Interactions in Deformed Special Relativity

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In deformed special relativity with commuting coordinates transforming according special relativity and deformed plane waves the field equations and interactions in coordinate space remain unchanged. However in momentum space Dirac and Weyl equations become deformed together with helicity spinors and Mandelstam variables resulting in deformed amplitudes for massive and massless particles.

1. Introduction

Deformed or doubly special relativity (DSR) is a modification of special relativity (SR) with two invariant scales, a velocity scale c (speed of light) and a fundamental length scale ℓ (proportional to the Planck length or inverse Planck mass), see the reviews in [1],[2].[3] and references therein. DSR theories obey an invariant deformed dispersion relation for energy and three-momentum and modified Lorentz transformations. References [4],[5],[6] investigated the possibility to use commuting coordinates transforming according SR with a standard field theory in coordinate space but with deformed plane waves [6]. With the aid of auxiliary variables transforming as in SR one can derive the deformed Lorentz transformations, momenta, massive and massless helicity spinors. An explicit representation of these helicity spinors together with momentum conservation and deformed Mandelstam variables is provided. With these quantities one can write down amplitudes in DSR as shown in several examples.

2. Commuting SR Coordinates

We use the metric $(+,-,-,-)$ and $\hbar = c = 1$. A generic DSR theory can be characterised by a deformed dispersion relation written in the form

$$f(p)^2 = F^2E^2 - G^2\boldsymbol{p}^2 = m^2 \qquad (1)$$

where E is the energy and $\boldsymbol{p}$ the three-momentum of the particle. Here $f^{\mu}(p) = \left(F\,p^0, G\,p^i\right)$ and the functions $F, G(E, \boldsymbol{p}^2, \ell)$ are assumed to preserve spatial rotational symmetry, see text below (2). ℓ is a fundamental length corresponding to a high energy $\ell \sim 1/M$, which may be the Planck mass or some lower scale, and one requires $F, G\big|_{\ell=0} = 1$ in order to obtain SR for low energies. A subtlety arises in the limit $P = |\boldsymbol{p}| = 0$: if F depends only on P^2 then obtains with $\mathrm{F(P = 0) = 1}$ from (1) $\mathrm{E(P = 0) = m_0}$ ($\mathrm{m_0}$ is the rest mass), if F depends only on E then one obtains with $\mathrm{E(P = 0) = m_0}$ from (1) $\mathrm{FE = F(m_0)m_0 = m}$ (m is the Casimir mass). This can be clearly seen from various DSR models in table 1, where again we denote $P = |\boldsymbol{p}|$. It can be proved that the last five entries in table 1 own the usual group property for two subsequent boosts see [12],[6] and appendix C. The models [9],[10],[11] are of course contained in the classes [12],[6].

Table 1: DSR models

DSR model $F^2E^2 - G^2P^2 = m^2$	functions F,G
κ-Poincare [7]	$F = 2\sinh(\ell E/2)/(\ell E)$, $G = \exp(\ell E/2)$
Herranz [8]	$F = (\exp(\ell E) - 1)/(\ell E)$, $G = 1$
Magueijo Smolin [9]	$F = G = (1 - \ell E)^{-1}$
Heuson [10]	$F = G = (1 - \ell^2 P^2)^{-1/2}$
Hinterleitner [11],[10]	$F = G = (1 - \ell^2 E^2)^{-1/2}$
Salesi et al. [12]	$F = G = (1 - \ell^n P^n)^{-1/n}$
Heuson [6]	$F = G = (1 - \ell^n E^n)^{-1/n}$

For a deformed dispersion relation in the form (1) one can always define a map to auxiliary (SR like) variables π^μ obeying a standard dispersion relation

$$\pi^\mu = \begin{pmatrix} \pi^0 & \pi^i \end{pmatrix} = f^\mu(p) = \begin{pmatrix} Fp^0 & Gp^i \end{pmatrix} \tag{2}$$
$$\pi^2 = (\pi^0)^2 - \boldsymbol{\pi}^2 = m^2$$

These auxiliary variables transform under ordinary Lorentz transformations and obey standard momentum conservation, which will be important for the setup of helicity spinors used in amplitudes. Helicity is defined as $h = \frac{\boldsymbol{\pi} \cdot \boldsymbol{S}}{|\boldsymbol{\pi}|} = \frac{G\,\boldsymbol{p} \cdot \boldsymbol{S}}{|G\,\boldsymbol{p}|} = \frac{\boldsymbol{p} \cdot \boldsymbol{S}}{|\boldsymbol{p}|}$ and for the modified dispersion relation (1) remains the same if $G > 0$. One can take this as an argument for considering the spatially isotropic modified dispersion relation in (1) and not an anisotropic relation with different factors G_i in front of every p^i.

Here we are mostly interested in the case $F = G$ with the property that the deformed Lorentz transformations can be written in a very compact form. Furthermore the speed of light equals 1 and is energy independent as can be seen by using $v^i = \pi^i / \pi^0 = Fp^i / Fp^0 = p^i / p^0$ [13]. In [6] we considered in the case $F = G$ commuting coordinates transforming according SR with no momentum dependence in their transformation, but with deformed plane waves. Field theory with gauge invariant interactions remains undeformed in coordinate space by using the commuting SR coordinates. The dispersion relation (1) for $F = G$ is invariant under the transformation of the momenta $p'^\mu = A\Lambda^\mu_\nu p^\nu$ for $F' = F/A$ derived from $F'^2 p'^2 = F'^2 A^2 p^2 = F^2 p^2$. The commuting SR coordinates are here denoted as x_μ and transform as $x'_\mu = \overline{\Lambda}^\nu_\mu x_\nu$, where $\Lambda, \overline{\Lambda}$ are standard, standard inverse Lorentz transformations. The boost and rotation generators are $M_{\mu\nu} = F(p_\nu x_\mu - p_\mu x_\nu)$ and the commutators between them are as usual, as can be seen by introducing the auxiliary SR like momenta $\pi_\mu = F\,p_\mu$. Noticing that $F' = F/A$ the invariants built from these coordinates and the momenta become

$$F^2 p^\mu p_\mu = inv,\; F\,p^\mu x_\mu = inv,\; x^\mu x_\mu = inv \tag{3}$$

The map in (2) can for example be used to derive from $\pi'^\mu = \Lambda^\mu_\nu \pi^\nu$ the deformed Lorentz transformations $p'^\mu = A\Lambda^\mu_\nu p^\nu$ [10]. Similarly one can derive the deformed algebra from the standard algebra of the auxiliary momenta and coordinates [10],[12]. The invariant deformed dispersion relation is $F^2 p^\mu p_\mu = m^2$ and plane waves are then deformed as $\exp(-iF\,p \cdot x)$.

For the following we denote the derivatives with respect to the commuting SR coordinates used in field equations as $\partial_\mu = \partial / \partial x^\mu$ and $\partial^\mu = \partial / \partial x_\mu$.

Now consider the general deformed dispersion relation in (1). The Klein Gordon equation in coordinate space is $(\partial^\mu \partial_\mu + m^2)\varphi(x) = 0$ and by inserting deformed plane waves $\varphi(x) = \varphi_0 \exp(-\mathrm{i}\, f(p)\cdot x)$ one gets again the deformed dispersion relation (1), where $f(p)\cdot x = F\, p^0 x_0 + G\, p^i x_i = F\, E\, t - G\, \boldsymbol{p}\cdot\boldsymbol{x}$. The Dirac equation in coordinate space is $(\mathrm{i}\,\gamma^\mu \partial_\mu - m)\psi(x) = 0$ and the deformed plane wave solutions for particles and antiparticles are $\psi_s(x) = u_s(p)\exp(-\mathrm{i}\, f(p)\cdot x)$, $\chi_s(x) = v_s(p)\exp(+\mathrm{i}\, f(p)\cdot x)$. The Dirac equation then becomes deformed in momentum space and solutions for the spinors u_s, v_s were described in [14],[6]. In Weyl representation with $\sigma^\mu = (1,\sigma^i)$, $\bar\sigma^\mu = (1,-\sigma^i)$ and σ^i the Pauli matrices one inserts the deformed plane waves in the standard Dirac equation in coordinate space to receive the deformed Dirac equation in momentum space, where $f(p)\cdot\sigma = F\, p_0\sigma^0 + G\, p_i\sigma^i$. Similar to the procedure in textbooks [16],[17] one obtains

$$\begin{pmatrix} -m & f(p)\cdot\sigma \\ f(p)\cdot\bar\sigma & -m \end{pmatrix} u_s(p) = 0\ ,\ \begin{pmatrix} -m & -f(p)\cdot\sigma \\ -f(p)\cdot\bar\sigma & -m \end{pmatrix} v_s(p) = 0 \tag{4}$$

One could also consider the Dirac equation in momentum space in auxiliary momenta $(\mathrm{i}\,\gamma^\mu \pi_\mu - m)\psi(\pi) = 0$. By inserting the map (2) one arrives again at (4).

3. Deformed spinor helicity

Amplitudes in particle scattering can be calculated most easily in the spinor helicity formalism see [18],[19],[20]. For massive particles we use the formalism developed in [21] and adopt the notation of [22],[23],[24]. From hereon we denote the auxiliary momenta in a more suggestive form as $p_f^\mu = \pi^\mu = \left(F\, p^0, G\, p^i\right)$ pointing to the physical momenta *p* and the deformation function *f*. They obey standard momentum conservation and depend on the physical momenta p^μ with the deformed dispersion relation (1) written shortly as $p_f^2 = m^2$. In spherical coordinates one can write the momenta and spinors by simply attaching an index called *f*. The formulas for momenta and spinors in DSR are obtained from the formulas in [22]-[24] by substituting $E \to FE$, $P \to GP$, $p \to p_f$. The momentum p_f is then given by

$$p_f^\mu = \left(FE \quad GP\sin(\theta)\cos(\phi) \quad GP\sin(\theta)\sin(\phi) \quad GP\cos(\theta)\right) \tag{5}$$

Denoting the deformed massive spinors as $\left| p^I \right\rangle_f$ and $\left| p^I \right]_f$, the corresponding momentum in bispinor form becomes $p_f\cdot\sigma = \left| p^I \right\rangle_f \left[p_I \right|_f$, $p_f\cdot\bar\sigma = -\left| p^I \right]_f \left\langle p_I \right|_f$ (for explicit representation and relations between them see appendix A). The helicity spinors are

$$\left| p^I \right\rangle_f = \left(\left| n \right\rangle_f \quad \left| p \right\rangle_f \right),\ \left| p^I \right]_f = \left(\left| p \right]_f \quad \text{-}\left| n \right]_f \right) \tag{6}$$

where $\left| p \right\rangle_f = \sqrt{FE+GP}\begin{pmatrix} -s^* \\ c \end{pmatrix}$, $\left| n \right\rangle_f = \sqrt{FE-GP}\begin{pmatrix} c \\ s \end{pmatrix}$, $\left| p \right]_f = \sqrt{FE+GP}\begin{pmatrix} c \\ s \end{pmatrix}$, $\left| n \right]_f = \sqrt{FE-GP}\begin{pmatrix} s^* \\ -c \end{pmatrix}$. For the DSR models with $F = G$ in table 1 one gets $p_f = F\,p$, $\left| p^I \right\rangle_f = \sqrt{F}\left| p^I \right\rangle$, $\left| p^I \right]_f = \sqrt{F}\left| p^I \right]$, where $\left| p^I \right\rangle$ and $\left| p^I \right]$ are the undeformed spinors. The deformed Weyl equations agreeing with the Dirac equation (4) above with $u_f^I = \left(\left| p^I \right\rangle_f \quad \left| p^I \right]_f \right)$ are now

$$p_f\cdot\bar\sigma \left| p^I \right\rangle_f = m\left| p^I \right]_f,\ p_f\cdot\sigma \left| p^I \right]_f = m\left| p^I \right\rangle_f \tag{7}$$

The deformed Lorentz transformations of two component spinors are by using the auxiliary momenta, see also [6]: $\Lambda_f^{|\rangle} = \frac{1}{\sqrt{m}}\sqrt{p_f \cdot \sigma}$ acting on angle spinors and $\Lambda_f^{|]} = \frac{1}{\sqrt{m}}\sqrt{p_f \cdot \bar{\sigma}}$ acting on square spinors. This gives

$$\Lambda_f^{|\rangle} = \frac{1}{\sqrt{m}}\frac{1}{\sqrt{2(FE+m)}}\begin{pmatrix} FE+m-GP(cc-ss^*) & -2GPcs^* \\ -2GPcs & FE+m+GP(cc-ss^*) \end{pmatrix},\ \Lambda_f^{|]} = \Lambda_f^{|\rangle}\Big|_{G\to -G} \tag{8}$$

where we used (A2) and

$$\sqrt{p_f \cdot \sigma} = (p_f \cdot \sigma + m)/\sqrt{2(FE+m)}\ ,\ \sqrt{p_f \cdot \bar{\sigma}} = (p_f \cdot \bar{\sigma} + m)/\sqrt{2(FE+m)} \tag{9}$$

The deformed spinors in a restframe (*RF*) derived from (A4) with $P = 0$ and $FE = m$ are

$$|\,p\,\rangle_f^{RF} = \sqrt{m}\begin{pmatrix} -s^* \\ c \end{pmatrix},\ |\,n\,\rangle_f^{RF} = \sqrt{m}\begin{pmatrix} c \\ s \end{pmatrix},\ |\,p\,]_f^{RF} = \sqrt{m}\begin{pmatrix} c \\ s \end{pmatrix},\ |\,n\,]_f^{RF} = \sqrt{m}\begin{pmatrix} s^* \\ -c \end{pmatrix} \tag{10}$$

Boosting them with Λ_f one finds the deformed spinors in (6) as is shown in appendix B, i.e. we get

$$|\,p\,\rangle_f = \Lambda_f^{|\rangle}|\,p\,\rangle_f^{RF},\ |\,n\,\rangle_f = \Lambda_f^{|\rangle}|\,n\,\rangle_f^{RF},\ \ |\,p\,]_f = \Lambda_f^{|]}|\,p\,]_f^{RF},\ |\,n\,]_f = \Lambda_f^{|]}|\,n\,]_f^{RF} \tag{11}$$

The auxiliary all outgoing momenta obeying standard momentum conservation are written as

$$\sum_i p_{if} = \sum_i |\,i^I\rangle_f\,[\,i_I\,|_f = 0 \tag{12}$$

The deformed Mandelstam variables are

$$s_{ij,f} = (p_{if} + p_{jf})^2 = m_i^2 + m_j^2 + 2p_{if}\cdot p_{jf} \tag{13}$$

For DSR models with $F = G$ one obtains $p_{if} \underset{F=G}{=} F_i\,p_i$, for momentum conservation $\sum_i p_{if} \underset{F=G}{=} \sum_i F_i p_i = 0$ and for the deformed Mandelstam variables $s_{ij,f} \underset{F=G}{=} m_i^2 + m_j^2 + F_iF_j\,2p_i\cdot p_j$. The deformed massive polarisation is defined as $\varepsilon_{if}^{IJ} = \frac{\sqrt{2}}{m}|\,i^I\rangle_f\,[i^J|_f \underset{F=G}{=} F_i\frac{\sqrt{2}}{m}|\,i^I\rangle[i^J|$, where upper indices should be symmetrised.

We now turn to massless spinor helicity in DSR. In the massless case dispersion relation (1) is $F^2E^2 - G^2P^2 = 0$ and gives $GP = FE$. The massless momentum is therefore $\underline{p}_f = p_f\cdot\sigma = 2FE\begin{pmatrix} ss^* & -cs^* \\ -cs & cc \end{pmatrix}$, $\bar{p}_f = p_f\cdot\bar{\sigma} = 2FE\begin{pmatrix} cc & cs^* \\ cs & ss^* \end{pmatrix}$. The corresponding massless spinors are then from (A4) (again $|\,i\,\rangle$ and $|\,i\,]$ are the undeformed spinors) given as

$$|\,i\,\rangle_f = \sqrt{2F_iE_i}\begin{pmatrix} -s_i^* \\ c_i \end{pmatrix} = \sqrt{F_i}\,|\,i\,\rangle,\ |\,i\,]_f = \sqrt{2F_iE_i}\begin{pmatrix} c_i \\ s_i \end{pmatrix} = \sqrt{F_i}\,|\,i\,] \tag{14}$$

$$\langle\,i\,|_f = \sqrt{2F_iE_i}\begin{pmatrix} c_i \\ s_i^* \end{pmatrix} = \sqrt{F_i}\,\langle\,i\,|,\ [\,i\,|_f = \sqrt{2F_iE_i}\begin{pmatrix} -s_i \\ c_i \end{pmatrix} = \sqrt{F_i}\,[\,i\,|$$

The n_i spinors vanish and we have $\underline{p}_{if} = |\, i \,\rangle_f [\, i \,|_f = F_i\, \underline{p}_i$, $\overline{p}_{if} = |\, i \,]_f \langle\, i \,|_f = F_i\, \overline{p}_i$.

The massless Weyl equations are $p_{if} \cdot \sigma \big|\, i^I \,\big]_f = 0$ and $p_{i,f} \cdot \overline{\sigma} \big|\, i^I \,\big\rangle_f = 0$. One might first think that one could omit the factors F_i, however the conserved momenta needed in amplitudes are the p_{if} and not the p_i. Momentum conservation for massless particles is

$$\sum_i p_{if}^{m=0} = \sum_i |\, i \,\rangle_f [\, i \,|_f = \sum_i F_i\, p_i = 0 \tag{15}$$

The deformed massless Mandelstam variables are

$$s_{ij,f}^{m=0} = (p_{if} + p_{jf})^2 = 2 p_{if} \cdot p_{jf} = F_i F_j\, 2 p_i \cdot p_j = F_i F_j s_{ij} \tag{16}$$

As can be seen easily the massless deformed polarisation agrees with the undeformed polarisation:

$$\varepsilon_{i,f}^{+} = \sqrt{2}\frac{|\, i \,]_f \langle\, r_i \,|_f}{\langle\, i \;\; r_i \,\rangle_f} = \sqrt{2}\frac{\sqrt{F_i}\,|\, i \,]\sqrt{F_{ri}}\,\langle\, r_i \,|}{\sqrt{F_i}\sqrt{F_{ri}}\,\langle\, i \;\; r_i \,\rangle} = \sqrt{2}\frac{|\, i \,]\langle\, r_i \,|}{\langle\, i \;\; r_i \,\rangle} = \varepsilon_i^{+}\;,\;\; \varepsilon_{i,f}^{-} = \varepsilon_i^{-}$$

3. Deformed Amplitudes

Based on the previous section we discuss how one can derive amplitudes in DSR from the amplitudes in SR. One simply has to replace the momenta and spinors in SR amplitudes by the deformed ones in (5) and (6) to obtain the deformed amplitudes. We discuss this in massless and massive amplitudes (omitting couplings and other factors) and comment shortly on loops.

Amplitudes with massless particles:

We begin with massless three particle amplitudes with particles i,j,k. Let us define $|\, i \,)_\sigma = \{|\, i \,\rangle \text{if } \sigma = -, |\, \mathrm{i} \,] \text{if } \sigma = +\}$ and similarly $(\, i \;\; j \,)_\sigma = \{\langle\, i \;\; j \,\rangle \text{ if } \sigma = -, [\, i \;\; j \,] \text{ if } \sigma = +\}$ [24]. Here σ is the sign of the total helicity $h = h_i + h_j + h_k$ of the amplitude and we get for the deformed amplitude

$$\mathcal{A}_{3f} = (\, i \;\; j \,)_{\sigma f}^{\sigma(h-2h_k)} (\, j \;\; k \,)_{\sigma f}^{\sigma(h-2h_i)} (\, k \;\; i \,)_{\sigma f}^{\sigma(h-2h_j)} = F_i^{\frac{\sigma}{2}(2h-2h_k-2h_j)} F_j^{\frac{\sigma}{2}(2h-2h_k-2h_i)} F_k^{\frac{\sigma}{2}(2h-2h_i-2h_j)} (\, i \;\; j \,)_{\sigma}^{\sigma(h-2h_k)} (\, j \;\; k \,)_{\sigma}^{\sigma(h-2h_i)} (\, k \;\; i \,)_{\sigma}^{\sigma(h-2h_j)}$$

and thereby

$$\mathcal{A}_{3f} = F_i^{\sigma h_i} F_j^{\sigma h_j} F_k^{\sigma h_k} (\, i \;\; j \,)_{\sigma}^{\sigma(h-2h_k)} (\, j \;\; k \,)_{\sigma}^{\sigma(h-2h_i)} (\, k \;\; i \,)_{\sigma}^{\sigma(h-2h_j)} = F_i^{\sigma h_i} F_j^{\sigma h_j} F_k^{\sigma h_k} \mathcal{A}_3 \tag{17}$$

If $\sigma = +$, then one has to use square brackets, if $\sigma = -$, then use angle brackets. Let us look at a few examples of massless 3 particle amplitudes:

$\mathcal{A}_{3f}(-1/2,+1/2,+1) = F_1^{-1/2} F_2^{+1/2} F_3^{1} [\, 2 \;\; 3 \,]^2 / [\, 1 \;\; 2 \,], (\sigma = +)$

$\mathcal{A}_{3f}(-1/2,+1/2,-1) = F_1^{+1/2} F_2^{-1/2} F_3^{+1} \langle\, 1 \;\; 3 \,\rangle^2 / \langle\, 1 \;\; 2 \,\rangle, (\sigma = -)$

$\mathcal{A}_{3f}(-1/2,+1/2,-2) = F_1^{+1/2} F_2^{-1/2} F_3^{+2} \langle\, 2 \;\; 3 \,\rangle^3 \langle\, 3 \;\; 1 \,\rangle / \langle\, 1 \;\; 2 \,\rangle^2, (\sigma = -)$

$\mathcal{A}_{3f}(-1,+1,+1) = F_1^{-1} F_2^{+1} F_3^{+1} [\, 3 \;\; 2 \,]^3 / [\, 1 \;\; 2 \,][\, 3 \;\; 1 \,], (\sigma = +)$

$\mathcal{A}_{3f}(-2,+2,-2) = F_1^{+2} F_2^{-2} F_3^{+2} \langle\, 3 \;\; 1 \,\rangle^6 / \langle\, 1 \;\; 2 \,\rangle^2 \langle\, 2 \;\; 3 \,\rangle^2, (\sigma = -)$

One sees that the prefactors of the deformed massless 3 particle amplitude are simply given by $F_i^{\sigma h_i}$ for every particle.

The deformed Parke Taylor formula [25] for n gluon scattering is then ($\mathcal{A}_n$ is the undeformed amplitude)

$$\mathcal{A}_{nf} = \frac{\langle i\ j\rangle_f^4}{\langle 1\ 2\rangle_f .. \langle n\ 1\rangle_f} = \frac{F_i^2 F_j^2}{F_1..F_n} \frac{\langle i\ j\rangle^4}{\langle 1\ 2\rangle .. \langle n\ 1\rangle} = \frac{F_i^2 F_j^2}{F_1..F_n} \mathcal{A}_n$$

As an example for massless quark, anti-quark, gluon, gluon scattering take [20]

$$\mathcal{A}_{4f}\left(-1/2,+1/2,-1,+1\right) = \frac{\langle 1\ 3\rangle_f^3 \langle 2\ 3\rangle_f}{\langle 1\ 2\rangle_f \langle 2\ 3\rangle_f \langle 3\ 4\rangle_f \langle 4\ 1\rangle_f} = F_1^{1/2} F_2^{-1/2} F_3^{+1} F_4^{-1} \mathcal{A}_4$$

The spin averaged cross section for $e^+e^- \to \gamma\gamma$ with massless particles given in [18] becomes in the deformed case

$$\left\langle |\mathcal{T}|^2 \right\rangle_f = 2e^4 \left(\left| \frac{s_{13f}}{s_{14f}} \right| + \left| \frac{s_{14f}}{s_{13f}} \right| \right) = 2e^4 \left(\left| \frac{F_3\, s_{13}}{F_4\, s_{14}} \right| + \left| \frac{F_4\, s_{14}}{F_3\, s_{13}} \right| \right)$$

In summary it seems that one can easily obtain massless amplitudes in DSR from undeformed amplitudes by using

$$\big|\, i\,\big)_{\sigma f} = \sqrt{F_i}\, \big|\, i\,\big)_\sigma ,\ \ p_{if} = F_i\, p_i ,\ \ s_{ij,f}^{m=0} = F_i\, F_j s_{ij} \tag{18}$$

We make a very short comment on amplitudes with loops. Since SR is valid for the auxiliary variables p_{if} , it should be possible to overtake the results obtained for loops there. Clearly much more work is needed in this context.
As an example we consider the 4-gluon 1-loop amplitude [20], which in SR can be written as $\mathcal{A}_4^{1-loop}\left(1^-,2^-,3^+,4^+\right) = st\mathcal{A}_4^{tree}\left(1^-,2^-,3^+,4^+\right) I_4\left(s,t\right)+\dots$ Here I_4 is a 4-point box integral containing (di-) logarithms of the Mandelstam variables and $\mathcal{A}_4^{tree} = \langle 1\ 2\rangle^4 / \langle 1\ 2\rangle\langle 2\ 3\rangle\langle 3\ 4\rangle\langle 4\ 1\rangle$. With (18) one can transfer this in the deformed case into $\mathcal{A}_{4f}^{1-loop} = F_1F_2F_1F_3 \dfrac{F_1F_2}{F_3F_4} st\mathcal{A}_4^{tree} I_{4f} + \dots$ The variables s, t in I_4 should be modified according (18).

Amplitudes with massive particles:

Now we consider some deformed amplitudes with massive particles. In three particle amplitudes with two equal massive particles $m_1 = m_2 = m$ and one massless particle $m_3 = 0$ one needs the so called x-factor [21],[22]. From $(p_{1f} + p_{3f})^2 = p_{2f}^2$ one obtains $2p_{1f} \cdot p_{3f} = \langle 3_f |p_{1f}| 3_f] = 0$ and therefore $\langle 3 |_f \propto p_{1f} | 3]_f$ or $m\, x_f^{+1} \langle 3 |_f = [3 |_f\, p_{1f}$.
Contracting with a massless auxiliary spinor $|\, \varsigma\, \rangle_f$ results in the deformed x-factor

$$x_f^{+1} = \frac{[3_f |p_{1f}| \varsigma_f\rangle}{m\langle 3_f\ \varsigma_f\rangle} = \frac{\sqrt{F_\varsigma}\sqrt{F_3}\,[3|p_{1f}|\varsigma\rangle}{m\sqrt{F_\varsigma}\sqrt{F_3}\,\langle 3\ \varsigma\,\rangle} \underset{F=G}{=} \frac{F_1\,[3|p_1|\varsigma\rangle}{m\langle 3\ \varsigma\,\rangle} = F_1 x^{+1} ,\ \ x_f^{-1} \underset{F=G}{=} \frac{F_1\langle 3|p_1|\varsigma]}{m[\,3\ \varsigma\,]} = F_1 x^{-1}$$

or together with $\sigma = \pm 1$ corresponding to the helicity of the massless boson

$$x_f^\sigma \underset{F=G}{=} F_1\, x^\sigma \tag{19}$$

Using the symmetrised x-factor would instead give the more complicated expressions

$$x_f^{+1} = \frac{\langle\varsigma| p_{1f} - p_{2f} |3]}{2m\langle\,\varsigma\ 3\,\rangle} \underset{F=G}{=} \frac{\langle\varsigma| F_1 p_1 - F_2 p_2 |3]}{2m\langle\,\varsigma\ 3\,\rangle} \text{ and } x_f^{-1} \underset{F=G}{=} \frac{\langle 3| F_1 p_1 - F_2 p_2 |\varsigma]}{2m[\,3\ \varsigma\,]}.$$

The amplitude with two equal mass fermions and a massless spin one boson (photon/gluon), where as usual $\big|\, \boldsymbol{i}\,\big)_\sigma = \big|\, i^I\,\big)_\sigma$, is in the deformed case with x_f^σ from (19): $\mathcal{A}_{3f}\left(\mathbf{1},\mathbf{2},3^\sigma\right) = \big(\,\mathbf{1}\ \ \mathbf{2}\,\big)_{-\sigma f}\, x_f^\sigma \underset{F=G}{=} \sqrt{F_1}\sqrt{F_2}F_1 \big(\,\mathbf{1}\ \ \mathbf{2}\,\big)_{-\sigma} x^\sigma$. In the case of a massless spin two boson (graviton) one obtains $\mathcal{A}_{3f}\left(\mathbf{1},\mathbf{2},3^{2\sigma}\right) = \big(\,\mathbf{1}\ \ \mathbf{2}\,\big)_{-\sigma f}\, x_f^{2\sigma} \underset{F=G}{=} \sqrt{F_1}\sqrt{F_2}F_1^2 \big(\,\mathbf{1}\ \ \mathbf{2}\,\big)_{-\sigma} x^{2\sigma}$.

The amplitude with two massive fermions and one massive spin one boson is

$$\mathcal{A}_{3f}\left(\mathbf{1},\mathbf{2},\mathbf{3}\right) = \big(\,\mathbf{2}\ \ \mathbf{3}\,\big)_{-\sigma f} \big(\,\mathbf{3}\ \ \mathbf{1}\,\big)_{\sigma f} \underset{F=G}{=} \sqrt{F_1}\sqrt{F_2}F_3 \big(\,\mathbf{2}\ \ \mathbf{3}\,\big)_{-\sigma} \big(\,\mathbf{3}\ \ \mathbf{1}\,\big)_\sigma$$

As example for the massive four-particle amplitude with s_f from (13) we take

$$\mathcal{A}_{4f}\left(e,\bar{e},\mu,\bar{\mu}\right) \underset{F=G}{=} \frac{e^2}{s_f}\sqrt{F_1F_2F_3F_4}\left(\langle\mathbf{1}\ \mathbf{3}\rangle[\mathbf{2}\ \mathbf{4}]+[\mathbf{1}\ \mathbf{3}]\langle\mathbf{2}\ \mathbf{4}\rangle+(1\leftrightarrow 2)\right)$$

So similarly to deformed amplitudes with only massless particles we get for DSR models with $F=G$ in the massive case prefactors according the rule

$$|\,\boldsymbol{i}\,)_{\sigma f} \underset{F=G}{=} \sqrt{F_i}\,|\,\boldsymbol{i}\,)_{\sigma}\,,\ p_{i\,f} \underset{F=G}{=} F_i\ p_i\,,\ s_{ij,f} \underset{F=G}{=} m_i^2+m_j^2+F_iF_j\,2p_i\cdot p_j \tag{20}$$

4. Summary

In summary we have investigated DSR theories based on commutative SR coordinates with deformed plane waves. The field theory and interaction structure in space-time remains unchanged in these coordinates, while the Lorentz transformations and spinor solutions in momentum space are modified. The physical energy and three momentum of a particle are E and $\boldsymbol{p}$ obeying the deformed dispersion relation (1). Using the map to the auxiliary momenta in (2) one can investigate momenta, deformed helicity spinors with their deformed Lorentz transformations and by boosting from a restframe one obtains deformed massive helicity spinors. We also considered deformed massless helicity spinors, which must differ from the undeformed ones since momentum conservation is only valid for the auxiliary momenta. Based on this one can derive deformed amplitudes in DSR by substituting the equations (18) or (20) in the massless or massive amplitudes obtained in SR. The deformed amplitudes now are depending on the deformation functions F and G as one would expect from the beginning. Several examples for deformed amplitudes in the massless and massive case are shown.

The serious question arising immediately is of course, how can one check the connection with nature? In a recent review on quantum gravity phenomenology [26] many possibilities for testing DSR are suggested. There also may be a relation to the recent work [27] proposing that the deformation affects only interactions between elementary particles and thus avoids some conceptual problems in DSR theories.

Appendix A: Momenta and helicity spinors in DSR

Here we provide an explicit representation for massive spinors (spin spinors) in spherical coordinates used in [22],[23],[24] adapted to DSR based on momentum

$$p_f^\mu = \begin{pmatrix} FE & GP\sin(\theta)\cos(\phi) & GP\sin(\theta)\sin(\phi) & GP\cos(\theta) \end{pmatrix} \tag{A1}$$

Using the Pauli matrices σ^i and $\sigma^\mu = (1,\sigma^i)$, $\bar{\sigma}^\mu = (1,-\sigma^i)$ the momentum can be written in bispinor form as $\underline{p}_f = p_{f\alpha\dot{\alpha}} = p_{f\mu}\sigma^\mu = p_f \cdot \sigma$ and $\bar{p}_f = p_f^{\dot{\alpha}\alpha} = p_{f\mu}\bar{\sigma}^\mu = p_f \cdot \bar{\sigma}$ where as usual $c = \cos(\theta/2)$, $s = sin(\theta/2)exp(i\cdot\phi)$, $s^* = sin(\theta/2)exp(-i\cdot\phi)$ and we have $cc + ss^* = 1$. Note that $P = |\boldsymbol{p}|$.

$$\underline{p}_f = p_f \cdot \sigma = \begin{pmatrix} FE - GP(cc-ss^*) & -2GPcs^* \\ -2GPcs & FE + GP(cc-ss^*) \end{pmatrix},\ \bar{p}_f = p_f \cdot \bar{\sigma} = \begin{pmatrix} FE + GP(cc-ss^*) & 2GPcs^* \\ 2GPcs & FE - GP(cc-ss^*) \end{pmatrix} \tag{A2}$$

One can check that $det(\underline{p}_f) = det(\bar{p}_f) = m^2$ gives the deformed dispersion relation (1). In DSR models with $F = G$ one obtains simply $p_f^\mu = F\,p^\mu$, $p_f \cdot \sigma = F\,p \cdot \sigma$ and $p_f \cdot \bar{\sigma} = F\,p \cdot \bar{\sigma}$. For several particles $i = 1..n$ we denote the massive spinors as $|\,\mathrm{i}^\mathrm{I}\rangle_\mathrm{f} = |\,\mathrm{p}_\mathrm{i}^\mathrm{I}\,\rangle_\mathrm{f}$ (I=1,2=SU(2) little group index) and their contractions as $\langle\,\mathrm{i}^\mathrm{I}\ \mathrm{j}^\mathrm{J}\rangle_\mathrm{f}$ etc. Lowercase index spinors are obtained by $|\,\mathrm{i}_\mathrm{I}\rangle_\mathrm{f} = \epsilon_{\mathrm{IJ}}\,|\,\mathrm{i}^\mathrm{I}\rangle_\mathrm{f}$ (ϵ_{IJ} is the usual Levi-Civita tensor) and mirrored spinors by $|\ \rangle \to \langle\ |$, similarly for square brackets. The spinors are then explicitly

$$\begin{aligned} &|\,\mathrm{i}^\mathrm{I}\rangle_\mathrm{f} = \left(|\,\mathrm{n_i}\rangle_\mathrm{f}\ \ |\,\mathrm{i}\rangle_\mathrm{f}\right),\ \langle\mathrm{i}^\mathrm{I}|_\mathrm{f} = \left(\langle\,\mathrm{n_i}|_\mathrm{f}\ \ \langle\,\mathrm{i}\,|_\mathrm{f}\right),\ |\,\mathrm{i}^\mathrm{I}]_\mathrm{f} = \left(|\,\mathrm{i}\,]_\mathrm{f}\ \ -|\,\mathrm{n_i}]_\mathrm{f}\right),\ [\mathrm{i}^\mathrm{I}\,|_\mathrm{f} = \left([\,\mathrm{i}\,|_\mathrm{f}\ \ -[\mathrm{n_i}|_\mathrm{f}\right) \\ &|\,\mathrm{i_I}\rangle_\mathrm{f} = \left(-|\,\mathrm{i}\,\rangle_\mathrm{f}\ \ |\,\mathrm{n_i}\rangle_\mathrm{f}\right),\ \langle\,\mathrm{i_I}|_\mathrm{f} = \left(-\langle\,\mathrm{i}\,|_\mathrm{f}\ \ \langle\,\mathrm{n_i}|_\mathrm{f}\right),\ |\,\mathrm{i_I}]_\mathrm{f} = \left(|\,\mathrm{n_i}]_\mathrm{f}\ \ |\,\mathrm{i}\,]_\mathrm{f}\right),\ [\,\mathrm{i_I}\,|_\mathrm{f} = \left([\,\mathrm{n_i}|_\mathrm{f}\ \ [\,\mathrm{i}\,|_\mathrm{f}\right) \end{aligned} \tag{A3}$$

$$\begin{aligned} &|\,\mathrm{i}\,\rangle_\mathrm{f} = \sqrt{\mathrm{F_iE_i + G_iP_i}}\begin{pmatrix} -\mathrm{s_i^*} \\ \mathrm{c_i} \end{pmatrix},\ |\,\mathrm{n_i}\rangle_\mathrm{f} = \sqrt{\mathrm{F_iE_i - G_iP_i}}\begin{pmatrix} \mathrm{c_i} \\ \mathrm{s_i} \end{pmatrix},\ \langle\,\mathrm{i}\,|_\mathrm{f} = \sqrt{\mathrm{F_iE_i + G_iP_i}}\begin{pmatrix} \mathrm{c_i} \\ \mathrm{s_i^*} \end{pmatrix},\ \langle\,\mathrm{n_i}\,|_\mathrm{f} = \sqrt{\mathrm{F_iE_i - G_iP_i}}\begin{pmatrix} \mathrm{s_i} \\ -\mathrm{c_i} \end{pmatrix} \\ &|\,\mathrm{i}\,]_\mathrm{f} = \sqrt{\mathrm{F_iE_i + G_iP_i}}\begin{pmatrix} \mathrm{c_i} \\ \mathrm{s_i} \end{pmatrix},\ |\,\mathrm{n_i}]_\mathrm{f} = \sqrt{\mathrm{F_iE_i - G_iP_i}}\begin{pmatrix} \mathrm{s_i^*} \\ -\mathrm{c_i} \end{pmatrix},\ [\,\mathrm{i}\,|_\mathrm{f} = \sqrt{\mathrm{F_iE_i + G_iP_i}}\begin{pmatrix} -\mathrm{s_i} \\ \mathrm{c_i} \end{pmatrix},\ [\,\mathrm{n_i}\,|_\mathrm{f} = \sqrt{\mathrm{F_iE_i - G_iP_i}}\begin{pmatrix} \mathrm{c_i} \\ \mathrm{s_i^*} \end{pmatrix} \end{aligned} \tag{A4}$$

In DSR models with $F = G$ one gets $|\,\mathrm{i}^\mathrm{I}\rangle_\mathrm{f} = \sqrt{\mathrm{F_i}}\,|\,\mathrm{i}^\mathrm{I}\rangle_\mathrm{f}$, and similar relations for the other spinors. One can express the momentum in (A2) in terms of the spin spinors

$$\underline{\mathrm{p}}_{\mathrm{i\,f}} = |\,\mathrm{i}^\mathrm{I}\rangle_\mathrm{f}[\,\mathrm{i_I}\,|_\mathrm{f} = |\,\mathrm{i}\,\rangle_\mathrm{f}[\,\mathrm{i}\,|_\mathrm{f} + |\mathrm{n_i}\rangle_\mathrm{f}[\mathrm{n_i}|_\mathrm{f},\ \bar{\mathrm{p}}_{\mathrm{i\,f}} = -|\,\mathrm{i}^\mathrm{I}\,]_\mathrm{f}\langle\,\mathrm{i_I}\,|_\mathrm{f} = |\,\mathrm{i}\,]_\mathrm{f}\langle\,\mathrm{i}\,|_\mathrm{f} + |\,\mathrm{n_i}\,]_\mathrm{f}\langle\,\mathrm{n_i}\,|_\mathrm{f} \tag{A5}$$

One can also check that the relations for massive spinors in [22],[23],[24] are fulfilled but now with an index *f* on momenta and spin spinors and we note here several of them

$$\begin{aligned} &[\mathrm{i}^\mathrm{I}\ \mathrm{i}^\mathrm{J}]_\mathrm{f} = \mathrm{m_i}\epsilon^{\mathrm{IJ}} = -\langle\,\mathrm{i}^\mathrm{I}\ \mathrm{i}^\mathrm{J}\rangle_\mathrm{f},\ [\,\mathrm{i_I}\ \mathrm{i_J}]_\mathrm{f} = -\mathrm{m_i}\epsilon_{\mathrm{IJ}} = -\langle\,\mathrm{i_I}\ \mathrm{i_J}\rangle_\mathrm{f} \\ &[\,\mathrm{i}^\mathrm{I}\ \mathrm{i_J}]_\mathrm{f} = -\mathrm{m_i}\delta_\mathrm{J}^\mathrm{I} = -\langle\,\mathrm{i}^\mathrm{I}\ \mathrm{i_J}\rangle_\mathrm{f},\ [\,\mathrm{i}^\mathrm{I}\ \mathrm{i_I}]_\mathrm{f} = -2\mathrm{m_i} = -\langle\,\mathrm{i}^\mathrm{I}\ \mathrm{i_I}\rangle_\mathrm{f} \\ &|\,\mathrm{i}^\mathrm{I}]_\mathrm{f}[\,\mathrm{i_I}\,|_\mathrm{f} = \mathrm{m_i}\delta_\mathrm{a}^\mathrm{b} = -|\,\mathrm{i}^\mathrm{I}\rangle_\mathrm{f}\langle\,\mathrm{i_I}|_\mathrm{f},\ \langle\,\mathrm{i}\ \mathrm{n_i}\rangle_\mathrm{f} = \mathrm{m_i} = -[\,\mathrm{i}\ \mathrm{n_i}\,]_\mathrm{f} \\ &|\,\mathrm{i}^\mathrm{I}\rangle_\mathrm{f}[\,\mathrm{i_I}\,|_\mathrm{f} = \underline{\mathrm{p}}_{\mathrm{i\,f}},\ -|\,\mathrm{i}^\mathrm{I}\,]_\mathrm{f}\langle\,\mathrm{i_I}|_\mathrm{f} = \bar{\mathrm{p}}_{\mathrm{i\,f}} \\ &\mathrm{p_{i\,f}}\cdot\bar{\sigma}|\,\mathrm{i}^\mathrm{I}\rangle_\mathrm{f} = \mathrm{m_i}\,|\,\mathrm{i}^\mathrm{I}\,]_\mathrm{f},\ \mathrm{p_{i\,f}}\cdot\sigma|\,\mathrm{i}^\mathrm{I}\,]_\mathrm{f} = \mathrm{m_i}\,|\,\mathrm{i}^\mathrm{I}\rangle_\mathrm{f} \end{aligned} \tag{A6}$$

Appendix B: Boost from a restframe

We begin with showing $|\,p\rangle_f = \Lambda_f^{|\rangle}\,|\,p\rangle_f^{RF}$. By inserting (A4), (8), (10) one obtains

$$\sqrt{FE+GP}\begin{pmatrix}-s^* \\ c\end{pmatrix} \overset{!}{=} \frac{1}{\sqrt{m}}\frac{1}{\sqrt{2(FE+m)}}\begin{pmatrix}FE+m-GP(cc-ss^*) & -2GPcs^* \\ -2GPcs & FE+m+GP(cc-ss^*)\end{pmatrix}\sqrt{m}\begin{pmatrix}-s^* \\ c\end{pmatrix} \text{ or with } cc+ss^*=1$$

$$\sqrt{2(FE+m)}\sqrt{FE+GP}\begin{pmatrix}-s^* \\ c\end{pmatrix} = \begin{pmatrix}-s^*\left(FE+m-GPcc+GPss^*+2GPcc\right) \\ c\left(2GPss^*+FE+m+GPcc-GPss^*\right)\end{pmatrix} = \begin{pmatrix}-s^*\left(FE+m+GP\right) \\ c\left(FE+m+GP\right)\end{pmatrix}.$$

Equating both vector components gives after squaring $2(FE+m)(FE+GP) = (FE+GP+m)^2$ and therefore $2F^2E^2 + 2FEGP + 2m(FE+GP) = F^2E^2 + G^2P^2 + 2FEGP + 2m(FE+GP) + m^2$. With $m^2 = F^2E^2 - G^2P^2$ one sees, that both sides are equal concluding the proof, which of course also holds in SR with $F = G = 1$.

For $|\,n\rangle_f = \Lambda_f^{|\rangle}\,|\,n\rangle_f^{RF}$ one obtains

$$\sqrt{FE-GP}\begin{pmatrix}c \\ s\end{pmatrix} \overset{!}{=} \frac{1}{\sqrt{m}}\frac{1}{\sqrt{2(FE+m)}}\begin{pmatrix}FE+m-GP(cc-ss^*) & -2GPcs^* \\ -2GPcs & FE+m+GP(cc-ss^*)\end{pmatrix}\sqrt{m}\begin{pmatrix}c \\ s\end{pmatrix} \text{ or}$$

$$\sqrt{2(FE+m)}\sqrt{FE-GP}\begin{pmatrix}c \\ s\end{pmatrix} = \begin{pmatrix}c(FE+m-GPcc+GPss^*-2GPss^*) \\ s(-2GPcc+FE+m+GPcc-GPss^*)\end{pmatrix} = \begin{pmatrix}c(FE+m-GP) \\ s(FE+m-GP)\end{pmatrix}.$$

Equating the components and squaring gives $2(FE+m)(FE-GP) = (FE-GP+m)^2$ or
$2F^2E^2 - 2FEGP + 2m(FE-GP) = F^2E^2 + G^2P^2 - 2FEGP + 2m(FE-GP) + m^2$ and again both sides coincide.

The proof for the square spinors $|\,p\,]_f = \Lambda_f^{|]}\,|\,p\,]_f^{RF}$, $|\,n\,]_f = \Lambda_f^{|]}\,|\,n\,]_f^{RF}$ is analogous.

Appendix C: Group property of DSR models

Here we show the proof of the group property in DSR classes [6],[12]. We begin with the class of models $F = G = (1-\ell^n E^n)^{-1/n}$. Two subsequent deformed Lorentz transformations are given as $p' = A\Lambda p$ and $p'' = A'\Lambda' p'$, where Λ, Λ' are standard Lorentz transformations. We introduce the additive rapidity ξ defined by $ch(\xi) = \gamma$, $sh(\xi) = \gamma v$, $th(\xi) = v$ and employ the hyperbolic relations $ch(\xi')ch(\xi) + sh(\xi')sh(\xi) = ch(\xi+\xi') = ch(\xi'')$ as well as $ch(\xi')sh(\xi) + sh(\xi')ch(\xi) = sh(\xi+\xi') = sh(\xi'')$. The factor A is from [6] $A = \left(1-\ell^n E^n + \ell^n\left(ch(\xi)E - sh(\xi)p_x\right)^n\right)^{-1/n}$ and the deformed Lorentz transformations are $E' = A\left(ch(\xi)E - sh(\xi)p_x\right)$, $p_x' = A\left(ch(\xi)p_x - sh(\xi)E\right)$, $p_y' = Ap_y$, $p_z' = Ap_z$. Their product should be $p'' = A'\Lambda' p' = A'\Lambda' A\Lambda p = A'A\,\Lambda'\Lambda p \overset{!}{=} A''\Lambda'' p$. It is well known that $\Lambda'' = \Lambda'\Lambda$, so we only have to show that $A'' = A'A$. From $F' = F/A$ we get $1-\ell^n E'^n = A^n\left(1-\ell^n E^n\right)$.

$$\begin{aligned}
A' &= \left(1-\ell^n E'^n + \ell^n\left(ch(\xi')E' - sh(\xi')p_x'\right)^n\right)^{-1/n} \\
&= \left(\left(1-\ell^n E^n\right)A^n + \ell^n A^n\left(ch(\xi')\left(ch(\xi)E - sh(\xi)p_x\right) - sh(\xi')\left(ch(\xi)p_x - sh(\xi)E\right)\right)^n\right)^{-1/n} \\
&= A^{-1}\left(1-\ell^n E^n + \ell^n\left(\left(ch(\xi')ch(\xi)+sh(\xi')sh(\xi)\right)E - \left(ch(\xi')sh(\xi)+sh(\xi')ch(\xi)\right)p_x\right)^n\right)^{-1/n} \\
&= A^{-1}\left(1-\ell^n E^n + \ell^n\left(ch(\xi'')E - sh(\xi'')p_x\right)^n\right)^{-1/n} = A^{-1}A''
\end{aligned}$$

So it follows $A'A = A''$ which concludes the proof.

For the class of models $F = G = (1-\ell^n P^n)^{-1/n}$ one has $A = \left(1-\ell^n P^n + \ell^n\left(\left(ch(\xi)p_x - sh(\xi)E\right)^2 + p_y^2 + p_z^2\right)^{n/2}\right)^{-1/n}$ [6] and from $F' = F/A$ one has $1-\ell^n P'^n = A^n\left(1-\ell^n P^n\right)$. This gives similar to above

$$A' = \left(1-\ell^n P'^n + \ell^n\left(\left(ch(\xi)p_x' - sh(\xi)E'\right)^2 + p_y'^2 + p_z'^2\right)^{n/2}\right)^{-1/n} = A^{-1}\left(1-\ell^n P^n + \ell^n\left(\left(ch(\xi'')p_x - sh(\xi'')E\right)^2 + p_y^2 + p_z^2\right)^{n/2}\right)$$

and again $A'A = A''$.

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