

# On Prime Cycles in Directed Graphs Built with Primality Test

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## Abstract

One of the most famous unsolved problems in mathematics is Collatz conjecture [1] which is claiming that all positive numbers subjected to simple  $3x + 1/2$  formula will eventually result in 1, with only one known cycle (1, 4, 2, 1) present in the calculations. This work is devoted to finding cycles in other interesting sequences of integer numbers, constructed with the use of some aspect of primality test.

## 1 Introduction

Cycles in integer sequences can be beautiful and surprising about their quantity and length. Famous illustration of this statement is definitely is Collatz conjecture which is claiming that all positive numbers subjected to simple  $3x+1$  (if  $x$  is odd) or  $/2$  (if  $x$  is even) formula will eventually result in 1, with only one known cycle (1, 4, 2, 1) present in the calculations. Collatz formula is taking advantage of divisibility by 2, the work presented in this paper is considering various sequences where primality of the term decides on argument or the next value in the sequence.

## 2 Plan for experiments

Experiments are planned against various sequences where integer primality plays a central role. In expedited cases the formula for sequence will take advantage of functions like:  $GPF(n)$  - the greatest prime factor of  $n$  and  $SOPF(n)$  - the smallest odd prime factor of  $n$ .  $P_i$  is the  $i$ -th prime. In presented results cycles of length  $n$  for sequence  $S_i$  are symbolized as a list  $[a_1, a_2, \dots, a_n]$ , where  $a_{j+1} = S(a_j)$  ( $1 \leq j < n$ ) and  $a_1 = S(a_n)$ . All terms of the sequences are primes. Experiments were run with framework [2].

For example, if sequence formula is  $S_1(P_i) = GPF(P_i + P_{i+1})$ , then we have:

$$\begin{aligned} S_1(2) &= GPF(2 + 3) = GPF(5) = 5, \\ S_1(3) &= GPF(3 + 5) = GPF(8) = 2, \\ S_1(5) &= GPF(5 + 7) = GPF(12) = 3, \\ S_1(7) &= GPF(7 + 11) = GPF(18) = 3, \\ S_1(11) &= GPF(11 + 13) = GPF(24) = 3, \\ S_1(13) &= GPF(13 + 17) = GPF(30) = 5. \end{aligned}$$

These calculations are depicting a cycle in this sequence:  $S_1(S_1(S_1(2))) = 2$ .

## 3 Results of experiments

Experiments were executed against first 50000 terms of each sequence. They allowed to find interesting cycles (Table 1). If graph (drawn as a graphical representation

of  $S_i$ ) is connected and has one cycle, then such case is similar to Collatz conjecture where all numbers subjected to the formula eventually land in one cycle.

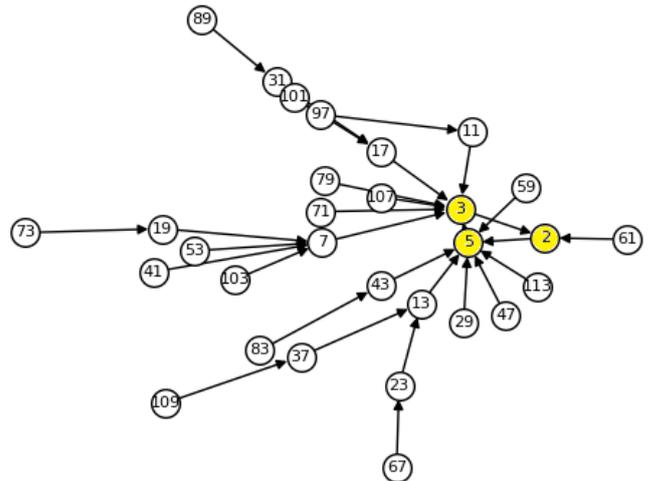


Figure 1:  $S_1$  and its first 30 terms - example of connected graph (one cycle: [2, 5, 3] found so far)

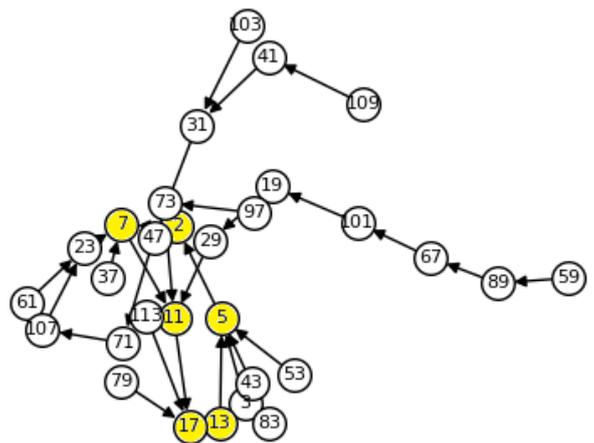


Figure 2:  $S_6$  and its first 30 terms (one cycle [2, 7, 11, 17, 13, 5] found so far)

## 4 Summary and future work

Work allowed to find few interesting cycles. Directed graphs used to visualised sequences also revealed interesting shapes. Future work may be focused on checking further sequences. Three classes of sequences are interesting: class A - sequences producing one cycle; class B - sequences

producing more than one cycle; class C - sequences without known cycle. Similarly to Collatz conjecture, the very interesting part of this analysis is actual proof, not empirical confirmation only.

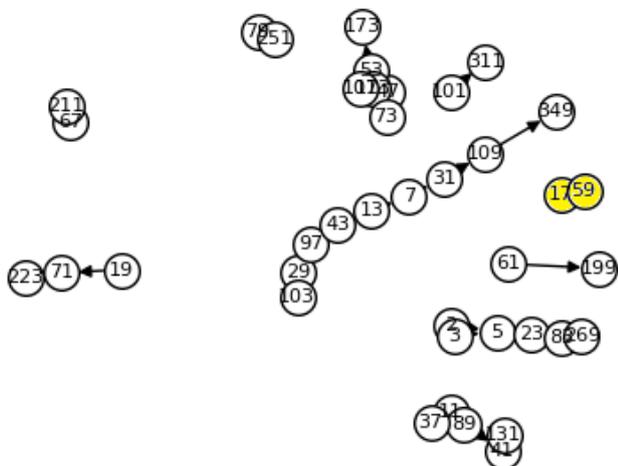


Figure 3:  $S_9$  and its first 30 terms - example of disconnected graph (one cycle: [17, 59] found so far)

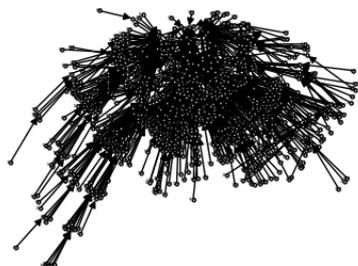


Figure 4:  $S_1$  and its first 1000 terms



Figure 5:  $S_2$  and its first 1000 terms

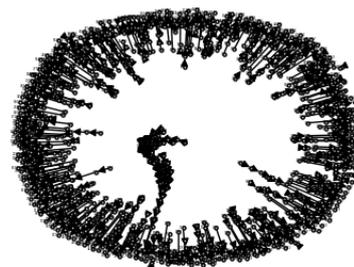


Figure 6:  $S_3$  and its first 1000 terms

## References

- [1] Weisstein, Eric W. "Collatz Problem." From MathWorld—A Wolfram Resource. <https://mathworld.wolfram.com/CollatzProblem.html>
- [2] Library for various operations on primes. <https://github.com/mbarylsk/primes/>

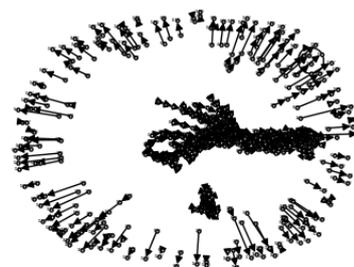


Figure 7:  $S_4$  and its first 1000 terms

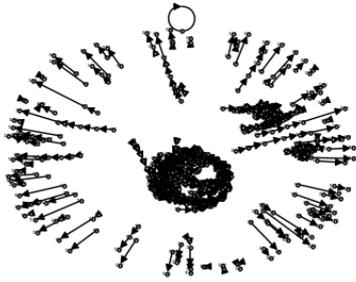


Figure 8:  $S_5$  and its first 1000 terms

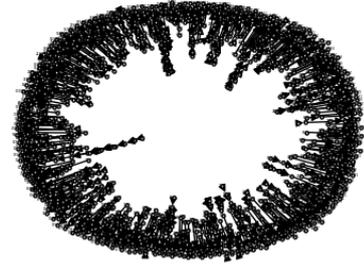


Figure 11:  $S_8$  and its first 1000 terms

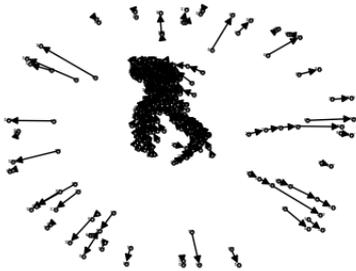


Figure 9:  $S_6$  and its first 1000 terms



Figure 12:  $S_9$  and its first 1000 terms

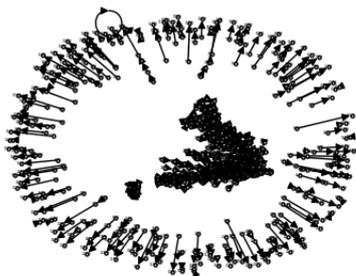


Figure 10:  $S_7$  and its first 1000 terms

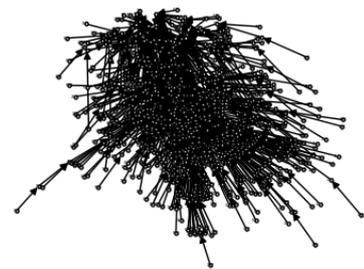


Figure 13:  $S_{10}$  and its first 1000 terms



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Figure 14:  $S_{11}$  and its first 1000 terms

Table 1: Sequences  $S_j$  subjected to experiments, results after 50000 iterations

j	Formula for $S_j(P_i)$	# of cycles found	cycles found
1	$GPF(P_i + P_{i+1})$	1	[2, 5, 3]
2	$GPF(2 \times P_i + 1)$	1	[3, 7, 5, 11, 23, 47, 19, 13]
3	$GPF(P_i \times P_{i+1} + 1)$	1	[2287, 479]
4	$GPF(2 \times P_i + 3)$	3	[3], [73, 149, 43, 89, 181], [61, 5, 13, 29]
5	$GPF(2 \times P_i + 5)$	4	[5], [739, 1483, 2971, 313, 631, 181, 367], [67, 139, 283, 571, 37, 79, 163, 331, 29, 7, 19, 43, 13, 31], [11, 3]
6	$GPF(3 \times P_i + 1)$	1	[2, 7, 11, 17, 13, 5]
7	$GPF(3 \times P_i + 2)$	3	[2], [389, 167, 503, 1511, 907], [131, 79, 239, 719, 127, 383, 1151, 691, 83, 251, 151, 13, 41, 5, 17, 53, 23, 71, 43]
8	$GPF(P_i^2 + 1)$	1	[89, 233]
9	$GPF(P_i + P_{i+1} + P_{i+2})$	3	[17, 59], [41, 131, 37, 11], [97, 43, 13, 7, 31, 109, 349, 1061, 103, 29]
10	$GPF(P_i + 1)$	1	[2, 3]
11	$GPF(P_i + 2)$	2	[2], [3, 5, 7]