The Langenscheidt's Pocket Russian Dictionary and The Graphical Law

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Abstract

We study the words of the Langenscheidt's Pocket Russian Dictionary, 2012 Reprint. We draw the natural logarithm of the number of words, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised. We conclude that the dictionary can be characterised by the magnetisation curve, BP(4,$\beta H = 0.01$), in the Bethe-Peierls approximation of the Ising model, in the presence of four nearest neighbours and little external magnetic field, H, with $\beta H = 0.01$. β is $\frac{1}{k_B T}$ where, T is temperature and k_B is the tiny Boltzmann constant.
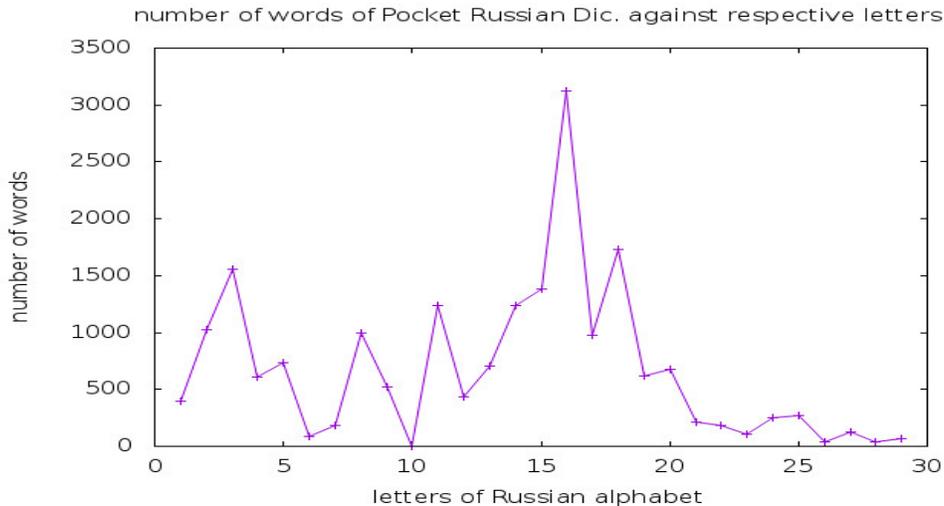
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FIG. 1. The vertical axis is the number of words of the Langenscheidt's Pocket Russian Dictionary, [1]. The horizontal axis is the letters of the Russian alphabet. Letters are represented by the sequence number in the alphabet as it appears in the figure, fig.2.

I. INTRODUCTION

"And Quiet Flows the Don"....the title of a novel by Mikhail Sholokhov.

Flows also the Lena, flows the Volga, flows the Russian language. Written in the Russian Cyrilic script, the Russian language is the medium of conversation for many many people in this world. Here in this paper, we study the Russian language. We study the words. We study the Russian words appearing in the Langenscheidt's Pocket Russian Dictionary, [1]. We count the words of the Langenscheidt's Pocket Russian Dictionary, 2012 Reprint, [1], one by one. The result is the figure, fig.2, the table, tableI. To visualise we plot the number of words against the respective letters as it appears in the figure, fig.2, in the adjoining figure, fig.1.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
397	1027	1559	609	739	90	182	995	523	2	1234	432	701	1236	1386
16	17	18	19	20	21	22	23	24	25	26	27	28	29	
3120	976	1733	621	681	208	188	104	248	267	43	122	34	70	

TABLE I. Words of the Langenscheidt's Pocket Russian Dictionary: the odd rows represent letters of the Russian alphabet, in the serial order, represented by their sequence number in the figure, fig.2, the even rows represent the numbers of corresponding words.
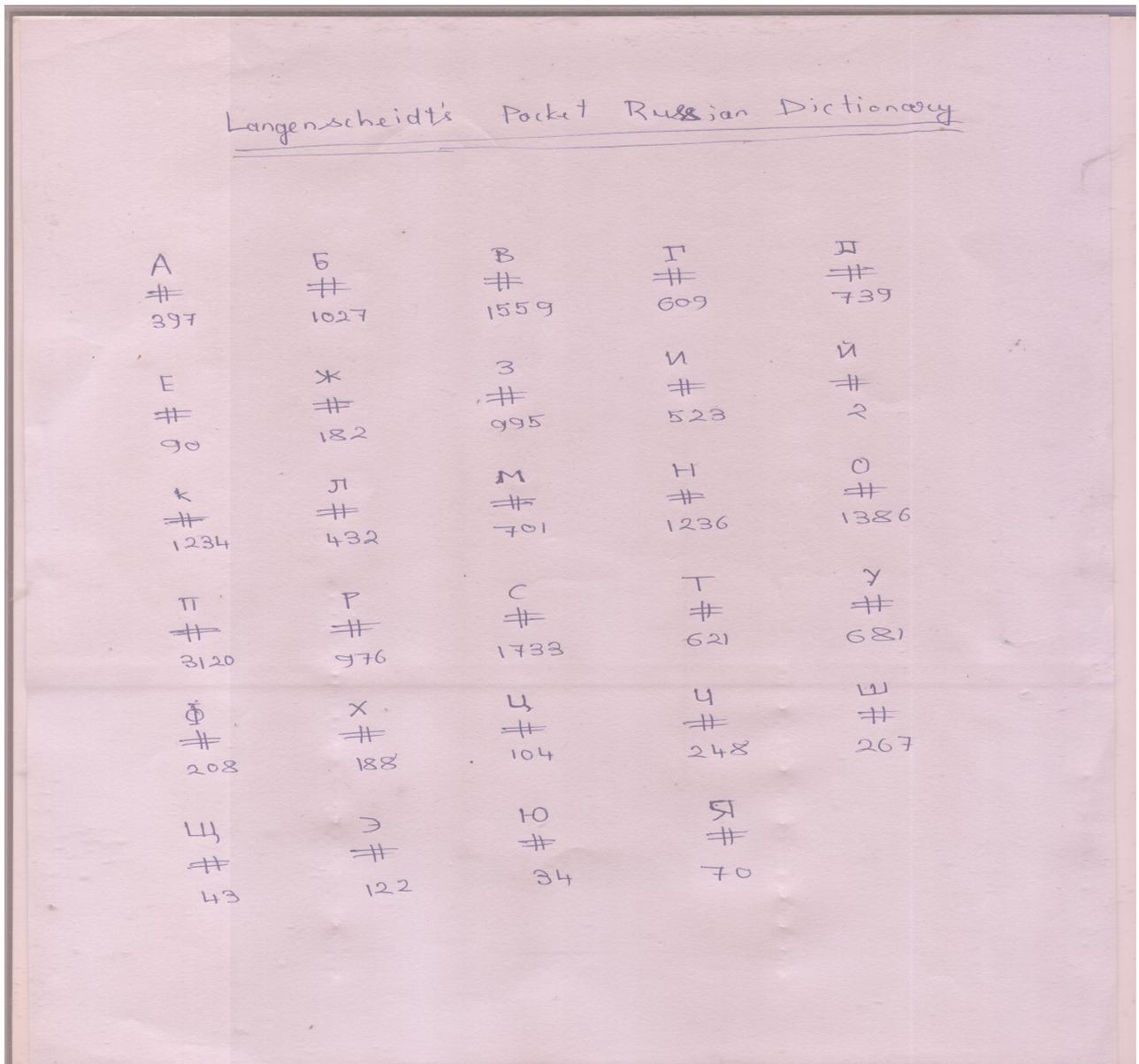

FIG. 2. Numbers of words starting with various letters of the Russian alphabet

Next on to the Graphical Law, we proceed in the rest of the paper. We have started considering magnetic field pattern in [2], in the languages we converse with. We have studied there, a set of natural languages, [2] and have found existence of a magnetisation curve under each language. We have termed this phenomenon as the Graphical Law.

Then, we moved on to investigate into, [3], dictionaries of five disciplines of knowledge and found existence of a curve magnetisation under each discipline. This was followed by finding of the graphical law behind the bengali language,[4] and the basque language[5]. This was pursued by finding of the graphical law behind the Romanian language, [6], five more disciplines of knowledge, [7], Onsager core of Abor-Miri, Mising languages,[8], Onsager Core of Romanised Bengali language,[9], the graphical law behind the Little Oxford English Dictionary, [10], the Oxford Dictionary of Social Work and Social Care, [11], the Visayan-English Dictionary, [12], Garo to English School Dictionary, [13], Mursi-English-Amharic Dictionary, [14] and Names of Minor Planets, [15], A Dictionary of Tibetan and English, [16], Khasi English Dictionary, [17], Turkmen-English Dictionary, [18], Websters Universal Spanish-English Dictionary, [19], A Dictionary of Modern Italian, [20], Langenscheidt's German-English Dictionary, [21], Essential Dutch dictionary by G. Quist and D. Strik, [22], Swahili-English dictionary by C. W. Rechenbach, [23], Larousse Dictionnaire De Poche for the French, [24], the Onsager's solution behind the Arabic, [25], the graphical law behind Langenscheidt Taschenwörterbuch Deutsch-Englisch / Englisch-Deutsch, Völlige Neubearbeitung, [26], the graphical law behind the NTC's Hebrew and English Dictionary by Arie Comey and Naomi Tsur, [27], the graphical law behind the Oxford Dictionary Of Media and Communication, [28], the graphical law behind the Oxford Dictionary Of Mathematics, Penguin Dictionary Of Mathematics, [29], the Onsager's solution behind the Arabic Second part, [30], the graphical law behind the Penguin Dictionary Of Sociology, [31], behind the Concise Oxford Dictionary Of Politics, [32], a Dictionary Of Critical Theory by Ian Buchanan, [33], the Penguin Dictionary Of Economics, [34], the Concise Gojri-English Dictionary by Dr. Rafeeq Anjum, [35], A Dictionary of the Kachin Language by Rev.O.Hanson, [36], A Dictionary Of World History by Edmund Wright, [37], Ekagi-Dutch-English-Indonesian Dictionary by J. Steltenpool, [38], A Dictionary of Plant Sciences by Michael Allaby, [39], respectively. The graphical law was pursued more in Along the side of the Onsager's solution, the Ekagi language ,[40], Along the side of the Onsager's solution, the Ekagi language-Part Three, [41], Oxford Dictionary of Biology by Robert S. Hine and the Graphical law, [42], A Dictionary of the Mikir Language

by G. D. Walker and the Graphical law, [43], A Dictionary of Zoology by Michael Allaby and the Graphical Law, [44], Dictionary of all Scriptures and Myths by G. A. Gaskell and the Graphical Law, [45], Dictionary of Culinary Terms by Philippe Pilibossian and the Graphical law, [46], A Greek and English Lexicon by H.G.Liddle et al simplified by Didier Fontaine and the Graphical law, [47], Learner's Mongol-English Dictionary and the Graphical law, [48], Complete Bulgarian-English Dictionary and the Graphical law, [49], A Dictionary of Sindhi Literature by Dr. Motilal Jotwani and the Graphical Law, [50], Penguin Dictionary of Physics, the Fourth Edition, by John Cullerne, and the Graphical law, [51], Oxford Dictionary of Chemistry, the seventh edition and the Graphical Law, [52], A Burmese-English Dictionary, Part I-Part V, by J. A. Stewart and C. W. Dunn et al, head words and the Graphical Law, [53], The Graphical Law behind the head words of Dictionary Kannada and English written by W. Reeve, revised, corrected and enlarged by Daniel Sanderson, [54], Sanchayita and the Graphical Law, [55], Samsad Bangla Abhidan and The Graphical Law, [56], Bangiya Sabdakosh and The Graphical Law, [57], Samsad Bengali-English Dictionary and The Graphical Law, [58], Rudyard Kipling's Verse and the Graphical Law, [59], W. B. Yeats, The Poems and the Graphical Law, [60], The Penguin Encyclopedia of Places by W. G. Moore and the Graphical law, [61], The Poems of Tennyson and the Graphical Law, [62], Khasi-Jaintia Jaids(Surnames) and the Graphical law, [63], Age, Amplitude of accommodation and the Graphical law, [64], Dictionary of Ayurveda by Dr. Ravindra Sharma and the Graphical law, [65], The Practical Sanskrit-English Dictionary by Vaman Shivram Apte and The Graphical Law, [66], respectively.

The planning of the paper is as follows. In the next section, we describe the Graphical Law analysis of words of the Langenscheidt's Pocket Russian Dictionary, [1]. The section III, we give an introduction to the standard curves of magnetisation of Ising model. The section IV is Acknowledgment. The last section is Bibliography.

II.   THE GRAPHICAL LAW ANALYSIS

For the purpose of exploring graphical law, we assort the letters according to the number of words, in the descending order, denoted by f and the respective rank, [67], denoted by k. k is a positive integer starting from one. Moreover, the minimum non-zero number of words is two. Hence, we attach a limiting word number one. The limiting rank is maximum rank

plus one, here it is thirty. As a result both $\frac{lnf}{lnf_{max}}$ and $\frac{lnk}{lnk_{lim}}$ varies from zero to one. Then we tabulate in the adjoining table,II, and plot $\frac{lnf}{lnf_{max}}$ against $\frac{lnk}{lnk_{lim}}$ in the figure fig.4. We then ignore the letter with the highest of words, tabulate in the adjoining table,II,and redo the plot, normalising the lnfs with lnf_{n-max}, and starting from $k = 2$ in the figure fig.5. Normalising the lnfs with lnf_{2n-max}, we tabulate in the adjoining table,II, and starting from $k = 3$ we draw in the figure fig.6. Normalising the lnfs with lnf_{3n-max} we record in the adjoining table,II, and plot starting from $k = 4$ in the figure fig.7. In this way we obtain up to the figure fig.8.

k	lnk	lnk/lnk_{lim}	f	lnf	lnf/lnf_{max}	lnf/lnf_{n-max}	lnf/lnf_{2n-max}	lnf/lnf_{3n-max}	lnf/lnf_{4n-max}	lnf/lnf_{5n-max}
1	0	0	3120	8.046	1	Blank	Blank	Blank	Blank	Blank
2	0.69	0.203	1733	7.458	0.927	1	Blank	Blank	Blank	Blank
3	1.10	0.324	1559	7.352	0.914	0.986	1	Blank	Blank	Blank
4	1.39	0.409	1386	7.234	0.899	0.970	0.984	1	Blank	Blank
5	1.61	0.474	1236	7.120	0.885	0.955	0.968	0.984	1	Blank
6	1.79	0.526	1234	7.118	0.885	0.954	0.968	0.984	0.9997	1
7	1.95	0.574	1027	6.934	0.862	0.930	0.943	0.959	0.974	0.974
8	2.08	0.612	995	6.903	0.858	0.926	0.939	0.954	0.970	0.970
9	2.20	0.647	976	6.883	0.855	0.923	0.936	0.951	0.967	0.967
10	2.30	0.676	739	6.605	0.821	0.886	0.898	0.913	0.928	0.928
11	2.40	0.706	701	6.553	0.814	0.879	0.891	0.906	0.920	0.921
12	2.48	0.729	681	6.524	0.811	0.875	0.887	0.902	0.916	0.917
13	2.56	0.753	621	6.431	0.799	0.862	0.875	0.889	0.903	0.903
14	2.64	0.776	609	6.412	0.797	0.860	0.872	0.886	0.901	0.901
15	2.71	0.797	523	6.260	0.778	0.839	0.851	0.865	0.879	0.879
16	2.77	0.815	432	6.068	0.754	0.814	0.825	0.839	0.852	0.852
17	2.83	0.832	397	5.984	0.744	0.802	0.814	0.827	0.840	0.841
18	2.89	0.850	267	5.587	0.694	0.749	0.760	0.772	0.785	0.785
19	2.94	0.865	248	5.513	0.685	0.739	0.750	0.762	0.774	0.775
20	3.00	0.882	208	5.338	0.663	0.716	0.726	0.738	0.750	0.750
21	3.04	0.894	188	5.236	0.651	0.702	0.712	0.724	0.735	0.736
22	3.09	0.909	182	5.204	0.647	0.698	0.708	0.719	0.731	0.731
23	3.14	0.924	122	4.804	0.597	0.644	0.653	0.664	0.675	0.675
24	3.18	0.935	104	4.644	0.577	0.623	0.632	0.642	0.652	0.652
25	3.22	0.947	90	4.500	0.559	0.603	0.612	0.622	0.632	0.632
26	3.26	0.959	70	4.248	0.528	0.570	0.578	0.587	0.597	0.597
27	3.30	0.971	43	3.761	0.467	0.504	0.512	0.520	0.528	0.528
28	3.33	0.979	34	3.526	0.438	0.473	0.480	0.487	0.495	0.495
29	3.37	0.991	2	0.693	0.86	0.093	0.094	0.096	0.097	0.097
30	3.40	1	1	0	0	0	0	0	0	0

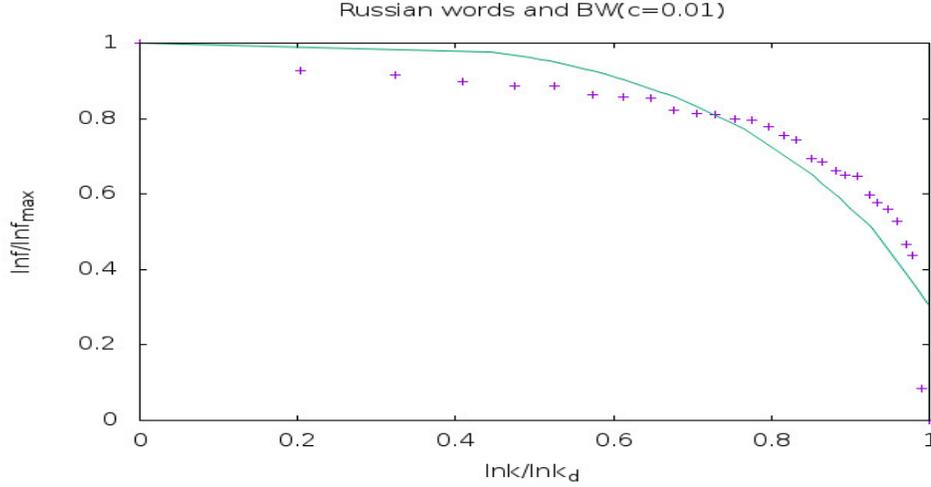TABLE II. The Langenscheidt's Pocket Russian Dictionary words: ranking, natural logarithm, normalisations

FIG. 3. The vertical axis is $\frac{lnf}{lnf_{max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The $+$ points represent the words of the Langenscheidt's Pocket Russian Dictionary, with the fit curve being the Bragg-Williams curve in the presence of external magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$.
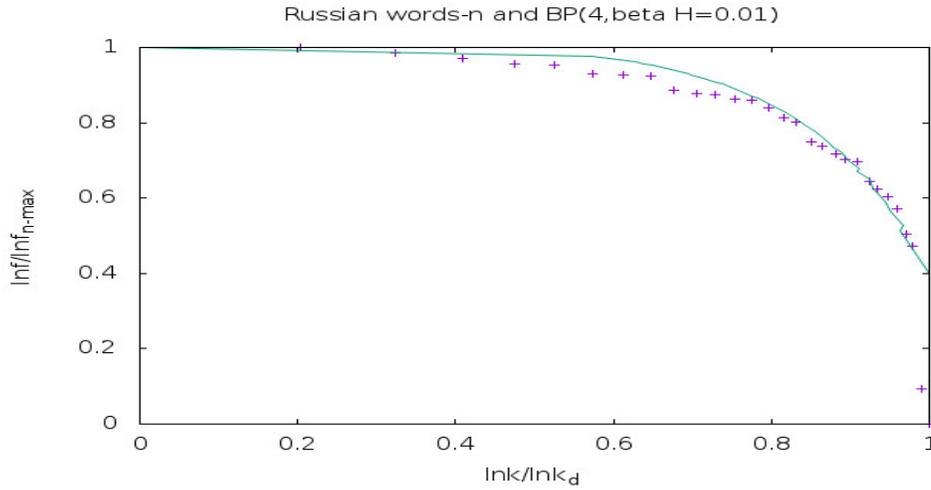

FIG. 4. The vertical axis is $\frac{lnf}{lnf_{n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The $+$ points represent the words of the Langenscheidt's Pocket Russian Dictionary, with the fit curve, BP$(4, \beta H = 0.01)$, being the Bethe-Peierls curve in the presence of four nearest neighbours and external magnetic field, $m = 0.005$ or, $\beta H = 0.01$.
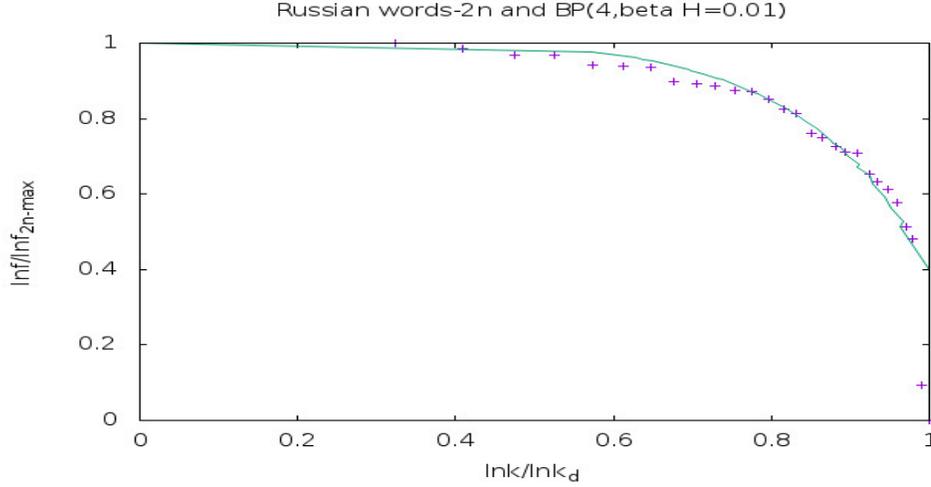
FIG. 5. The vertical axis is $\frac{lnf}{lnf_{2n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Langenscheidt's Pocket Russian Dictionary, with the fit curve, BP(4,$\beta H = 0.01$), being the Bethe-Peierls curve in the presence of four nearest neighbours and external magnetic field, $m = 0.005$ or, $\beta H = 0.01$.
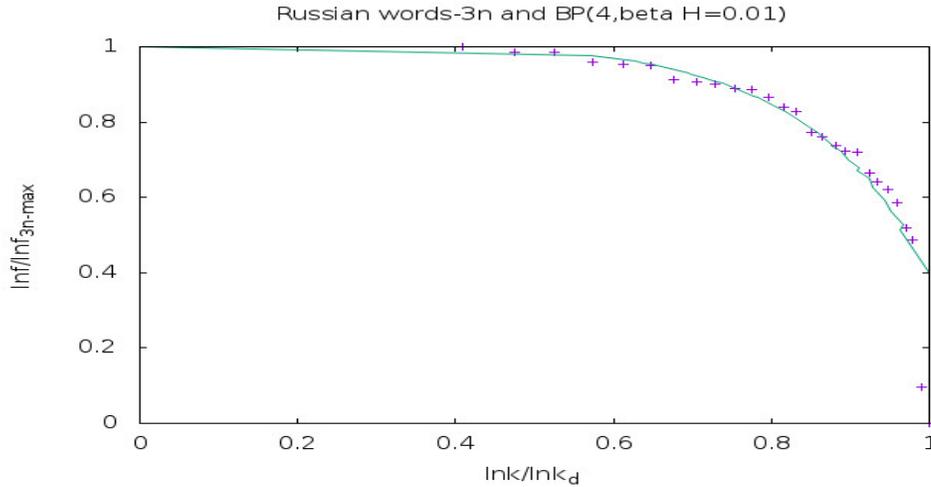

FIG. 6. The vertical axis is $\frac{lnf}{lnf_{3n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Langenscheidt's Pocket Russian Dictionary, with the fit curve, BP(4,$\beta H = 0.01$), being the Bethe-Peierls curve in the presence of four nearest neighbours and external magnetic field, $m = 0.005$ or, $\beta H = 0.01$.
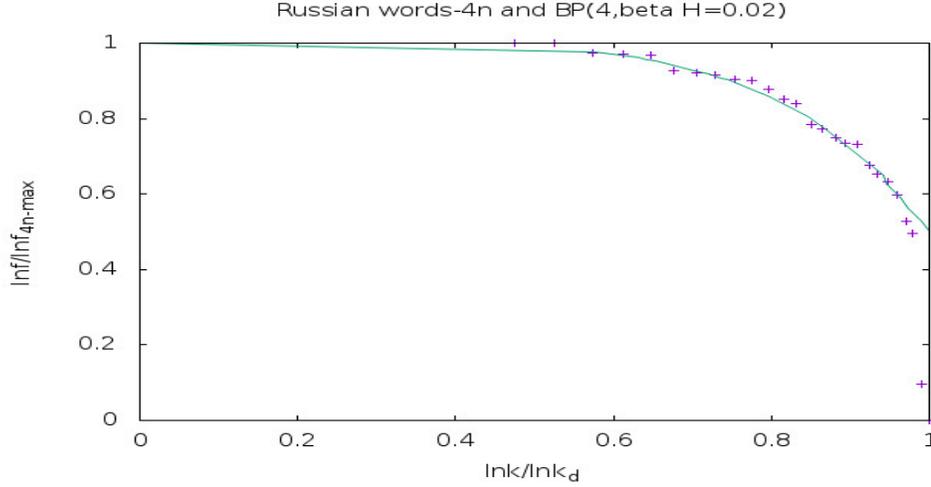
FIG. 7. The vertical axis is $\frac{lnf}{lnf_{4n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Langenscheidt's Pocket Russian Dictionary, with the fit curve, BP(4,$\beta H = 0.02$), being the Bethe-Peierls curve in the presence of four nearest neighbours and external magnetic field, $m = 0.01$ or, $\beta H = 0.02$.

FIG. 8. The vertical axis is $\frac{lnf}{lnf_{5n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Langenscheidt's Pocket Russian Dictionary, with the fit curve, BP(4,$\beta H = 0.02$), being the Bethe-Peierls curve in the presence of four nearest neighbours and external magnetic field, $m = 0.01$ or, $\beta H = 0.02$.
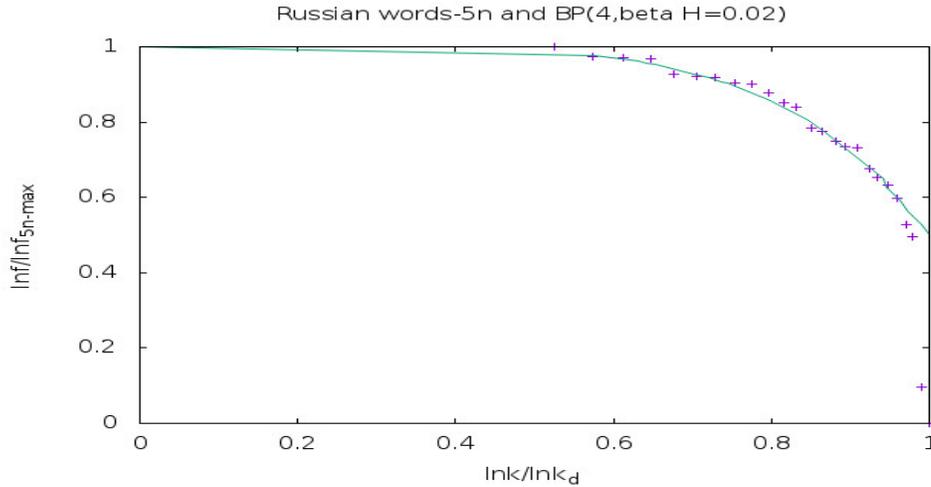
A. conclusion

From the figures (fig.3-fig.8), we observe that there is a curve of magnetisation, behind the words of the Langenscheidt's Pocket Russian Dictionary,[1]. This is the magnetisation curve in the Bethe-Peierls approximation of the Ising model, BP(4,$\beta H = 0.01$), in the presence of four nearest neighbours and little external magnetic field, $m = 0.005$ or, $\beta H = 0.01$

Moreover, the associated correspondence is,

$$\frac{ln f}{ln f_{3n-max}} \longleftrightarrow \frac{M}{M_{max}},$$

$$ln k \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [74].

III.   APENDIX: MAGNETISATION

A.   Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like para magnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N}\Sigma_i\sigma_i$, where σ_i is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where N_+ is the number of up spins, N_- is the number of down spins. $L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1 + L)$ and $N_- = \frac{N}{2}(1 - L)$. Magnetisation or, net magnetic moment , M is $\mu\Sigma_i\sigma_i$ or, $\mu(N_+ - N_-)$ or, μNL, $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$. $\frac{M}{M_{max}}$ is

referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[68], for the lattice of spins, setting μ to one, is $-\epsilon\Sigma_{n.n}\sigma_i\sigma_j - H\Sigma_i\sigma_i$, where n.n refers to nearest neighbour pairs. The difference $\triangle E$ of energy if we flip an up spin to down spin is, [69], $2\epsilon\gamma\bar{\sigma} + 2H$, where γ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_-}{N_+}$ equals $exp(-\frac{\triangle E}{k_B T})$, [70]. In the Bragg-Williams approximation,[71], $\bar{\sigma} = L$, considered in the thermal average sense. Consequently,

$$ln\frac{1+L}{1-L} = 2\frac{\gamma\epsilon L + H}{k_B T} = 2\frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2\frac{L + c}{\frac{T}{T_c}} \tag{1}$$

where, $c = \frac{H}{\gamma\epsilon}$, $T_c = \gamma\epsilon/k_B$, [72]. $\frac{T}{T_c}$ is referred to as reduced temperature.

Plot of L vs $\frac{T}{T_c}$ or, reduced magentisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [69]. W. L. Bragg was a professor of Hans Bethe. Rudolf Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudolf Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field

In the approximation scheme which is improvement over the Bragg-Williams, [68],[69],[70],[71],[72], due to Bethe-Peierls, [73], reduced magnetisation varies with reduced temperature, for γ neighbours, in absence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \tag{2}$$

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe
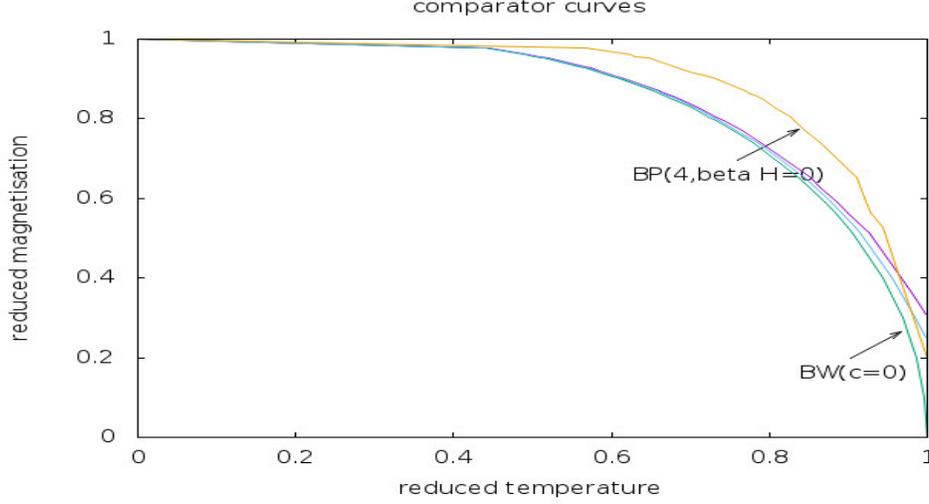
FIG. 9. Reduced magnetisation vs reduced temperature curves, for the Bragg-Williams approximation, in the absence (BW(c=0)) and in the presence (BW(c=0.005), BW(c=0.01)) of magnetic field, $c = 0$, $c = \frac{H}{\gamma\epsilon} = 0.005$, $c = \frac{H}{\gamma\epsilon} = 0.01$, outwards; and in the Bethe-Peierls approximation, BP(4,βH=0), in the absence of magnetic field, for four nearest neighbours (outer in the top).

data s generated from the equation(1) and the equation(2) in the table, III, and curves of magnetisation plotted on the basis of those data s. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.9. Empty spaces in the table, III, mean corresponding point pairs were not used for plotting a line.

BW(c=0)	BW(c=0.005)	BW(c=0.01)	BP(4,$\beta H = 0$)	reduced magnetisation
0	0	0	0	1
0.435	0.437	0.439	0.563	0.978
0.439	0.441	0.443	0.568	0.977
0.491	0.493	0.495	0.624	0.961
0.501	0.504	0.507	0.630	0.957
0.514	0.517	0.519	0.648	0.952
0.559	0.562	0.565	0.654	0.931
0.566	0.569	0.573	0.7	0.927
0.584	0.587	0.590	0.7	0.917
0.601	0.604	0.607	0.722	0.907
0.607	0.610	0.613	0.729	0.903
0.653	0.658	0.661	0.770	0.869
0.659	0.663	0.666	0.773	0.865
0.669	0.674	0.678	0.784	0.856
0.679	0.684	0.688	0.792	0.847
0.701	0.705	0.709	0.807	0.828
0.723	0.728	0.732	0.828	0.805
0.732	0.736	0.743	0.832	0.796
0.753	0.758	0.766	0.845	0.772
0.779	0.784	0.788	0.864	0.740
0.838	0.844	0.853	0.911	0.651
0.850	0.858	0.864	0.911	0.628
0.870	0.877	0.885	0.923	0.592
0.883	0.891	0.899	0.928	0.564
0.899	0.908	0.918		0.527
0.905	0.914	0.926	0.941	0.513
0.944	0.956	0.968	0.965	0.400
		0.985		0.350
		0.998		0.310
0.969	0.985		0.965	0.300
	0.998			0.250
0.987			1	0.200
0.997			1	0.100
1			1	0

TABLE III. Datas for Reduced temperature[for the Bragg-Williams approximation, in the absence (BW(c=0)) and in the presence (BW(c=0.005), BW(c=0.01)) of magnetic field, $c = 0$, $c = \frac{H}{\gamma\epsilon} = 0.005$, $c = \frac{H}{\gamma\epsilon} = 0.01$ respectively and in the Bethe-Peierls approximation, BP(4,βH=0), in the absence of magnetic field, for four nearest neighbours] vs reduced magnetisation. Reduced temperature is drawn along the x-axis and Reduced magnetisation is drawn along the y-axis. In gnuplot the command is plot ".dat" using 1:2 with line; 1 standing for x-axis and 2 standing for y-axis datas.

C. Bethe-peierls approximation in presence of four nearest neighbours, in the presence of external magnetic field

In the Bethe-Peierls approximation scheme , [73], reduced magnetisation varies with reduced temperature, for γ neighbours, in presence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{3}$$

Derivation of this formula ala [73] is given in the appendix of [7].

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For four neighbours,

$$\frac{0.693}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{4}$$

In the following, we describe datas in the table, IV, generated from the equation(4) and curves of magnetisation plotted on the basis of those datas. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.06$. calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.05$. calculated from the equation(4). BP(m=0.02) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.04$. calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.02$. calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approxima- tion, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.01$. calculated from the equation(4). The data set is used to plot fig.10. Empty spaces in the table, IV, mean corresponding point pairs were not used for plotting a line.

BP(m=0.03)	BP(m=0.025)	BP(m=0.02)	BP(m=0.01)	BP(m=0.005)	reduced magnetisation
0	0	0	0	0	1
0.583	0.580	0.577	0.572	0.569	0.978
0.587	0.584	0.581	0.575	0.572	0.977
0.647	0.643	0.639	0.632	0.628	0.961
0.657	0.653	0.649	0.641	0.637	0.957
0.671	0.667		0.654	0.650	0.952
	0.716			0.696	0.931
0.723	0.718	0.713	0.702	0.697	0.927
0.743	0.737	0.731	0.720	0.714	0.917
0.762	0.756	0.749	0.737	0.731	0.907
0.770	0.764	0.757	0.745	0.738	0.903
0.816	0.808	0.800	0.785	0.778	0.869
0.821	0.813	0.805	0.789	0.782	0.865
0.832	0.823	0.815	0.799	0.791	0.856
0.841	0.833	0.824	0.807	0.799	0.847
0.863	0.853	0.844	0.826	0.817	0.828
0.887	0.876	0.866	0.846	0.836	0.805
0.895	0.884	0.873	0.852	0.842	0.796
0.916	0.904	0.892	0.869	0.858	0.772
0.940	0.926	0.914	0.888	0.876	0.740
	0.929			0.877	0.735
	0.936			0.883	0.730
	0.944			0.889	0.720
	0.945				0.710
	0.955			0.897	0.700
	0.963			0.903	0.690
	0.973			0.910	0.680
				0.909	0.670
	0.993			0.925	0.650
		0.976	0.942		0.651
	1.00				0.640
		0.983	0.946	0.928	0.628
		1.00	0.963	0.943	0.592
			0.972	0.951	0.564
			0.990	0.967	0.527
				0.964	0.513
			1.00		0.500
				1.00	0.400
					0.300
					0.200
					0.100
					0

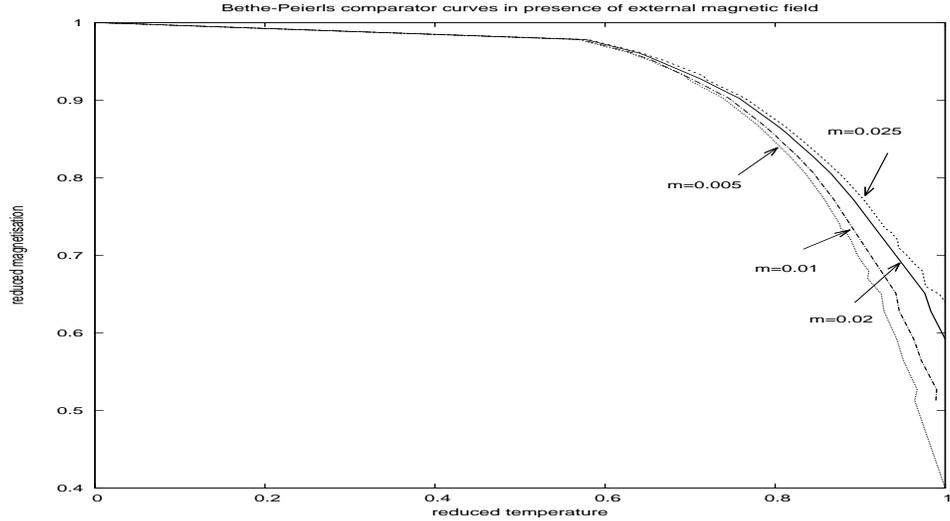TABLE IV. Bethe-Peierls approx. in presence of little external magnetic fields

FIG. 10. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

IV. ACKNOWLEDGMENT

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