

A Dictionary of British Surnames by P. H. Reaney and the Graphical Law

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Abstract

We study A Dictionary of British Surnames by P. H. Reaney. We draw the natural logarithm of the number of entries, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised. We conclude that the Dictionary can be characterised by $BP(4, \beta H = 0.01)$ i.e. a magnetisation curve for the Bethe-Peierls approximation of the Ising model with four nearest neighbours in the presence of external magnetic field, H , with $\beta H = 0.01$. β is $\frac{1}{k_B T}$ where, T is temperature and k_B is the tiny Boltzmann constant.

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I. INTRODUCTION

Few days back in a rainy day, the author was travelling in a rickshaw in his hometown in Calcutta. The author asked the rickshaw puller what is your surname. He replied that his surname is Daptari. Daptar means department. The author asked back "was your father working in a department?" The rickshaw puller told "yes". Then the rickshaw puller narrated the following tale. The title of his grandfather was Mondal. He was from somewhere in West Bengal. The father of the rickshaw puller went to work in a department in a central government office in Cuttack, in Orissa. The rickshaw puller was there in his childhood days. There their surname has changed to Daptari. The general story of generation, evolution, transmutation of surnames probably goes the same way throughout our world. The British empire was not an exception. The British Surnames may be divided into four groups as follows: local Surnames, Surnames of Relationship, Surnames of Occupation or, Offices and Nicknames, [1]. In A Dictionary of British Surnames,[1], P. H. Reaney, the author catalogued, described, traced the histories in cases from 1200 AD onwards, of the British Surnames. Many surnames refer to river, hill, road, marshland, battle, store, Jesus Christ etc. Deformation also is a dominant factor in generating Surnames.

We study magnetic field pattern behind the entries of this dictionary, [1], in this article. We count the entries to probe for the magnetic field pattern. We have started considering magnetic field pattern in [2], in the languages we converse with. We have studied there, a set of natural languages, [2] and have found existence of a magnetisation curve under each language. We have termed this phenomenon as the Graphical Law. Then, we moved on to investigate, [3], into dictionaries of five disciplines of knowledge and found the existence of a curve of magnetisation under each discipline. This was followed by finding of the graphical law in references from [4] to [75].

We describe how the graphical law is hidden within A Dictionary of British Surnames by P. H. Reaney, [1]. in this article. The planning of the paper is as follows. We give an introduction to the standard curves of magnetisation of Ising model in the section II. In the section III, we describe analysis of the entries of A Dictionary of British Surnames by P. H. Reaney [1]. The section IV is Acknowledgment. The last section is Bibliography.

II. MAGNETISATION

A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like para magnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N}\sum_i\sigma_i$, where σ_i is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where N_+ is the number of up spins, N_- is the number of down spins. $L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1 + L)$ and $N_- = \frac{N}{2}(1 - L)$. Magnetisation or, net magnetic moment , M is $\mu\sum_i\sigma_i$ or, $\mu(N_+ - N_-)$ or, μNL , $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$. $\frac{M}{M_{max}}$ is

referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[77], for the lattice of spins, setting μ to one, is $-\epsilon \sum_{n.n} \sigma_i \sigma_j - H \sum_i \sigma_i$, where n.n refers to nearest neighbour pairs. The difference ΔE of energy if we flip an up spin to down spin is, [78], $2\epsilon\gamma\bar{\sigma} + 2H$, where γ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_-}{N_+}$ equals $exp(-\frac{\Delta E}{k_B T})$, [79]. In the Bragg-Williams approximation,[80], $\bar{\sigma} = L$, considered in the thermal average sense. Consequently,

$$\ln \frac{1+L}{1-L} = 2 \frac{\gamma\epsilon L + H}{k_B T} = 2 \frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2 \frac{L + c}{\frac{T}{T_c}} \quad (1)$$

where, $c = \frac{H}{\gamma\epsilon}$, $T_c = \gamma\epsilon/k_B$, [81]. $\frac{T}{T_c}$ is referred to as reduced temperature.

Plot of L vs $\frac{T}{T_c}$ or, reduced magnetisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [78]. W. L. Bragg was a professor of Hans Bethe. Rudolf Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudolf Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field

In the approximation scheme which is improvement over the Bragg-Williams, [77],[78],[79],[80],[81], due to Bethe-Peierls, [82], reduced magnetisation varies with reduced temperature, for γ neighbours, in absence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}} \quad (2)$$

$\ln \frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe

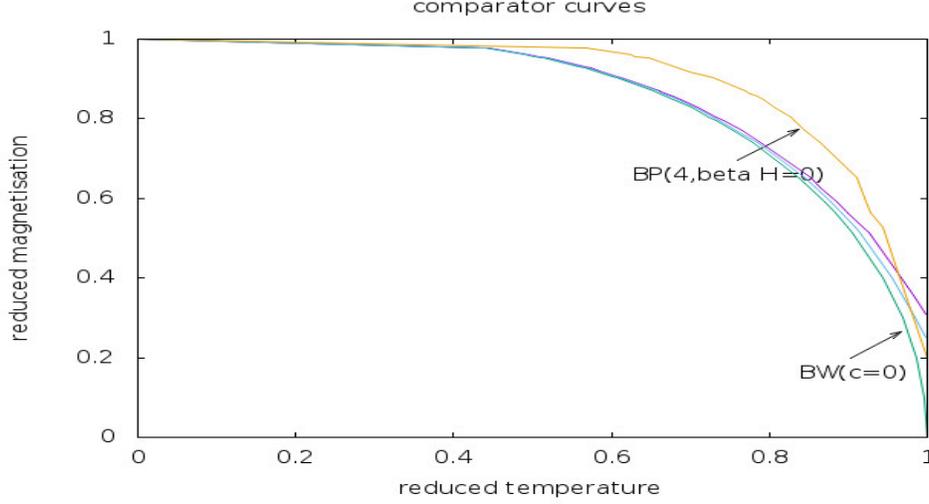


FIG. 1. Reduced magnetisation vs reduced temperature curves, for the Bragg-Williams approximation, in the absence (BW($c=0$)) and in the presence (BW($c=0.005$), BW($c=0.01$)) of magnetic field, $c = 0$, $c = \frac{H}{\gamma\epsilon} = 0.005$, $c = \frac{H}{\gamma\epsilon} = 0.01$, outwards; and in the Bethe-Peierls approximation, BP(4, $\beta H=0$), in the absence of magnetic field, for four nearest neighbours (outer in the top).

data s generated from the equation(1) and the equation(2) in the table, I, and curves of magnetisation plotted on the basis of those data s. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.1. Empty spaces in the table, I, mean corresponding point pairs were not used for plotting a line.

reduced temperature, $\frac{T}{T_c}$				$\frac{M}{M_{max}}$,
BW(c=0)	BW(c=0.005)	BW(c=0.01)	BP(4, $\beta H = 0$)	reduced magnetisation
0	0	0	0	1
0.435	0.437	0.439	0.563	0.978
0.439	0.441	0.443	0.568	0.977
0.491	0.493	0.495	0.624	0.961
0.501	0.504	0.507	0.630	0.957
0.514	0.517	0.519	0.648	0.952
0.559	0.562	0.565	0.654	0.931
0.566	0.569	0.573	0.7	0.927
0.584	0.587	0.590	0.7	0.917
0.601	0.604	0.607	0.722	0.907
0.607	0.610	0.613	0.729	0.903
0.653	0.658	0.661	0.770	0.869
0.659	0.663	0.666	0.773	0.865
0.669	0.674	0.678	0.784	0.856
0.679	0.684	0.688	0.792	0.847
0.701	0.705	0.709	0.807	0.828
0.723	0.728	0.732	0.828	0.805
0.732	0.736	0.743	0.832	0.796
0.753	0.758	0.766	0.845	0.772
0.779	0.784	0.788	0.864	0.740
0.838	0.844	0.853	0.911	0.651
0.850	0.858	0.864	0.911	0.628
0.870	0.877	0.885	0.923	0.592
0.883	0.891	0.899	0.928	0.564
0.899	0.908	0.918		0.527
0.905	0.914	0.926	0.941	0.513
0.944	0.956	0.968	0.965	0.400
		0.985		0.350
		0.998		0.310
0.969	0.985		0.965	0.300
	0.998			0.250
0.987			1	0.200
0.997			1	0.100
1			1	0

TABLE I. Datas for Reduced temperature[for the Bragg-Williams approximation, in the absence (BW(c=0)) and in the presence (BW(c=0.005), BW(c=0.01)) of magnetic field, $c = 0$, $c = \frac{H}{\gamma\epsilon} = 0.005$, $c = \frac{H}{\gamma\epsilon} = 0.01$ respectively and in the Bethe-Peierls approximation, BP(4, $\beta H=0$), in the absence of magnetic field, for four nearest neighbours] vs reduced magnetisation. Reduced temperature data set(say, data set BW(c=0)) is drawn along the x-axis and the corresponding Reduced magnetisation data set is drawn along the y-axis. In gnuplot the command is plot ".dat" using 1:2 with line; 1 standing for x-axis and 2 standing for y-axis datas.[For example, for drawing BW(c=0), ".dat" file, say denoted as "0.dat", contains BW(c=0) data set in first column and reduced magnetisation data set in second column. Moreover, after (0.944,0.400), next pair of points will be (0.969,0.300), then (0.987,0.200), .and so on in the "0.dat" file.]

C. Bethe-peierls approximation in presence of four nearest neighbours, in presence of external magnetic field

In the Bethe-Peierls approximation scheme , [82], reduced magnetisation varies with reduced temperature, for γ neighbours, in presence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{\gamma-1}{\gamma}}}{e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; \text{factor} = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (3)$$

Derivation of this formula Ala [82] is given in the appendix of [7].

$\ln \frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For four neighbours,

$$\frac{0.693}{\ln \frac{e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{\gamma-1}{\gamma}}}{e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; \text{factor} = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (4)$$

In the following, we describe data s in the table, II, generated from the equation(4) and curves of magnetisation plotted on the basis of those data s. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.06$. calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.05$. calculated from the equation(4). BP(m=0.02) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.04$. calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.02$. calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.01$. calculated from the equation(4). The data set is used to plot fig.2. Empty spaces in the table, II, mean corresponding point pairs were not used for plotting a line.

reduced temperature, $\frac{T}{T_c}$					$\frac{M}{M_{max}}$
BP(m=0.03)	BP(m=0.025)	BP(m=0.02)	BP(m=0.01)	BP(m=0.005)	reduced magnetisation
0	0	0	0	0	1
0.583	0.580	0.577	0.572	0.569	0.978
0.587	0.584	0.581	0.575	0.572	0.977
0.647	0.643	0.639	0.632	0.628	0.961
0.657	0.653	0.649	0.641	0.637	0.957
0.671	0.667		0.654	0.650	0.952
	0.716			0.696	0.931
0.723	0.718	0.713	0.702	0.697	0.927
0.743	0.737	0.731	0.720	0.714	0.917
0.762	0.756	0.749	0.737	0.731	0.907
0.770	0.764	0.757	0.745	0.738	0.903
0.816	0.808	0.800	0.785	0.778	0.869
0.821	0.813	0.805	0.789	0.782	0.865
0.832	0.823	0.815	0.799	0.791	0.856
0.841	0.833	0.824	0.807	0.799	0.847
0.863	0.853	0.844	0.826	0.817	0.828
0.887	0.876	0.866	0.846	0.836	0.805
0.895	0.884	0.873	0.852	0.842	0.796
0.916	0.904	0.892	0.869	0.858	0.772
0.940	0.926	0.914	0.888	0.876	0.740
	0.929			0.877	0.735
	0.936			0.883	0.730
	0.944			0.889	0.720
	0.945				0.710
	0.955			0.897	0.700
	0.963			0.903	0.690
	0.973			0.910	0.680
				0.909	0.670
	0.993			0.925	0.650
		0.976	0.942		0.651
	1.00				0.640
		0.983	0.946	0.928	0.628
		1.00	0.963	0.943	0.592
			0.972	0.951	0.564
			0.990	0.967	0.527
				0.964	0.513
			1.00		0.500
				1.00	0.400
					0.300
					0.200
					0.100
					0

TABLE II. Bethe-Peierls approx. in presence of little external magnetic fields

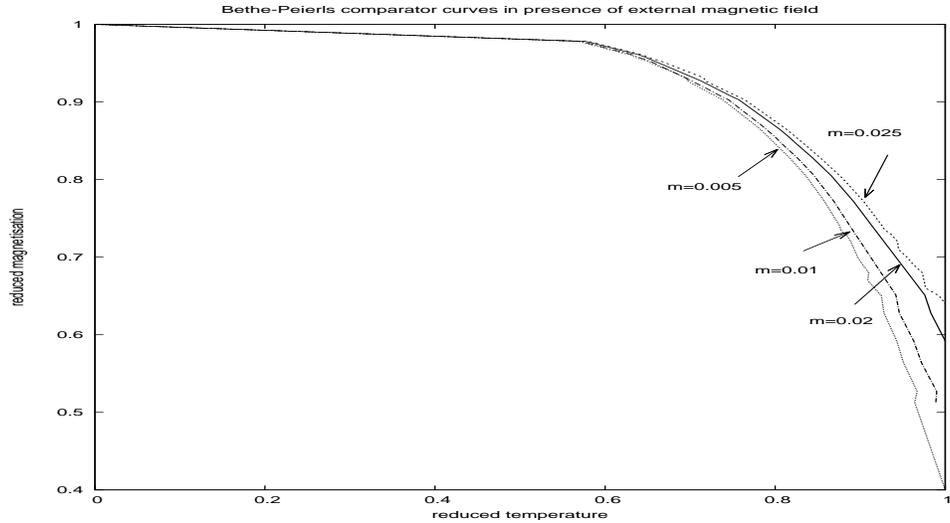


FIG. 2. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
936	1979	1395	986	419	714	1016	1531	158	347	348	836	1222	322	231	1190	42	824	1780	849	75	268	1281	0	54	9

TABLE III. The entries of A Dictionary of British Surnames by P. H. Reaney along the English letters

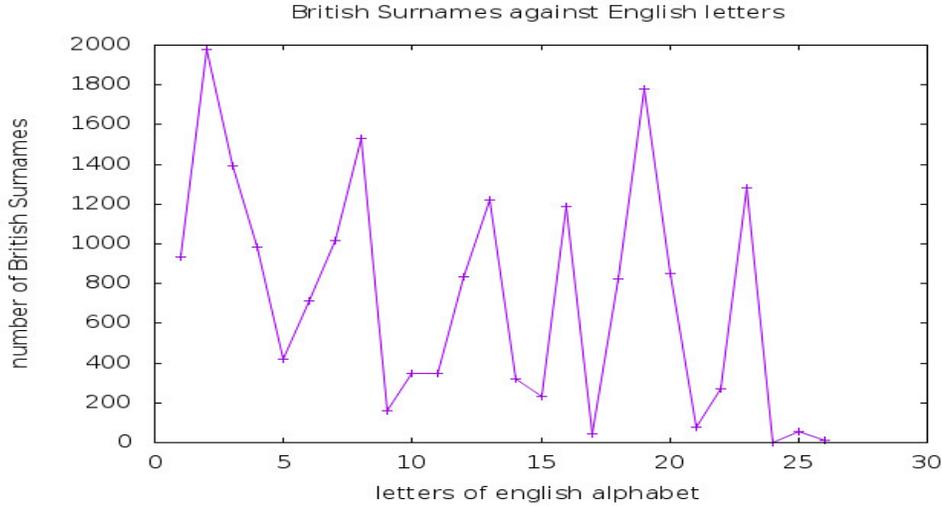


FIG. 3. The vertical axis is the number of entries of A Dictionary of British Surnames by P. H. Reaney, [1], and the horizontal axis is the respective letters. Letters are represented by the sequence number in the alphabet or, dictionary sequence,[1].

III. ANALYSIS OF THE ENTRIES OF A DICTIONARY OF BRITISH SURNAMES

We count the entries of A Dictionary of British Surnames by P. H. Reaney, [1], starting with different letters. The result is the table, III. Highest number of entries, one thousand nine hundred seventy nine, starts with the letter B followed by entries numbering one thousand seven hundred eighty beginning with S, one thousand five hundred thirty one with the letter H etc. To visualise we plot the number of entries against respective letters in the dictionary sequence, [1], in the figure fig.3.

For the purpose of exploring graphical law, we assort the letters according to the number of entries, in the descending order, denoted by f and the respective rank, denoted by k . k is a positive integer starting from one. The lowest value of f is nine. Hence we attach a

k	lnk	lnk/ lnk_{lim}	f	lnf	lnf/ lnf_{max}	lnf/ lnf_{nmax}	lnf/ lnf_{2nmax}	lnf/ lnf_{3nmax}	lnf/ lnf_{4nmax}	lnf/ lnf_{5nmax}
1	0	0	1979	7.590	1	Blank	Blank	Blank	Blank	Blank
2	0.69	0.212	1780	7.484	0.986	1	Blank	Blank	Blank	Blank
3	1.10	0.337	1531	7.334	0.966	0.980	1	Blank	Blank	Blank
4	1.39	0.426	1395	7.241	0.954	0.968	0.987	1	Blank	Blank
5	1.61	0.494	1281	7.155	0.943	0.956	0.976	0.988	1	Blank
6	1.79	0.549	1222	7.108	0.936	0.950	0.969	0.982	0.993	1
7	1.95	0.598	1190	7.082	0.933	0.946	0.966	0.978	0.990	0.996
8	2.08	0.638	1016	6.924	0.912	0.925	0.944	0.956	0.968	0.974
9	2.20	0.675	986	6.894	0.908	0.921	0.940	0.952	0.964	0.970
10	2.30	0.706	936	6.842	0.901	0.914	0.933	0.945	0.956	0.963
11	2.40	0.736	849	6.744	0.889	0.901	0.920	0.931	0.943	0.949
12	2.48	0.761	836	6.729	0.887	0.899	0.918	0.929	0.940	0.947
13	2.56	0.785	824	6.714	0.885	0.897	0.915	0.927	0.938	0.945
14	2.64	0.810	714	6.571	0.866	0.878	0.896	0.907	0.918	0.924
15	2.71	0.831	419	6.038	0.796	0.807	0.823	0.834	0.844	0.849
16	2.77	0.850	348	5.852	0.771	0.782	0.798	0.808	0.818	0.823
17	2.83	0.868	347	5.849	0.771	0.782	0.798	0.808	0.817	0.823
18	2.89	0.887	322	5.775	0.761	0.772	0.787	0.798	0.807	0.812
19	2.94	0.902	268	5.591	0.737	0.747	0.762	0.772	0.781	0.787
20	3.00	0.920	231	5.442	0.717	0.727	0.742	0.752	0.761	0.766
21	3.04	0.933	158	5.063	0.667	0.677	0.690	0.699	0.708	0.712
22	3.09	0.948	75	4.317	0.569	0.577	0.589	0.596	0.603	0.607
23	3.14	0.963	54	3.989	0.526	0.533	0.544	0.551	0.558	0.561
24	3.18	0.975	42	3.738	0.492	0.499	0.510	0.516	0.522	0.526
25	3.22	0.988	9	2.197	0.289	0.294	0.300	0.303	0.307	0.309
26	3.26	1	1	0	0	0	0	0	0	0

TABLE IV. The entries of A Dictionary of British Surnames by P. H. Reaney: ranking, natural logarithm, normalisations

limiting f equal to one. The corresponding rank, k, denoted as k_{lim} is twenty six. As a result both $\frac{lnf}{lnf_{max}}$ and $\frac{lnk}{lnk_{lim}}$ varies from zero to one. Then we tabulate in the adjoining table, IV and plot $\frac{lnf}{lnf_{max}}$ against $\frac{lnk}{lnk_{lim}}$ in the figure fig.4. We then ignore the letter with the highest of entries, tabulate in the adjoining table, IV and redo the plot, normalising the $lnfs$ with next-to-maximum $lnf_{nextmax}$, and starting from $k = 2$ in the figure fig.5. This program then we repeat up to $k = 6$, resulting in figures up to fig.9.

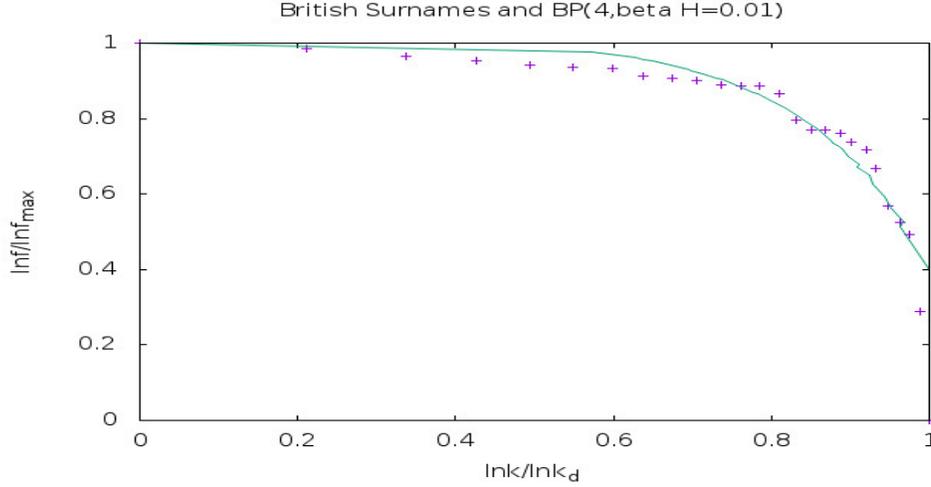


FIG. 4. The vertical axis is $\frac{\ln f}{\ln f_{max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the entries of A Dictionary of British Surnames by P. H. Reaney with the fit curve being the Bethe-Peierls curve, $BP(4, \beta H = 0.01)$, in the presence of four nearest neighbours and little external magnetic field, $m = 0.005$ or, $\beta H = 0.01$.

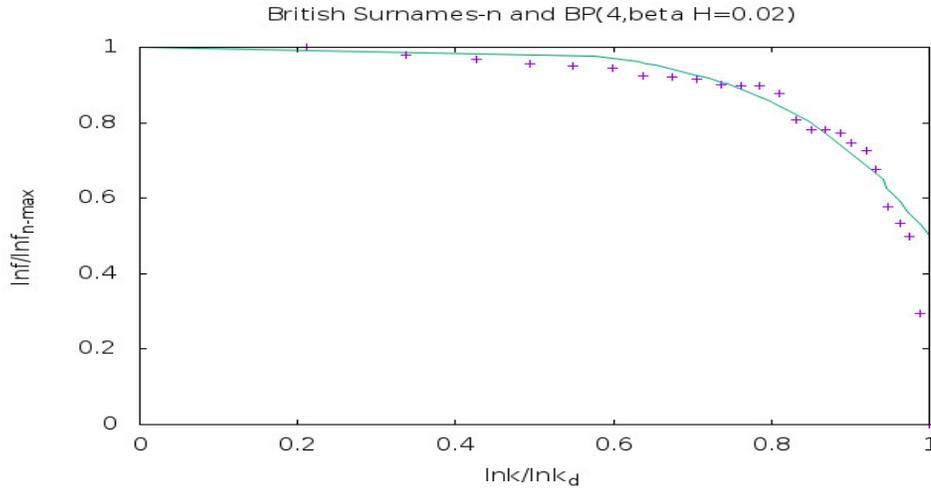


FIG. 5. The vertical axis is $\frac{\ln f}{\ln f_{n-max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the entries of A Dictionary of British Surnames by P. H. Reaney with the fit curve being the Bethe-Peierls curve, $BP(4, \beta H = 0.02)$, in the presence of four nearest neighbours and little external magnetic field, $m = 0.01$ or, $\beta H = 0.02$.

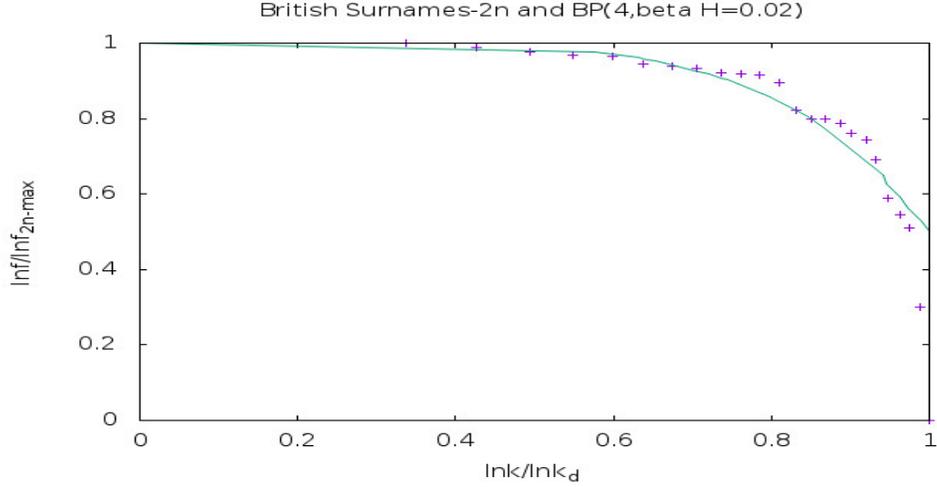


FIG. 6. The vertical axis is $\frac{\ln f}{\ln f_{2n-max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the entries of A Dictionary of British Surnames by P. H. Reaney with the fit curve being the Bethe-Peierls curve, $BP(4, \beta H = 0.02)$, in the presence of four nearest neighbours and little external magnetic field, $m = 0.01$ or, $\beta H = 0.02$.

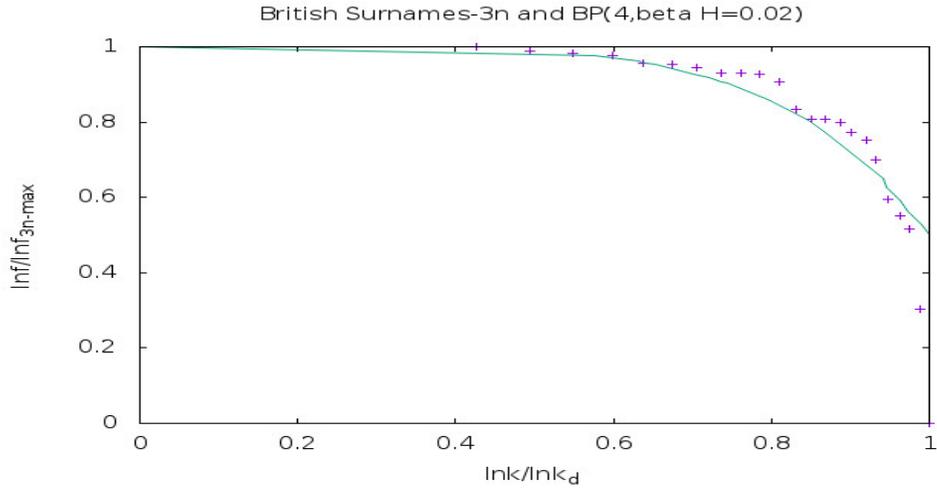


FIG. 7. The vertical axis is $\frac{\ln f}{\ln f_{3n-max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the entries of A Dictionary of British Surnames by P. H. Reaney with the fit curve being the Bethe-Peierls curve, $BP(4, \beta H = 0.02)$, in the presence of four nearest neighbours and little external magnetic field, $m = 0.01$ or, $\beta H = 0.02$.

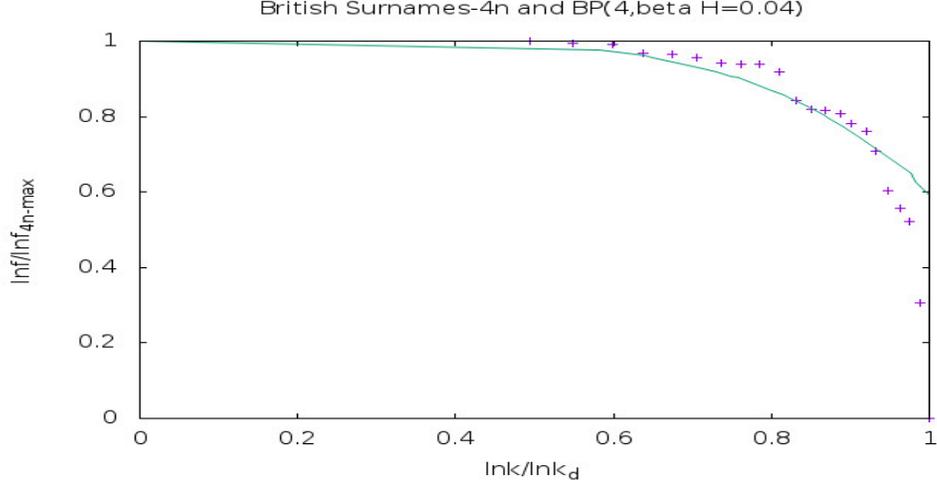


FIG. 8. The vertical axis is $\frac{\ln f}{\ln f_{4n-max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the entries of A Dictionary of British Surnames by P. H. Reaney with the fit curve being the Bethe-Peierls curve, $BP(4, \beta H = 0.04)$, in the presence of four nearest neighbours and little external magnetic field, $m = 0.02$ or, $\beta H = 0.04$.

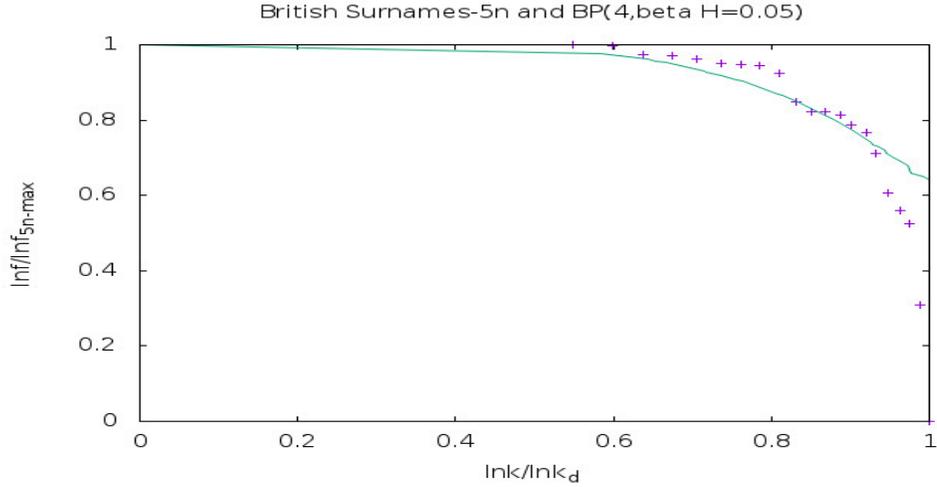


FIG. 9. The vertical axis is $\frac{\ln f}{\ln f_{5n-max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the entries of A Dictionary of British Surnames by P. H. Reaney with the fit curve being the Bethe-Peierls curve, $BP(4, \beta H = 0.05)$, in the presence of four nearest neighbours and little external magnetic field, $m = 0.025$ or, $\beta H = 0.05$.

A. conclusion

From the figures (fig.4-fig.9), we observe that there is a curve of magnetisation, behind the entries of A Dictionary of British Surnames by P. H. Reaney, [1], This is the magnetisation curve, $BP(4, \beta H = 0.01)$, in the presence of four nearest neighbours and little external magnetic field, $m = 0.005$ or, $\beta H = 0.01$. Moreover, the associated correspondence is,

$$\frac{\ln f}{\ln f_{max}} \longleftrightarrow \frac{M}{M_{max}}, \quad \ln k \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [83].

This result should be seen in the background of the references, [10] and [6] respectively.

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