Exact Approximations of Physical Constants using the Figures Φ, π, 144 and 666

- Version 3 -

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1) Abstract:

In this report Approximations of selected Physical Constants are presented, which results mostly are far within the tolerance of the Constants - that is the reason of the attribute *exact* in the title - and which often show a similar form with repeating figures. Besides the Quotient of the Golden Ratio and the Circle Figure π especially the figures 144 and 666 have to be named referring the used figures at these approximations. Because of their interplay the author calls them the Versatile Four.

The author firstly became aware of the figure 666 by simple mathematical relations with input data of earth, moon and sun, which is described in chapter 2. Gradually the author noticed that the figure 666 cooperates well with the figure 144.

The assumption, that the figures 144 and 666 in connection with the Circle Figure π and the Golden Ratio Φ are suitable to describe also Physical Constants, lead to the approximations, which can be read in the extensive chapter 3. The figures 144 and 666 are often used performing Fine-Tuning Terms for example with the form [1 ± x/(144*666)], which further are used as the basis of selected exponents. The selected quantities x and the selected exponents naturally have to be conclusive figures or terms.

2) Approximations of Data of our Celestial Bodies Earth[(1)], Moon[(2)] and Sun[(3)]:

Seven Approximations for the figure 666 dependent on data of Earth and Moon:

In the following diameters Ø are given without unit km and the Rotation Times RT without unit day.

$$Ø_{Earth} / \sqrt{RT_{Earth}} = 12756.27 / \sqrt{365.256} = \textbf{667.460} \qquad \text{(Appr-1)}$$
$$Ø_{Moon} / \sqrt{RT_{Moon}} = 3476 / \sqrt{27.3217} = \textbf{665.007} \qquad \text{(Appr-2)}$$
$$Ø_{Earth} * \sqrt{RT_{Moon}} / 100 = \textbf{666.772} \qquad \text{(Appr-3)}$$
$$Ø_{Moon} * \sqrt{RT_{Earth}} / 100 = \textbf{664.322} \qquad \text{(Appr-4)}$$
$$(0.1 * Ø_{Earth})^{(1/1.1)} = \textbf{665.863} \qquad \text{(Appr-5)}$$
$$(0.1 * Ø_{Moon})^{(1/0.9)} = \textbf{665.920} \qquad \text{(Appr-6)}$$
$$20 * \sqrt{(RT_{Earth} + RT_{Moon}^2)} = 20 * \sqrt{(365.256 + 27.3217^2)} = \textbf{666.853} \qquad \text{(Appr-7)}$$

Mean Value MV_{666} of the seven 666-close result values:
$$MV_{666} = (667.460 + 665.007 + 666.772 + 664.322 + 665.863 + 665.920 + 665.853) / 7 =$$
$$= \textbf{666.028}$$

Mathematical formulas with the figures 144 and 666, which lead to data of Earth, Moon and Sun:

Approximations (Appr-5) and (Appr-6) can be transformed to:
$$666^{0.1*9} = 347.64 \quad [\approx 0.1 * Ø_{Moon}];$$
$$666^{0.1*11} = 1275.9 \quad [\approx 0.1 * Ø_{Earth}];$$
Further:
$$666^{0.1*(11-9)*9.11} = 139434.4 \quad [\approx 0.1 * Ø_{Sun} = 0.1 * 1392684 \text{ (km)}]$$

Approximation of the diameters of Moon, Earth and Sun using the figure 144 (as basis) and the Circle Figure π and the figures 9 and 11 (at the exponents):
$$Appr_{144a} = 144^{[(\sqrt{\pi}+10)/10]} = 144^{1.1772454} = 347.474 \quad [\approx 0.1 * Ø_{Moon}] \qquad \text{(Appr-8)}$$
$$Appr_{144b} = Appr_{144a}^{(11/9)} = 144^{1.4388555} = 1275.18 \quad [\approx 0.1 * Ø_{Earth}] \qquad \text{(Appr-9)}$$
$$Appr_{144c} = Appr_{144b}^{[(11-9)*9.11/11]} = 144^{2.3832679} = 139301.6 \quad [\approx 0.1 * Ø_{Sun}] \qquad \text{(Appr-10)}$$

Approximation of the Circle Figure π (Rotation Times are used per unit day):

$$Pi_{Appr} = 2 * \sqrt{[\sqrt{\text{Ø}_{Earth}} / RT_{Earth} + \sqrt{\text{Ø}_{Moon}} / RT_{Moon}]} = 3.141415 \quad [\approx \pi = 3.14159\ldots]$$

Approximation of the root of Φ (Golden Ratio Φ: $\Phi = (5+5*\sqrt{5})/10 = 1.618033989$):

$$W\Phi_{Appr} = \sqrt{(\text{Ø}_{Earth} * RT_{Moon} + \text{Ø}_{Moon} * RT_{Earth})} / 1000 = 1.272066 \quad [\approx \sqrt{\Phi} = 1.27201965]$$

The last two formulas work with the same input data of earth and moon and possess a form, which is harmonic and actually not too difficult to find (By that, probably someone might have found them before!).

Distance (Big Half Axle) Earth to Sun[1] without unit km:

$$4.8^{12} = 149.587 * 10^6$$

Compare the relative big deviations by small changes of the basis:

$$(0.999 * 4.8)^{12} = 4.7952^{12} = 147.802 * 10^6;$$
$$(0.999^{-1} * 4.8)^{12} = 4.8048^{12} = 151.394 * 10^6$$

$$666^2 * 60 / \sqrt{RT_{Earth}} = 666^2 * 60 / \sqrt{365.256} = 1392519 \quad [\approx 0.1 * \text{Ø}_{Sun} = 0.1 * 1392684 \text{ (km)}]$$

Distance (Big Half Axle) Earth-Moon without unit km: $620^2 = 384400$ [Remarkable: a square value!]

$$0.5 * 620^2 * (\sqrt{3} + 1/\sqrt{3}) - 0.5 * 620 = \mathbf{(666.0006659)^2}$$

The author gives the four figures Phi, Pi, 144 and 666 the marking "The Four Versatile Figures". Also the figures 9, 11 and 99 (=9*11), respectively are helpful figures not only referring the approximations of Physical Constants. Examples are the already listed relations of page 1:

$$10 * 666^{11/10} \approx \text{Ø}_{Earth} \quad \text{and} \quad 10 * 666^{9/10} \approx \text{Ø}_{Moon} \quad \text{and} \quad 10 * 666^{(11-9)*9.11/10} \approx \text{Ø}_{Sun}$$

Besides these six Figures (Phi, π, 144, 666, 9 and 11) the full numbers from 2 to 12, the figure 48 and Multiplies of 11 and 111 in connection with 10-Powers are often used. As it is shown later, also with help of the figures 1.286 and 14.146 (=11*1.286) Approximations of Physical Constants are listed, which results often are far within their tolerance.

Approximation for the Rotation Duration of the Earth [per unit day]:

$$10 * \sqrt{(2*666 + 6.66/\pi)} = 365.25607 \qquad \qquad \mathbf{(RT_{Earth})}$$

Upper approximation will be changed around. Within the operator √ the term (2*666) is now located in the denominator and the term 6.66/π is inverted, whose denominator is now multiplied by the figure 10. Operator Minus is applied at the the second term instead of operator Plus (another kind of inversion). Approximation for the Rotation Duration of the Moon [per unit day]:

$$10 * \sqrt{[10000/(2*666) - \pi/66.6]} = 27.31362$$

One can further increase the just presented Extraordinary. Please keep in mind the figures 2, 666 and 10000, which are used at the first term within the operator √ of the last equation. The last equation is widened by a third term within the operator √, at which also the just mentioned figures are used, and leads to the following equation:

$$10 * \sqrt{[10000/(2*666) - \pi/66.6 + (666/10000)^2]} = 27.321735 \qquad \qquad \mathbf{(RT_{Moon})}$$

It seems to be like a paper chase, but chasing figures and formulas with help of previously used figures and formulas!

Please look again at the two Equations **(RT_{Earth})** and **(RT_{Moon})** with their inversive terms to each other. Consider that the result values are only valid during a certain earthly time period and that the relations were found in this period. If the development of the earth with the moon or mankind itself had changed just a little bit different, the result values never would have been possessed this correctness to the existing values, neither in the past nor in the future!

Another formula for the Rotation Duration of the Moon [per unit day] dependent on the Rotation Duration of the Earth and on the term "$\Phi*\pi^2$" (see term "$\Phi^2*\pi$" at formula RT_{Earth2} below) is the following one:

$$\sqrt{[2 * RT_{Earth} + \Phi * \pi^2 - (\Phi * \pi^2)^{-2}]} = 27.321739 \qquad \textbf{(RT}_{\textbf{Moon}}\textbf{2)}$$

There are still two other remarkable approximations for the Rotation Duration of the Earth [per unit day], whose results have a small deviation to each other:

$$RT_{Earth1} = 10^{0.5} * 120 - 1.2^2 * \pi^2 - 0.5 * \pi^{-4} \qquad = 365.255956 \qquad \textbf{(RT}_{\textbf{Earth}}\textbf{1)}$$
$$RT_{Earth2} = 10 * 4.44 * \Phi^2 * \pi + (10 / 2) * 6.66 / 444 = 365.255958 \qquad \textbf{(RT}_{\textbf{Earth}}\textbf{2)}$$

Deviation is: $\textbf{RT}_{\textbf{Earth2}} - \textbf{RT}_{\textbf{Earth1}} = \textbf{1.867} * \textbf{10}^{\textbf{-6}}$

Is it random or an aspect of a creation act or is it the case, that one can describe very well the data of our celestial bodies, the Physical Constants (see next chapter) or any physical data using the Versatile Figures by similar constructed formulas?
Further it isn't easy finding those exact formulas using a transzendental figure, even it is wellknown!
Because of these assumptions isn't worth for deeper investigations about the connection of the Versatile Figures by expert mathematicians, who have the possibilities applying special softwares and their skills?

3) Approximations for Physical Constants by use of figures Φ, π, 144 and 666:

Please keep in mind, that even very small changes of many of the used input values at the exponents lead to approximation results, which are outside the tolerance range of the respective Physical Constants.
The result values of the approximations for the Physical Constants are written mostly without SI Units.

Fine Structure Constant α (α^{-1} = 137.035999084)[4.1], [4.2]:

Around the Fine Structure Constant α there is some kind of magic (rough approximation):

$$100 * 999 / (9*9*9) = 144 * 666 / (0.96 * 9*9*9) = 137.037037037 = \alpha_{\#0}^{-1} \qquad (\alpha0)$$

An approximation $\alpha_{\#}^{-1}$ dependent on the figure 137.036 is given by the following relation:

$$\alpha_{\#1}^{-1} = 137.036 * (1 - 6.66 * 10^{-9}) \qquad = 137.035999087340 \qquad (\alpha1)$$
$$137.036 = 0.999*144 - 6.66 - 0.4^2 = 2*66.6 + 2*1.44 + 1.1 - 0.144 = 144 + 1.286 - 3*11/4$$

$$\alpha_{\#2}^{-1} = 1 * \pi^4 + 4 * \pi^2 + 1 * \pi^{-2} + 5 * \pi^{-4} - 4 * \pi^{-6} = 137.035999087382 \qquad (\alpha2)$$
$$1 * 10^1 + 4 * 10^0 + 1 * 10^{-1} + 5 * 10^{-2} - 4 * 10^{-3} = 14.146 \qquad (F1)$$

Result value 14.146 of Equation (F1), which is derived by the multiplicators in front of the π-terms of Equation ($\alpha2$) and by use of decreasing 10-powers, is used not only at Formula ($\alpha3$). Please see the result close to figure "1.4146 (=14.146/10)" at the last page. A comprehensive explanation of Equation ($\alpha2$) and (F1) is readable in the report[5] of the author.

The difference of Approximation **($\alpha1$)** to the one **($\alpha2$)** delivers the **extremely small value 4.2*10⁻¹¹**.
This value is 1000 times smaller than the tolerance range (= $2 * 21 * 10^{-9}$) for the Reciprocal α^{-1} of the Fine Structure Constant.
Remarkable referring the figure 14.146, which is derived by Equation (F1), is:

$$14.146 = 11 * 1.286; \qquad 1286 * 777 = 999222 \ [999 - 222 = 777]; \quad 14.146 = 2*6.66 + 4*0.144 + 1/4;$$
$$\ln(\text{Ø}_{sun}) = \ln(1392684) = 14.146743;$$
$$44.444 / (1.2 * \Phi^2) \qquad = 14.146748;$$
$$\textbf{\Phi} + \boldsymbol{\pi} + \textbf{1.44} + \textbf{6.66} = \textbf{12.8596} \quad [\approx 10*1.286]; \text{ see another approximation of figure 1.286 at last page.}$$

Other Approximations of the Reciprocal of the Fine Structure Constant α:

$$\alpha_{\#3}^{-1} = (1 - 1/144/666)^{1/8.88} * 14.146 * 1.44 * 6.66 / 0.99 = 137.035999088345 \qquad (\alpha3)$$

The Main Term 137.03616 of Equation ($\alpha3$) can be described as follows:

$$14.146 * 1.44 * 6.66 / 0.99 = 137.03616 = 14.146 * 6.66 * (1 + 3/6.6)$$

$\alpha_{\#4}^{-1} = 137.036 - (1/20/\Phi)^4 = 137.035999088\,137$ (α4)

System with the figures 4 und 20 of Eq. (α4): $(1/20)^4 = \mathbf{0.625} * 10^{-5}$; $6.66/1.44 = \mathbf{4 + 0.625}$
Remarkable: $1.44/6.66 = 6*6*6*10^{-3} + 6*6*6*10^{-6} + 6*6*6*10^{-9} + 6*6*6*10^{-12} + \ldots$

Deviation of Formula (α3) to the one (α4) amounts only to $\mathbf{2.1 * 10^{-10}}$.

The part within the bracket of Formula (α5-mt) is widened by a third term and delivers Formula (α5).
Main Term: $14.146 * 6.66 * [1 + 3/6.6] = 137.03616$ (α5-mt)

$\alpha_{\#5}^{-1} = 14.146 * 6.66 * [1 + 3/6.6 - 1/(2*0.66*666^2)] = 137.035999089\,089$ (α5)

$\alpha_{\#6}^{-1} = \alpha_0^{-1} * (1 - 666 * 1.12244 * \pi^{-2} * 10^{-7}) = 137.035999089\,079$ (α6)

The difference of approximation (**α6**) [with $\alpha_0^{-1} = 100*999/(9*9*9)$] to the one (**α5**) is the smallest one referring the α^{-1}-approximations and delivers the **extremely small value $-9.95*10^{-12}$**.
Remarkable: $11+22+44 = 77$; $1.1/1.12244 = 0.\mathbf{98}0007840$; $\mathbf{98} = 2*7*7$; $\mathbf{784} = 16*7*7$

"Hans de Vries Formula"[6] with Euler Figure e (= 2.7182818) and Circle Figure π:

$\alpha_1 = \Gamma\alpha_1^2 * e^{-\pi*\pi/2}$ with
$\quad \Gamma\alpha_1 = 1 + \alpha_1/(2*\pi)^0 * (1 + \alpha_1/(2*\pi)^1 * (1 + \alpha_1/(2*\pi)^2 * (1 + \alpha_1/(2*\pi)^3 * (1 + \ldots))))$

Equation α_1 is only iteratively, but sufficient exactly to solve. Therefore the quantity $\mathbf{\Gamma\alpha_0}$ is used:

$\mathbf{\alpha_{HdV}^{-1} = \Gamma\alpha_0^{-2} * e^{\pi*\pi/2}} = 137.035999096$ (α-HdV)
$\quad \mathbf{\Gamma\alpha_0} = 1 + \alpha_0/(2*\pi)^0 * (1 + \alpha_0/(2*\pi)^1 * (1 + \alpha_0/(2*\pi)^2 * (1 + \alpha_0/(2*\pi)^3 * (1 + \ldots))))$

An appropriate exact input value for α_0^{-1} (α_0^{-1} between 137.035999084 and 137.035999110) is required to get the above result value (with 9 digits behind the decimal point) for α_{HdV}^{-1}.

Another formula with serie form:

$\alpha_{\#7}^{-1} = \Gamma\alpha_1 * [2 * \pi^{e*e/2}] = 137.035999088\,323$ (α7)
$\quad \Gamma\alpha_1 = 1 - \alpha_1/Z_1^1 * (1 - \alpha_1/Z_1^2 * (1 - \alpha_1/Z_1^3 * (1 - \alpha_1/Z_1^4 * (1 - \alpha_1/Z_1^5 * (1 + \ldots)))))$ (α7-1)
$\quad\quad$ with $\mathbf{Z_1 = 3.4111777} = 2*17*10^{-1} + 111777*10^{-7}$ (Z1)

An appropriate exact input value for α_1^{-1} (α_1^{-1} between 137.0359990880 and 137.0359990884) is required to get the above exact value (with 11 digits behind the decimal point) for $\alpha_{\#7}^{-1}$.

Connection of the figures: figures 17 and 111777, both consist of figure 1 and figure 7.
Connection of Z_1 to figure e: $\mathbf{Z_{1e}} = 2*e - (0.9^2 + 4*0.034) * (0.9^2 + 1.1^3) = 3.411177657$ ($\approx \mathbf{Z_1}$)
$\quad\quad Z_1$ to figure π: $\mathbf{Z_{1\pi}} = 2*\pi - (2^2 * 7^2 * 11^2) * (11^3 + 13^2 - 17^2) * 10^{-7} = 3.411177707$ ($\approx \mathbf{Z_1}$)
$\quad\quad\quad\quad$ Quantity $\mathbf{Z_{1\pi}}$ is dependent on the Serie Primes 7, 11, 13, 17
$\quad\quad\quad\quad Z_1^{0,666} * 5.55 = 12.566296$ ($\approx 4*\pi = 12.566371$)

The deviation of approximation (**α7**) to the one (**α3**) delivers the extremely small value $\mathbf{-2.21*10^{-11}}$.
Connection to figure 888 of Equation (α3) and figures 0.66 and 1.12244 of Equations (α5) and (α6):
$\quad 888*2^7 - 17*111 = 111777$ and $888 = 111+777$; $1.12244/0.66 = 1.7 + (2/3)*10^{-3}$

At the "Hans de Vries"-Formula (**α-HdV**) the figure 2 is used two times at the exponent as well as at the Equation (**α7**). The sites - basis and exponent - of figures π and e are reversed at Equation (α7) contrary to the "Hans de Vries"-Formula (**α-HdV**).

Formulas for the Plancks Constant h with basis "144*666" and crooked number at the Exponent, which leads to a value close to the figure 6.66:

$h = 6.62607015 * 10^{-34}$ J s[(4.3)];
$h_{wU} = 6.62607015 * 10^{-34}$; wU: without SI Units

$h_{\#0} = (144*666)^{-6.659942071}$ J s;   Exponent 6.659942071 is perceptibly close to the figure 6.66   (h0)

$h_{\#1} = 0.999^{-1} * (144*666)^{-6.66} / (1 + 1/66.6)^{1/(6.66*6.66)}$ J s = $6.62607010 * 10^{-34}$ J s   (h1)

Approximation (h2) for the Plancks Constant, which uses the figure 1.286 at the exponent and as multiplicator, is very close to the set value h_{wU} and also to the result value of Equation (h4) at page 7.

$h_{\#2} = (144*666)^{-6.66} * [1 + 1000/(1.286*144*666)]^{1.286*14.4*66.6 / (12*1286 - 444)} =$
$= 6.6216685222 * 10^{-34} * 1.0006647309 = 6.626070149969 * 10^{-34}$ (h2)

Approximation (h3) for the Plancks Constant, which uses prime numbers at the exponent, is also close to the set value h_{wU}:

$h_{\#3} = (144*666)^{-6.66} / [1 - (0.999/144/666)^{23*47*59 / 100000}] =$
$= 6.6260701494 * 10^{-34}$ (h3)

Connection of the primes at the exponent: 23+2*12=47;   47+12=59;   23+47+59=3*43;   47+59=2*53
Fine-Tuning Term: the exponent term "23*47*59 / 100000" is now located within the rectangular brackets.

Light velocity[(4.4)] c without SI Units with semi-serie form:

c = 299792458 m/s
c_{wU} = 299792458;   wU: without SI Units

$c_{\#1} = 144^3 + 666^3 + 3*(144^2 + 666^2) +$
$+ 6*(144^1 + 666^1) +$
$+ 9*(144^{0.5} + 666^{0.5}) +$
$+ 12*(144^{0.25} + 666^{0.25}) = 299792458.79$ (c1)
Deviation: $c_{\#1} - c_{wU} = 0.79$

Remarkable: with the first link of above Equation (c1) this good approximation for the sun diameter in unit km is yielded: **$3*(144^2 + 666^2)$ km = 1392876 km**   [≈ Ø$_{Sun}$]

Exact Formula for the light velocity c_{wU}:
$c_{\#2} = 144^3 + 666^3 + 3*(144^2 + 666^2) + (40/9)*144 + 7*666 = 299792458$ (c2)
This is the exact value, deviation Zero !!!
System behind it: multiplicator 7 with 666 and figures 40 and 9 with 144: $7^2 = 40 + 9$

[**A short insert:**
Formula (c1) is changed to a formula with serie character, which result corresponds to Equation (α0). As listed in the following one has only a single basis (namely 10), the multipliers within the rectangular bracket increase by 3 starting from 3 and the exponents decrease by 1 starting from 2:
$\alpha_{\#S}^{-1} = 0.1*10^3 + 0.1*[3*10^2 + 6*10^1 + 9*10^0 + 12*10^{-1} + 15*10^{-2} + 18*10^{-3} + 21*10^{-4} + …] =$
$= 100*999 / (9*9*9) = 137.037037037… ≈ \alpha^{-1}$ (α: Fine Structure Constant)

A serie formula with the basis 11 and with the above exponents is:
$3*11^2 + 6*11^1 + 9*11^0 + 12*11^{-1} + 15*11^{-2} + 18*11^{-3} + 21*11^{-4} + … = 439.23 = 3*11^4 / 100$

One has to proove these serie formulas with a more exact software as with a regular spreadsheet program. Regarding the multipliers there is a similarity of the approximations (c1) and the above formula for $\alpha_{\#S}^{-1}$, the last-named equation possesses the same result as Equation (α0). See Page 3.]

Very close approximation with three times the figure 9942:
$c_{\#3} = (8*144*666)^{1.44} * 0.9942^{-1} * [1 + 1/(1.44*9942)^{2/0.9942}] = 299792458.04$ (c3)
Deviation: $c_{\#3} - c_{wU} = 0.04$
Connection to the figure 9942: 9942 - 12*144 - 12*666 = 666/3

Formula for the Gravitation Constant G $[= (6.67430 \pm 0.00015) * 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}]^{(4.5)}$:

   $G_{wU} = 6.67430(15) * 10^{-11}$ (wU: without SI Units)

   $G_{\#1} = 1 / [4.8^8 * 666^2 * (2 - 1.2^{-1} * 0.999^{-1} * 666^{1/8})] = 6.6743025 * 10^{-11}$ (G1)

System behind it: number 8 two times at the exponents, multiples of the numbers 1.2 and 111!
Remarkable the relations: $1.2^{-1} * 0.999^{-1} * 666^2 = 37 * 10^4$ $[36 * 37 = 2 * 666]$
                $1.2^{-1} * 0.999^{-1} * 666^{-1/8} = 0.370099916$ $[\approx 37 * 10^{-2} + 999 * 10^{-7}]$
                $1.2^{-1} * 0.999^{-1} * 666^{1/8} = 1.88012855$ $[2 - 1.88012855 \approx 12 * 10^{-2}]$
                $4.8^8 * 666^2 = 1.2499089 * 10^{11}$ $[\approx 1.25 * 10^{11}]$

Another Approximation with the figures 4.625 (=6.66/1.44), 36 and 37 (36*37=2*666) is:

   $G_{\#2} = [1.44 * (144/0.99)^{4.625}]^{-1} * [1 + 1.44/(6.66 * 10^4)]^{-1600 * 36/37} = 6.67430016 * 10^{-11}$ (G2)
   Main term: $[1.44 * (144/0.99)^{4.625}]^{-1} = 6.90277563 * 10^{-11}$

The deviation of $G_{\#2}$ to G_{wU} amounts to $1.6 * 10^{-18}$, which is only 1/938 of the tolerance $+15 * 10^{-16}$.

There is an interesting Equation (G-EK) for the Gravitation Constant G of Dr. Endre Kereszturi[7]. The result with added units (Meter **m** and second **s**) is very exact referring the tolerance:

   $G_{EK} = h^5 * \alpha^2 / [(c^2 * m_e^6) * (4 * \pi)^3] \text{ m}^{-5} \text{ s} = 6.6743017 * 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ (G-EK)

The result value $6.6743017 * 10^{-11}$ (without SI Units) is as mentioned before very exact, however the units "$\text{m}^5 \text{ s}^{-1}$" have to be solved at the Equation (G-EK). Quantity m_e is the Electron mass[4.6].
The author is convinced, that this result, which is far within the tolerance, isn't random, even the units do not agree to the ones of the Gravitation Constant G.
His theory to this topic is, that there might be a plausible relation in the following form:

   $G_{\#3} = [h^5 * \alpha^2 / (c^2 * m_e^6) / (4 * \pi)^3] * T_{EK} = 6.6743017 * 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ (G3)

        with Term $T_{EK} = lr^4 * c_G * F_G = 1 \text{ m}^5 \text{ s}^{-1}$ or $T_{EK} = lr^5 / t_G * F_G = 1 \text{ m}^5 \text{ s}^{-1}$

lr: Length or Radius in m; for example the Electron Radius r_e
c_G: Velocity in m s^{-1}: for example the Light Velocity c
t_G: Time in s: for example the Planck-Time
F_G: Factor without SI Units, which follows for example a formula $F_G(\alpha, \pi, 144, 666 \text{ etc.})$.

To this topic a formula in the following form can be given:

   $lr^4 * c * (666/\pi)^{16} * 666^{4.8} = r_e^2 * r_p^2 * c * (666/\pi)^{16} * 666^{4.8} = 1.00001844 \text{ m}^5 \text{ s}^{-1}$ (G4)

with $lr^4 = r_e^2 * r_p^2$;   Proton Radius r_p according to Pohl[8]: $r_p = 0.84087 * 10^{-15} \text{ m}$
and   c: Light Velocity[4.4]   and r_e: Electron Radius[4.7]

   $G_{\#5} = G_{EK} / [r_e^2 * r_p^2 * c * (666/\pi)^{16} * 666^{4.8}] = 6.67418 * 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ (G5)

The deviation of Equation G5 to the set value G amounts to "$-12.1 * 10^{-16}$", which is about 80% of the tolerance "$-15 * 10^{-16}$". Values are given without SI Units of the Gravitation Constant.

For a Proton Radius $r_{p\#}$, which delivers exactly the full number 1 as result of Equation (G4), one performs the following formula in dependence of the Electron Radius r_e[4.7] and Light velocity c:

   $r_e^2 * r_{p\#}^2 * c * (666/\pi)^{16} * 666^{4.8} = 1 \text{ m}^5 \text{ s}^{-1}$
   $r_{p\#} = (r_e * \sqrt{c})^{-1} * (666/\pi)^{-8} * 666^{-2.4} \text{ m}^{2.5} \text{ s}^{-0.5} = 0.840862 * 10^{-15} \text{ m}$

There is another formula using the Electron Radius r_e and the Light Velocity c, which exactly fulfills Equation (G-EK) and which is partly dependent on prime numbers:

   $r_e^4 * c = (2.8179403262 * 10^{-15} \text{ m})^4 * 299792458 \text{ m s}^{-1} = 1.89037459 * 10^{-50} \text{ m}^5 \text{ s}^{-1}$
   $r_e^4 * c * 529 = r_e^4 * c * 23^2 = 1.0000082 * 10^{-47} \text{ m}^5 \text{ s}^{-1}$

Approximation for above relation without SI Units: $\pi^{-85085/900} = 1.0000079 * 10^{-47}$

Relative deviation of the just won figures: $(1.0000079 - 1.0000082) / 1.0000082 = -2.74 * 10^{-7}$

$$G_{\#6} = G_{EK} * [\pi^{-85085/900} / (r_e^4 * c * 529)] = [h^5 * \alpha^2 / (c^2 * m_e^6) / (4*\pi)^3] * [\pi^{-85085/900} / (r_e^4 * c * 23^2)] =$$
$$= [6.6743017 * 10^{-11} \text{ m}^8 \text{ kg}^{-1} \text{ s}^{-3}] * [1.0000079 * 10^{-47} / (1.0000082 * 10^{-47} \text{ m}^5 \text{ s}^{-1})] =$$
$$= 6.67429988 * 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \tag{G6}$$

Five prime numbers – starting with the number 5 and the next four prime numbers – multiplied by each other deliver the number 85085, which is part of the term "-85085/900" of the above exponent with its basis π. At Figure 900 ($= 2^2 * 3^2 * 5^2$) one can find the primes 2, 3 and 5.

$85085 = 5 * 7 * 11 * 13 * 17$ $(= 1000*5*17 + 5*17)$; 5 and 17 are the first and the last prime numbers

Sums of three successive prime numbers result again to prime numbers:
$5 + 7 + 11 = 23$ (see above: number 23^2 (=529) used as multiplicator at Equation (G6))
$7 + 11 + 13 = 31$ and
$11 + 13 + 17 = 41$

Sum: $23+31+41 = 95 = 5 * 19$; with the numbers 19 und 23 there are seven successive prime numbers!

Approximation of the Elementar Charge e without SI Unit:

$e = 1.602176634 * 10^{-19}$ C[(4.8)]
$$e_{wU} = 1.602176634 * 10^{-19} \qquad \text{(wU: without SI Unit)} \tag{e0}$$

$$e_{\#1} = 666^{-6.66} * [(1 + 1/(144*666)]^{3*666+0.666/3} = 1.6021766334 * 10^{-19} \tag{e1}$$
Isn't it peculiar – the form (5 times figure 666) as well as the accuracy?

The best approximation for the Elementar Charge e is won by use of the Term "$4.8 * \pi * \Phi$ (=24.3994)", used as basis as well at the exponent term:

$$e_{\#2} = (4.8*\pi*\Phi)^{-0.555222*4.8*\pi*\Phi} * [1 - 1/(0.666888*144*666)]^{66.6-12} =$$
$$= 1.6021766338 * 10^{-19} \tag{e2}$$

The wanted exponent of the basis $[1 - 1/(0.666888*144*666)]$ of the Fine Tuning Term, by which the most exact result value ($1.6021766340 * 10^{-19}$) is reached for the Elementar Charge e, takes the value 54.59999:
$54.59999 = 66.59999 - 12$.
The figures 555222 and 666888 with their six-digit form NNNMMM (N and M are full numbers between 1 and 9) multiplied by 10-powers fit to the next section with the figure 111222.

Approximations of several Physical Constants (without SI Units) by use of Figure 111222:

Electron radius r_e (=$2.8179403262(13) * 10^{-15}$ m)[(4.7)]:
$$r_{e\#1} = (4*\Phi*\pi)^{-11.1222} / [1 + 11.1222/(144*666)]^{(2*11.1222 + 44.4444)/99} = 2.8179403265 * 10^{-15} \tag{re1}$$

Gravitation Constant with exponent 66.6888 (=2*11.1222 + 44.4444): see also exponent 66.6888 above
$$G_{\#7} = 1 / [(4*\Phi*\pi)^{(1.11222 + 6.66666)} * [1 - 1/(1.44*6.66)]^{1/66.6888}] = 6.67429979 * 10^{-11} \tag{G7}$$

Result without SI Units ($=6.67429988*10^{-11}$) of Equation (G6) is close to one of Equation (G7).

Plancks Constant with the shortly before used figure serie 666888 at the exponent at Equation (h4):
$$h_{\#4} = (144*666)^{-6.66} / [1 - 1/(144*666)]^{5*5*17/6.66888} = 6.626070149967 * 10^{-34} \tag{h4}$$

See the use of figures 5 and 17 at the exponent of Equation (G6) and further please compare the result value of Equation (h4) with the set value[(4.3)] $6.62607015 * 10^{-34}$ (without SI Units).

Result of Equation (h4) is close to one of Equation (h2): $h_{\#2} = 6.626070149969 * 10^{-34}$

Coulomb Constant[(4.9)] k_C dependent on Planck Constant h without SI Units (h_{wU}):

$h_{wU} = 6.62607015 * 10^{-34}$; $k_{CwU} = 8.9875517922 * 10^9$;   wU: without SI Units

Main Term: $h_{wU}^{-0.3} = 8.9871748496 * 10^9$

$k_{C\#1} = h_{wU}^{-0.3} * [1 + 1/(3*1.11222)]^{16/100000} = 8.9875517928 * 10^9$ (kc1)

Deviation: $(k_{C\#} - k_{CwU}) = 0.579$

Each result of the last four formulas (r_e1), (G7), (h4) and (k_C1) is within the tolerance; Tolerance ±1.4 is given for the Coulomb Constant k_C.

Remarkable in this context are still the following formulas:

$1.44 * 6.66 * 3 * 1.11222 = 31.999904$ [≈ 32 = 2*16; see figure 16 at above exponent]

$32 / (1.44 * 6.66 * 3) = 1.11222333444556$

Mass Ratio Neutron[(9)] to Proton[(10)]:

$MR_{Ne-Pr\#1} = 0.999^{-1} * [1 - 1000/(144*666)]^{-4/111.222} = 1.00137841939$ (MR_{Ne-Pr}1)

$MR_{Ne-Pr} = m_n / m_p = 1.00137841931$ with

Proton Mass: $m_p = 1.67262192369 * 10^{-27}$ kg;   Neutron Mass: $m_n = 1.67492749804 * 10^{-27}$ kg

Mass of Electron[(4.6)] without unit kg:

$m_{e\#} = [(12.5 - 1) / 12.5] * (144 * 666 / 0.999 / 3)^{-6.66} / [1 - 1/(144*666)]^{6.66/(1250+12.5)} =$
$= 9.1093837015 * 10^{-31}$ (m_e1)

The figure 12.5 occurs two times at the exponents and also two times at the first term!
The term (144 * 666 / 0.999 / 3) can be written by the term "$T_x = 2^5 * 10^3 (= 32000)$".
Connection to the figure 125: $5^3 = 125$ (Figures 5 und 3 are exponents of term T_x)

Formula of **Mass Ratio Neutron[(9)] to Electron[(4.6)]** (=1838.68366173) with main term MR_{Pi}:

$MR_{Pi} = (2*\pi)^4 + (2*\pi)^3 + (2*\pi)^2 -$
$- (2*\pi)^1 - (2*\pi)^0 - (2*\pi)^{-1} +$
$+ (2*\pi)^{-2} + (2*\pi)^{-3} + (2*\pi)^{-4} = 1838.66175070$ (MR1)

System: the first and the last three terms are positive, the three mid terms negative. Exponents from +4 down to -4, next exponent always by 1 decreasing.

Very exact Approximation (MR2) for the mass ratio with the Fine Tuning Term $(1 - 1/144/666)^{8/7}$ is:

$MR_{Ne-El\#} = MR_{Pi} / [1 - 1/(144*666)]^{8/7} = 1838.68366169$ (MR2)

System with figure 2, the first three exponents of Equation (MR1) and figures 7 and 8:

$2 * (2^4 + 2^3 + 2^2) = 7 * 8 = 56$ [See the three terms with figure 2 and their exponents at (MR4)]

System with figure 3 and α: $MR_{Ne-El\#} = MR_{Pi} + [3^1*\alpha^1 + 3^{-1}*\alpha^2 + 3^1*\alpha^3] = 1838.68366168$ (MR3)
Result of Equation (MR3) is very close to the one of Equation (MR2)!

Formula of **Mass Ratio Proton[(10)] to Electron[(4.6)]** (=1836.15267343) with main term MR_{Pi3}:

Main Term MR_{Pi3} with figures 7 und 8 and figure 2.4:

$MR_{Pi3} = (2*\pi)^4 + (2*\pi)^3 + (2*\pi)^2 - 0.5*(7+8) - 2.4 = 1836.17408759$ (MR4)

Very exact approximation for this mass ratio:

$MR_{Pr-El\#} = MR_{Pi3} * [1 - (1/144/666)^{0.99+2.4/10000}] = 1836.15267349$ (MR5)

System (paper chase, but with figures): $0.5*(7+8) = 10*0.99 - 2.4 (= 7.5)$;
Remarkable: $(m_p/m_e)^{2/3} = 149.947487$ [≈ 150 = 100 * 3/2];   used Ratio 2/3 is often listed at formulas of the last page!

Mass Ratio Neutron[9] to Proton[10] (=1.001 378 41931) **dependent on the light velocity c:**

$(m_n / m_p)^{0.5} * c_{wU} * 10^{-8} = 2.999990067$ (nearly 3); c_{wU}: Light Velocity c without SI Units

$(m_n / m_p)_{\#2} = [(3 * 10^8 / c_{wU}) * (1 - (1/144/666)^{1.1})]^2 = 1.001\,378\,418\,98$ (MR$_{Ne-Pr}$2)

Mass Ratio Tauon[11] to Electron[4.6]:

$MR_{\tau/e} = m_\tau / m_e = 0.999^{-2*0.99} * (2*\pi)^{0.999*4.44} = 3477.2429$ (MRτ/e)

Exponent term $0.999*4.44$ is equal the term $6.66*0.666$.

Mass Ratio Myon[12] to Electron[4.6] is:

$MR_{\mu/e} = m_\mu / m_e = 0.999^{-2/1.14} * (2*\pi)^{2.9} = 0.999^{-2*0.99/(1+0.1*1.286)} * (2*\pi) * (2*\pi)^{1.9} =$
$= 206.7682821$ (MRμ/e)

Remarkable referring the exponents of basis 0.999 at the last two Equations: there are connections, which lead to the figure 0.99 as well to figure 1.9 at the exponents of Equation (MRτ/e) and (MRμ/e) by use of the figure 1.286:

$0.99 = (1 + 0.1*1.286) / 1.14$ and $1000 * (1 + 0.1*1.286) / (6*99) = 1.9$; $19 * 6 = 114$

As proof of these connections an approximation for the reciprocal of the Fine Structure Constant is given, at which also the just named figures 0.99, 1.286 and 1.9 come to use:

$\alpha_{\#8}^{-1} = 0.99 * (\pi^4 + \pi^3 + \pi^2) / (1 - 1.286/10^4)^{4*1.9} = 137.0359990732$ (α8)

If the term "$4*1.9$" at the exponent of Equation (α8) is tiny little changed to the terms "$4*(1.9 \pm 1*10^{-6})$", the results of Equation (α8) lie outside the tolerance! Isn't that impressive?
Compare the exponents of basis π at Equation (α8) with the exponents at Equation (MR4); see page 8!

Atomic Mass of Helium[13] in Unit u dependent on light velocity c:

$m_{uHe} = 4.002602$ u   (Tolerance: $\pm 2*10^{-6}$ u)

$m_{uHe\#1} = (12 * 10^8 / c) * (1 - 1/144/666)^4$ u m s^{-1} = 4.0026022 u (m$_{uHe}$1)
    Deviation: $m_{uHe\#1} - m_{uHe} = 1.96 * 10^{-7}$ u

$m_{uHe\#2} = (12 * 10^8 / c) * (1 - 1/144/666)^{4.002602}$ u m s^{-1} = 4.00260209 u (m$_{uHe}$2)
    Deviation: $m_{uHe\#2} - m_{uHe} = 8.74 * 10^{-8}$ u
    Remarkable: Formula (m$_{uHe}$2) with the exponent term "m_{uHe} u^{-1}" is more exact as Form. (m$_{uHe}$1)!

Another exact formulas for the atomic mass of Helium:

$m_{uHe\#3} = (4/3 * 10^{-8} * c) * (1 - 10/144/666)^{-12.86/0.999}$ u m^{-1} s = 4.002602008 u (m$_{uHe}$3)
    Deviation: $m_{uHe\#3} - m_{uHe} = 7.67 * 10^{-9}$ u
Remarkable referring the used exponent "-12.86/0.999": $(1.286/0.999)^{1.44} / 0.999 = 1.4400156$ [≈ 1.44]

$m_{uHe\#4} = (14.146 * 1.286^{8.5}) * [e^2 / (m_e * r_e) / c] * (1 - 1/144/666)^{14.146-10}$ u S =
    $= 120.000177 * [9999999.99457 / 299792458] * 0.99995677$ u =
    $= 4.002602005$ u (m$_{uHe}$4)
    Deviation: $m_{uHe\#4} - m_{uHe} = 5.3 * 10^{-9}$ u
Exponent 8.5 (= 0.5*17) is used also at Equation (G6) and at Equation (h4), both presented at page 7.
Term "$e^2 / (m_e * r_e) = 9999999.99457$ C^2 kg^{-1} m^{-1}" is derived by Equation (μ0), which is listed below!

$m_{uHe\#5} = 4 * (1 - 1/128.6)^{-1/12}$ u = 4.00260299 u (m$_{uHe}$5)
    Deviation: $m_{uHe\#5} - m_{uHe} = 9.9 * 10^{-7}$ u

$$m_{uHe\#6} = (2 * 0.666 * 10^{-8} * c) * (1 - 1/128.6)^{-1/[2*(1+0.666)]} \text{ u m}^{-1}\text{ s} = 4.00260214 \text{ u} \quad (m_{uHe}6)$$

Deviation: $m_{uHe\#6} - m_{uHe} = 1.4 * 10^{-7}$ u

The Reciprocal μ_0^{-1} of the Magnetic Field Constant[4.10] multiplied by the term "$4*\pi$" (It's well known):

$\mu_{0wU} = 1.25663706212(19) * 10^{-6}$; wU: without SI Units $C^{-2} \text{ kg}^1 \text{ m}^1$; tolerance $\pm19*10^{-17}$

$$\mu_0^{-1} * 4 * \pi = e^2 / (m_e * r_e) = 9999999.9946 \text{ C}^2 \text{ kg}^{-1} \text{ m}^{-1} \quad [\approx 10^7] \quad (\mu 0)$$

$$\mu_{0\#1} = 4 * \pi * 10^{-7} = 1.25663706144 * 10^{-6} \quad (\mu 1)$$

Formula (μ1) is multiplied by the term $[1 + (1/6.66)^{10*9/8}]$, which leads to a very accurate result:

$$\mu_{0\#2} = 4 * \pi * 10^{-7} * [1 + (1/6.66)^{10*9/8}] = 1.2566370621200 * 10^{-6} \quad (\mu 2)$$

Deviation: $\mu_{0oE} - \mu_{0\#2} = 3.3 * 10^{-20}$

System of Formula (μ2) with figures 9 and 8 at the exponent referring to the figure 144:

$2 * (9 * 8) = 144!$

Proton Radius r_P:

Proton Radius r_P according to Pohl[8]:

$r_P = 8.4087(39) * 10^{-16}$ m (max: $8.4126*10^{-16}$ m; min: $8.4048*10^{-16}$ m)

$r_e/r_P = 2.8179403262 * 10^{-15}$ m $/ 8.4087 * 10^{-16}$ m $= 3.3512200$

A simple approximation of the Proton Radius r_P, which is dependent on the Electron Radius r_e, on the Fine Structure Constant α and the mass ratio Electron to Proton, can be written as follows:

$$r_{P\#1} = 4 * r_e * (m_e / m_p) * \alpha^{-1} = 8.412356 * 10^{-16} \text{ m} \quad (r_P1)$$

The value of approximation (r_P1) lies within the tolerance. Approximation (r_P1) corresponds to the value, which is given by the Elementarkörpertheorie© (Proton Radius) of Dirk Freyling[14] with its form:

$$r_P = 2 * h / (\pi * c * m_p) = 8.412356 * 10^{-16} \text{ m} \quad (r_P Ek)$$

Formula (r_P1) is multiplied by the term ($1-m_e/m_p$), by that its result value is closer to the set value of Pohl ($=8.4087*10^{-16}$ m):

$$r_{P\#2} = 4 * r_e * (m_e / m_p) * \alpha^{-1} * (1 - m_e/m_p) = 8.4078 * 10^{-16} \text{ m} \quad (r_P2)$$

$$r_{P\#3} = [\pi / (1.44*2^3)] * r_e * (m_p / m_e)^{2/3} * \alpha = 8.4088 * 10^{-16} \text{ m} \quad (r_P3)$$

One considers: $2^3 / (6.66/0.999) = 1.2$; Figures 2 and 3 are combined two times at Equat. (r_P3)

$$r_{P\#4} = \sqrt{\sigma_e} / (10*0.22290) - r_e = r_e * [\sqrt{(\pi*8/3)} / (10*0.22290) - 1] = 8.4122 * 10^{-16} \text{ m} \quad (r_P4)$$

Thomson Cross Section[4.11] σ_e $[= (8/3)*\pi*r_e^2]$

Weinberg Angle[4.12] ($\sin^2$) is: $0.22290(30)$

$$r_{P\#5} = (14.146 - 8)^{-0.666} * r_e = (6.146)^{-0.666} * r_e = 8.408706 * 10^{-16} \text{ m} \quad (r_P5)$$

$$r_{P\#6} = (14.146 / T_{ln})^{-0.666} * r_e = (14.146 / 2.3016622)^{-0.666} * r_e = 8.40871 * 10^{-16} \text{ m} \quad (r_P6)$$

Derivation of the term T_{ln} ($=2.3016622$) of Equation (r_P6):

$\text{Ratio}_{e-p\#} = \sqrt{19} / (2.3009001 - 1) = 3.350679$ with $2.3009001 = \ln(1000) / \ln(r_e/r_P) - 3.4111777$

$T_{ln} = \ln(1000) / \ln(\text{Ratio}_{e-p\#}) - 3.4111777 = 2.3016622$

see use of figure 3.4111777 at page 4 ($=Z_1$) and page 13; Primes of Figure 23009($=7*19*173$):

$29=173-144$; $180=173+7$; $137=173+3*7-3*19$; see Figures at Eq. (α-17) and (α-18), page 14

Approximation (r$_p$6) isn't to solve directly, because figure 2.3009 of the quantity Ratio$_{e-p\#}$ is dependent on the set value r$_p$. This formula is listed as an evidence, that the use of figure 3.4111777 as input quantity at Equation (α7-1) isn't a single case. See also the use of figure 3.4111777 at Formula (α9) at the next page!
Remarkable: $\ln(23) / \ln(3.4111777) * 9 = 22.99766$ [$\approx 229977 / 10000$; $22 + 77 = 99$]

Results of Equations (r$_p$5) and (r$_p$6) are located very close to the set value r$_p$ (=8.4087*10^{-16} m).

$$r_{p\#7} \;=\; 0.333^{1.1} * r_e \;=\; 8.40660 * 10^{-16} \text{ m} \qquad\qquad (r_p7)$$

$$r_{p\#8} \;=\; \sqrt{[16 / (10*\sqrt{17}*\sqrt{19})]} * r_e \;=\; 8.4080 * 10^{-16} \text{ m} \qquad\qquad (r_p8)$$

$$r_{p\#9} \;=\; \alpha^{-0.5} * \sqrt{[30 / (11*13*17*19)]} * r_e \;=\; 8.4070 * 10^{-16} \text{ m} \qquad\qquad (r_p9)$$
$$\text{Connection of the used figures:} \quad 30 = 0.5 * (11 + 13 + 17 + 19) \quad [11, 13, 17, 19: \text{primes in serie}]$$

$$r_{p\#10} \;=\; [\alpha^{-0.5} / (9*\sqrt{19})] * r_e \;=\; 8.40872 * 10^{-16} \text{ m} \qquad\qquad (r_p10)$$
$$\text{Prime figure 19 is repeatedly used in this section. See Equations } (r_p6), (r_p8) \text{ and } (r_p9)!$$

$$r_{p\#11} \;=\; \alpha^{-0.5} * (1.286 + 1.6) * (144*666*\pi / 34111777) * r_e \;=\; 8.40868 * 10^{-16} \text{ m} \qquad\qquad (r_p11)$$
$$\text{See term "16/10=1.6" within the root operator at Equation } (r_p8)!$$

Modification of the following Equation: $r_e/r_p + r_p/r_e = 3.649619$

Analog upper modification term "$r_e/x_p + x_p/r_e$" leads to an equation of second order with the quantity x_p (=$r_{p\#12}$) to find. Value 3.649619 is close to the one of the term "$500*\alpha$" (=3.648676). The multiplicator "$(1+m_e/m_p)^{0.5}$" is added to get an accuracy within the tolerance. The equation of second order amounts to:

$$x^2 \;-\; 500 * \alpha * (1 + m_e/m_p)^{0.5} * r_e * x \;+\; r_e^2 \;=\; 0 \quad \text{--->} \quad x = r_{p\#12} = 8.40856 * 10^{-16} \text{ m} \qquad (r_p12)$$

Approximation T$_{N\#}$ (without unit K) of the Norm Temperature T$_N$ (=273.15 K)
$$T_{N\#} = 2 * \alpha^{-2} / (4*\pi^3 + \pi^2 + \pi^1 + \pi^{-1} + \pi^{-2} + \pi^{-3} + \pi^{-4}) = 273.150202$$

See used Full Numbers 4, 3, 2, 1 and -1, -2; -3; -4 within the brackets of the denominator! Sum is Zero!
Sum of Full Numbers at the numerator is also Zero: $2 + (-2) = 0$!

Approximation p$_{N\#}$ (without unit Pa) of the Norm Pressure p$_N$ (=101325 Pa)
$$p_{N\#} = 10^6 / \pi^2 / [1 - (1/144/666)^{0.888}] = 101321.1836 / [1 - (1/144/666)^{0.888}] = 101325.0017$$
See also the use of figure 888 at the exponent of the Fine-Tuninig Term of Equation (α2); see page 3:
$$\alpha_{\#2}^{-1} \;=\; (1 - 1/144/666)^{1/8.88} * 14.146 * 1.44 * 6.66 / 0.99 \;=\; 137.035999088\,345 \qquad (\alpha2)$$
Main Term at Equation (α2): $14.146 * 1.44 * 6.66 / 0.99 \;=\; 137.03616 \;=\; 1.286 * 12 * \mathbf{8.88}$

Product "Norm Pressure times Norm Temperature":
$$PR_{PnTn} \;=\; p_N * T_N \text{ Pa}^{-1} \text{ K}^{-1} \;=\; 101325 \text{ Pa} * 273.15 \text{ K Pa}^{-1} \text{ K}^{-1} \;=\; 27676923.8$$
$$PR_{PnTn\#} \;=\; 11 * \alpha^{-3} \;-\; 33 * \alpha^{-2} \;-\; 77 * \alpha^{-1} \;=\; 27676931.5$$
$$PR_{PnTn\#} - PR_{PnTn} = 7.795; \qquad \text{Value is a small deviation related to quantity } PR_{PnTn}$$

Conspicious: difference of multiplicators in front of the α-terms is each time:
$$11 - (-33) = -33 - (-77) = 44 \qquad \text{furthermore} \qquad |11 - 33 - 77| \;=\; 99 \;=\; 9 * 11$$

Norm Pressure and Norm Temperature are fixed Norm Data for our Planet. By that aren't the harmony of the Quantity PR$_{PnTn\#}$, which is besides full numbers only dependent on an Universal Constant (namely the Fine Structure Constant α), and its connection to these earthly Norm Data remarkable.

Norm Temperature $T_{N\#}$ - dependent on Product $PR_{PnTn\#}$ as a function of the Fine Structure Constant α, on molar Volume V_m and on Gas Constant R_m - is derived with help of the Gas Equation "$p_N*V_m/R_m=T_N$":

$$T_{N\#} = [(V_m / R_m) * PR_{PnTn\#} \ \ Pa \ K]^{0.5} = 273.15004 \ K; \quad p_{N\#} = T_{N\#} * R_m / V_m = 101325.014$$

The following approximations for the Reciprocal α^{-1} of the Fine Structure Constant are a broadening of the Equation (α_{HR})[15] of R. Heyrovska, which is dependent on the Golden Ratio Φ:

$$\alpha_{HR}^{-1} = 360 / \Phi^2 - 2 / \Phi^3 = 137.035628095 \tag{αHR}$$

At Equation (α_{HR}) the ratio of the two multiplicators (360 and 2) is 1/180 (= 2/360).
The Ratio of the Koide Formula[16] is close to the ratio 2/3, which can be seen at the last page. The ratio of the values (2 at the first term and 3 at the second term) of the two - adjacent - Φ-exponents is 3/2 (=1.5).
Considering these ratios for an third term T_{3HR} one gets the proportionality related to the second term:

$$T_{3HR} \sim (2/180) * (1/ \Phi^{3*1.5}) = 1 / (90 * \Phi^{4.5})$$

$$\alpha_{\#9}^{-1} = 360 / \Phi^2 - 2 / \Phi^3 + (3.4^{1.2*\Phi*\Phi-1} / 3.4111777^{1.2*\Phi*\Phi}) / (90 * \Phi^{4.5}) =$$
$$= 137.035999080 \tag{α9}$$

See the Exponent Exp_Φ (= $(3/4)^{1.2*\Phi*\Phi}$) with its exponent term "$1.2*\Phi^2$" at formulas listed at the last page.

Considering the three figures $2 - 3 - 5$ of the Fibonacci Serie $0 - 1 - 1 - 2 - 3 - 5 - 8 - 13 - 21 - \dots$, one gets the proportionality for a third term T_{3HR} derived by the sum of the two previous exponents "2 - 3":

$$T_{3HR} \sim 1/ \Phi^{2+3} = 1/ \Phi^5$$

$$\alpha_{\#10}^{-1} = 360 / \Phi^2 - 2 / \Phi^3 + 1 / (1.286 * 3^3 * 7 * \Phi^5) = 137.035999082 \tag{α10}$$

At the third term of Formula (α10) the three single-digit primes 3, 5, 7 and the known figure 1.286 are used. Term "$1.286*3^3*7*\Phi^5$" corresponds to the term "$90*0.1286*3*7*\Phi^5$ (= $90*2.7006*\Phi^5$)".
With figures 2.7004 and 2.7008, respectively instead of figure 2.7006 the results of Equation (α10) are outside the tolerance.

Considering the three figures serie $2 - 3 - 4$ one gets the proportionality for an third term T_{4HR} derived by ascending exponent value 1 and the ratio 1/180 related to the second term of Equation (α_{HR}):

$$T_{4HR} \sim (2/180) * (1/ \Phi^{3+1}) = 1 / (90 * \Phi^4)$$

$$\alpha_{\#11}^{-1} = 360 / \Phi^2 - 2 / \Phi^3 + 0.228855 / (90 * \Phi^4) = 137.035999089 \tag{α11}$$

Connection of figure 228855: $22 + 66 = 88$; $22 + 0.5*66 = 55$

Remarkable is the connection of figure 228855 to the used figures (without decimal points) of Equation (α10):

$$228855 = (360/2) * 1286 - 3 * 5^3 * 7$$

At page 2 the Formula of the Golden Ratio Φ is presented:

$$\Phi = (5+5*\sqrt{5})/10 = 1.618033989$$

As one can see, the Golden Ratio Φ is connected to the figure 5. Please keep in mind: $10*\Phi = (5+5*\sqrt{5})$

Another rough value for the Reciprocal of the Fine Structure Constant is written to:

$$\alpha_{\#12rough}^{-1} = 555 / 4 - 5 * 5.55 / (10 * \Phi) = 137.034956 \tag{α12r}$$

The numerator of the second term of Equation (α12r) contains four times the figure 5, which - the figure 4 - is the denumerator of the first term.
Besides figure 555 the figures Φ, 4, 5 and 10 are used at Equation (α12r).

The next Equation (α12a) uses besides the Golden Ratio Φ the figures 100, 144, 666 and 999.

$$\alpha_{\#12a}^{-1} = \alpha_{\#12rough}^{-1} * [1 + 1 / (100 * \Phi)]^{1/[0.999 * (144+666)]} = 137.035999094 \tag{α12a}$$

Remarkable: $0.999 * (144+666) = 9.99 * 9*9$ [5 times the figure 9 is given at the right side of equation]

Some Equations more:

$$\alpha_{\#12b}^{-1} = \alpha_{\#12rough}^{-1} * [1 + \{\Phi / (2 * 5.55)\}^{6.12}] = 137.035999101 \tag{α12b}$$

Connection to Figures of Equation (α12r): $612 + 10 * 5 + 4 = 666;\quad 666 * 5 / (10 - 4) = 555$

$$\alpha_{\#12c}^{-1} = \alpha_{\#12rough}^{-1} * [1 + 2 * \Phi / (1000 * 555)]^{999/765} = 137.035999104 \tag{α12c}$$

Connection of used Figures: $666 + 99 = 765$

$$\alpha_{\#12d}^{-1} = \alpha_{\#12rough}^{-1} / [1 - 1 / (100 * 555 * 5 * \Phi)]^{0.555 * 6.16} = 137.035999095 \tag{α12d}$$

Connection to Figures of Equation (α12r): $616 + 10*5 = 666$

$$\alpha_{\#12e}^{-1} = \alpha_{\#12rough}^{-1} / [1 - 5 * \Phi / (1000 * 555)]^{51.712/99} = 137.035999096 \tag{α12e}$$

Remarkable: $51712 = 512 * 101;\quad 512 = 8 * 8 * 8;\quad 3 * 137 + 101 = 512;\quad 512 + (99+101)/2 = 612$ (see α12b)
At Equation (α12d) the term "$5 * \Phi$" stands in denominator, at the Equation (α12e) in the numerator.

The results of Equation (α12a), (α12d) and (α12e) are very close to the result value of the Hans de Vries Formula (=137.035999096). See the connection with the Figures 3.4111777 and 137.035999096 below! All of the results of the Equations (α12a) to (α12e) are within the rolerance range of the Fine Structure Constant according to CODATA2018.

Figure 3.4111777: firstly used at the Serie Formula (α7), page 4 (Figure 44.444 at term MT_1, next page)
$$3.4111777^{137/(5 * \pi)} - \sqrt{2} = 44445.414091 - 1.414214 = 44443.999877 \quad [\approx 44444]$$

With the Hans de Vries value (α^{-1}=137.035999096) set in the exponent of upper Equation one gets:
$$3.4111777^{137.035999096/(5 * \pi)} - 128 + 1.4146 = 44570.576644 - 128 + 1.4146 = 44443.99124 \quad [\approx 44444]$$
$$128 = 2^7;\quad 44444 * 10^{-4} / (0.3 * 4 * \Phi * \Phi) - 3 / 4 * 10^{-4} = 1.414600216$$

Figure 128: $128 / 0.99^{2.16 * \pi} = 137.03416$ $[6*6*6 = 216]$; see also Equation (α13) at next page
Figure 14.146 ($= 10 * 1.4146$) derived at page 3; see Equation (F1)

Figure 3.4111777 in combinations with Equation (α12r):

$$\alpha_{\#12f}^{-1} = \alpha_{\#12rough}^{-1} * [1 + (3.4111777 / 5.55)^{100/4.13}] = 137.035999109 \tag{α12f}$$

Connection of the figures: $555 - 413 - 17 = 5 * 5 * 5 = 125;\quad 17 + 4 * 99 = 413$

$$\alpha_{\#12g}^{-1} = \alpha_{\#12rough}^{-1} * [1 + 1 / (\mathbf{34}.111777 * 555)]^{99.9/693} = 137.035999095 \tag{α12g}$$

Connection of the figures: $7 * 99 = 693;\quad 999 - 693 = \mathbf{9} * \mathbf{34}$ (figure 34 is part of figure 34.111777)

$$\alpha_{\#12h}^{-1} = \alpha_{\#12rough}^{-1} * [1 + 1 / (10 * \mathbf{34}.111777 * 555 * \Phi)]^{0.999 * \mathbf{34}/(\mathbf{9} * \Phi)} = 137.035999107 \tag{α12h}$$

Connection of the figures: $999 - \mathbf{9} * \mathbf{34} = 693$ (see figures 9, 34 and 693 at the formula before)

$$\alpha_{\#12i}^{-1} = \alpha_{\#12rough}^{-1} * [1 + 1 / (34.111777 * 555)]^{0.999 * 0.1443} = 137.035999094 \tag{α12i}$$

Connection of the figures: $1443 - 999 = 444;\quad 1443 - 693 = 6 * 5 * 5 * 5;\quad 693 - 666 = 3 * 3 * 3$ (see 693 before)

$$\alpha_{\#12j}^{-1} = \alpha_{\#12rough}^{-1} * [1 + \Phi / (10 * 34.111777 * 555)]^{1/1.1224433} = 137.035999096 \tag{α12j}$$

Connection of the figures: $34111777 - 3 * 11224433 = 438478;\quad 438478 - 666^2 + (144/2)^2 - 9^2 - 5^2 = 0$

$$\alpha_{\#12k}^{-1} = \alpha_{\#12rough}^{-1} / [1 - 1 / (34.111777 * 555)]^{144/999} = 137.035999065 \tag{α12k}$$

The result value lies within the tolerance range of CODATA2018.

$$\alpha_{\#12l}^{-1} = \alpha_{\#12rough}^{-1} * \{1 + [\Phi * 3.4111777 / (10^3 * 555)]^{3.4111777/3.3338888}\} = 137.035999105 \tag{α12l}$$

Primes from 3 to 29: $8888 - 3333 = 5555;\quad [5555 - 555 + 55 - 5] - 3 - (5 * 7 * 11 * 13) - 17 - 19 + 23 - 29 = 0$

There are 6 Equations of the just presented Equations, which are dependent on the rough term $\alpha_{\#12rough}$ and of which the results are very close to the value of the Hans de Vries Formula (=137.035999096).

Serie Equation by use of the Multiplication Term "1 - 8/(3*44.444)", of the Figures 12 and 128 and of the start value α_0 (α_0^{-1} = 137.035999110) for the Reciprocal of the Fine Structure Constant:

$MT_1 = 1 - \alpha_0 * 8/(3*44.444) = 0.9995621545$ [8/3: Fraction of the Thomson Cross Section σ_e]

$\mathbf{\Gamma_{MT1}} = 1 + MT_1 * (\alpha_0^1/(2*\pi)^0 + \alpha_0^2/(2*\pi)^1 + \alpha_0^3/(2*\pi)^3 + \alpha_0^4/(2*\pi)^6 + \alpha_0^5/(2*\pi)^{10} + \alpha_0^6/(2*\pi)^{15} + \ldots) =$
$= 1.007302630526$

$\alpha_{\#13}^{-1} = 128 * \mathbf{\Gamma_{MT1}}^{\,12\,/\,1.28} = 137.035999110$ (α13)

The result value is very close to the result values of Equations (α12f) and (α12h) and lies barely outside the tolerance range of CODATA2018. Relations with the Figure 44444 can be seen at the page before.

Serie Equation by use of the Multiplication Term "1 - α_0 * 7.7 / 128", of the Figures 12 and 128 and of the start value α_0 (α_0^{-1} = 137.035999093) for the Reciprocal of the Fine Structure Constant:

$MT_2 = 1 - \alpha_0 * 7.7 / 128 = 0.9995610186$

$\mathbf{\Gamma_{MT2}} = 1 + MT_2 * (\alpha_0^1/(2*\pi)^0 + \alpha_0^2/(2*\pi)^1 + \alpha_0^3/(2*\pi)^2 + \alpha_0^4/(2*\pi)^3 + \alpha_0^5/(2*\pi)^4 + \alpha_0^6/(2*\pi)^5 + \ldots) =$
$= 1.007302630512$

$\alpha_{\#14}^{-1} = 128 * \mathbf{\Gamma_{MT2}}^{\,12\,/\,1.28} = 137.035999092$ (α14)

See Figure 77 at Equation (α-17) and Figure 128 at Equation (α-18)

The result value is very close to the one of the Hans de Vries Formula (=137.035999096) and lies inside the tolerance range of CODATA2018.
With an exponent term "(12/1.28) * (1 ±1.2*10^{-8})" the result values of Equation (α14) lie outside the tolerance range of CODATA2018 and also of CODATA2022. Isn't that astonishing?

Equations with Figure 137:
In Literature (17) of Hua-Fang Wu the following Formulas are given:

$137 = 1*2^5 + 2*2^4 + 3*2^3 + 5*2^2 + 8*2^1 + 13*2^0$ [Fibonacci-Serie starts with Figures 1 and 2]

$\alpha_{HFW1}^{-1} = 137 / [1 - (\Phi/100)^2] = 137.035876458$ (α-HFW1)

$\alpha_{\#15}^{-1} = 137 / [1 - (\Phi/100)^2 - (\Phi/52.6)^4] = 137.035999190$ (α15)

Connection of the used (changed to full) figures 137, 100, 2, 526, 4:   526 / 2 + 137 - 4 * 100 = 0
The result value of Equation (α15) is within the tolerance range of CODATA2022.

$\alpha_{\#16}^{-1} = 137 / [1 - (\Phi/99.9)^2 - 5.38 * (\Phi/99.9)^4] = 137.035999086$ (α16)

Connection with figure 666 and 128:   666 - 538 = 128;   see use of figure 128 at Equation (α14)

Another Formula in Literature (17) is given to:

$\alpha_{HFW2}^{-1} = \pi / \{29 * \cos[\pi / 137] * \tan[\pi / (29 * 137)]\} = 137.035999787$ (α-HFW2)

$\alpha_{\#17}^{-1} = \pi * [1 - 1/(77*137^3)] / \{29 * \cos[\pi / 137] * \tan[\pi / (29 * 137)]\} = 137.035999095$ (α-17)

Connections: Figures 77 and 128 (= 29 + 99) are used at term MT_2, which is part of Equation (α14)

$\alpha_{\#18}^{-1} = \pi * [1 - 1/(2*29*180*137^2)] / \{29 * \cos[\pi / 137] * \tan[\pi / (29 * 137)]\} =$
$= 137.035999087$ (α-18)

Relations, which deliver result values with the form xxyyzz... using full figures x, y, z ... from 1 to 9:

$(77^2 - 29*137) = 1956;$ $\sqrt{1956} = 44.2266887750;$ $137 - 128/2 - 29 = 44;$
$\sqrt{1956} - 44 = 0.2266887750;$ $1 / (\sqrt{1956} - 44) = 4.411334439$
the Reciprocal "1 / ($\sqrt{1956}$ - 44)" possesses the same form as the starting figure!

Connections: $180 + 2*137 - 2*29 = 4*99;$ $180 + 2*137 + 2*29 = 4*128 = 512;$ $666 - 512 = 2*77$

4) Version of CODATA

At the time point of performing this report the Physical Constants according to CODATA 2018 were valid. In the year 2024 a new version of Physical Constants is acknowledged by the CODATA team, namely the version CODATA2022.
If one compars the approximations – especially the one's for the Fine Structure Constant, for the Electron mass and the Proton mass – with the data of the Physical Constants according to CODATA2022, many of these approximations lie now outside the tolerance range.
The pendulum around an exact value for the Fine Structure Constant has been swinging from one value to another for decades. By that the CODATA 2022 value for the Fine Structure Constant is not the final one.

The approximations presented in this report are valid for the data of the CODATA Version, which were given at the time point of their performing. A time scale is also valid for the Rotation Times and Distances of our Celestial Bodies Earth and Moon (See page 1 to 3), however this time scale is far bigger.

The author wants to present the following suggestion: one simply uses the value of the exceptional "Hans de Vries"-Formula α_{Hdv} as the Reference value for the Fine Structure Constant ($\alpha_{Hdv}^{-1} = 137.035999096$; see page 4). By that in the future the natural scienctists can work with fixed values of the Fine Structure Constant and furthermore of Physical Constants, which are dependent on the Fine Structure Constant α. The result value of the "Hans de Vries"-Formula is correct to CODATA 2018 and very close to other Approximations. A choice of these simple constructed Formulas are presented in the following:

$$\alpha_{\#1}^{-1} = 137.036 * (1 - 6.66 * 10^{-9}) = 137.035999087\,340 \qquad (\alpha 1)$$

$$\alpha_{\#2}^{-1} = 1 * \pi^4 + 4 * \pi^2 + 1 * \pi^{-2} + 5 * \pi^{-4} - 4 * \pi^{-6} = 137.035999087\,382 \qquad (\alpha 2)$$

$$\alpha_{\#4}^{-1} = 137.036 - (1 / 20 / \Phi)^4 = 137.035999088\,137 \qquad (\alpha 4)$$

If one takes a look at the form of the "Hans de Vries"-Formula, one can interprete its result value as a Constant, which is consistent by itself.

5) Conclusion

Did the reader ever see mathematical formulas for Physical Constants, which results correspond to the real values (many refer to CODATA2018) in such exact respects as presented in this report. And this exactness is performed with similar formula forms and repeating figures? That leads to the logical question: Can this be random?
Please look again at the Equation ($m_{uHe}2$) for the atomic mass of Helium, at which the set value (result value) is used as exponent and leads to a more exact result as by use of the full number, which is close to the set value.
Or look again at the approximations for the Reciprocal of the Fine Structure Constant, which are structured differently, but lead very closely to the same results, and which are far within the tolerance.

Another question is permissible in this context: Is there an Universal Equation Formula or Matrix for the Physical Constants, which is dependent on the four Versatile Figures Φ, π, 144 and 666?
The author is convinced, that further results - achieved by investigations to this topic performed by use of special mathematical software - will come to light in the future, which will support these assumptions.

Further it would be interesting to know, which answers does an appropriate Artificial Intelligence give to these questions in its consideration of the informations given in this report?
One can try to find this Universal Equation Formula or Matrix - if existing – by special mathematical softwares, which is connected to Artificial Intelligence, with input parameters *Φ, π, 144 and 666*.

Literature and wikipedia.de- or other Internet-Entries:

The data of the physical Constants and the data of the celestial bodies of our sun system are taken in the mayority from the entries of Wikipedia Germany. The physical constants given in the corresponding entries refer mostly to CODATA2018.

(1) Wikipedia.de-Entry "Erde"; Status March 2024
   1. David R. Williams: Earth Fact Sheet. (https://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html)
      In: NASA.gov. 20. April 2020, abgerufen am 9. Mai 2020 (englisch).
   2. Solar System Exploration: Planet Compare. (https://solarsystem.nasa.gov/planet-compare/)
      In: NASA.gov. Abgerufen am 10. Mai 2020 (englisch).

(2) Wikipedia.de-Entry "Mond"; Status March 2024
   1. David R. Williams: Moon Fact Sheet. (https://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html)
      In: NASA.gov. 13. Januar 2020, abgerufen am 16. Mai 2020 (englisch).

(3) Wikipedia.de-Entry "Sonne"; Status March 2024
   1. David R. Williams: Sun Fact Sheet. (https://nssdc.gsfc.nasa.gov/planetary/factsheet/sunfact.html)
      In: NASA.gov. 23. Februar 2018, abgerufen am 15. August 2021 (englisch).
   2. Marcelo Emilio et al.: Measuring the Solar Radius from Space during the 2003 and 2006 Mercury Transits. In: Astrophysical Journal Bd. 750, Nr. 2, bibcode:2012ApJ...750..135E (https://ui.adsabs.harvard.edu/abs/2012ApJ...750..135E), doi:10.1088/0004-637X/750/2/135 (https://doi.org/10.1088/0004-637X%2F750%2F2%2F135)

(4) Wikipedia.de-Entry "Physikalische Konstante"; Status May 2024
   (4.1) Fine Structure Constant α:
      25. CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Value?alph) NIST, abgerufen am 20. April 2020 (englisch, Wert für die Feinstrukturkonstante)
   (4.2) Reciprocal of Fine Structure Constant α^{-1}:
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   (4.3) Plancks Constant h:
      17. CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Value?h) NIST, abgerufen am 3. Juni 2019 (englisch, Wert für die Planck-Konstante in der Einheit Js).
   (4.4) Light Velocity c:
      12. CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Value?c) NIST, abgerufen am 3. Juni 2019 (englisch, Wert für die Lichtgeschwindigkeit).
   (4.5) Gravitation Constant G:
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   (4.6) Electron Mass m_e:
      41. CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Value?me) NIST, abgerufen am 3. Juni 2019 (englisch, Wert für die Elektronenmasse in Kilogramm).
   (4.7) Electron Radius r_e:
      45. CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Value?re) NIST, abgerufen am 3. Juni 2019 (englisch, Wert für den klassischen Elektronenradius).
   (4.8) Electron Charge e:
      13. CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Value?e) NIST, abgerufen am 3. Juni 2019 (englisch, Wert für die Elementarladung).
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   (4.10) Magnetic Field Constant μ_0:
      66. CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Value?gn) NIST, abgerufen am 21. März 2022 (englisch, Wert für die Normfallbeschleunigung).
   (4.11) Thomson Cross Section σ_e:
      46. CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Value?sigmae) NIST, abgerufen am 19. Oktober 2021 (englisch, Wert für den Thomson-Wirkungsquerschnitt).

(4.12) Weinberg Angle:
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          abgerufen am 18. Oktober 2021 (englisch, Wert für den schwachen Mischungswinkel.

(5) Mathematical Formulas, which follow to the Koide-Formula, using the Input Values "Data of the
   Celestial Bodies of our Sun System and Physical Constants", and the Connection of the Results;
   Andreas Ball;   Published at viXra: viXra submission 2405.0080v1; 2024-05-15

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   An exact formula for the Electro Magnetic coupling constant. (fine structure constant) by Hans de Vries

(7) LAUFENDE KOPPLUNGSKONSTANTEN ZEIGEN DEN WEG ZUR WELTFORMEL
   DER NATURKONSTANTEN; Dr. Endre Kereszturi; 12.12.2010
   [The Formula for the Gravitation Constant is listed at page 16 with Equation (19)]

(8) Muonic hydrogen and the proton radius puzzle; Randolf Pohl.
   Published at arXiv: 1301.0905v2, doi: 10.1146/annurev-nucl-102212-170627
   [The Proton Radius is listed at page 3 and page 58]

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   1. Die Angaben über die Teilcheneigenschaften (Infobox) sind, wenn nicht anders angegeben, entnommen
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(12) Wikipedia.de-Entry "Myon";   Status March 2024
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(14) Elementarkörpertheorie [EK-Theorie]; Dirk Freyling, May 2012
   © 1986 2006 2012 Dirk Freyling; Internet Presence under www.kinkynature.com/ektheorie

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   Bruno R. Galeffi. Published at viXra: 2010.0253 *submitted on 2020-10-31.*
   Reference [11] R Heyrovska; "Golden Ratio Based Fine Structure Constant and Rydberg Constant
      for Hydrogen Spectra"; International Journal of Sciences; Volume 2, Issue May 2013

(16) Wikipedia.de-Entry "Yoshio Koide";   Status March 2024
   1. Piotr Żenczykowski: Elementary Particles and Emergent Phase Space. WORLD SCIENTIFIC, 2013,
     ISBN 978-981-4525-68-8, S. 66-68, doi:10.1142/8918 (https://doi.org/10.1142/8918).

(17) Discovery Of The Solution To The Fibonacci Sequence Of The Fine Structure Constant 137 ";
   Hua-Fang Wu;   Published at viXra: 2408.0097v1 submitted on 25.08.2024.

<u>Data of the equatorial Diameters D of Earth, Moon and Sun, of the Rotation Times RT and of the Distances (big Half Axles) of Earth to Sun and to Moon, respectively:</u>

$\varnothing_{Earth}$ = 12756.27 [km][1]; $\varnothing_{Moon}$ = 3476 [km][2]; $\varnothing_{Sun}$ = 1392684 [km][3];

RT_{Earth} = 365.256 [days][1]; RT_{Moon} = 27.3217 [days][2];

$Dist_{E-S}$ = 149.6 * 10^6 [km][1]; $Dist_{E-M}$ = 384400 [km][2]

<u>Used Data of Physical Constants:</u>

Atomic Mass of Helium[13]:	4.002 602(2) u
Coulomb Constant k_C[4.9]:	8.987 551 7922(14) * 10^9 V m C^{-1}
Electron Charge e[4.8]:	1.602 176 634 * 10^{-19} C
Fine Structure Constant α[4.1]:	7.297 352 5693(11) * 10^{-3}
Reciprocal of Fine Structure Constant $1/\alpha$[4.2]:	137.035 999 084(21)
Gravitation Constant G[4.5]:	6.67430(15) * 10^{-11} m^3 kg^{-1} s^{-2}
Light velocity c[4.4]:	299 792 458 m/s
Magnetic Field Constant μ_0[4.10]:	1.256 637 062 12(19) * 10^{-6} kg m C^{-2}
Mass of Electron m_e[4.6]:	9.109 383 7015(28) * 10^{-31} kg
Mass of Neutron m_n[9]:	1.674 927 498 04(95) * 10^{-27} kg
Mass Ratio Neutron/Electron $MR_{n/e}$[9]:	1838.683 661 73(89)
Mass Ratio Neutron/Proton $MR_{n/p}$:	1.001 378 41931
Mass of Protons m_p[10]:	1.672 621 923 69(51) * 10^{-27} kg
Mass Ratio Proton/Electron $MR_{p/e}$[10]:	1836.152 673 43(11)
Mass of Myon m_μ[12]:	1.883 531 627(42) * 10^{-28} kg
Mass Ratio Myon/Electron $MR_{\mu/e}$[12]:	206.768 2830(46)
Mass of Tauon m_τ[11]:	3.167 54(21) * 10^{-27} kg
Mass Ratio of Tauon/Electron $MR_{\tau/e}$[11]:	3477.23(23)
Plancks Constant h[4.3]:	6.626 070 15 * 10^{-34} J s
Radius of Electron r_e[4.7]:	2.817 940 3262(13) * 10^{-15} m
Radius of Proton r_p[8]:	0.84087(39) * 10^{-15} m
Thomson Cross Section σ_e[4.11]:	6.652 458 7321(60) * 10^{-29} m^2
Weinberg Angle[4.12]:	0.22290(30)

The figures in the brackets behind the data descibe the uncertainty referring the last places of the given value.[4]

<u>Selected Modifications[5] of the Koide Formula[16] of the Japanese Physician Yoshio Koide[16]:</u>

$(m_e + m_\mu + m_\tau) / (\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2$ = 0.66666056 [≈ 2/3]   (Koide Formula)

$Exp = (3/4)^2 = (0.75)^2 = 0.5625$

$(m_e + m_\mu + m_\tau) / (m_e^{Exp} + m_\mu^{Exp} + m_\tau^{Exp})^{1/Exp}$ = 0.7500633 [≈ 0.75 = $(2/3)^{-1}$ / 2]

$Exp_\Phi = (3/4)^{(1.2*\Phi*\Phi)} = 0.40503017$

$(m_e + m_\mu + m_\tau) / (m_e^{Exp_\Phi} + m_\mu^{Exp_\Phi} + m_\tau^{Exp_\Phi})^{1/Exp_\Phi}$ = 0.50001

$[(m_e + m_\mu + m_\tau) / m_e] / [(m_e^{Exp_\Phi} + m_\mu^{Exp_\Phi} + m_\tau^{Exp_\Phi}) / m_e^{Exp_\Phi}]$ = 99.99994 [≈ 100]

$(m_e + m_p + m_n) / \sqrt{(m_e^2 + m_p^2 + m_n^2)}$ = 1.414598 [≈ 0.1 * 14.146]

$\Phi^{2/3} + e^{2/3} + \pi^{2/3} + 1.44^{2/3} + 6.66^{2/3}$ = 1.286 + 9.000028 = 1.286028 + 9   [1.286 = 14.146/11]

Next used figures π, 4 and 6 are quantities for determination of the circle surface and sphere volume:

$Exp_a = 0.72559092$ [≈ $\Phi^{-2/3} = 0.72556263$]; Exponent Exp_a is derived by Set Result Value 2/3

$(\pi + 4 + 6) / (\pi^{Exp_a} + 4^{Exp_a} + 6^{Exp_a})^{1/Exp_a}$ = 0.66666666 [Set Result Value 2/3]

$Exp_b = 1 / Exp_a = 1 / 0.72559092 = 1.37818704$ [≈ $\Phi^{2/3}$ = 1.3782408; with Exponent 2/3]

$(\pi + 4 + 6) / (\pi^{Exp_b} + 4^{Exp_b} + 6^{Exp_b})^{1/Exp_b}$ = 1.333358 [≈ 4/3 = 2 * 2/3]