 \quad (28)$$

We reached the conclusion of the dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions:

$$4 \cdot a_s^2 \cdot a^2 \cdot a_G \cdot NA^2 = i^{4i} \quad (29)$$

$$a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^4 \cdot a_G \cdot NA^2 = i^{8i} \quad (30)$$

The figure 10 below shows the geometric representation of the dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions.



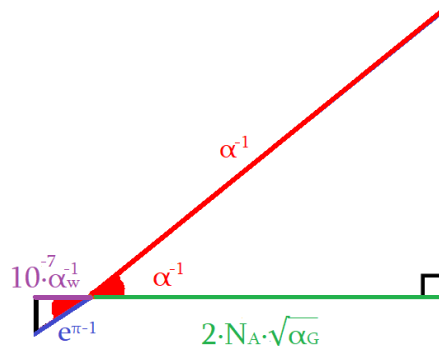
**Figure 10.** Geometric representation of the dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions

We reached the conclusion of the dimensionless unification of the weak nuclear, the gravitational and electromagnetic forces:

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2 = i^{4i} \cdot e^2 \quad (31)$$

$$10^{14} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G \cdot NA^2 = i^{8i} \quad (32)$$

The figure 11 below shows the geometric representation of the dimensionless unification of the weak nuclear, the gravitational and the electromagnetic interactions.



**Figure 11.** Geometric representation of the dimensionless unification of the weak nuclear, the gravitational and the electromagnetic interactions

Resulting the unity formula that connect the strong coupling constant  $a_s$ , the weak coupling constant  $a_w$ , the fine-structure constant  $a$  and the gravitational coupling constant  $a_G(p)$  for the proton:

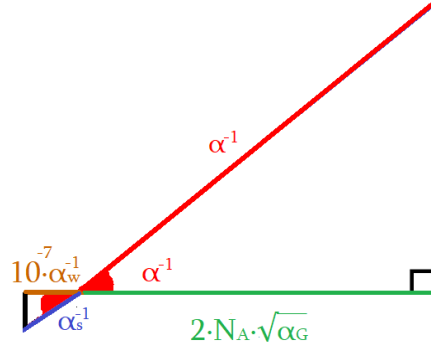
$$4 \cdot 10^{14} \cdot NA^2 \cdot a_w^2 \cdot a^2 \cdot a_G(p) = \mu^2 \cdot a_s^2 \quad (33)$$

We reached the conclusion of the dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions:

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 \quad (34)$$

$$8 \cdot 10^7 \cdot N_A^2 \cdot \alpha_w \cdot \alpha^2 \cdot \alpha_G = \alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) \quad (35)$$

The figure 12 below shows the geometric representation of the dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions.



**Figure 12.** Geometric representation of the dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions

From these expressions resulting the unity formulas that connects the strong coupling constant  $\alpha_s$ , the weak coupling constant  $\alpha_w$ , the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton, the Avogadro's number  $N_A$ , the gravitational coupling constant  $\alpha_G$  of the electron, the gravitational coupling constant of the proton  $\alpha_G(p)$ , the strong coupling constant  $\alpha_s$  and the weak coupling constant  $\alpha_w$ :

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 \quad (36)$$

$$\mu^2 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G(p) \cdot N_A^2 \quad (37)$$

$$\mu \cdot N_1 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^3 \cdot N_A^2 \quad (38)$$

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 \quad (39)$$

$$\mu^3 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1 \quad (40)$$

$$\mu \cdot \alpha_s = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha_G \cdot G(p)^2 \cdot N_A^2 \cdot N_1^2 \quad (41)$$

$$\mu \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_G(p) \cdot N_A^2 \cdot N_1 \quad (42)$$

These equations are applicable for all energy scales. In [15] and [16] we found the expressions for the gravitational constant:

$$G = (2e\alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (43)$$

$$G = i^{4i} (2\alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (44)$$

$$G = i^{4i} e^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (45)$$

$$G = \alpha_s^2 (2 \cdot 10^7 \alpha_w \alpha_{NA})^{-2} \frac{\hbar c}{m_e^2} \quad (46)$$

It presented the theoretical value of the Gravitational constant  $G=6.67448 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$ . This value is very close to the 2018 CODATA recommended value of gravitational constant and two experimental measurements from a research group announced new measurements based on torsion balances. They ended up measuring  $6.674184 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$  and  $6.674484 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$ -of-swinging and angular acceleration methods, respectively.

## 2. Dimensionless unification of atomic physics and cosmology

In [17] and [18] resulting in the dimensionless unification of atomic physics and cosmology. The relevant constant in atomic physics is the fine-structure constant  $\alpha$ , which plays a fundamental role in atomic physics and quantum electrodynamics. The analogous constant in cosmology is the gravitational fine-structure constant  $\alpha_g$ . It plays a fundamental role in cosmology. The mysterious value of the gravitational fine-structure constant  $\alpha_g$  is an equivalent way to express the biggest issue in theoretical physics. The mysterious value of the gravitational fine-structure constant  $\alpha_g$  is an equivalent way to express the biggest issue in theoretical physics. The gravitational fine structure constant  $\alpha_g$  is defined as:

$$\alpha_g = \frac{l_{pl}^3}{r_e^3} = \frac{\sqrt{\alpha_G^3}}{\alpha^3} = \sqrt{\frac{\alpha_G^3}{\alpha^6}} = 1.886837 \times 10^{-61} \quad (47)$$

The expression that connects the gravitational fine-structure constant  $\alpha_g$  with the golden ratio  $\phi$  and the Euler's number  $e$  is:

$$\alpha_g = \frac{4e}{3\sqrt{3}\phi^5} \times 10^{-60} = 1,886837 \times 10^{-61} \quad (48)$$

Resulting the unity formula for the gravitational fine-structure constant  $\alpha_g$ :

$$\alpha_g = (2 \cdot e \cdot \alpha^2 \cdot NA)^{-3} \quad (49)$$

$$\alpha_g = i^{6i} \cdot (2 \cdot \alpha_s \cdot \alpha^2 \cdot NA)^{-3} \quad (50)$$

$$\alpha_g = i^{6i} \cdot e^3 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot \alpha^3 \cdot NA)^{-3} \quad (51)$$

$$\alpha_g = (10^7 \cdot \alpha_w \cdot \alpha_G^{1/2} \cdot e^{-1} \cdot \alpha_s^{-1} \cdot \alpha^{-1})^3 \quad (52)$$

$$\alpha_g^2 = (10^{14} \cdot \alpha_w^2 \cdot \alpha_G \cdot e^{-2} \cdot \alpha_s^{-2} \cdot \alpha^{-2})^3 \quad (53)$$

$$\alpha_g = 10^{21} \cdot i^{6i} \cdot \alpha_w^3 \cdot \alpha_G^{3/2} \cdot \alpha_s^{-6} \cdot \alpha^{-3} \quad (54)$$

So the unity formulas for the gravitational fine-structure constant  $\alpha_g$  are:

$$\alpha_g^2 = 10^{42} \cdot i^{12i} \cdot \alpha_w^6 \cdot \alpha_G^3 \cdot \alpha_s^{-12} \cdot \alpha^{-6} \quad (55)$$

The cosmological constant  $\Lambda$  is presumably an enigmatic form of matter or energy that acts in opposition to gravity and is considered by many physicists to be equivalent to dark energy. Nobody really knows what the cosmological constant is exactly, but it is required in cosmological equations in order to reconcile theory with our observations of the universe. Resulting the dimensionless unification of the atomic physics and the cosmology:

$$|pl|^2 \cdot \Lambda = (2 \cdot e \cdot \alpha^2 \cdot NA)^{-6} \quad (56)$$

$$|pl|^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot \alpha_s \cdot \alpha^2 \cdot NA)^{-6} \quad (57)$$



$$|pl|^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot \alpha^3 \cdot N_A)^{-6} \quad (58)$$

$$e^6 \cdot \alpha_s^6 \cdot \alpha^6 \cdot |pl|^2 \cdot \Lambda = 10^{42} \cdot \alpha_G^3 \cdot \alpha_w^6 \quad (59)$$

$$\alpha_s^{12} \cdot \alpha^6 \cdot |pl|^2 \cdot \Lambda = 10^{42} \cdot i^{12i} \cdot \alpha_G^3 \cdot \alpha_w^6 \quad (60)$$

For the cosmological constant  $\Lambda$  equals:

$$\Lambda = \left(2e\alpha^2 N_A\right)^{-6} \frac{c^3}{G\hbar} \quad (61)$$

$$\Lambda = i^{12i} (2\alpha_s \alpha^2 N_A)^{-6} \frac{c^3}{G\hbar} \quad (62)$$

$$\Lambda = i^{12i} e^6 (2 \cdot 10^7 \alpha_w \alpha^3 N_A)^{-6} \frac{c^3}{G\hbar} \quad (63)$$

$$\Lambda = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 \alpha^2}\right)^3 \frac{c^3}{G\hbar} \quad (64)$$

$$\Lambda = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4}\right)^3 \frac{c^3}{G\hbar} \quad (65)$$

In [19] we found the Equations of the Universe:

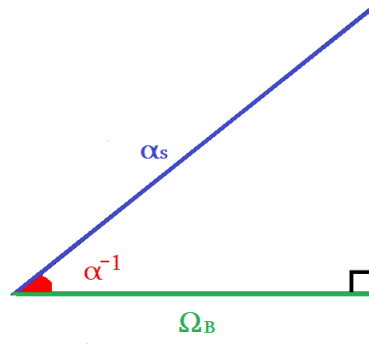
$$\frac{\Lambda G\hbar}{c^3} = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4}\right)^3 \quad (66)$$

$$e^{6\pi} \frac{\Lambda G\hbar}{c^3} = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4}\right)^3 \quad (67)$$

For the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{2e^2 \varphi^{-5}}{3^3 \pi \varphi^5} \times 10^{-120} \quad (68)$$

In [20] , [21] and [22] we proved that the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions will propose a possible solution for the density parameter of baryonic matter, dark matter and dark energy. The sum of the contributions to the total density parameter  $\Omega_0$  at the current time is  $\Omega_0 = 1.02 \pm 0.02$ . Current observations suggest that we live in a dark energy dominated Universe with  $\Omega_\Lambda = 0.73$ ,  $\Omega_D = 0.23$  and  $\Omega_B = 0.04$ . The figure 13 shows the Geometric representation of the density parameter for the baryonic matter.



**Figure 13.** Geometric representation of the the density parameter for the baryonic matter

The assessment of baryonic matter at the current time was assessed by WMAP to be  $\Omega_B=0.044\pm 0.004$ . From the dimensionless unification of the fundamental interactions the density parameter for the normal baryonic matter is:

$$\Omega_B=e^{-\pi}=i^{2i}=0.0432=4.32\% \quad (69)$$

From Euler's identity for the density parameter of baryonic matter apply:

$$\Omega_B^i+1=0 \quad (70)$$

$$\Omega_B^i=i^2 \quad (71)$$

$$\Omega_B^{2i}=1 \quad (72)$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_B=e^{-1}\cdot a_s \quad (73)$$

$$\Omega_B=a_w^{-1}\cdot a_s^2\cdot 10^{-7} \quad (74)$$

$$\Omega_B=2^{-1}\cdot a_s\cdot (e^{i/a}+e^{-i/a}) \quad (75)$$

$$\Omega_B=2\cdot N_A\cdot a_s\cdot a\cdot a_G^{1/2} \quad (76)$$

$$\Omega_B=2^{-1}\cdot e^{-1}\cdot 10^7\cdot a_w\cdot (e^{i/a}+e^{-i/a}) \quad (77)$$

$$\Omega_B=2\cdot 10^7\cdot N_A\cdot e^{-1}\cdot a_w\cdot a\cdot a_G^{1/2} \quad (78)$$

$$\Omega_B=10^{-7}\cdot a_g^{1/3}\cdot a_s^2\cdot a\cdot a_w^{-1}\cdot a_G^{-1/2} \quad (79)$$

In [23] we presented the solution for the Density Parameter of Dark Energy. From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda=2\cdot e^{-1}=0.73576=73.57\% \quad (80)$$

So apply:

$$2\cdot R_d^2=e\cdot L_H^2 \quad (81)$$

Also from the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda=2\cdot \cos a^{-1} \quad (82)$$

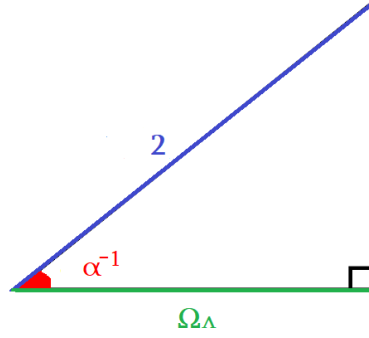
So apply the expression:

$$\cos \alpha^{-1} = \frac{\Omega_{\Lambda}}{2} \quad (83)$$

So the beautiful equation for the density parameter for dark energy is:

$$\Omega_{\Lambda} = e^{i/a} + e^{-i/a} \quad (84)$$

The figure 22 shows the geometric representation of the density parameter for dark energy.

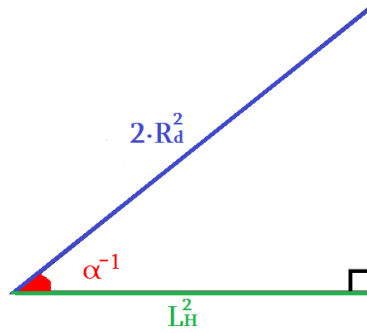


**Figure 14.** Geometric representation of the the density parameter for the dark energy

So apply the expression:

$$\cos \alpha^{-1} = \frac{L_H^2}{2R_d^2} \quad (85)$$

The figure 15 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.



**Figure 15.** Geometric representation of the relationship between the de Sitter radius and the Hubble length

From the dimensionless unification of the fundamental interactions for the density parameter of dark energy apply:

$$\Omega_{\Lambda} = 2 \cdot i^{2i} \cdot a_s^{-1} \quad (86)$$

$$\Omega_{\Lambda} = 2 \cdot 10^{-7} a_s \cdot a_w^{-1} \quad (87)$$

$$\Omega_{\Lambda} = 2 \cdot e \cdot 10^{-7} \cdot i^{2i} \cdot a_w^{-1} \quad (88)$$

$$\Omega_{\Lambda} = 2 \cdot 10^{-7} \cdot a_s \cdot a_w^{-1} \quad (89)$$

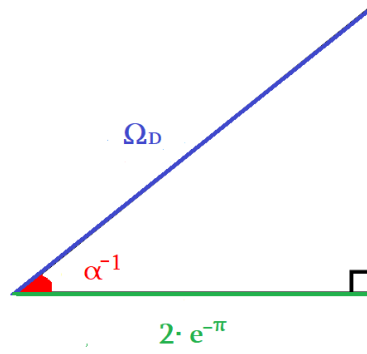
$$\Omega_{\Lambda} = 4 \cdot a \cdot a_G^{1/2} \cdot N_A \quad (90)$$

$$\Omega_{\Lambda} = i^{8i} \cdot a^{-2} \cdot a_s^{-4} \cdot a_G^{-1} \cdot N_A^{-2} \quad (91)$$

$$\Omega_{\Lambda} = 10^7 \cdot i^{4i} \cdot a^{-1} \cdot a_w^{-1} \cdot a_G^{-1/2} \cdot N_A^{-1} \quad (92)$$

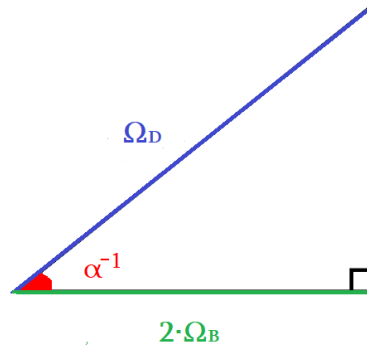
$$\Omega\Lambda=8 \cdot 10^7 \cdot NA^2 \cdot a_w \cdot a^2 \cdot a_G \cdot a_s^{-1} \quad (93)$$

The figure 16 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.



**Figure 16.** Geometric representation of the density parameter of dark matter.

The figure 17 shows the geometric representation of the relationship between the density parameter of dark and baryonic matter.



**Figure 17.** Geometric representation of the relationship between the density parameter of dark and baryonic matter.

Current observations suggest that we live in a dark energy dominated Universe with density parameters for dark matter  $\Omega_D=0.23$ . From the dimensionless unification of the fundamental interactions the density parameter for dark matter is:

$$\Omega_D=2 \cdot e^{1-\pi}=2 \cdot e^{-i^{2i}}=0.2349=23.49\% \quad (94)$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_D=2 \cdot a_s \quad (95)$$

$$\Omega_D=2 \cdot 10^7 \cdot e^{-1} \cdot a_w \quad (96)$$

$$\Omega_D=2 \cdot (i^{2i} \cdot 10^7 \cdot a_w)^{1/2} \quad (97)$$

$$\Omega_D=4 \cdot i^{2i} \cdot (e^{i/a} + e^{-i/a})^{-1} \quad (98)$$

$$\Omega_D=10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) \quad (99)$$

$$\Omega_D=4 \cdot 10^7 \cdot a_w \cdot a \cdot a_G^{1/2} \cdot NA \quad (100)$$

$$\Omega_D=16 \cdot 10^7 \cdot NA^2 \cdot a_w \cdot a^2 \cdot a_G \cdot (e^{i/a} + e^{-i/a})^{-1} \quad (101)$$

The relationship between the density parameter of dark matter and baryonic matter is:

$$\Omega_D=2 \cdot e \cdot \Omega_B \quad (102)$$

The relationship between the density parameter of dark energy, dark matter and baryonic matter is:

$$\Omega_D \cdot \Omega_\Lambda = 4 \cdot \Omega_B \quad (103)$$

From the dimensionless unification of the fundamental interactions the sum of the contributions to the total density parameter  $\Omega_0$  at the current time is:

$$\Omega_0 = \Omega_B + \Omega_D + \Omega_\Lambda = e^{-n} + 2 \cdot e^{1-n} + 2 \cdot e^{-1} = 1.0139 \quad (104)$$

In [24] we proposed a possible solution for the Equation of state in cosmology. From the dimensionless unification of the fundamental interactions the state equation  $w$  has value:

$$w = -24 \cdot e^{-n} = -24 \cdot i^{2i} = -1.037134 \quad (105)$$

### 3. Unification of the Microcosm and the Macrocosm

In [25] , [26] and [27] we presented the law of the gravitational fine-structure constant  $\alpha_g$  followed by ratios of maximum and minimum theoretical values for natural quantities. This theory uses quantum mechanics, cosmology, thermodynamics, and special and general relativity. Length  $l$ , time  $t$ , speed  $v$  and temperature  $T$  have the same min/max ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{max}} = \frac{t_{min}}{t_{max}} = \frac{v_{min}}{v_{max}} = \frac{T_{min}}{T_{max}} \quad (106)$$

Energy  $E$ , mass  $M$ , action  $A$ , momentum  $P$  and entropy  $S$  have another min/max ratio, which is the square of  $\alpha_g$ :

$$\alpha_g^2 = \frac{E_{min}}{E_{max}} = \frac{M_{min}}{M_{max}} = \frac{A_{min}}{A_{max}} = \frac{P_{min}}{P_{max}} = \frac{S_{min}}{S_{max}} \quad (107)$$

Force  $F$  has min/max ratio which is  $\alpha_g^4$ :

$$\alpha_g^4 = \frac{F_{min}}{F_{max}} \quad (108)$$

Mass density has min/max ratio which is  $\alpha_g^5$ :

$$\alpha_g^5 = \frac{\rho_{min}}{\rho_{max}} \quad (109)$$

Also apply the expressions:

$$\frac{l_{max}}{M_{max}} = \frac{l_{pl}}{m_{pl}} \quad (110)$$

$$\left( \frac{l_{max}}{l_{min}} \right)^2 = \frac{M_{max}}{m_{min}} \quad (111)$$

In [28] we presented the Unification of the Microcosm and the Macrocosm. For the minimum mass  $M_{min}$  apply:

$$M_{min} = \frac{m_{pl}^2}{M_{max}} = \alpha_g m_{pl} = \frac{\alpha_G}{\alpha^3} m_e = \frac{\sqrt[3]{\alpha_g^2}}{\alpha} m_e \quad (112)$$

$$M_{min} = (2 \cdot e \cdot NA)^{-2} \cdot \alpha^{-1} \cdot m_e = 4.06578 \times 10^{-69} \text{ kg} \quad (113)$$

The expressions for the mass of the observable universe  $M_U$  are:

$$M_U = \alpha^{-1} \cdot \alpha_g^{-4/3} \cdot m_e = \alpha^3 \cdot \alpha_G^{-2} \cdot m_e = (2 \cdot e \cdot \alpha^2 \cdot NA)^2 \cdot N_1 \cdot m_p = \mu \cdot \alpha \cdot N_1^2 \cdot m_p \quad (114)$$

For the value of the mass of the observable universe  $M_U$  apply  $M_U = 1.153482 \times 10^{53}$  kg. The expressions who calculate the number of protons in the observable universe are:

$$N_{Edd} = \frac{M_U}{m_p} = \mu \alpha N_1^2 = \frac{N_1}{\alpha_g^{\frac{2}{3}}} = \left(2e\alpha^2 N_A\right)^2 N_1 = \left(\frac{r_e}{l_{pl}}\right)^2 N_1 = 6.9 \times 10^{79} \quad (115)$$

In [29] and [30] we presented the Dimensionless theory of everything. The new formula for the Planck length  $l_{pl}$  is:

$$l_{pl} = a\sqrt{a_G}\alpha_0 \quad (116)$$

The fine-structure constant is universal scaling factor:

$$\alpha = \frac{2\pi r_e}{\lambda_e} = \frac{\lambda_e}{2\pi\alpha_0} = \frac{r_e}{l_{pl}} \frac{m_e}{m_{pl}} = \sqrt{\frac{r_e}{\alpha_0}} \quad (117)$$

Also the gravitational coupling constant is universal scaling factor:

$$\alpha_G = \frac{m_e^2}{m_{pl}^2} = \frac{\alpha_{G(p)}}{\mu^2} = \frac{\alpha}{\mu N_1} = \frac{\alpha^2}{N_1^2 \alpha_{G(p)}} = \left(\frac{2\pi l_{pl}}{\lambda_e}\right)^2 = \left(\alpha \frac{l_{pl}}{r_e}\right)^2 = \left(\frac{l_{pl}}{\alpha\alpha_0}\right)^2 \quad (118)$$

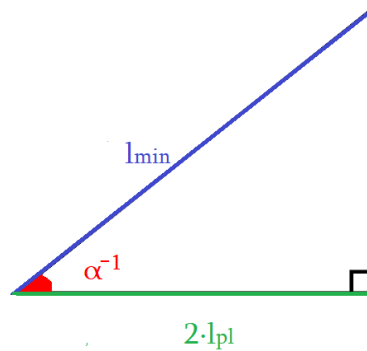
A smallest length in nature thus implies that there is no way to define exact boundaries of objects or elementary particles. Max Planck proposed natural units that indirectly discovered the lowest-level properties of free space, all born from equations that simplified the mathematics of physics equations. The fundamental unit of length in this unit system is the Planck length  $l_{pl}$ . The smallest components will never be seen with the human eye as it is orders of magnitudes smaller than an atom. Thus, it will never be directly observed but it can be deduced by mathematics. We proposed to be a lattice structure, in which its unit cells have sides of length  $2 \cdot e \cdot l_{pl}$ . Perhaps for the minimum distance  $l_{min}$  apply:

$$l_{min} = 2 \cdot e \cdot l_{pl} = 2 \cdot e^n \cdot a_s \cdot l_{pl} \quad (119)$$

From expressions apply:

$$\cos \alpha^{-1} = \frac{2l_{pl}}{l_{min}} \quad (120)$$

The figures 18 below show the geometric representation of the fundamental unit of length.

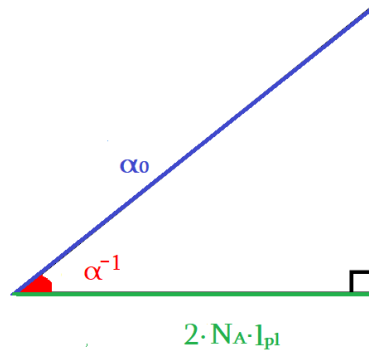


**Figure 18.** Geometric representation of the fundamental unit of length.

For the Bohr radius  $\alpha_0$  apply:

$$\alpha_0 = N_A \cdot l_{min} = 2 \cdot e \cdot N_A \cdot l_{pl} \quad (121)$$

The figures 19 below show the geometric representation of the relationship between the Bohr radius and the Planck length.



**Figure 19.** Geometric representation of the relationship between the Bohr radius and the Planck length.

We will use this expression and the new formula for the Planck length  $l_p$  to resulting the unity formula that connects the fine-structure constant  $\alpha$  and the gravitational coupling constant  $\alpha_G$ :

$$2eN_A\alpha\sqrt{\alpha_G} = 1 \quad (122)$$

Therefore the unity formula that connect the fine-structure constant  $\alpha$ , the gravitational coupling constant  $\alpha_G$  and the Avogadro's number  $N_A$  is:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1 \quad (123)$$

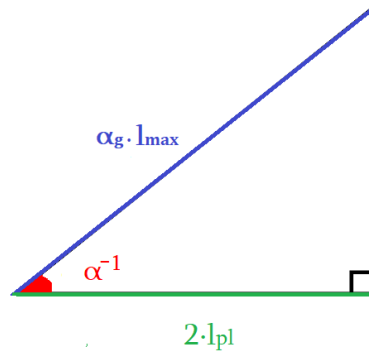
Length  $l$  has the max/min ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{max}} \quad (124)$$

The maximum distance  $l_{max}$  corresponds to the distance of the universe:

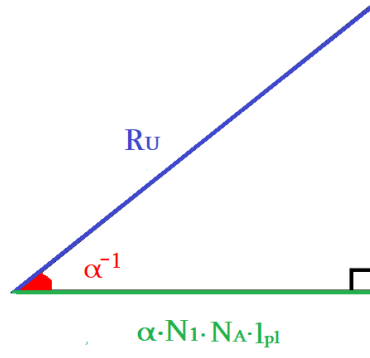
$$l_{max} = \alpha_g^{-1} \cdot l_{min} = 4.657 \times 10^{26} \text{ m} \quad (125)$$

The figure 20 shows the geometric representation of the relationship between the maximum distance and the Planck length.



**Figure 20.** Geometric representation of the relationship between the maximum distance and the Planck length.

The figure 21 shows the geometric representation of the relationship between the radius of the universe with the Planck length.



**Figure 21.** Geometric representation of the relationship between the radius of the universe with the Planck length

In [31] we presented the New Large Number Hypothesis of the universe. The diameter of the observable universe will be calculated to be equal to the product of the ratio of electric force to gravitational force between electron and proton on the reduced Compton wavelength of the electron:

$$2 \cdot R_U = N_1 \cdot \lambda_c \quad (126)$$

So apply the expression:

$$R_U = e \cdot \alpha \cdot N_1 \cdot N_A \cdot l_{pl} \quad (127)$$

The expressions for the radius of the observable universe are:

$$R_U = \frac{\alpha N_1}{2} \alpha_0 = \frac{N_1}{2\alpha} r_e = \frac{1}{2\mu\alpha_G} r_e = \frac{m_{pl}^2 r_e}{2m_e m_p} = \frac{\hbar c r_e}{2Gm_e m_p} = \frac{\alpha \hbar}{2Gm_e^2 m_p} \quad (128)$$

We Found the value of the radius of the universe  $R_U = 4.38 \times 10^{26}$  m. The expressions for the radius of the observable universe are:

$$T_U = \frac{R_U}{c} = \frac{N_1 r_e}{2\alpha c} = \frac{r_e}{2\mu\alpha_G c} = \frac{\alpha N_1 \alpha_0}{2c} = \frac{\alpha \hbar}{2c G m_e^2 m_p} = \frac{\hbar r_e}{2G m_e m_p} \quad (129)$$

For the value of the age of the universe apply  $T_U = 1.46 \times 10^{18}$  s. The expressions for the relationship between the mass of the observable universe  $M_U$  with the radius of the universe  $R_U$  are:

$$\frac{M_U}{R_U^2} = 4\alpha\mu^2 \frac{m_e}{r_e^2} \quad (130)$$

$$\frac{M_U}{m_p} = \alpha\mu \left( \frac{2R_U}{r_e} \right)^2 \quad (131)$$

The gamma rhythm is a pattern of neuronal oscillations whose frequency ranges from 25 Hz to 100 Hz although 40 Hz is typical. Gamma frequency oscillations are present during wakefulness and REM sleep. The time quantum in the brain  $t_B$ , the smallest unit of time that related to the 40 Hz oscillation of the gamma rate:

$$\frac{t_B}{t_{pl}} = \sqrt[3]{\alpha_g^2} \quad (132)$$



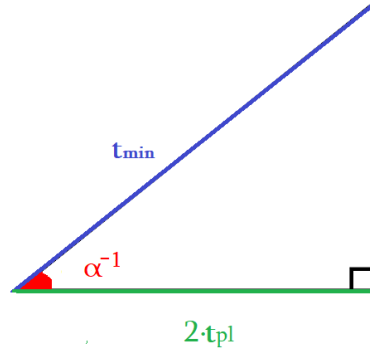
For the minimum distance  $l_{min}$  apply  $l_{min}=2 \cdot e \cdot l_{pl}$ . So for the minimum time  $t_{min}$  apply:

$$t_{min} = \frac{l_{min}}{c} = \frac{2el_{pl}}{c} = 2et_{pl} \quad (133)$$

From expressions apply:

$$\cos \alpha^{-1} = \frac{2t_{pl}}{t_{min}} \quad (134)$$

The figures 22 below show the geometric representation of the fundamental unit of time.



**Figure 22.** Geometric representation of the fundamental unit of time.

In the papers [32] was presented the theoretical value for the Hubble Constant. The formulas for the Hubble Constant are:

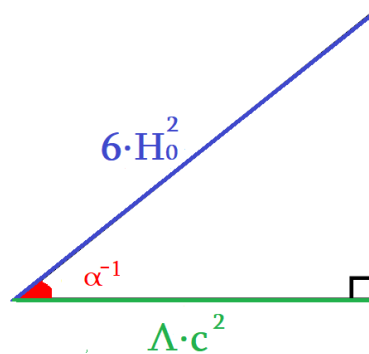
$$H_0 = c \sqrt{\frac{e}{6} \Lambda} \quad (135)$$

$$H_0 = \frac{\alpha_g}{t_{pl}} \sqrt{\frac{e}{6}} \quad (136)$$

These equations calculate the theoretical value of the Hubble Constant:

$$H_0 = 2.355683 \times 10^{-18} \text{ s}^{-1} = 72.69 \text{ (km/s)/Mpc} \quad (137)$$

The figure 24 shows the geometric representation of the relationship between the Hubble constant and cosmological constant.



**Figure 23.** Geometric representation of the relationship between the Hubble constant and cosmological constant.

Also apply the expression:

$$\frac{G\hbar H_0^2}{c^5} = \frac{e}{6}\alpha_g^2 \quad (138)$$

$$\frac{G\hbar H_0^2}{c^5} = \frac{1}{6e^5(2\alpha^2 N_A)^6} \quad (139)$$

$$\frac{G\hbar H_0^2}{c^5} = \frac{e}{48(e^\pi \alpha_s \alpha^2 N_A)^3} \quad (140)$$

$$\frac{G\hbar H_0^2}{c^5} = \frac{10^{42}}{6e^5} \left( \frac{\alpha_w^2 \alpha_G}{\alpha_s^2 \alpha^2} \right)^3 \quad (141)$$

$$\frac{6G\hbar H_0^2}{ec^5} = \left( \frac{10^{14} \alpha_w^2 \alpha_G}{e^{2\pi} \alpha^2 \alpha_s^4} \right)^3 \quad (142)$$

$$6e^{6\pi} \frac{G\hbar H_0^2}{c^5} = e \left( \frac{10^{14} \alpha_w^2 \alpha_G}{\alpha^2 \alpha_s^4} \right)^3 \quad (143)$$

$$6e^{5\pi} \frac{G\hbar H_0^2}{c^5} = \frac{1}{\alpha_s^{11}} \left( \frac{10^{14} \alpha_w^2 \alpha_G}{\alpha^2} \right)^3 \quad (144)$$

The Equations of the Universe are:

$$6e^{5\pi} \frac{G\hbar H_0^2}{c^5} = 10^{42} \frac{\alpha_w^6 \alpha_G^3}{\alpha^6 \alpha_s^{11}} \quad (145)$$

$$e^{7\pi} \frac{G\hbar \Lambda^2}{cH_0^2} = 6 \cdot 10^{42} \frac{\alpha_w^6 \alpha_G^3}{\alpha^6 \alpha_s^{13}} \quad (146)$$

#### 4. Conclusions

We presented new exact formula for the fine-structure constant  $\alpha$  in terms of the golden angle, the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1} = 360 \cdot \phi^{-2} - 2 \cdot \phi^{-3} + (3 \cdot \phi)^{-5}$$

We propose in a simple and accurate expression for the fine-structure constant  $\alpha$  in terms of the Archimedes constant  $\pi$ :

$$\alpha^{-1} = 2 \cdot 3 \cdot 11 \cdot 41 \cdot 43^{-1} \cdot \pi \cdot \ln 2$$

We propose the exact mathematical expressions for the proton to electron mass ratio:

$$7 \cdot \mu^3 = 165^3 \cdot \ln^{11} 10$$

We present the exact mathematical expressions that connect the proton to electron mass ratio and the fine-structure constant:

$$9 \cdot \mu - 119 \cdot \alpha^{-1} = 5 \cdot (\varphi + 42)$$

The new formula for the Planck length  $l_{pl}$  is:

$$l_{pl} = a \sqrt{a_G} \alpha_0$$

The new formula for the Avogadro's number  $N_A$  is:

$$N_A = \left( 2e\alpha\sqrt{\alpha_G} \right)^{-1}$$

The mathematical formulas that connect dimensionless physical constants are:

$$\alpha_G(p) = \mu^2 \cdot \alpha_G$$

$$\alpha = \mu \cdot N_1 \cdot \alpha_G$$

$$\alpha \cdot \mu = N_1 \cdot \alpha_G(p)$$

$$\alpha^2 = N_1^2 \cdot \alpha_G \cdot \alpha_G(p)$$

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1$$

$$\mu^2 = 4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G(p) \cdot N_A^2$$

$$\mu \cdot N_1 = 4 \cdot e^2 \cdot \alpha^3 \cdot N_A^2$$

$$4 \cdot e^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = 1$$

$$\mu^3 = 4 \cdot e^2 \cdot \alpha \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1$$

$$\mu^2 = 4 \cdot e^2 \cdot \alpha_G \cdot \alpha_G(p)^2 \cdot N_A^2 \cdot N_1^2$$

$$\mu = 4 \cdot e^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_G(p) \cdot N_A^2 \cdot N_1$$

We reached the conclusion of the simple unification of the nuclear and the atomic physics:

$$10 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha})^{1/2} = 13 \cdot i$$

We presented the recommended value for the strong coupling constant:

$$\alpha_s = \frac{\text{Euler's number}}{\text{Gerford's constant}} = \frac{e}{e^\pi} = e^{1-\pi} = 0,11748..$$

It presented the dimensionless unification of the fundamental interactions. We calculated the unity formulas that connect the coupling constants of the fundamental forces. The dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$e \cdot a_s = 10^7 \cdot a_w$$

$$a_s^2 = i^{2i} \cdot 10^7 \cdot a_w$$

The dimensionless unification of the strong nuclear and electromagnetic interactions:

$$a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot i^{2i}$$

The dimensionless unification of the weak nuclear and electromagnetic interactions:

$$10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot e \cdot i^{2i}$$

The dimensionless unification of the strong nuclear, the weak nuclear and electromagnetic interactions:

$$10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot a_s$$

The dimensionless unification of the gravitational and the electromagnetic interactions:

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1$$

$$16 \cdot a^2 \cdot a_G \cdot N_A^2 = (e^{i/a} + e^{-i/a})^2$$

The dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions:

$$4 \cdot a_s^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i}$$

$$a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^4 \cdot a_G \cdot N_A^2 = i^{8i}$$

The dimensionless unification of the weak nuclear, the gravitational and the electromagnetic interactions:

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i} \cdot e^2$$

$$10^{14} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G \cdot N_A^2 = i^{8i}$$

The dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions:

$$a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot N_A^2$$

$$8 \cdot 10^7 \cdot N_A^2 \cdot a_w \cdot a^2 \cdot a_G = a_s \cdot (e^{i/a} + e^{-i/a})$$

From these expressions resulting the unity formulas that connects the strong coupling constant  $a_s$ , the weak coupling constant  $a_w$ , the proton to electron mass ratio  $\mu$ , the fine-structure constant  $a$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton, the Avogadro's number  $N_A$ , the gravitational coupling constant  $a_G$  of the electron, the gravitational coupling constant of the proton  $a_{G(p)}$ , the strong coupling constant  $a_s$  and the weak coupling constant  $a_w$ :

$$a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot N_A^2$$

$$\mu^2 \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_{G(p)} \cdot N_A^2$$

$$\mu \cdot N_1 \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^3 \cdot N_A^2$$

$$a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot \mu \cdot a_G^2 \cdot N_A^2 \cdot N_1$$

$$\mu^3 \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1$$

$$\mu \cdot a_s = 4 \cdot 10^{14} \cdot a_w^2 \cdot a_G \cdot G(p)^2 \cdot N_A^2 \cdot N_1^2$$

$$\mu \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot a_G \cdot a_G(p) \cdot N_A^2 \cdot N_1$$

We found the formula for the Gravitational constant:

$$G = (2e\alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

$$G = i^{4i} (2\alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

$$G = i^{4i} e^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

$$G = \alpha_s^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

It presented the theoretical value of the Gravitational constant  $G = 6.67448 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$ . This value is very close to the 2018 CODATA recommended value of gravitational constant and two experimental measurements from a research group announced new measurements based on torsion balances. They ended up measuring  $6.674184 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$  and  $6.674484 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$ -of-swinging and angular acceleration methods, respectively.

We calculated the expression that connects the gravitational fine structure constant with the four coupling constants:

$$\alpha_g^2 = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3$$

Perhaps the gravitational fine structure constant is the coupling constant for the fifth force. It presented that the gravitational fine structure constant is a simple analogy between atomic physics and cosmology. Resulting the dimensionless unification of the atomic physics and the cosmology:

$$|p|^2 \cdot \Lambda = (2 \cdot e \cdot a^2 \cdot N_A)^{-6}$$

$$|p|^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-6}$$

$$|p|^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^{-6}$$

$$e^6 \cdot a_s^6 \cdot a^6 \cdot |p|^2 \cdot \Lambda = 10^{42} \cdot a_G^3 \cdot a_w^6$$

$$a_s^{12} \cdot a^6 \cdot |p|^2 \cdot \Lambda = 10^{42} \cdot i^{12i} \cdot a_G^3 \cdot a_w^6$$

For the cosmological constant equals:

$$\Lambda = (2e\alpha^2 N_A)^{-6} \frac{c^3}{G\hbar}$$

$$\Lambda = i^{12i} (2\alpha_s a^2 N_A)^{-6} \frac{c^3}{G\hbar}$$

$$\Lambda = i^{12i} e^6 (2 \cdot 10^7 \alpha_w a^3 N_A)^{-6} \frac{c^3}{G\hbar}$$

$$\Lambda = 10^{42} \left( \frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 \alpha^2} \right)^3 \frac{c^3}{G \hbar}$$

$$\Lambda = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \frac{c^3}{G \hbar}$$

The Equation of the Universe is:

$$\frac{\Lambda G \hbar}{c^3} = 10^{42} i^{12i} \left( \frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3$$

We presented the law of the gravitational fine-structure constant  $\alpha_g$  followed by ratios of maximum and minimum theoretical values for natural quantities. Length  $l$ , time  $t$ , speed  $v$  and temperature  $T$  have the same min/max ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{max}} = \frac{t_{min}}{t_{max}} = \frac{v_{min}}{v_{max}} = \frac{T_{min}}{T_{max}}$$

Energy  $E$ , mass  $M$ , action  $A$ , momentum  $P$  and entropy  $S$  have another min/max ratio, which is the square of  $\alpha_g$ :

$$\alpha_g^2 = \frac{E_{min}}{E_{max}} = \frac{M_{min}}{M_{max}} = \frac{A_{min}}{A_{max}} = \frac{P_{min}}{P_{max}} = \frac{S_{min}}{S_{max}}$$

Force  $F$  has min/max ratio which is  $\alpha_g^4$ :

$$\alpha_g^4 = \frac{F_{min}}{F_{max}}$$

Mass density has min/max ratio which is  $\alpha_g^5$ :

$$\alpha_g^5 = \frac{\rho_{min}}{\rho_{max}}$$

Perhaps for the minimum distance  $l_{min}$  apply:

$$l_{min} = 2 \cdot e \cdot |p|$$

The maximum distance  $l_{max}$  is:

$$l_{max} = LH = c \cdot H_0^{-1} = \alpha_g^{-1} \cdot l_{min}$$

For the minimum mass  $M_{min}$  apply:

$$M_{min} = \frac{m_{pl}^2}{M_{max}} = \alpha_g m_{pl} = \frac{\alpha_G}{\alpha^3} m_e = \frac{\sqrt[3]{\alpha_g^2}}{\alpha} m_e$$

From the dimensionless unification of the fundamental interactions we discover a new simple Large Number Hypothesis which calculates the Mass, the Age and the Radius of the universe. The expressions for the mass of the observable universe are:

$$MU = a^{-1} \cdot a_g^{-4/3} \cdot m_e = a^3 \cdot a_G^{-2} \cdot m_e = (2 \cdot e \cdot a^2 \cdot N_A)^2 \cdot N_1 \cdot m_p = \mu \cdot a \cdot N_1^2 \cdot m_p = 1.153482 \times 10^{53} \text{ kg}$$

The expressions who calculate the number of protons in the observable universe are:

$$N_{Edd} = \frac{M_U}{m_p} = \mu \alpha N_1^2 = \frac{N_1}{\alpha_g^{2/3}} = \left(2e\alpha^2 N_A\right)^2 N_1 = \left(\frac{r_e}{l_{pl}}\right)^2 N_1 = 6.9 \times 10^{79}$$

The diameter of the observable universe will be calculated to be equal to the ratio of electric force to gravitational force between electron and proton on the reduced Compton wavelength of the electron:

$$2 \cdot R_U = N_1 \cdot \lambda_c$$

The expressions for the radius of the observable universe are:

$$R_U = \frac{\alpha N_1}{2} \alpha_0 = \frac{N_1}{2\alpha} r_e = \frac{1}{2\mu\alpha_G} r_e = \frac{m_{pl}^2 r_e}{2m_e m_p} = \frac{\hbar c r_e}{2Gm_e m_p} = \frac{\alpha \hbar}{2Gm_e^2 m_p}$$

We Found the value of the radius of the universe  $R_U = 4.38 \times 10^{26}$  m. The expressions for the radius of the observable universe are:

$$T_U = \frac{R_U}{c} = \frac{N_1 r_e}{2\alpha c} = \frac{r_e}{2\mu\alpha_G c} = \frac{\alpha N_1 \alpha_0}{2c} = \frac{\alpha \hbar}{2c G m_e^2 m_p} = \frac{\hbar r_e}{2G m_e m_p}$$

For the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{2e^2 \varphi^{-5}}{3^3 \pi \varphi^5} \times 10^{-120}$$

Perhaps for the minimum time  $t_{min}$  apply:

$$t_{min} = 2 \cdot e \cdot t_{pl}$$

We proved the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions propose a possible solution for the density parameters of baryonic matter, dark matter and dark energy:

$$\Omega_B = e^{-n} = i^{2i} = 0.0432 = 4.32\%$$

$$\Omega_\Lambda = 2 \cdot e^{-1} = 0.7357 = 73.57\%$$

$$\Omega_D = 2 \cdot e^{1-n} = 2 \cdot e \cdot i^{2i} = 0.2349 = 23.49\%$$

The sum of the contributions to the total density parameter at the current time is  $\Omega_0 = 1.0139$ . It is surprising that Plato used a dodecahedron as the quintessence to describe the cosmos. A positively curved universe is described by elliptic geometry, and can be thought of as a three-dimensional hypersphere, or some other spherical 3-manifold, such as the Poincaré dodecahedral space, all of which are quotients of the 3-sphere. These results prove that the weather space is finite. The state equation  $w$  has value:

$$w = -24 \cdot e^{-n} = -24 \cdot i^{2i} = -1.037134$$

For as much as  $w < -1$ , the density actually increases with time. The Equations of the Universe are:

$$6e^{5\pi} \frac{G\hbar H_0^2}{c^5} = 10^{42} \frac{\alpha_w^6 \alpha_G^3}{\alpha^6 \alpha_s^{11}}$$

$$e^{7\pi} \frac{G\hbar \Lambda^2}{cH_0^2} = 6 \cdot 10^{42} \frac{\alpha_w^6 \alpha_G^3}{\alpha^6 \alpha_s^{13}}$$

## References

- [1] Pellis, Stergios, Unification Archimedes constant  $\pi$ , golden ratio  $\phi$ , Euler's number  $e$  and imaginary number  $i$  (October 10, 2021)  
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.3975869>
- [2] Pellis, Stergios, Exact formula for the Fine-Structure Constant  $\alpha$  in Terms of the Golden Ratio  $\phi$  (October 13, 2021)  
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4160769>
- [3] Pellis, Stergios, Fine-Structure Constant from the Golden Angle, the Relativity Factor and the Fifth Power of the Golden Mean (September 5, 2022)  
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4247343>
- [4] Pellis, Stergios, Exact expressions of the fine-structure constant (October 20, 2021)  
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.3975861>
- [5] Pellis, Stergios, Fine-structure constant from the Archimedes constant (October 11, 2022)  
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4245208>
- [6] Pellis, Stergios, Fine-Structure Constant from the Madelung Constant (July 27, 2022)  
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4174644>
- [7] Pellis, Stergios, Exact mathematical expressions of the proton to electron mass ratio (October 10, 2021)  
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.3967998>
- [8] Pellis, Stergios, Unity formula that connect the fine-structure constant and the proton to electron mass ratio (November 8, 2021)  
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.3963425>
- [9] Pellis, Stergios, Exact mathematical formula that connect 6 dimensionless physical constants (October 17, 2021)  
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.3963427>
- [10] Pellis, Stergios, Theoretical value for the strong coupling constant (January 1, 2022)  
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.3998175>
- [11] Pellis, S. (2023) Unity Formulas for the Coupling Constants and the Dimensionless Physical Constants. Journal of High Energy Physics, Gravitation and Cosmology, 9, 245-294.  
<https://doi.org/10.4236/jhepgc.2023.91021>
- [12] Pellis, Stergios, Dimensionless Unification of the Fundamental Interactions (August 27, 2022)  
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4201780>
- [13] Pellis, Stergios, Unification of the fundamental interactions (2022)  
DOI: 10.13140/RG.2.2.12296.70405
- [14] Pellis, Stergios, Unification of the Fundamental Forces (2022)  
DOI: 10.13140/RG.2.2.33651.60967
- [15] Pellis, Stergios, Theoretical Value of the Gravitational Constant (May 7, 2023)  
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4442074>
- [16] Pellis, Stergios, Formula for the Gravitational constant (January 2023)  
DOI: 10.13140/RG.2.2.19656.60166
- [17] Pellis, Stergios, Dimensionless Solution for the Cosmological Constant (September 14, 2022)  
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4219292>
- [18] Pellis, Stergios, Unification of atomic physics and cosmology (2022)  
DOI: 10.13140/RG.2.2.11493.88804
- [19] Pellis, Stergios, The Equation of the Universe (2023)  
DOI: 10.13140/RG.2.2.24768.40960
- [29] Pellis, Stergios, Poincaré Dodecahedral Space Solution of The Shape of The Universe (December 31, 2022)  
Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4316617>
- [21] Pellis, Stergios, The Shape of The Universe (March 2023).



doi: 10.13140/RG.2.2.16034.09922

[22] Pellis, Stergios, Solution for the cosmological parameters (December 2022)

DOI: 10.13140/RG.2.2.23917.67047/2

[23] Pellis, Stergios, Solution for the Density Parameter of Dark Energy (December 25, 2022)

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4406279>

[24] Pellis, Stergios, Equation of state in cosmology (December 2022)

DOI: 10.13140/RG.2.2.17952.25609/1

[25] Pellis, Stergios, Maximum and Minimum Values for Natural Quantities (December 10, 2022)

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4306280>

[26] Pellis, Stergios, The coupling constant for the fifth force (April 2023)

DOI: 10.13140/RG.2.2.32481.99686

[27] Pellis, Stergios, Gravitational fine-structure constant (December 2022)

DOI: 10.13140/RG.2.2.34770.43206/2

[28] Pellis, Stergios, Unification of the Microcosm and the Macrocosm (April 24, 2023)

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4429653>

[29] Pellis, Stergios, Dimensionless theory of everything (June 5, 2023)

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4469668>

[30] Pellis, Stergios, The Dimensionless Equations of the Universe (June 23, 2023)

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4488688>

[31] Pellis, Stergios, New Large Number Hypothesis of the universe (August 1, 2023)

Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4529088>

[32] Pellis, Stergios, Theoretical value for the Hubble Constant (August 2023)

DOI: 10.13140/RG.2.2.24461.10729/3