The Urdu-Hindi Shabdakosh by Muhammad Sajjad Osmani, Sudhindra Kumar and The Graphical Law

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Abstract

We study the Urdu head entries of the Urdu-Hindi Shabdakosh by Muhammad Sajjad Osmani, Sudhindra Kumar. We draw the natural logarithm of the number of Urdu head entries, normalised, starting with a Hindi letter vs the natural logarithm of the rank of the Hindi letter, normalised. We conclude that the Dictionary can be characterised by BP(4,$\beta H = 0$), the magnetisation curve for the Bethe-Peierls approximation of the Ising model with four nearest neighbours in the absence of external magnetic field, H, with $\beta H = 0$. β is $\frac{1}{k_B T}$ where, T is temperature and k_B is the tiny Boltzmann constant.

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a	ā	i	ī	u	ū	ṛ	e	ai	o	au	ka	kha	ga	gha	ṅa	ca	cha	ja	jha	ña	ṭa
1682	775	1393	122	289	12	0	124	57	13	55	2249	1877	1746	87	0	737	94	2932	108	0	91
ṭha	ḍa	ḍha	ṇa	ta	tha	da	dha	na़	pa	pha	ba	bha	ma	ya	ra	la	va	śa	ṣa	sa	ha
30	77	29	0	2662	1624	0	0	1919	1222	1331	3071	0	4443	501	1275	829	874	1498	0	2715	1944

TABLE I. The Urdu-Hindi Shabdakosh head entries: the odd rows represent letters of the Hindi alphabet,[2], in the serial order; the even rows represent the number of head entries of the Urdu-Hindi Shabdakosh, [1].

I. INTRODUCTION

Muhammad Sajjad Osmani, Sudhindra Kumar of Patrachar Pathyakram Abong Anuvarti Shiksa Vidyalay, Delhi University, have composed Urdu-Hindi Shabdakosh in 2003, [1]. They have written Urdu words in the Hindi alphabet, [2]. This is in the spirit of Romanisation. How do the Graphical Law and the magnetisation curves appear in this case. We expect that the curves will not go as close to the Urdu words as in the the Standard Urdu-English Dictionary by Abdul Haq, [3], [4]. Expectation is correct as we find as we go along this paper. We count one by one all the head entries, in this Dictionary,[1]. The result is the table, I. To visualise we plot the number of head entries against the respective Hindi letters in the sequence,[2], in the adjoining figure, fig.1.

Looking for the Graphical Law in this dictionary, we proceed narrating the development. We have started considering magnetic field pattern in [5], in the languages we converse with. We have studied there, a set of natural languages, [5] and have found existence of a magnetisation curve under each language. We have termed this phenomenon as the Graphical Law. Then, we moved on to investigate into, [6], dictionaries of five disciplines of knowledge and found existence of a curve magnetisation under each discipline. This was followed by finding of the graphical law in references from [7] to [75]. The latest one was the paper,[4].

The planning of the paper is as follows. In the next section, we describe the Graphical Law analysis of the Urdu head entries of the Urdu-Hindi Shabdakosh, [1]. The section III, we give an introduction to the standard curves of magnetisation of Ising model. The section IV is Acknowledgment. The last section is Bibliography.

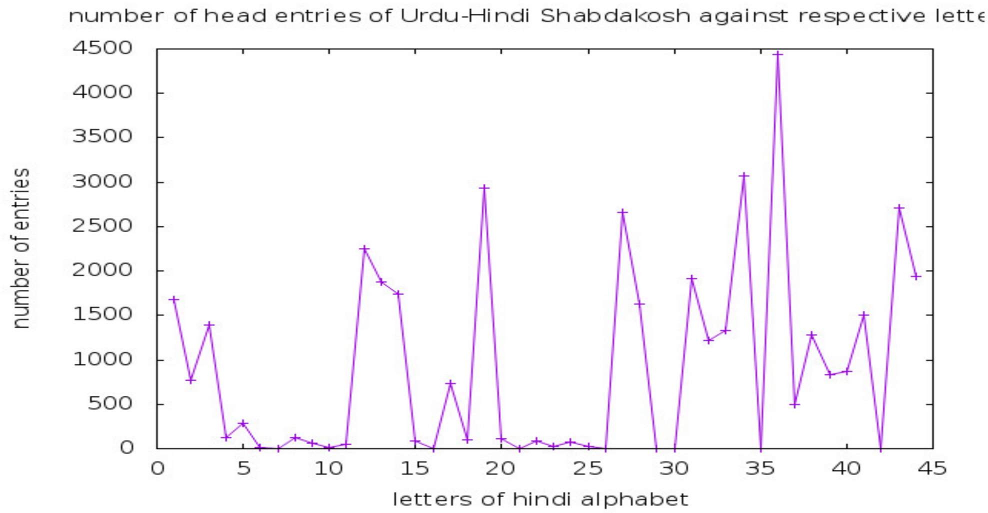

FIG. 1. The vertical axis is the number of Urdu head entries of the Urdu-Hindi Shabdakosh,[1]. The horizontal axis is the letters of the Hindi alphabet,[2], in the serial order. Letters are represented by the sequence number in the alphabet as it appears in the dictionary, [2].

II. THE GRAPHICAL LAW ANALYSIS

For the purpose of exploring graphical law, we assort the letters according to the number of Urdu head entries, in the descending order, denoted by f and the respective rank, [76], denoted by k. k is a positive integer starting from one. Moreover, the minimum non-zero number of head entries is twelve. Hence, we attach a limiting head entry number one. The limiting rank is maximum rank plus one, here it is thirty seven. As a result both $\frac{lnf}{lnf_{max}}$ and $\frac{lnk}{lnk_{lim}}$ varies from zero to one. Then we tabulate in the adjoining table,II, and plot $\frac{lnf}{lnf_{max}}$ against $\frac{lnk}{lnk_{lim}}$ in the figure fig.2. We then ignore the letter with the highest number of head entries, tabulate in the adjoining table,II,and redo the plot, normalising the lnfs with lnf_{n-max}, and starting from $k=2$ in the figure fig.3. Normalising the lnfs with lnf_{2n-max}, we tabulate in the adjoining table,II, and starting from $k=3$ we draw in the figure fig.4. Normalising the lnfs with lnf_{3n-max} we record in the adjoining table,II, and plot starting from $k=4$ in the figure fig.5. In this way we obtain up to the figure fig.7.

k	lnk	lnk/lnk_{lim}	f	lnf	lnf/lnf_{max}	lnf/lnf_{n-max}	lnf/lnf_{2n-max}	lnf/lnf_{3n-max}	lnf/lnf_{4n-max}	lnf/lnf_{5n-max}
1	0	0	4443	8.399	1	Blank	Blank	Blank	Blank	Blank
2	0.69	0.191	3071	8.030	0.956	1	Blank	Blank	Blank	Blank
3	1.10	0.305	2932	7.983	0.950	0.994	1	Blank	Blank	Blank
4	1.39	0.385	2715	7.907	0.941	0.985	0.990	1	Blank	Blank
5	1.61	0.446	2662	7.887	0.939	0.982	0.988	0.997	1	Blank
6	1.79	0.496	2249	7.718	0.919	0.961	0.967	0.976	0.979	1
7	1.95	0.540	1944	7.573	0.902	0.943	0.949	0.958	0.960	0.981
8	2.08	0.576	1919	7.560	0.900	0.941	0.947	0.956	0.959	0.980
9	2.20	0.609	1877	7.537	0.897	0.939	0.944	0.953	0.956	0.977
10	2.30	0.637	1746	7.465	0.889	0.930	0.935	0.944	0.946	0.967
11	2.40	0.665	1682	7.428	0.884	0.925	0.930	0.939	0.942	0.962
12	2.48	0.687	1624	7.393	0.880	0.921	0.926	0.935	0.937	0.958
13	2.56	0.709	1498	7.312	0.871	0.911	0.916	0.925	0.927	0.947
14	2.64	0.731	1393	7.239	0.862	0.901	0.907	0.916	0.918	0.938
15	2.71	0.751	1331	7.194	0.857	0.896	0.901	0.910	0.912	0.932
16	2.77	0.767	1275	7.151	0.851	0.891	0.896	0.904	0.907	0.927
17	2.83	0.784	1222	7.108	0.846	0.885	0.890	0.899	0.901	0.921
18	2.89	0.801	874	6.773	0.806	0.843	0.848	0.857	0.859	0.878
19	2.94	0.814	829	6.720	0.800	0.837	0.842	0.850	0.852	0.871
20	3.00	0.831	775	6.653	0.792	0.829	0.833	0.841	0.844	0.862
21	3.04	0.842	737	6.603	0.786	0.822	0.827	0.835	0.837	0.856
22	3.09	0.856	501	6.217	0.740	0.774	0.779	0.786	0.788	0.806
23	3.14	0.870	289	5.666	0.675	0.706	0.710	0.717	0.718	0.734
24	3.18	0.881	124	4.820	0.574	0.600	0.604	0.610	0.611	0.625
25	3.22	0.892	122	4.804	0.572	0.598	0.602	0.608	0.609	0.622
26	3.26	0.903	108	4.682	0.557	0.583	0.586	0.592	0.594	0.607
27	3.30	0.914	94	4.543	0.541	0.566	0.569	0.575	0.576	0.589
28	3.33	0.922	91	4.511	0.537	0.562	0.565	0.571	0.572	0.584
29	3.37	0.934	87	4.466	0.532	0.556	0.559	0.565	0.566	0.579
30	3.40	0.942	77	4.344	0.517	0.541	0.544	0.549	0.551	0.563
31	3.43	0.950	57	4.043	0.481	0.503	0.506	0.511	0.513	0.524
32	3.47	0.961	55	4.007	0.477	0.499	0.502	0.507	0.508	0.519
33	3.50	0.970	30	3.401	0.405	0.424	0.426	0.430	0.431	0.441
34	3.53	0.978	29	3.367	0.401	0.419	0.422	0.426	0.427	0.436
35	3.56	0.986	13	2.565	0.305	0.319	0.321	0.324	0.325	0.332
36	3.58	0.992	12	2.485	0.296	0.309	0.311	0.314	0.315	0.322
37	3.61	1	1	0	0	0	0	0	0	0

TABLE II. Urdu head entries of the Urdu-Hindi Shabdakosh: ranking,natural logarithm, normalisations

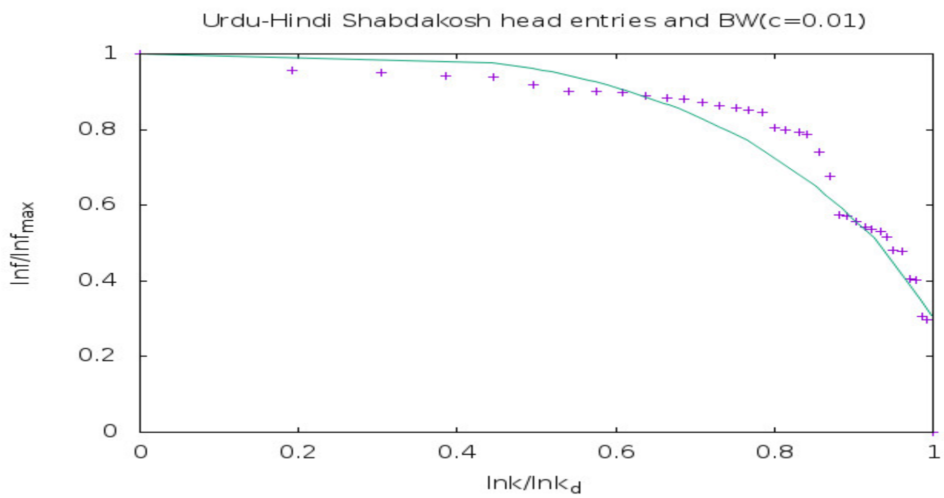

FIG. 2. The vertical axis is $\frac{lnf}{lnf_{max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the Urdu-Hindi Shabdakosh head entries, [1], with the fit curve, BW(c=0.01), being the Bragg-Williams curve in the presence of external magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$.

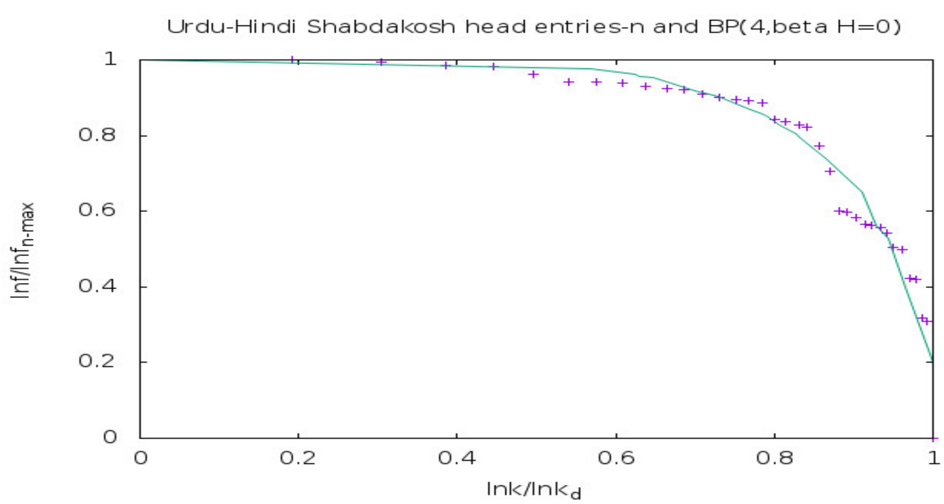

FIG. 3. The vertical axis is $\frac{lnf}{lnf_{n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the Urdu-Hindi Shabdakosh head entries, [1], with the fit curve, BP(4,$\beta H = 0$), being the Bethe-Peierls curve in the presence of four nearest neighbours and in the absence of external magnetic field.

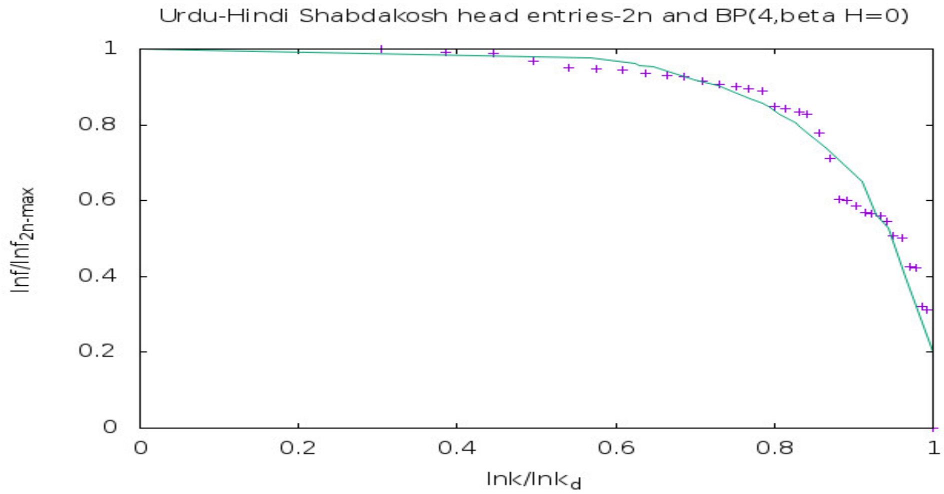

FIG. 4. The vertical axis is $\frac{lnf}{lnf_{2n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the Urdu-Hindi Shabdakosh head entries, [1], with the fit curve, BP(4,$\beta H = 0$), being the Bethe-Peierls curve in the presence of four nearest neighbours and in the absence of external magnetic field.

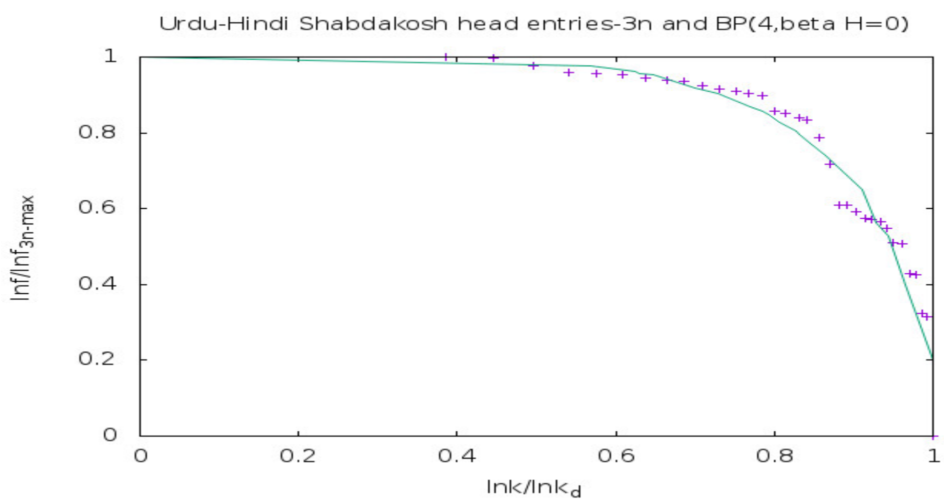

FIG. 5. The vertical axis is $\frac{lnf}{lnf_{3n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the Urdu-Hindi Shabdakosh head entries, [1], with the fit curve, BP(4,$\beta H = 0$), being the Bethe-Peierls curve in the presence of four nearest neighbours and in the absence of external magnetic field.

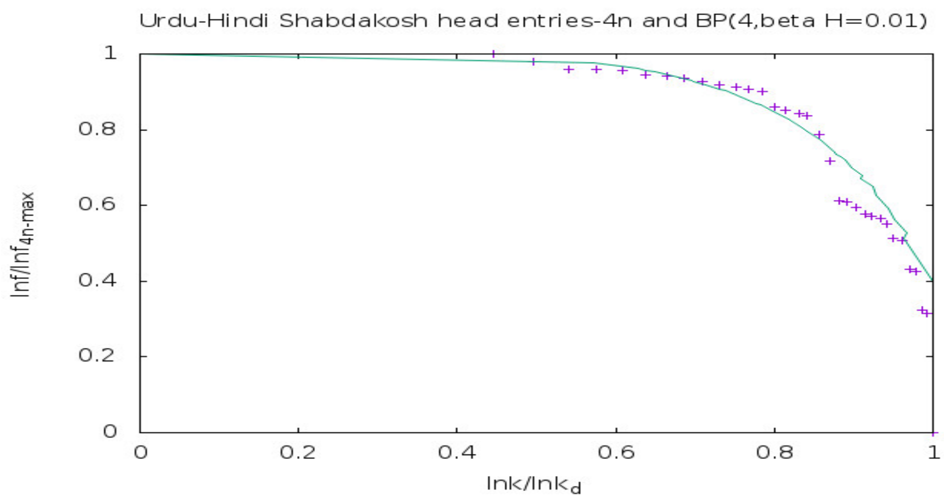

FIG. 6. The vertical axis is $\frac{lnf}{lnf_{4n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the Urdu-Hindi Shabdakosh head entries, [1], with the fit curve, BP(4,$\beta H = 0.01$), being the Bethe-Peierls curve in the presence of four nearest neighbours and external magnetic field, $m = 0.005$ or, $\beta H = 0.01$.

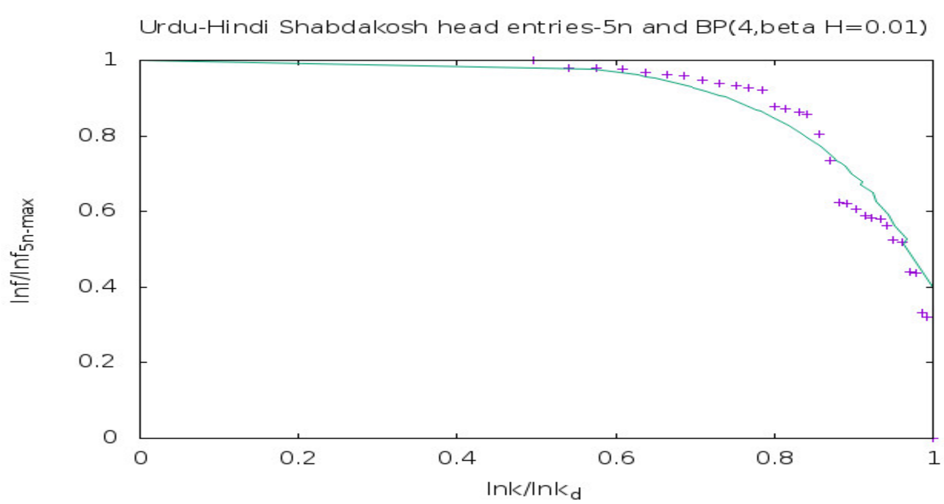

FIG. 7. The vertical axis is $\frac{lnf}{lnf_{5n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the Urdu-Hindi Shabdakosh head entries, [1], with the fit curve, BP(4,$\beta H = 0.01$), being the Bethe-Peierls curve in the presence of four nearest neighbours and external magnetic field, $m = 0.005$ or, $\beta H = 0.01$.

A. conclusion

From the figures (fig.2-fig.7), we observe that there is a curve of magnetisation, behind the head entries of the Urdu-Hindi Shabdakosh,[1]. This is the magnetisation curve, BP(4,$\beta H =$ 0), in the Bethe-Peierls approximation of the Ising model, in the presence of four nearest neighbours and in the absence of external magnetic field.

Moreover, the associated correspondence is,

$$\frac{lnf}{lnf_{n-max}} \longleftrightarrow \frac{M}{M_{max}},$$

$$lnk \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [83].

III. APENDIX: MAGNETISATION

A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like para magnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N}\Sigma_i \sigma_i$, where σ_i is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where N_+ is the number of up spins, N_- is the number of down spins. $L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1+L)$ and $N_- = \frac{N}{2}(1-L)$. Magnetisation or, net magnetic moment , M is $\mu\Sigma_i\sigma_i$ or, $\mu(N_+ - N_-)$ or, $\mu N L$, $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$. $\frac{M}{M_{max}}$ is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[77], for the lattice of spins, setting μ to one, is $-\epsilon\Sigma_{n.n}\sigma_i\sigma_j - H\Sigma_i\sigma_i$, where n.n refers to nearest neighbour pairs. The difference $\triangle E$ of energy if we flip an up spin to down spin is, [78], $2\epsilon\gamma\bar{\sigma} + 2H$, where γ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_-}{N_+}$ equals $exp(-\frac{\triangle E}{k_B T})$, [79]. In the Bragg-Williams approximation,[80], $\bar{\sigma} = L$, considered in the thermal average sense. Consequently,

$$ln\frac{1+L}{1-L} = 2\frac{\gamma\epsilon L + H}{k_B T} = 2\frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2\frac{L+c}{\frac{T}{T_c}} \quad (1)$$

where, $c = \frac{H}{\gamma\epsilon}$, $T_c = \gamma\epsilon/k_B$, [81]. $\frac{T}{T_c}$ is referred to as reduced temperature.
Plot of L vs $\frac{T}{T_c}$ or, reduced magentisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [78]. W. L. Bragg was a professor of Hans Bethe. Rudolf Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudolf Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field

In the approximation scheme which is improvement over the Bragg-Williams, [77],[78],[79],[80],[81], due to Bethe-Peierls, [82], reduced magnetisation varies with reduced temperature, for γ neighbours, in absence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (2)$$

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe

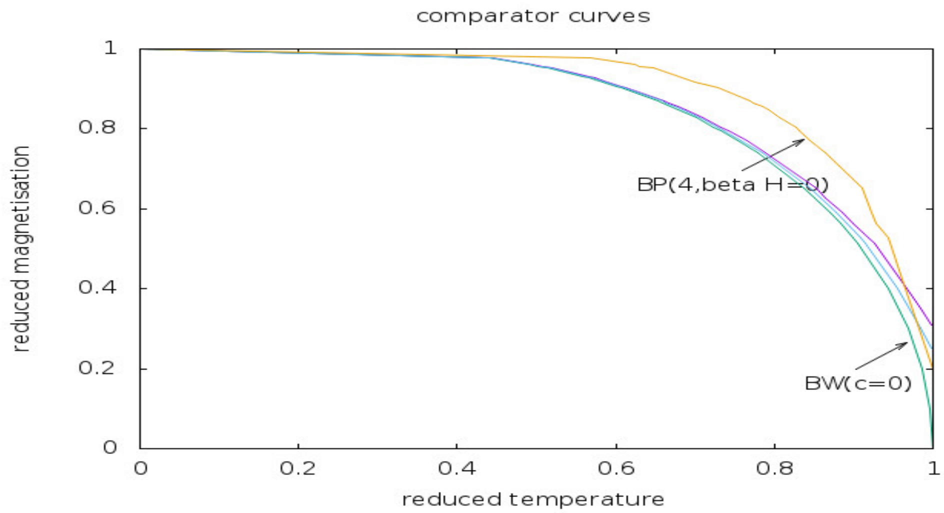

FIG. 8. Reduced magnetisation vs reduced temperature curves, for the Bragg-Williams approximation, in the absence (BW(c=0)) and in the presence (BW(c=0.005), BW(c=0.01)) of magnetic field, $c = 0$, $c = \frac{H}{\gamma\epsilon} = 0.005$, $c = \frac{H}{\gamma\epsilon} = 0.01$, outwards; and in the Bethe-Peierls approximation, BP(4,βH=0), in the absence of magnetic field, for four nearest neighbours (outer in the top).

data s generated from the equation(1) and the equation(2) in the table, III, and curves of magnetisation plotted on the basis of those data s. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.8. Empty spaces in the table, III, mean corresponding point pairs were not used for plotting a line.

reduced temperature, $\frac{T}{T_c}$				$\frac{M}{M_{max}}$,
BW(c=0)	BW(c=0.005)	BW(c=0.01)	BP(4,βH = 0)	reduced magnetisation
0	0	0	0	1
0.435	0.437	0.439	0.563	0.978
0.439	0.441	0.443	0.568	0.977
0.491	0.493	0.495	0.624	0.961
0.501	0.504	0.507	0.630	0.957
0.514	0.517	0.519	0.648	0.952
0.559	0.562	0.565	0.654	0.931
0.566	0.569	0.573	0.7	0.927
0.584	0.587	0.590	0.7	0.917
0.601	0.604	0.607	0.722	0.907
0.607	0.610	0.613	0.729	0.903
0.653	0.658	0.661	0.770	0.869
0.659	0.663	0.666	0.773	0.865
0.669	0.674	0.678	0.784	0.856
0.679	0.684	0.688	0.792	0.847
0.701	0.705	0.709	0.807	0.828
0.723	0.728	0.732	0.828	0.805
0.732	0.736	0.743	0.832	0.796
0.753	0.758	0.766	0.845	0.772
0.779	0.784	0.788	0.864	0.740
0.838	0.844	0.853	0.911	0.651
0.850	0.858	0.864	0.911	0.628
0.870	0.877	0.885	0.923	0.592
0.883	0.891	0.899	0.928	0.564
0.899	0.908	0.918		0.527
0.905	0.914	0.926	0.941	0.513
0.944	0.956	0.968	0.965	0.400
		0.985		0.350
		0.998		0.310
0.969	0.985		0.965	0.300
	0.998			0.250
0.987			1	0.200
0.997			1	0.100
1			1	0

TABLE III. Datas for Reduced temperature[for the Bragg-Williams approximation, in the absence (BW(c=0)) and in the presence (BW(c=0.005), BW(c=0.01)) of magnetic field, $c = 0$, $c = \frac{H}{\gamma\epsilon} = 0.005$, $c = \frac{H}{\gamma\epsilon} = 0.01$ respectively and in the Bethe-Peierls approximation, BP(4,βH=0), in the absence of magnetic field, for four nearest neighbours] vs reduced magnetisation. Reduced temperature data set(say, data set BW(c=0)) is drawn along the x-axis and the corresponding Reduced magnetisation data set is drawn along the y-axis. In gnuplot the command is plot ".dat" using 1:2 with line; 1 standing for x-axis and 2 standing for y-axis datas.[For example, for drawing BW(c=0), ".dat" file, say denoted as "0.dat", contains BW(c=0) data set in first column and reduced magnetisation data set in second column. Moreover, after (0.944,0.400), next pair of points will be (0.969,0.300), then (0.987,0.200), .and so on in the "0.dat" file.]

C. Bethe-peierls approximation in presence of four nearest neighbours, in the presence of external magnetic field

In the Bethe-Peierls approximation scheme , [82], reduced magnetisation varies with reduced temperature, for γ neighbours, in presence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}}=\frac{T}{T_c};factor=\frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \quad (3)$$

Derivation of this formula ala [82] is given in the appendix of [10].

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma=4$ is 0.693. For four neighbours,

$$\frac{0.693}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}}=\frac{T}{T_c};factor=\frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \quad (4)$$

In the following, we describe datas in the table, IV, generated from the equation(4) and curves of magnetisation plotted on the basis of those datas. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H=0.06$. calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H=0.05$. calculated from the equation(4). BP(m=0.02) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H=0.04$. calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H=0.02$. calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H=0.01$. calculated from the equation(4). The data set is used to plot fig.9. Empty spaces in the table, IV, mean corresponding point pairs were not used for plotting a line.

reduced temperature, $\frac{T}{T_c}$					$\frac{M}{M_{max}}$,
BP(m=0.03)	BP(m=0.025)	BP(m=0.02)	BP(m=0.01)	BP(m=0.005)	reduced magnetisation
0	0	0	0	0	1
0.583	0.580	0.577	0.572	0.569	0.978
0.587	0.584	0.581	0.575	0.572	0.977
0.647	0.643	0.639	0.632	0.628	0.961
0.657	0.653	0.649	0.641	0.637	0.957
0.671	0.667		0.654	0.650	0.952
	0.716			0.696	0.931
0.723	0.718	0.713	0.702	0.697	0.927
0.743	0.737	0.731	0.720	0.714	0.917
0.762	0.756	0.749	0.737	0.731	0.907
0.770	0.764	0.757	0.745	0.738	0.903
0.816	0.808	0.800	0.785	0.778	0.869
0.821	0.813	0.805	0.789	0.782	0.865
0.832	0.823	0.815	0.799	0.791	0.856
0.841	0.833	0.824	0.807	0.799	0.847
0.863	0.853	0.844	0.826	0.817	0.828
0.887	0.876	0.866	0.846	0.836	0.805
0.895	0.884	0.873	0.852	0.842	0.796
0.916	0.904	0.892	0.869	0.858	0.772
0.940	0.926	0.914	0.888	0.876	0.740
	0.929			0.877	0.735
	0.936			0.883	0.730
	0.944			0.889	0.720
	0.945				0.710
	0.955			0.897	0.700
	0.963			0.903	0.690
	0.973			0.910	0.680
				0.909	0.670
	0.993			0.925	0.650
		0.976	0.942		0.651
	1.00				0.640
		0.983	0.946	0.928	0.628
		1.00	0.963	0.943	0.592
			0.972	0.951	0.564
			0.990	0.967	0.527
				0.964	0.513
			1.00		0.500
				1.00	0.400
					0.300
					0.200
					0.100
					0

TABLE IV. Bethe-Peierls approx. in presence of little external magnetic fields

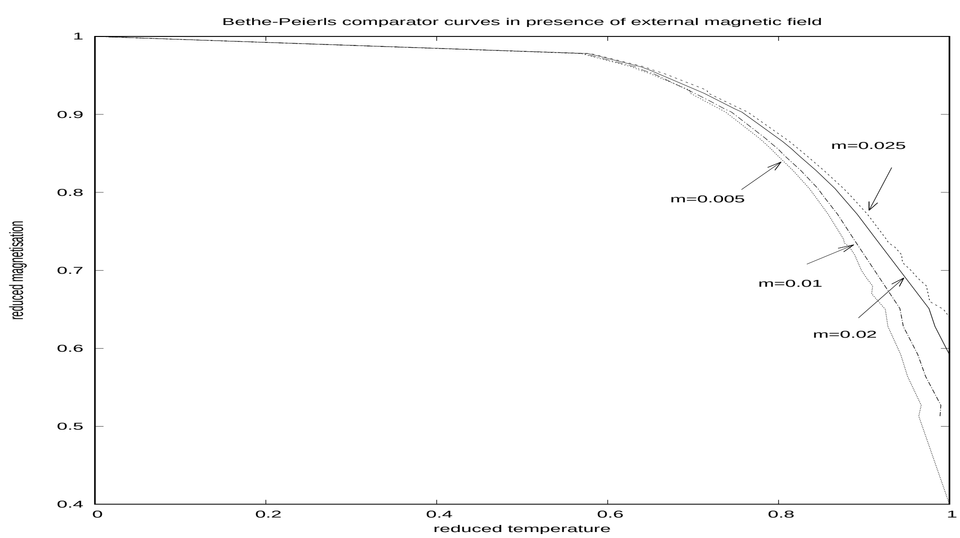

FIG. 9. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

IV. ACKNOWLEDGMENT

We have used gnuplot for plotting the figures in this paper.

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