Resolved: Inconsistency of Special Relativity

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Abstract

Symmetric clocks paradox using two clocks in a special setup, in which both clocks are in inertial movement on the entire duration of experiment. This version include the correction of an error of synchronization that existed in the earlier versions, with this correction special relativity remain consistent in this symmetric case.

1 The Clocks Paradox

The clocks paradox also known as the twins paradox is a direct consequence of the symmetry of the relativity principle in the context of constant light speed principle. One consequence of special relativity, among others, is the time dilation witch imply that in a relative moving inertial frame with respect to a reference frame, the coordinate time intervals become larger compared with the proper time intervals in the moving frame and consequently the clocks run slower than the clocks from the reference frame. The theory of relativity claim that this is a real physical effect which affect the proper time of the relative moving system compared with the proper time of the reference system which is equal with the coordinate time. Also a number of experiments seem to indicate that this is a real effect. In consequence a relation between the proper time intervals counted by two clocks in relative movement to each other must exist and this relation must be consistent in any valid analysis of special relativity, from any valid inertial frame.

The root of the problem is that while relativity principle is active, we cannot have a sense of which is in motion, instead any group of inertial systems can be considered in motion relative to each other. Because the relativistic kinetic effects are dependent on this relative velocities, is not dificult to see that this situation may lead to inconsistent arbitrary predictions, since the reference frame can be arbitrarily chosen.

1.1 The Classic Twins Paradox

Is the well known case of twins paradox, or clocks paradox, the original two clocks (twins) variant is pretty useless because imply accelerations and fall outside the scope of special relativity, which leave room for various interpretations. Lets suppose we have two clocks A and B, initialy both clocks are in the same reference frame having the same state of motion. The clocks counters are cleared to 0 and the clock B is accelerated at the speed v with respect to the clock A which remain in the same state of motion. After a while the clock B is accelerated again and it is turning back with the same speed v with respect to the clock A, until it reach the clock A and the clocks counters are compared. Analyzing the problem from the clock A reference frame, which is a valid inertial reference frame on the entire duration of experiment, will result that the clock B has lag behind the clock A due to the kinetic time dilation caused by the moving of B with respect to A (ignoring the effects of accelerations). However the same analysis can be made from the clock B reference frame, which see that the clock A is moving with respect to B and consequently the clock A will lag behind the clock B due to time dilation. However the problem is that the clock B experience accelerations and change reference frames on the duration of experiment and consequently is not a valid reference frame from the point of view of special relativity. As a result this case cannot be considered a clear paradox of special relativity.

2 Symmetric Clocks Paradox

In this case the acceleration is eliminated with the purpose to create a version where both clocks reside in inertial systems on the entire duration of experiment and are equally entitled to be used as reference frames. To be able to define a relation of simultaneity between the two clocks, we will send light pulses between them, similar to the method used by Einstein in its 1905 paper [1], since the light propagation speed is an invariant in special relativity.

Lets suppose we have two very long rods, every rod have a clock at one end and a marker at the other end. The marker (e.g. a small magnet) can be sensed by an appropriate sensor embedded in each clock, when the clock pass near it. Also both clocks can sense the proximity of the other clock by an appropiate sensor ambedded in each clock. Now these two rods are already in motion with respect to each other with the velocity v on an approaching trajectory with the clocks in the front of movement direction, like in figure 1. We arbitrary name them rod A and rod B, however the analysis is symmetrical.

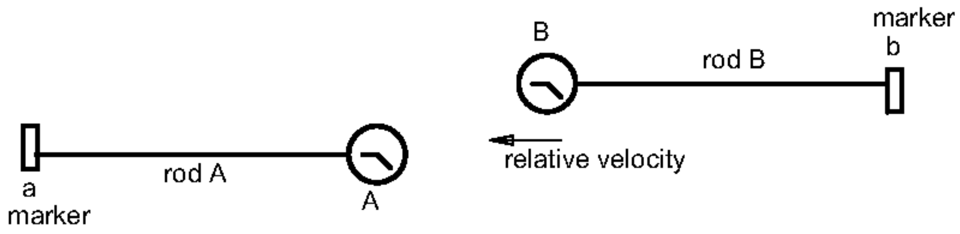

Figure 1: Initial setup of rod A and rod B

After a while both clocks arrive in the proximity of each other, like in figure 2, which represent the zero syncronization moment. This moment is simultaneous for both clocks, they having virtually the same position in space. At this moment both clocks reset their counters to zero, so we will call it the zero moment. This is the starting moment of our experiment.

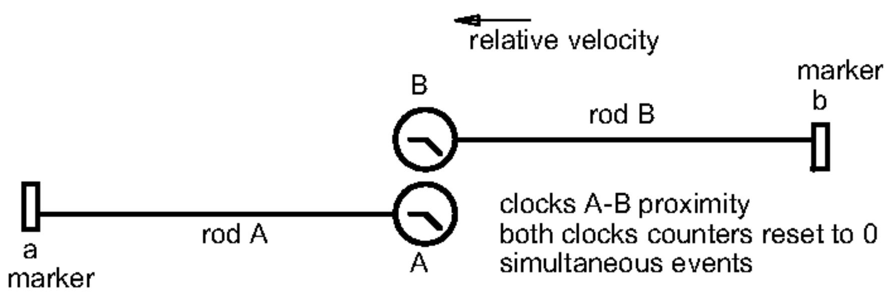

Figure 2: Zero syncronization moment

After a time interval the clock B will arrive in the proximity of marker a (this is event E1), like in figure 3. In a similar way the clock A will arrive in the proximity of marker b (this is event E2). There is no simultaneity between events E1 and E2 in our imaginary experiment.

When the clock B arrive in the proximity of marker a , two simultaneous events happens: first a light pulse is send toward the clock A by the clock B and second the clock B memorize its counter. Similar but unrelated events

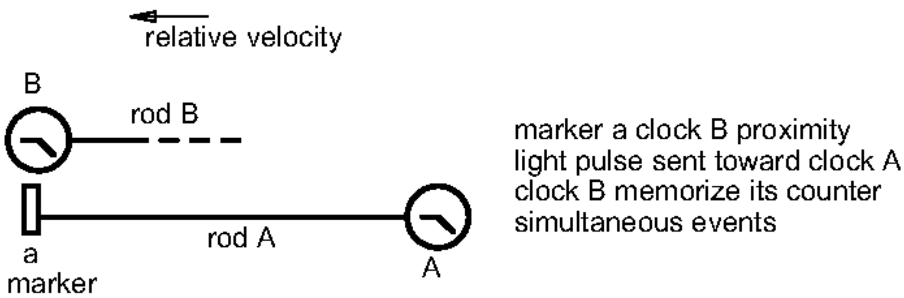

Figure 3: Marker a proximity moment

happens when the clock A arrive in the proximity of marker b , it will sent a light pulse toward clock B. The light pulse sent from the position of marker a will arrive at clock A after a delay, expressed as proper time of clock A equal with L/c where L is the lenght of the rods. When receiveing this light pulse, the clock A will memorize its counter and then will substract from this memorized value the known value of the light pulse delay that is invariant for the clock A. In this way the clock A have the value of its own counter at the moment of marker a clock B proximity, moment simultaneous with the memorize of the clock B counter. In consequence the clocks A and B proper times, accumulated between the zero moment and event E1, can be compared. Similar but unrelated events happen in the clock B when it receive the light pulse from the clock A. This setup will allow the comparition of the proper times of clocks between moments that are simultaneous in both systems(rods), respectively the zero moment and the marker-clock proximity moment. For the rod A the relevant proximity moment is marker a-clockB proximity (E1), while for the rod B is marker b-clockA proximity (E2).

After this the experiment ends, the memorized values can be compared using any practical method, by radio communication, or by bringing the clocks together, the experiment being over now. As can be observed, both clocks (and their corresponding rod) remain in an inertial moving state on the entire relevant duration of experiment, in consequence both clocks are entitled to be used as refrence frame. We will use

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

2.1 The event E1 case

Interval 0 to event E1 using rod A as reference frame. The clock A will count the proper time of itself until the clock B reach the marker a , as

$$\tau_{A1} = \frac{L}{v} = t_{A1} \tag{1}$$

The proper time counted by the clock B until it reach the marker a, considering the time t_{A1} dilation for the moving clock B, will be

$$\tau_{B1} = \frac{t_{A1}}{\gamma} = \frac{L}{v\gamma} \tag{2}$$

Interval 0 to event E1 using rod B as reference frame. The clock B (now used as reference) will count the proper time of itself until it reach the marker a , considering the lenght of the moving rod A contracted, as

$$\tau_{B1} = \frac{L}{\gamma v} = t_{B1} \tag{3}$$

which is identic with (2). Considering the time t_{B1} dilation for the moving rod A, the marker a will count its proper time as

$$\tau_{a1} = \frac{t_{B1}}{\gamma} = \frac{L}{\gamma^2 v} \tag{4}$$

However (this is the mistake made in earlier versions), in frame B, the rod A is moving, and because rod A is moving, its internal clocks are not synchronized with each other. This is standard relativity for any moving rod. The front moving clock A always reads a time that is leading the rear marker a, viewed from the frame B that now is reference. According to the special relativity, the time offset between two clocks separated by a proper length L moving at speed v is:

$$\triangle\tau = \frac{vL}{c^2} \tag{5}$$

with this the proper time counted by clock A until clock B reach the marker a become:

$$\tau_{A1} = \frac{L}{\gamma^2 v} + \frac{vL}{c^2} = \frac{L}{c^2}\left(\frac{c^2 - v^2}{v} + v\right) = \frac{L}{v} \tag{6}$$

which is identic with (1).

Because the proper time intervals are counted between the same two points which are simultaneous in both frames, zero moment and event E1, result that the clock A run faster than the clock B, independent of how we choose the reference frame. This result is clearly consistent with reality and experiment, indicating that special relativity remain consistent in a symmetric setup.

2.2 The event E2 case

Interval 0 to event E2 using rod B as reference frame. The clock B will count the proper time of itself until the clock A reach the marker b , as

$$\tau_{B2} = \frac{L}{v} = t_{B2} \tag{7}$$

The proper time counted by the clock A until it reach the marker b, considering the time t_{B2} dilation for the moving clock A, will be

$$\tau_{A2} = \frac{t_{B2}}{\gamma} = \frac{L}{v\gamma} \tag{8}$$

Interval 0 to event E2 using rod A as reference frame. The clock A (now used as reference) will count the proper time of itself until it reach the marker b , considering the lenght of the moving rod B contracted, as

$$\tau_{A2} = \frac{L}{\gamma v} = t_{A2} \tag{9}$$

which is identic with (8). Considering the time t_{A2} dilation for the moving rod B, the marker b will count its proper time as

$$\tau_{b2} = \frac{t_{A2}}{\gamma} = \frac{L}{\gamma^2 v} \tag{10}$$

Like in the case for event E1, in frame A, the rod B is moving, and because rod B is moving, its internal clocks are not synchronized with each other. The front moving clock B always reads a time that is leading the rear marker b, viewed from the frame A that now is reference. The proper time counted by clock B until clock A reach the marker b become:

$$\tau_{B2} = \frac{L}{\gamma^2 v} + \frac{vL}{c^2} = \frac{L}{c^2}\left(\frac{c^2 - v^2}{v} + v\right) = \frac{L}{v} \tag{11}$$

which is identic with (7).

Because the proper time intervals are counted between the same two points which are simultaneous in both frames, zero moment and event E2, result that the clock A run faster than the clock B, independent of how we choose the reference frame. This result is clearly consistent with reality and experiment, indicating that special relativity remain consistent in a symmetric setup.

3 Conclusion

All these results show that, when the principle of relativity is rigorously applied, the theory of special relativity remain consistent about what is happened with the two clocks in this setup that is still asymmetric but with no accelerations. Also one may rise the objection that the time intervals involved in event E1 shows that clock B run slower than A, while the time intervals involved in event E2 shows the opposite. This is true, but the two events E1 and E2 are not simultaneous events and no sinchronization is defined between them, as a result the time intervals involved in the events E1 and E2 can not be compared directly since in special relativity the sense of time is not absolute. The setup can be slightly modified into a truly symmetric version, like the 4-J variant presented in [8], in such a case the clocks will run at the same speed and the prediction will remain consistent.

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