Does gravity work according to Mach's principle[1]?

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Abstract

Background: Mach's Principle posits that the inertia of a body is influenced by the sum total of matter in the universe. This paper explores the relationship between gravitational potential energy and rest mass within the framework of Mach's Principle, utilizing a thought experiment involving the elevation of mass on Earth.

Methods: The study employs a theoretical approach, beginning with a thought experiment that demonstrates the change in an object's rest mass due to alterations in gravitational potential energy. Subsequently, a mathematical model is developed to express rest mass as a function of distance from a massive central body, incorporating adjustments for observed astronomical phenomena such as the precession of Mercury's orbit.

Conclusion: The findings affirm the principle that an object's rest mass is influenced by the gravitational potential of all other masses in the universe, aligning with Mach's Principle. The study underscores the need for revisiting the definitions of fundamental units of measurement such as the second and the meter, in light of gravitational dependency. The implications for astronomical theories, including gravitational redshift and the assessment of quasar emissions, are discussed, suggesting a potential reevaluation of existing models.

Let's conduct a thought experiment in the Earth's reference frame. We have an immobile one-kilogram weight at ground level. Its total energy is its rest mass equal to 1 [kg], which corresponds to 89 875 517 873 681 764 [J]. (We assume that the weight is cooled to absolute zero, meaning it has no kinetic energy due to temperature). This mass consists of the rest masses of all atoms, i.e., protons, neutrons, and electrons. Let's take this weight and carry it to the tenth floor of some building, i.e., 30 [m] above ground level. We will do work against gravity amounting to about 294 [J]. Therefore, on the tenth floor, the total energy of the weight will be 89 875 517 873 682 058 [J] (an increase of 294 [J]). This weight on the tenth floor is still at rest, so its rest mass has increased, and since the number of atoms remains the same, the rest mass of protons, neutrons, and electrons has increased.

The conclusion from this experiment is as follows: the change in the potential energy (E_p) of an object changes its rest mass $\boldsymbol{m_r}$, thus the **rest mass is potential energy**. Therefore, we can express Einstein's equation $\boldsymbol{E = mc^2}$ as an identity: $\boldsymbol{E_p \equiv m_r}$. Meanwhile, $\boldsymbol{c^2}$ in Einstein's equation is merely a coefficient for converting one unit of energy to another unit of energy (kilograms to joules), similar to coefficients used for converting calories to joules, electron volts to joules, etc. Of course, in generating the rest mass of subatomic particles, all force fields of potential interactions are involved. More precisely, this topic has been discussed in the book "The New Applications of Special Theory of Relativity" [2].

The best example of rest mass being potential energy is nuclear reactions with the release of heat, where a portion of the rest mass (potential energy) of atomic nuclei is converted into heat (kinetic energy). Moreover, all chemical reactions that release heat occur at the expense of the rest mass of the substrates. However, in this case (unlike nuclear reactions), the loss of mass is so small that it is practically difficult to measure. This loss of mass can only be calculated using the formula $\boldsymbol{E = mc^2}$. Similarly, it applies to the heat of fusion and the heat of solidification. These heats are also released at the expense of the rest mass (potential energy). Additionally, the binding energy of, for example, an electron in a hydrogen atom results from the loss of rest mass of the electron and proton. (The rest mass of a hydrogen atom is smaller by the value of the binding energy than the sum of the rest masses of a free electron and a free proton).

In the thought experiment described above, we concluded that lifting the weight to a height of 30 [m] results in an increase in its rest mass. Let's try to find the mathematical relationship of the rest mass of the test body $\boldsymbol{m_r}$ as a function of its distance $\boldsymbol{r}$ from the massive central body, denoted by $\boldsymbol{M_r}$. We assume that the mass $\boldsymbol{M_r}$ is several orders of magnitude larger than $\boldsymbol{m_r}$ and that these are the only two objects that exist. We will utilize the law of universal gravitation, which is generally known and looks as follows:

$$\boldsymbol{F = G\frac{m_1\, m_2}{r^2}} \tag{1}$$

The same can be expressed slightly differently:

$$\boldsymbol{F = G\, m_1\, m_2\, f_G(r)} \tag{2}$$

where $\boldsymbol{f_G(r)}$ is a function of distance $\boldsymbol{r}$, expressed in meters, of two attracting masses:

$$\boldsymbol{f_G(r) = \frac{1}{r^2}} \tag{3}$$

Already in the 19th century, based on astronomical observations of the planets in the Solar System, physicists realized that function (3) is just an approximation of the original function Nature employs, because in the case of Mercury, certain minimal deviations of its orbit from the orbit predicted by Newton's law were observed. Assuming that within the range of Mercury's orbit, a more precise approximation of the function $\boldsymbol{f_G(r)}$ is, for example, the expression $\boldsymbol{r^{-2.00000016}}$, the precession of Mercury's orbit can be explained without resorting to the general theory of relativity. Therefore, it should be assumed that the function $\boldsymbol{f_G(r)}$ is dimensionless and the gravitational constant has dimensions $[\frac{m}{kg\, s^2}]$ [2].

When the rest masses $\boldsymbol{m_r}$ and $\boldsymbol{M_r}$ are expressed in joules and $\boldsymbol{f_G(r)}$ is treated as a dimensionless function with the argument given in meters, and instead of the constant $\boldsymbol{G}$, we use:

$$\boldsymbol{k_G = \frac{1}{c^4} G = 8.26245 \times 10^{-45} \left[\frac{1}{Nm^2}\right]} \tag{4}$$

we can express formula (2) as:

$$\boldsymbol{F = k_G M_r m_r f_G\left(r\right)} \tag{5}$$

Note: The above value of the constant $\boldsymbol{k_G}$ and its unit result from converting the dimensions of the constant $\boldsymbol{G}$ $\left[\frac{m^3}{kg\, s^2}\right]$ from kilograms to joules $(1[kg] = c^2[Nm])$ *and seconds to meters* $(1[s] = c[m])$*, and from the necessity of removing the numerator* $[m^2]$*, since we assumed that the function $\boldsymbol{f_G(r)}$ is dimensionless.*

Let's express how the potential energy, i.e., the rest mass $\boldsymbol{m_r}$, changes with distance $\boldsymbol{r}$:

$$\boldsymbol{dE_p = dm_r = F dr = k_G m_r M_r f_G(r) dr} \tag{6}$$

after transformation:

$$\boldsymbol{\frac{1}{m_r} dm_r = k_G M_r f_G(r) dr} \tag{7}$$

that is:

$$\boldsymbol{\int \frac{1}{m_r} dm_r = k_G M_r \int f_G(r) dr + C} \tag{8}$$

$\boldsymbol{C}$ - integration constant. After integrating the left side:

$$\boldsymbol{\ln\left(m_r\right) = k_G M_r \int f_G(r) dr + C} \tag{9}$$

So the formula for rest mass is as follows:

$$\boldsymbol{m_r = e^{k_G M_r \int f_G(r) dr + C}} \tag{10}$$

$$m_r = e^C e^{k_G M_r \int f_G(r)dr} \tag{11}$$

Let's denote $e^C = m_n$ (where C is uppercase, not to be confused with lowercase c), where m_n will be referred to as the nominal gravitational charge or nominal rest mass. This is the rest mass of an object in the absence of a gravitational field (at infinity), or in other words, the potential energy of the object arising from all other potential fields if the gravitational field were excluded. Ultimately, we obtain the formula for the rest mass (potential energy) of an object with a nominal rest mass m_n in the gravitational field of an object with a rest mass M_r:

$$m_r = m_n e^{k_G M_r \int f_G(r)dr} \tag{12}$$

Now, we should determine the conditions that the function $f_G(r)$ must satisfy. The first condition arises from the fact that we want the entire potential energy (rest mass) to convert into kinetic energy during annihilation. This is the situation when a particle encounters its antiparticle. Therefore, for $r \to 0$, the rest mass from equation (12) should be zero. This implies that for $r \to 0$, the integral $\int f_G(r)dr$ should tend to minus infinity, because then $m_s = m_n e^{-\infty} = 0$.

The second requirement is that at infinity, the rest mass from equation (12) should be equal to the nominal mass, so for $r \to \infty$, the integral $\int f_G(r)dr$ should tend to zero, because then $m_s = m_n e^0 = m_n$.

We see that among many functions that satisfy these conditions, the Newtonian function also fulfills them: $\int \frac{1}{r^2}dr = -\frac{1}{r}$. The above conditions are satisfied by the entire class of functions:

$$f_G(r) = \frac{1}{r^n} \tag{13}$$

where n is a real number greater than 1. Also, functions of the type:

$$f_G(r) = r^{-n(r)} \tag{14}$$

fulfill these conditions, assuming that the real function $n(r) > 1$ for the entire domain of definition.

Now, let's introduce the concept of gravitational field potential. The expression:

$$\Phi_G(r) = e^{k_G M_r \int f_G(r)dr} \tag{15}$$

we will call it the gravitational field potential of an object with rest mass M_r. As we can see, the gravitational potential is a dimensionless quantity contained in the closed interval $\langle 0\,, 1\rangle$. Currently, it is accepted that the gravitational potential is measured in $\frac{J}{kg}$. Considering the concept presented at the beginning of the discussion, that the joule and the kilogram are units of the same physical quantity - energy, it follows that the ratio of these units is a dimensionless quantity, similar to a map scale. Converting kilograms to joules, $\mathbf{1}\,[\boldsymbol{kg}] = \mathbf{89\,875\,517\,873\,681\,764}\,[\boldsymbol{J}]$, we obtain $\frac{\boldsymbol{J}}{\boldsymbol{kg}} = \frac{\mathbf{1}}{\mathbf{89\,875\,517\,873\,681\,764}}$.

Note: Even in Newton's time, it was agreed that the gravitational field potential at infinity is represented by the number zero. As a result of this agreement, the gravitational field potential outside of infinity is represented by negative numbers. Mass, energy, potential, temperature, etc. are scalar quantities, so from the perspective of mathematics and physics, they cannot take negative values. Of course, it's possible to agree that some nonzero level of a scalar quantity is represented by the number zero, then automatically lower levels will be conventionally represented by negative numbers, but it's important to remember that this is just a convention. An example is the accepted convention regarding the zero point of the Celsius temperature scale. Placing the gravitational potential in the interval $\langle \mathbf{0}\,,\mathbf{1} \rangle$ *is a return to normalcy (without arbitrary zero levels), where the maximum potential value is at infinity and zero potential for zero distance.*

In the general case, we can specify that in a gravitational field, the rest mass of an object (potential energy) is expressed by the formula:

$$\boldsymbol{m_r = m_n \Phi_G} \tag{16}$$

Where $\boldsymbol{\Phi_G}$ is the resultant gravitational potential arising from all other massive bodies present within the field of view of an object with rest mass $\boldsymbol{m_r}$. Thus, we can express the formula for the rest mass of any object in the Universe as:

$$\boldsymbol{m_r = m_n e^{k_G \sum_{i=1}^{N} M_{ir} \int f_G(r_i) dr}} \tag{17}$$

Where $\boldsymbol{N}$ is the number of all massive objects located within the light cone of a given object (the entire observable Universe). $\boldsymbol{M_{ir}}$ is the rest mass of the i-th object, and $\boldsymbol{r_i}$ is the distance from the i-th object (measured in the reference frame of that distant object).

The same can be expressed differently:

$$\boldsymbol{m_r = m_n \prod_{i=1}^{N} \Phi_{iG}} \tag{18}$$

Where $\boldsymbol{\Phi_{iG}}$ is the gravitational potential arising from the i-th object. **Formula (17) or (18) is nothing but the Mach's principle expressed in a mathematical form**[1]. Notice that if in formula (17) one of the integrals

$\int f_G(r_i)dr$ under the summation in the exponent tends to $-\infty$, the rest mass disappears (annihilates). Therefore, when two point-like objects, for example, an electron and a positron, with the same set of charges but opposite signs, are at the same point, their rest masses annihilate, and all their potential energy is converted into the kinetic energy of photons.

Now let's consider the mechanism of gravitational force formation. We assumed that the source of rest mass is the potential fields of all interactions, so in order for gravitational force to be created, the distribution of rest masses in space must somehow modify these potential fields. Moving an atom to a point of higher gravitational potential increases its rest mass (potential energy), so its energy levels should undergo the same increase, meaning the ratio of the energy levels of the same atom placed in locations with different gravitational potentials is the same as the ratio of those potentials, and thus is equal to the ratio of their rest masses. We are not surprised by the influence of electric and magnetic fields on the energy levels of atoms, suffice it to mention the Zeeman and Stark effects. Why should the gravitational field be an exception?

When we lift an "atomic clock," energy levels spread out, which is why "clocks" on GPS satellites orbiting "tick" slightly faster than the same "clock" on the Earth's surface. Why is "atomic clock" in quotation marks? Because it is not a precise timekeeping device (clock) as its frequency depends on gravitational potential, and the gravitational potential at a given location constantly changes, due to factors such as the constantly changing distance from the Sun and Moon, as well as due to Earth's tectonic and tidal movements. Therefore, "atomic clocks" are very well suited for measuring the ratio of gravitational potentials at two different locations. This ratio is equal to the ratio of frequencies of two identical "atomic clocks." Meanwhile, the proper time of an object depends solely on the course of its worldline.

If we assume that within the Solar System, Newton's function $\frac{1}{r^2}$ is a very good approximation of the original function $f_G(r)$, then the formula for the ratio of gravitational potentials for two different distances r_1 and r_2 from the center of the Earth looks as follows:

$$\frac{\Phi_{G_2}}{\Phi_{G_1}} = e^{k_G M_Z\left(\frac{1}{r_1}-\frac{1}{r_2}\right)} \tag{19}$$

where M_Z denotes the rest mass of the Earth.

The ratio of frequencies of two identical stationary "atomic clocks" located at distances r_2 and r_1 from the center of the Earth is the same as given by formula (19).

(Note! The expression $\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ *in the exponent in formula (19) has the dimension of meters, not the inverse of meters, because it is the result of integrating the dimensionless function* $f_G(r) = \frac{1}{r^2}$ *over distance).*

According to the General Theory of Relativity (GTR), the ratio of clock frequencies in such positions is determined by the so-called Schwarzschild metric[3]*:*

$$\frac{f_2}{f_1} = \sqrt{\frac{r_1\left(r_2 c^2 - 2GM_Z\right)}{r_2\left(r_1 c^2 - 2GM_Z\right)}} \tag{20}$$

where f_1 *and* f_2 *denote the frequencies of clocks located at distances* r_1 *and* r_2 *from the center of the Earth. These two formulas do not take into account the presence of other celestial bodies (e.g., the Sun, the Moon), although considering multiple moving celestial bodies in formula (19) poses no difficulties (see Mach's formula (17)). On the other hand, the General Theory of Relativity (GTR) does not provide analytical solutions for such situations.*

Let's compare formulas (19) and (20) for two stationary "atomic clocks," one located on the surface of the Earth at a distance $r_1 = 6\,371\,000\,[m]$ *from the center of the Earth, and the other at a distance* $r_2 = 26\,554\,000\,[m]$*, which corresponds to the orbits of GPS satellites.*

For formula (20), we assume :

the mass of the Earth $M_Z = 5.97219 \times 10^{24}\ [kg]$

the gravitational constant $G = 6.674083 \times 10^{-11}\left[\frac{m^3}{kg\, s^2}\right]$

the speed of light $c = 299\,792\,458\left[\frac{m}{s}\right]$

However, for formula (19):

$M_Z = c^2 \times 5.97219 \times 10^{24}\ [Nm]$

$k_G = \frac{G}{c^4}\left[\frac{1}{Nm^2}\right]$

From formula (19): $\frac{\Phi_{G_2}}{\Phi_{G_1}} = 1.000\,000\,000\,529\,092\,953\,674$

From formula (20): $\frac{f_2}{f_1} = 1.000\,000\,000\,529\,092\,954\,130$

These two numbers differ only at the nineteenth significant digit, so using the simpler and more universal formula (19) practically yields the same results as formula (20). However, in situations where we deal with more extreme conditions, such as near a black hole, the results obtained from formulas (19) and (20) differ to a greater extent. The numbers given above have been experimentally confirmed (albeit with less precision).

From the above considerations, it follows that the currently accepted definition of a second, established based on the frequency of the hyperfine transition in cesium 133 atoms in their unaltered ground state, is imprecise, as the frequency of this transition depends on the gravitational potential resulting from the location on Earth and the positions of nearby celestial bodies (the Sun and the Moon). The same reservations apply to the definition of a meter, as it is based on the speed of light in a vacuum, and this speed is expressed in [m/s].

The change in energy levels of atoms depending on the gravitational potential is responsible for the so-called gravitational redshift[4], so estimating the velocities of distant astronomical objects based on this shift may be subject to an unknown degree of error. For example, the emission (or absorption) spectra of quasars may originate from gases that lie very close to supermassive black holes, and here the contribution of gravitational potential to the redshift value can be very significant.

Adopting the postulate that the predominant part of the redshift of quasars is due to the so-called gravitational redshift would explain a lot. For example, it could solve the puzzle of the strange coincidence of four objects: the galaxy NGC 7603, the galaxy PGC 71041, and two quasars. All these objects appear as if they were involved in some single collision, but they differ significantly in their redshifts.

It would also be necessary to verify the opinion about the gigantic energies supposedly emitted by quasars, as well as the large distances from which they are visible. According to current views, it is estimated that the brightest quasars emit energy in the form of photons equivalent to the mass of the Sun per year, often accompanied by a large amount of mass ejected through jets. Additionally, one must account for the mass absorbed by the quasar itself and include the energy emitted in the form of neutrinos. Therefore, it is difficult to imagine how such a huge amount of mass reaches the quasar, while simultaneously overcoming very high radiation pressure. The notion that quasars existed only in the early stages of the Universe's evolution, because they are not observed in our immediate vicinity, may turn out to be incorrect.

References

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