

# Santali and The Graphical Law

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## Abstract

We study the Santali head entries of two Santali to English dictionaries. These are Campbell's Santali-English Dictionary, third edition, edited by R. M. Macphail and A Santal Dictionary by Rev. P. O. Bodding, respectively. We draw the natural logarithm of the number of head entries, normalised(unnormalised), starting with a letter vs the natural logarithm of the rank of the letter, normalised. We conclude that Santali, the language of the Santal tribe, underlie a magnetisation curve of a Spin-Glass in the presence of little external magnetic field.

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## I. INTRODUCTION

”Santali language is spoken by almost a million and a half people, who are mainly distributed over a strip of Bengal extending for about 350 miles from the Ganges to the Baitarni, bisected by the meridian of Bhagalpur, or, 87<sup>0</sup> East longitude, and comprising the following districts- Bhagalpur, Monghyr, the Santal Pargannas, Birbhum, Bankura, Hazaribagh, Manbhum, Midnapur, Singbhum, Mayurbhanj and Balasore. Santali belongs to the Munda or, Kolarian family of languages and has reached a much higher stage of development than any other language or, dialect of the family to which it belongs. The Northern Santali is more polished and taken as standard. Santali not possessing an alphabet of its own, the Roman character has been adopted to it. This with diacritical marks suits the language extremely well.”....Rev. A. Campbell in the preface to the first edition, 1899,[1].

”The word Santal is used by foreigners about the Santals and may be used by Santals to foreigners who are thought not to know better. The word is explained by the Santals themselves as meaning ”one who belongs to Sant or, Sāat, or, Sāt, a country in the Midnapur district( the present Silda pargana)”. The name may also be connected with Santbhum( also Samantabhum) in the Bankura district. Sāt is probably an abbreviation of Skr. samanta, boundary; the meaning might thus be a ”borderman”.”...Rev. P. O. Bodding in [2].

As a tribe Santals are divided into twelve septs or, clans. Originally they were seven clans, five clans got added later. Hembrom is the noble clan. Kisku is the ruling clan. According to the Santal traditions, when the ancestors of the Santals were in Champa, the Kiskus were ruling. Sorens were in olden times the ”soldiers” of the Kisku rapaj, therefore they are often referred to as Soren siphai. Other clans are Hāsdaś, Murmu, Maṇḍi, Ṭuḍu, Baske, Besra, Cōrē, Pāuriḷ or, Pāuliḷ and Bedea. The Besra clan is among the five clans which got added later. The Bedea clan is the lost clan(p.596 Vol.4 of [2]). Each clan should be subdivided into twelve subsepts or subclans, though there are more. Every Santal sept has a subsept styled Nij like Nij. Baske. These subsepts are deemed to be descendants of the original sept or, clan from which all other subsepts or, subclans like Biṭol Baske, Maṅjhi-khil Baske etc. have sprung, [2]. Santals are always in a move towards the east, where the Sun rises. During movement they borrow words from their current neighbours, deform the words or, keep those as those were, keep the meaning as those were or, deform those, sometimes to the extent of attaching opposite meanings before absorbing in their ever increasing vocabulary.

Interestingly, Nola disom refers to a country south of the present country of the Santals, mentioned in the story of their wanderings; Nola Jharia refers to a country near the Nola country, where the Santals stayed for some time, [2].

In this paper, we study Santali, the language of the Santal tribe. Specifically we go through two dictionaries of Santali. We study the Santali head entries as those appear in the two dictionaries. These are Campbell's Santali-English Dictionary, third edition, edited by R. M. Macphail, [1], and A Santal Dictionary by Rev. P. O. Bodding, [2], respectively. We count one by one all the Santali head entries in these two dictionaries, looking for the graphical law. We have started considering magnetic field pattern in [3], in the languages we converse with. We have studied there, a set of natural languages, [3] and have found existence of a magnetisation curve under each language. We have termed this phenomenon as the Graphical Law. Then, we moved on to investigate, [4], into dictionaries of five disciplines of knowledge and found the existence of a curve of magnetisation under each discipline. This was followed by finding of the graphical law in references from [5] to [85].

The planning of the paper is as follows. In the section II, We give an introduction to the standard curves of magnetisation of Ising model. In the section III, we describe the graphical law analysis of the head entries of the Campbell's Santali-English Dictionary, [1]. In the section IV, we describe the graphical law analysis of the head entries of A Santal Dictionary by P. O. Bodding, [2], in the alphabet scheme of Campbell. In the section V, we describe the graphical law analysis of the head entries of A Santal Dictionary by P. O. Bodding, [2], in the alphabet scheme of P. O. Bodding. The section VI is Conclusion. Sections VII and VIII are Acknowledgment and Bibliography respectively.

## II. MAGNETISATION

### A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like para magnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by  $L = \frac{1}{N}\sum_i\sigma_i$ , where  $\sigma_i$  is i-th spin, N being total number of spins. L can vary from minus one to one.  $N = N_+ + N_-$ , where  $N_+$  is the number of up spins,  $N_-$  is the number of down spins.  $L = \frac{1}{N}(N_+ - N_-)$ . As a result,  $N_+ = \frac{N}{2}(1 + L)$  and  $N_- = \frac{N}{2}(1 - L)$ . Magnetisation or, net magnetic moment ,  $M$  is  $\mu\sum_i\sigma_i$  or,  $\mu(N_+ - N_-)$  or,  $\mu NL$ ,  $M_{max} = \mu N$ .  $\frac{M}{M_{max}} = L$ .  $\frac{M}{M_{max}}$  is

referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[86], for the lattice of spins, setting  $\mu$  to one, is  $-\epsilon \sum_{n.n} \sigma_i \sigma_j - H \sum_i \sigma_i$ , where n.n refers to nearest neighbour pairs. The difference  $\Delta E$  of energy if we flip an up spin to down spin is, [87],  $2\epsilon\gamma\bar{\sigma} + 2H$ , where  $\gamma$  is the number of nearest neighbours of a spin. According to Boltzmann principle,  $\frac{N_-}{N_+}$  equals  $exp(-\frac{\Delta E}{k_B T})$ , [88]. In the Bragg-Williams approximation,[89],  $\bar{\sigma} = L$ , considered in the thermal average sense. Consequently,

$$\ln \frac{1+L}{1-L} = 2 \frac{\gamma\epsilon L + H}{k_B T} = 2 \frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2 \frac{L + c}{\frac{T}{T_c}} \quad (1)$$

where,  $c = \frac{H}{\gamma\epsilon}$ ,  $T_c = \gamma\epsilon/k_B$ , [90].  $\frac{T}{T_c}$  is referred to as reduced temperature.

Plot of  $L$  vs  $\frac{T}{T_c}$  or, reduced magnetisation vs. reduced temperature is used as reference curve. In the presence of magnetic field,  $c \neq 0$ , the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [87]. W. L. Bragg was a professor of Hans Bethe. Rudolf Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudolf Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

## **B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field**

In the approximation scheme which is improvement over the Bragg-Williams, [86],[87],[88],[89],[90], due to Bethe-Peierls, [91], reduced magnetisation varies with reduced temperature, for  $\gamma$  neighbours, in absence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}} \quad (2)$$

$\ln \frac{\gamma}{\gamma-2}$  for four nearest neighbours i.e. for  $\gamma = 4$  is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe

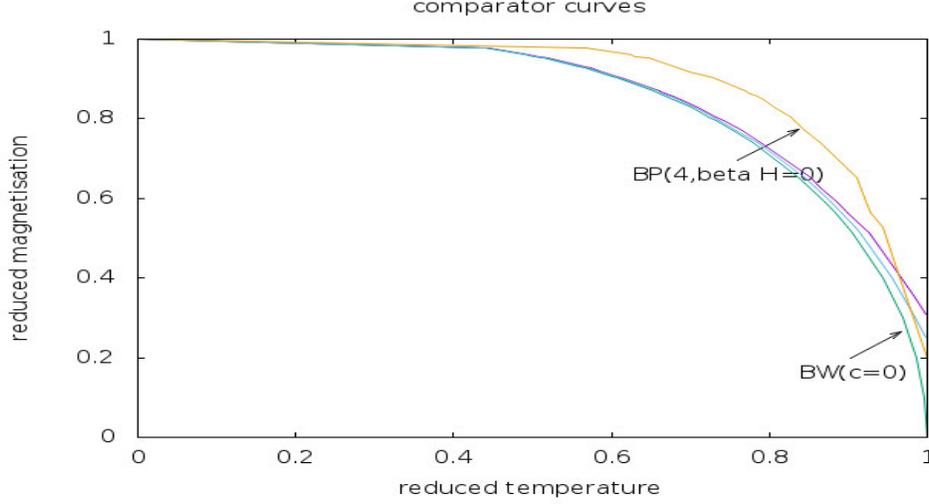


FIG. 1. Reduced magnetisation vs reduced temperature curves, for the Bragg-Williams approximation, in the absence (BW( $c=0$ )) and in the presence (BW( $c=0.005$ ), BW( $c=0.01$ )) of magnetic field,  $c = 0$ ,  $c = \frac{H}{\gamma\epsilon} = 0.005$ ,  $c = \frac{H}{\gamma\epsilon} = 0.01$ , outwards; and in the Bethe-Peierls approximation, BP(4, $\beta H=0$ ), in the absence of magnetic field, for four nearest neighbours (outer in the top).

data s generated from the equation(1) and the equation(2) in the table, I, and curves of magnetisation plotted on the basis of those data s. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.1. Empty spaces in the table, I, mean corresponding point pairs were not used for plotting a line.

BW(c=0)	BW(c=0.005)	BW(c=0.01)	BP(4, $\beta H = 0$ )	reduced magnetisation
0	0	0	0	1
0.435	0.437	0.439	0.563	0.978
0.439	0.441	0.443	0.568	0.977
0.491	0.493	0.495	0.624	0.961
0.501	0.504	0.507	0.630	0.957
0.514	0.517	0.519	0.648	0.952
0.559	0.562	0.565	0.654	0.931
0.566	0.569	0.573	0.7	0.927
0.584	0.587	0.590	0.7	0.917
0.601	0.604	0.607	0.722	0.907
0.607	0.610	0.613	0.729	0.903
0.653	0.658	0.661	0.770	0.869
0.659	0.663	0.666	0.773	0.865
0.669	0.674	0.678	0.784	0.856
0.679	0.684	0.688	0.792	0.847
0.701	0.705	0.709	0.807	0.828
0.723	0.728	0.732	0.828	0.805
0.732	0.736	0.743	0.832	0.796
0.753	0.758	0.766	0.845	0.772
0.779	0.784	0.788	0.864	0.740
0.838	0.844	0.853	0.911	0.651
0.850	0.858	0.864	0.911	0.628
0.870	0.877	0.885	0.923	0.592
0.883	0.891	0.899	0.928	0.564
0.899	0.908	0.918		0.527
0.905	0.914	0.926	0.941	0.513
0.944	0.956	0.968	0.965	0.400
		0.985		0.350
		0.998		0.310
0.969	0.985		0.965	0.300
	0.998			0.250
0.987			1	0.200
0.997			1	0.100
1			1	0

TABLE I. Datas for Reduced temperature[ for the Bragg-Williams approximation, in the absence (BW(c=0)) and in the presence (BW(c=0.005), BW(c=0.01)) of magnetic field,  $c = 0$ ,  $c = \frac{H}{\gamma\epsilon} = 0.005$ ,  $c = \frac{H}{\gamma\epsilon} = 0.01$  respectively and in the Bethe-Peierls approximation, BP(4, $\beta H=0$ ), in the absence of magnetic field, for four nearest neighbours] vs reduced magnetisation. Reduced temperature is drawn along the x-axis and Reduced magnetisation is drawn along the y-axis. In gnuplot the command is plot ".dat" using 1:2 with line; 1 standing for x-axis and 2 standing for y-axis datas.

### C. Bethe-peierls approximation in presence of four nearest neighbours, in presence of external magnetic field

In the Bethe-Peierls approximation scheme , [91], reduced magnetisation varies with reduced temperature, for  $\gamma$  neighbours, in presence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{1}{\gamma}}}{\text{factor} - 1}} = \frac{T}{T_c}; \text{factor} = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (3)$$

Derivation of this formula Ala [91] is given in the appendix of [8].

$\ln \frac{\gamma}{\gamma-2}$  for four nearest neighbours i.e. for  $\gamma = 4$  is 0.693. For four neighbours,

$$\frac{0.693}{\ln \frac{e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{1}{\gamma}}}{\text{factor} - 1}} = \frac{T}{T_c}; \text{factor} = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (4)$$

In the following, we describe data s in the table, II, generated from the equation(4) and curves of magnetisation plotted on the basis of those data s. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.06$ . calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.05$ . calculated from the equation(4). BP(m=0.02) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.04$ . calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.02$ . calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.01$ . calculated from the equation(4). The data set is used to plot fig.2. Similarly, we plot fig.3. Empty spaces in the table, II, mean corresponding point pairs were not used for plotting a line.

BP(m=0.03)	BP(m=0.025)	BP(m=0.02)	BP(m=0.01)	BP(m=0.005)	reduced magnetisation
0	0	0	0	0	1
0.583	0.580	0.577	0.572	0.569	0.978
0.587	0.584	0.581	0.575	0.572	0.977
0.647	0.643	0.639	0.632	0.628	0.961
0.657	0.653	0.649	0.641	0.637	0.957
0.671	0.667		0.654	0.650	0.952
	0.716			0.696	0.931
0.723	0.718	0.713	0.702	0.697	0.927
0.743	0.737	0.731	0.720	0.714	0.917
0.762	0.756	0.749	0.737	0.731	0.907
0.770	0.764	0.757	0.745	0.738	0.903
0.816	0.808	0.800	0.785	0.778	0.869
0.821	0.813	0.805	0.789	0.782	0.865
0.832	0.823	0.815	0.799	0.791	0.856
0.841	0.833	0.824	0.807	0.799	0.847
0.863	0.853	0.844	0.826	0.817	0.828
0.887	0.876	0.866	0.846	0.836	0.805
0.895	0.884	0.873	0.852	0.842	0.796
0.916	0.904	0.892	0.869	0.858	0.772
0.940	0.926	0.914	0.888	0.876	0.740
	0.929			0.877	0.735
	0.936			0.883	0.730
	0.944			0.889	0.720
	0.945				0.710
	0.955			0.897	0.700
	0.963			0.903	0.690
	0.973			0.910	0.680
				0.909	0.670
	0.993			0.925	0.650
		0.976	0.942		0.651
	1.00				0.640
		0.983	0.946	0.928	0.628
		1.00	0.963	0.943	0.592
			0.972	0.951	0.564
			0.990	0.967	0.527
				0.964	0.513
			1.00		0.500
				1.00	0.400
					0.300
					0.200
					0.100
					0

TABLE II. Bethe-Peierls approx. in presence of little external magnetic fields

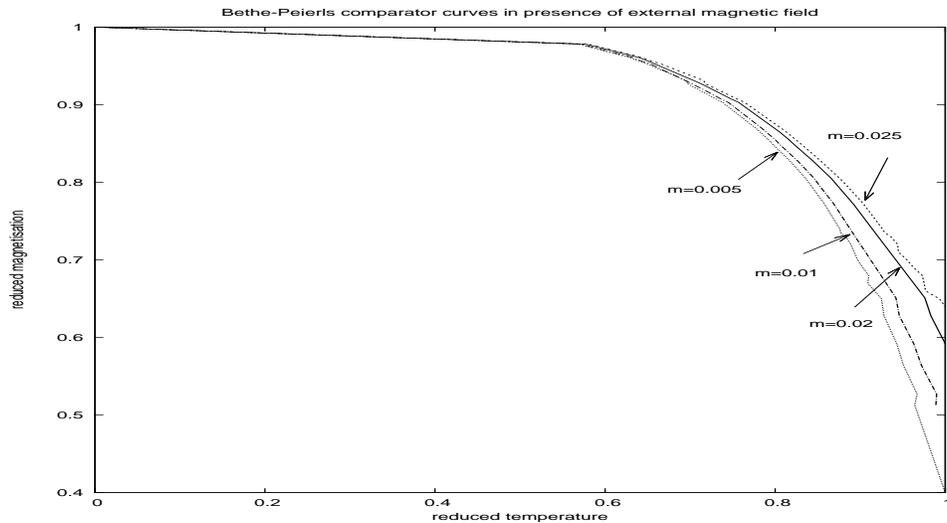


FIG. 2. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with  $\beta H = 2m$ .

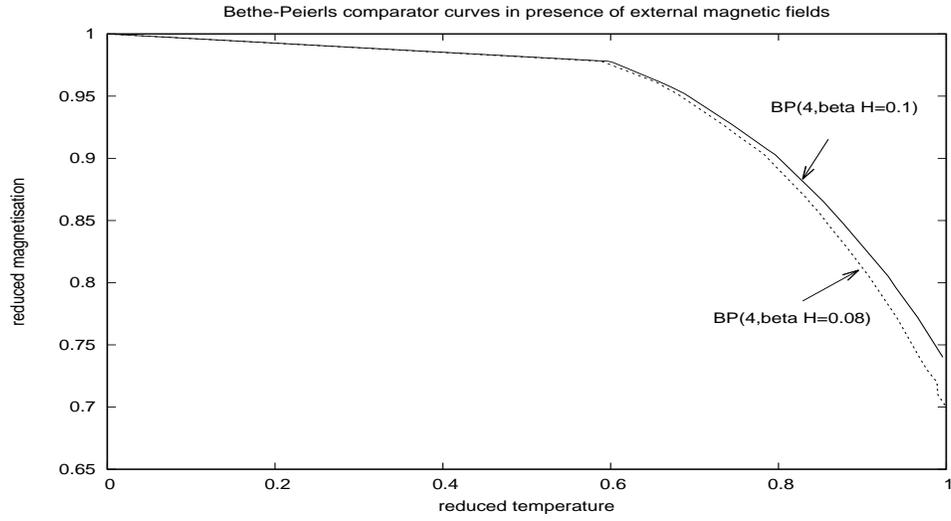


FIG. 3. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with  $\beta H = 2m$ .

## D. Spin-Glass

In the case coupling between ( among) the spins, not necessarily n.n, for the Ising model is( are) random, we get Spin-Glass. When a lattice of spins randomly coupled and in an external magnetic field, goes over to the Spin-Glass phase, magnetisation increases steeply like  $\frac{1}{T-T_c}$  i.e. like the branch of rectangular hyperbola, up to the the phase transition temperature, followed by very little increase,[95–97], in magnetisation, as the ambient temperature continues to drop.

Theoretical study of Spin Glass started with the paper by Edwards, Anderson,[98]. They were trying to explain two experimental results concerning continuous disordered freezing(phase transition) and sharp cusp in static magnetic susceptibility. This was followed by a paper by Sherrington, Kickpatrick, [99], who dealt with Ising model with interactions being present among all neighbours. The interaction is random, follows Gaussian distribution and does not distinguish one pair of neighbours from another pair of neighbours, irrespective of the distance between two neighbours. In presence of external magnetic field, they predicted in their next paper, [100], below spin-glass transition temperature a spin-glass phase with non-zero magnetisation. Almeida etal, [101], Gray and Moore, [102],finally Parisi, [103], [104] improved and gave final touch, [105], to their line of work. Parisi and collaborators, [106]-[110], wrote a series of papers in postscript, all revolving around a consistent assumption of constant magnetisation in the spin-glass phase in presence of little constant external magnetic field.

In another sequence of theoretical work, by Fisher etal,[111–113], concluded that for Ising model with nearest neighbour or, short range interaction of random type, spin-glass phase does not exist in presence of external magnetic field.

For recent series of experiments on spin-glass, the references, [114, 115], are the places to look into.

For an in depth account, accessible to a commoner, the series of articles by late P. W. Anderson in Physics Today, [116]-[122], is probably the best place to look into. For a book to enter into the subject of spin-glass, one may start at [123].

Here, in our work to follow, spin-glass refers to spin-glass phase of a system with infinite range random interactions.

A	B	C	D	E	G	H	I	J	K	L	M	N	O	P	R	S	T	U	W	Y
1153	2245	1363	1976	289	1427	1192	150	1206	2612	1462	1423	989	376	1559	892	1709	1764	406	3	7

TABLE III. Head entries of the Campbell’s Santali-English Dictionary: the first row represents letters of the alphabet ala Campbell in the serial order, the second row is the respective number of head entries of the Campbell’s Santali-English Dictionary, [1].

### III. CAMPBELL’S SANTALI-ENGLISH DICTIONARY: THE GRAPHICAL LAW ANALYSIS

Rev. A. Campbell worked among the Santals in Iskotland, Pokhuria. Campbell ’s Santali-English Dictionary, the first edition, was published in 1899 by the Santal Christian Council under the auspices of the Scotland Santal Mission. The dialect of the Dumka area of the Santal Parganas was taken as the standard. The second edition was published in 1933. After that appeared ”A Santal Dictionary by P. O. Bodding” in five volumes, published by the University of Oslo. This was consulted and many alterations made in the light of it, making the third edition, [1], appear in July 1953 by the editor, R. M. Macphail. We count all the head entries of this dictionary, [1], one by one from the beginning to the end, starting with different letters. The result is the following table, III. Highest number of head entries, two thousand six hundred twelve, starts with the letter K followed by head entries numbering two thousand two hundred forty five beginning with B, one thousand nine hundred seventy six with the letter D etc. To visualise we plot the number of head entries against the respective letters in the figure fig.4. For the purpose of exploring graphical law, we assort the letters according to the number of head entries, in the descending order, denoted by  $f$  and the respective rank, [124], denoted by  $k$ .  $k$  is a positive integer starting from one. Moreover, we attach a limiting rank,  $k_{lim}$ , and a limiting number of head entries. The limiting rank is maximum rank plus one, here it is twenty two and the limiting number of head entries is one. As a result both  $\frac{lnf}{lnf_{max}}$  and  $\frac{lnk}{lnk_{lim}}$  varies from zero to one. Then we tabulate in the adjoining table, IV, and plot  $\frac{lnf}{lnf_{max}}$  against  $\frac{lnk}{lnk_{lim}}$  in the figure fig.5.

We then ignore the letter with the highest number of head entries, tabulate in the adjoining table, IV, normalising the  $lnfs$  with next-to-maximum  $lnf_{n-max}$ , starting from  $k = 2$ , and plot  $\frac{lnf}{lnf_{n-max}}$  against  $\frac{lnk}{lnk_{lim}}$  in the figure fig.6, Normalising the  $lnfs$  with next-to-next-to-

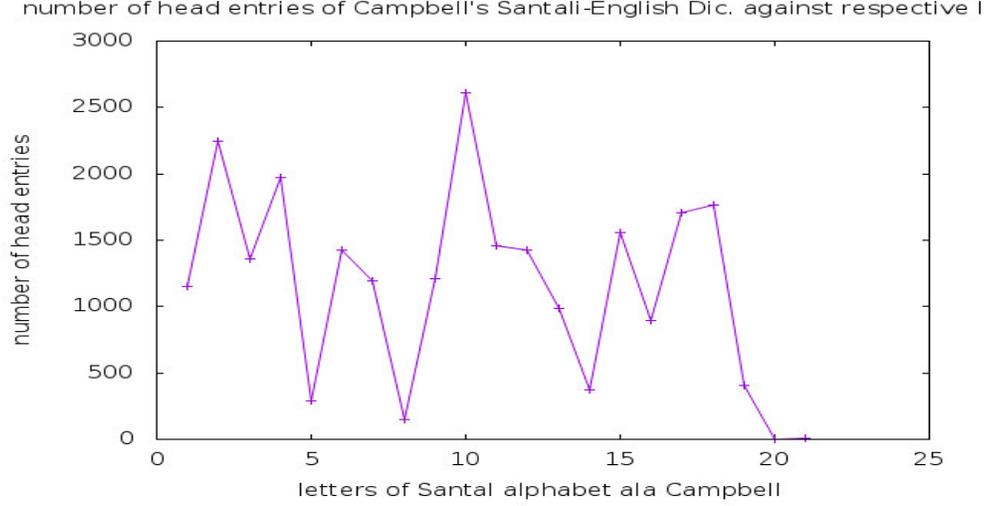


FIG. 4. Vertical axis is number of head entries of the Campbell's Santali-English Dictionary, [1]. Horizontal axis is the letters of the Santali alphabet, ala Campbell, [1]. Letters are represented by the sequence number in the alphabet.

maximum  $lnf_{2n-max}$ , we tabulate in the adjoining table, IV, and plot  $\frac{lnf}{lnf_{2n-max}}$  against  $\frac{lnk}{lnk_{lim}}$  in the figure fig.7, starting from  $k = 3$ . Normalising the  $lnfs$  with next-to-next-to-next-to-maximum  $lnf_{3n-max}$  we record in the adjoining table, IV, and plot  $\frac{lnf}{lnf_{3n-max}}$  against  $\frac{lnk}{lnk_{lim}}$  in the figure fig.8, starting from  $k = 4$ . Normalising the  $lnfs$  with 4n-maximum  $lnf_{4n-max}$  we record in the adjoining table, IV, starting from  $k = 5$ . Normalising the  $lnfs$  with 5n-maximum  $lnf_{5n-max}$  we record in the adjoining table, IV, starting from  $k = 6$ .

k	lnk	lnk/ $lnk_{lim}$	f	lnf	lnf/ $lnf_{max}$	lnf/ $lnf_{n-max}$	lnf/ $lnf_{2n-max}$	lnf/ $lnf_{3n-max}$	lnf/ $lnf_{4n-max}$	lnf/ $lnf_{5n-max}$
1	0	0	2612	7.868	1	Blank	Blank	Blank	Blank	Blank
2	0.69	0.223	2245	7.716	0.981	1	Blank	Blank	Blank	Blank
3	1.10	0.356	1976	7.589	0.965	0.984	1	Blank	Blank	Blank
4	1.39	0.450	1764	7.475	0.950	0.969	0.985	1	Blank	Blank
5	1.61	0.521	1709	7.444	0.946	0.965	0.981	0.996	1	Blank
6	1.79	0.579	1559	7.352	0.934	0.953	0.969	0.984	0.988	1
7	1.95	0.631	1462	7.288	0.926	0.945	0.960	0.975	0.979	0.991
8	2.08	0.673	1427	7.263	0.923	0.941	0.957	0.972	0.976	0.988
9	2.20	0.712	1423	7.261	0.923	0.941	0.957	0.971	0.975	0.988
10	2.30	0.744	1363	7.217	0.917	0.935	0.951	0.965	0.970	0.982
11	2.40	0.777	1206	7.095	0.902	0.920	0.935	0.949	0.953	0.965
12	2.48	0.803	1192	7.083	0.900	0.918	0.933	0.948	0.952	0.963
13	2.56	0.828	1153	7.050	0.896	0.914	0.929	0.943	0.947	0.959
14	2.64	0.854	989	6.897	0.877	0.894	0.909	0.923	0.927	0.938
15	2.71	0.877	892	6.793	0.863	0.880	0.895	0.909	0.913	0.924
16	2.77	0.896	406	6.006	0.763	0.778	0.791	0.803	0.807	0.817
17	2.83	0.916	376	5.930	0.754	0.769	0.781	0.793	0.797	0.807
18	2.89	0.935	289	5.666	0.720	0.734	0.747	0.758	0.761	0.771
19	2.94	0.951	150	5.011	0.637	0.649	0.660	0.670	0.673	0.682
20	3.00	0.971	7	1.946	0.247	0.252	0.256	0.260	0.261	0.265
21	3.04	0.984	3	1.099	0.140	0.142	0.145	0.147	0.148	0.149
22	3.09	1	1	0	0	0	0	0	0	0

TABLE IV. Head entries of the Campbell’s Santali-English Dictionary: ranking, natural logarithm, normalisations

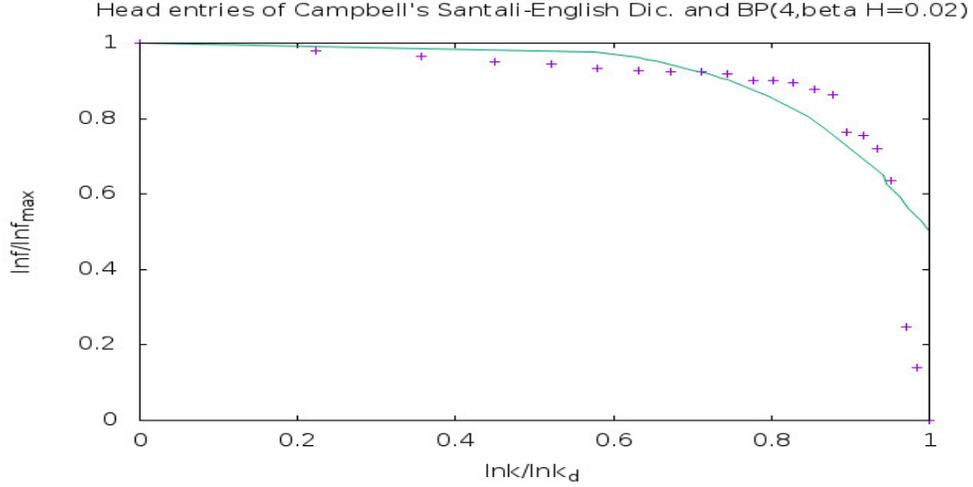


FIG. 5. The vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the head entries of the Campbell's Santali-English Dictionary with the fit curve being the Bethe-Peierls curve,  $BP(4, \beta H = 0.02)$ , in the presence of four nearest neighbours and little external magnetic field,  $m = 0.01$  or,  $\beta H = 0.02$ .

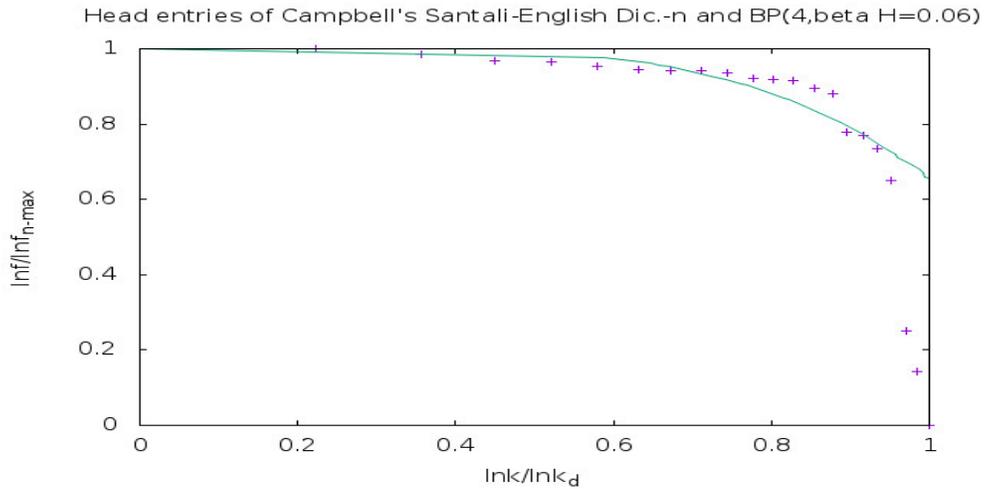


FIG. 6. The vertical axis is  $\frac{\ln f}{\ln f_{n-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the head entries of the Campbell's Santali-English Dictionary with the fit curve being the Bethe-Peierls curve,  $BP(4, \beta H = 0.06)$ , in the presence of four nearest neighbours and little external magnetic field,  $m = 0.03$  or,  $\beta H = 0.06$ .

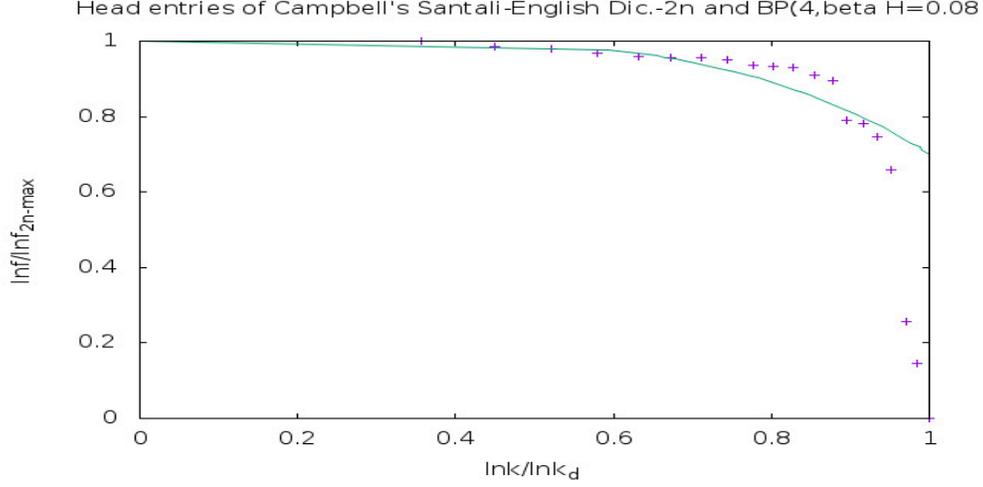


FIG. 7. The vertical axis is  $\frac{\ln f}{\ln f_{2n-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the head entries of the Campbell's Santali-English Dictionary with the fit curve being the Bethe-Peierls curve, BP(4,  $\beta H = 0.08$ ), in the presence of four nearest neighbours and little external magnetic field,  $m = 0.04$  or,  $\beta H = 0.08$ .

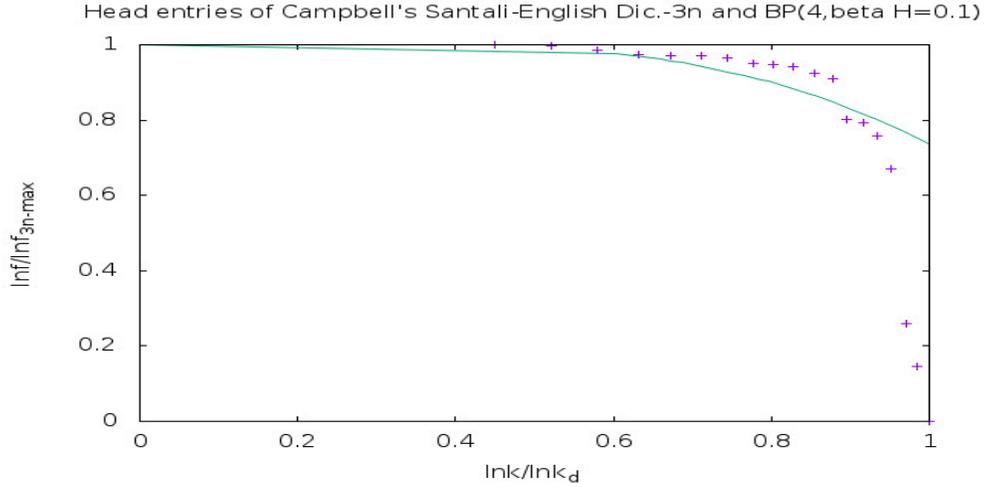


FIG. 8. The vertical axis is  $\frac{\ln f}{\ln f_{3n-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the head entries of the Campbell's Santali-English Dictionary with the fit curve being the Bethe-Peierls curve, BP(4,  $\beta H = 0.1$ ), in the presence of four nearest neighbours and little external magnetic field,  $m = 0.05$  or,  $\beta H = 0.1$ .

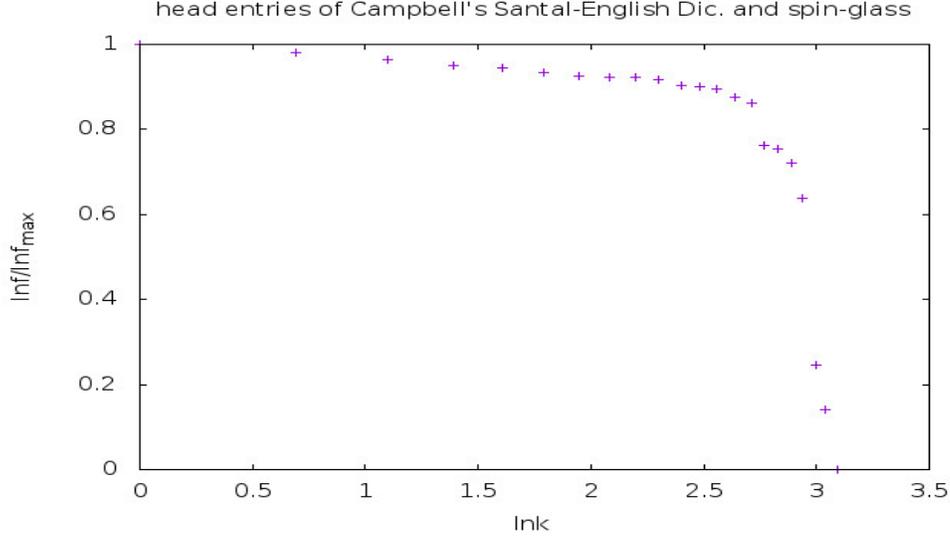


FIG. 9. The vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and the horizontal axis is  $\ln k$ . The + points represent the head entries of the Santali-English Dictionary of Campbell, [1], in the alphabet scheme of Campbell, [1].

### A. tentative conclusion

Matching of the plots in the figures fig.(5-8), with comparator curves i.e. the magnetisation curves of the Ising Model in various approximations, are with dispersions and dispersions do not reduce over higher orders of normalisations.

To explore for possible existence of spin-glass transition, in the presence of little external magnetic field,  $\frac{\ln f}{\ln f_{max}}$  is drawn against  $\ln k$  in the figure fig.9.

From the figure fig.9, we notice that the Santali head entries of the Santali-English Dictionary of Campbell, [1], is very well suited to be described to imbibe a magnetisation curve associated with spin-glass transition, in the presence of little external magnetic field.

A	B	C	D	E	G	H	I	J	K	L	M	N	O	P	R	S	T	U	W	Y
1597	3534	2252	3275	550	2067	2180	327	2037	4113	2149	1655	1069	736	2359	1297	2506	2590	619	3	13

TABLE V. Head entries of A Santal Dictionary by P. O. Bodding: the first row represents letters of the alphabet ala Campbell, [1] in the serial order, the second row is the respective number of head entries of A Santal Dictionary by P. O. Bodding, [2].

#### IV. A SANTAL DICTIONARY BY P. O. BODDING IN THE ALPHABET SCHEME OF CAMPBELL: THE GRAPHICAL LAW ANALYSIS

Rev. P. O. Bodding(1868-1936) was the most outstanding of the missionary scholars, who came to India in January, 1890, from Norway and worked among the Santals and studied their language and culture. He spent his twenty years to complete this masterpiece, [2]. First Volume was completed on 5-th July, 1932. Fifth volume was completed on 7-th April, 1936. Rev. L. O. Skrefsrud who wrote "A Grammar of the Santal Language" in Benares on 1873, collected over 13000 Santali words. Rev. L. O. Skrefsrud handed over this collection of words to Rev. P. O. Bodding, who in turn collected another block of over 13000 Santali words on his own, specially from the region east of Manbhum and southern part of the Santal Parganas and consulting the Santali-English Dictionary of Campbell, first edition. Rev. A. Campbell developed his vocabulary of Santali concentrating on Santals living in contact with Kolhes and consequently of Kherwari interest, majority being loan-words of Aryan i.e. Hindi or, Bihari origin, [2]. A Santal Dictionary by P. O. Bodding (7 parts in 5 Vols.) was reprinted in 2016 in India by Gyan Publishing House, [2], which carries a forward by K. S. Singh, Director General, Anthropological Survey of India. Rev. P. O. Bodding used his own scheme of alphabet. We count all the head entries of this dictionary, [2], one by one from the beginning to the end, starting with different letters in the scheme of alphabet of Rev. P. O. Bodding. The result is the table, VII, which we present in the next section, section V. Here in this section, we reduce Rev. P. O. Bodding's scheme of alphabet to that of Rev. A. Campbell. The result is the table, V. Highest number of head entries, four thousand one hundred thirteen, starts with the letter K followed by head entries numbering three thousand five hundred thirty four beginning with B, three thousand two hundred seventy five with the letter D etc. To visualise we plot the number of head entries

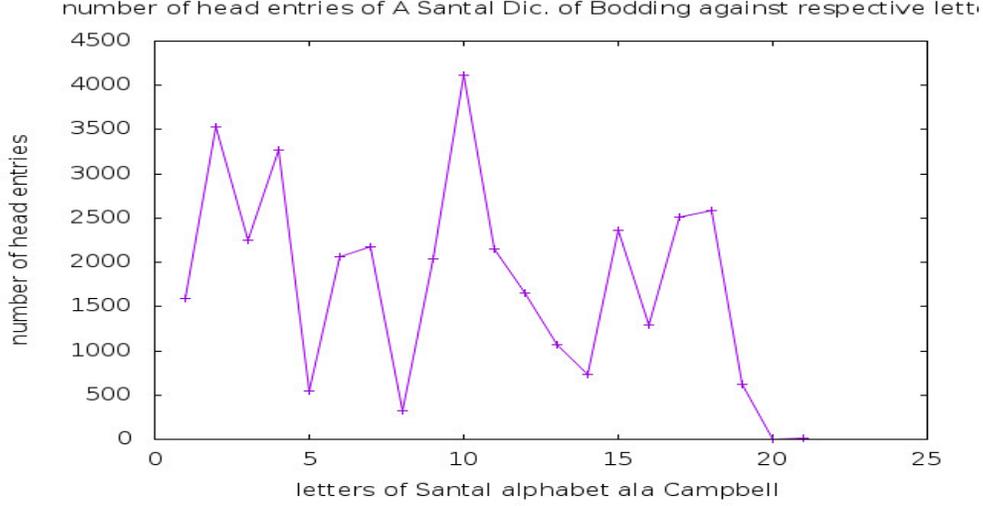


FIG. 10. Vertical axis is number of head entries of A Santal Dictionary by P. O. Bodding,[2]. Horizontal axis is the letters of the Santali alphabet ala Campbell, [1]. Letters are represented by the sequence number in the alphabet.

against the respective letters in the figure fig.10. For the purpose of exploring graphical law, we assort the letters according to the number of head entries, in the descending order, denoted by  $f$  and the respective rank, [124], denoted by  $k$ .  $k$  is a positive integer starting from one. Moreover, we attach a limiting rank,  $k_{lim}$ , and a limiting number of head entries. The limiting rank is maximum rank plus one, here it is twenty two and the limiting number of head entries is one. The limiting number of head entries being one, represent a letter which does not appear in the initial position of a word. As a result both  $\frac{lnf}{lnf_{max}}$  and  $\frac{lnk}{lnk_{lim}}$  varies from zero to one. Then we tabulate in the adjoining table, IV, and plot  $\frac{lnf}{lnf_{max}}$  against  $\frac{lnk}{lnk_{lim}}$  in the figure fig.11. We then ignore the letter with the highest number of head entries, tabulate in the adjoining table, VI, normalising the  $lnfs$  with next-to-maximum  $lnf_{n-max}$ , and plot  $\frac{lnf}{lnf_{n-max}}$  against  $\frac{lnk}{lnk_{lim}}$  in the figure fig.12, starting from  $k = 2$ . Normalising the  $lnfs$  with next-to-next-to-maximum  $lnf_{2n-max}$ , we tabulate in the adjoining table, VI, and plot  $\frac{lnf}{lnf_{2n-max}}$  against  $\frac{lnk}{lnk_{lim}}$  in the figure fig.13, starting from  $k = 3$ . Normalising the  $lnfs$  with next-to-next-to-next-to-maximum  $lnf_{3n-max}$  we record in the adjoining table, VI, and plot  $\frac{lnf}{lnf_{3n-max}}$  against  $\frac{lnk}{lnk_{lim}}$  in the figure fig.14, starting from  $k = 4$ . Normalising the  $lnfs$  with 4n-maximum  $lnf_{4n-max}$  we record in the adjoining table, VI, starting from  $k = 5$ . Normalising the  $lnfs$  with 5n-maximum  $lnf_{5n-max}$  we record in the adjoining table, VI.

k	lnk	lnk/ $lnk_{lim}$	f	lnf	lnf/ $lnf_{max}$	lnf/ $lnf_{n-max}$	lnf/ $lnf_{2n-max}$	lnf/ $lnf_{3n-max}$	lnf/ $lnf_{4n-max}$	lnf/ $lnf_{5n-max}$
1	0	0	4113	8.322	1	Blank	Blank	Blank	Blank	Blank
2	0.69	0.223	3534	8.170	0.982	1	Blank	Blank	Blank	Blank
3	1.10	0.356	3275	8.094	0.973	0.991	1	Blank	Blank	Blank
4	1.39	0.450	2590	7.859	0.944	0.962	0.971	1	Blank	Blank
5	1.61	0.521	2506	7.826	0.940	0.958	0.967	0.996	1	Blank
6	1.79	0.579	2359	7.766	0.933	0.951	0.959	0.988	0.992	1
7	1.95	0.631	2252	7.720	0.928	0.945	0.954	0.982	0.986	0.994
8	2.08	0.673	2180	7.687	0.924	0.941	0.950	0.978	0.982	0.990
9	2.20	0.712	2149	7.673	0.922	0.939	0.948	0.976	0.980	0.988
10	2.30	0.744	2067	7.634	0.917	0.934	0.943	0.971	0.975	0.983
11	2.40	0.777	2037	7.619	0.916	0.933	0.941	0.969	0.974	0.981
12	2.48	0.803	1655	7.412	0.891	0.907	0.916	0.943	0.947	0.954
13	2.56	0.828	1597	7.376	0.886	0.903	0.911	0.939	0.942	0.950
14	2.64	0.854	1297	7.168	0.861	0.877	0.886	0.912	0.916	0.923
15	2.71	0.877	1069	6.974	0.838	0.854	0.862	0.887	0.891	0.898
16	2.77	0.896	736	6.601	0.793	0.808	0.816	0.840	0.843	0.850
17	2.83	0.916	619	6.428	0.772	0.787	0.794	0.818	0.821	0.828
18	2.89	0.935	550	6.310	0.758	0.772	0.780	0.803	0.806	0.813
19	2.94	0.951	327	5.790	0.696	0.709	0.715	0.737	0.740	0.746
20	3.00	0.971	13	2.565	0.308	0.314	0.317	0.326	0.328	0.330
21	3.04	0.984	3	1.099	0.132	0.135	0.136	0.140	0.140	0.142
22	3.09	1	1	0	0	0	0	0	0	0

TABLE VI. Head entries of A Santal Dictionary by P. O. Bodding, [2],(ala Campbell scheme of alphabet, [1]): ranking,natural logarithm, normalisations

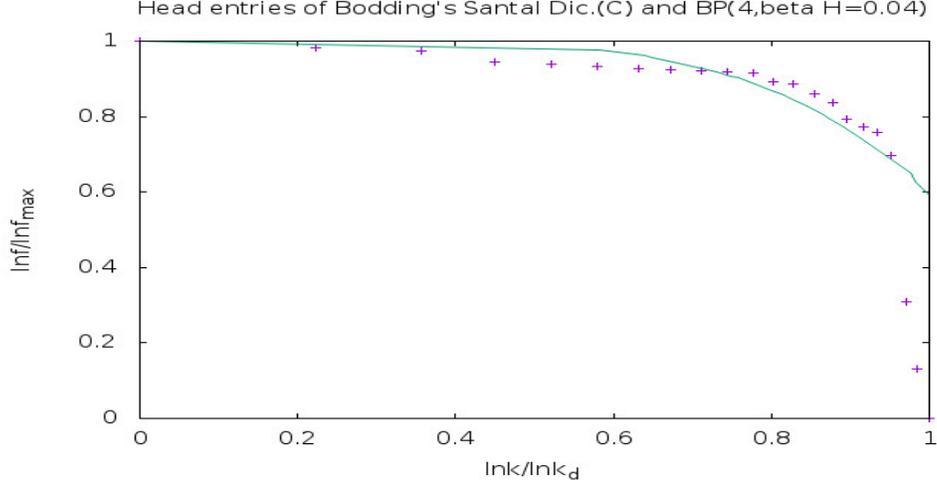


FIG. 11. The vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the head entries of the Santal Dictionary of P. O. Bodding in the alphabet scheme of Campbell, with the fit curve being the Bethe-Peierls curve,  $BP(4, \beta H = 0.04)$ , in the presence of four nearest neighbours and little external magnetic field,  $m = 0.02$  or,  $\beta H = 0.04$ .

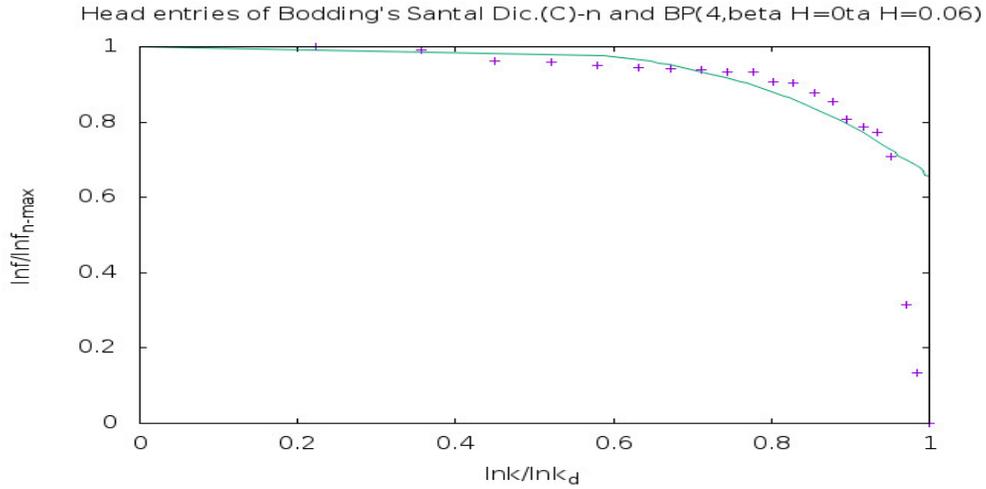


FIG. 12. The vertical axis is  $\frac{\ln f}{\ln f_{n-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the head entries of the Santal Dictionary of P. O. Bodding in the alphabet scheme of Campbell, with the fit curve being the Bethe-Peierls curve,  $BP(4, \beta H = 0.06)$ , in the presence of four nearest neighbours and little external magnetic field,  $m = 0.03$  or,  $\beta H = 0.06$ .

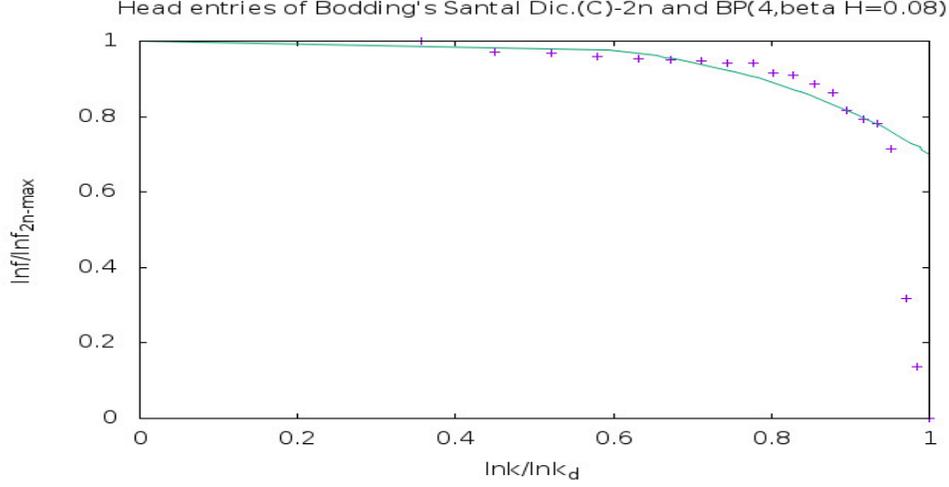


FIG. 13. The vertical axis is  $\frac{\ln f}{\ln f_{2n-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the head entries of the Santal Dictionary of P. O. Bodding in the alphabet scheme of Campbell, with the fit curve being the Bethe-Peierls curve,  $BP(4, \beta H = 0.08)$ , in the presence of four nearest neighbours and little external magnetic field,  $m = 0.04$  or,  $\beta H = 0.08$ .

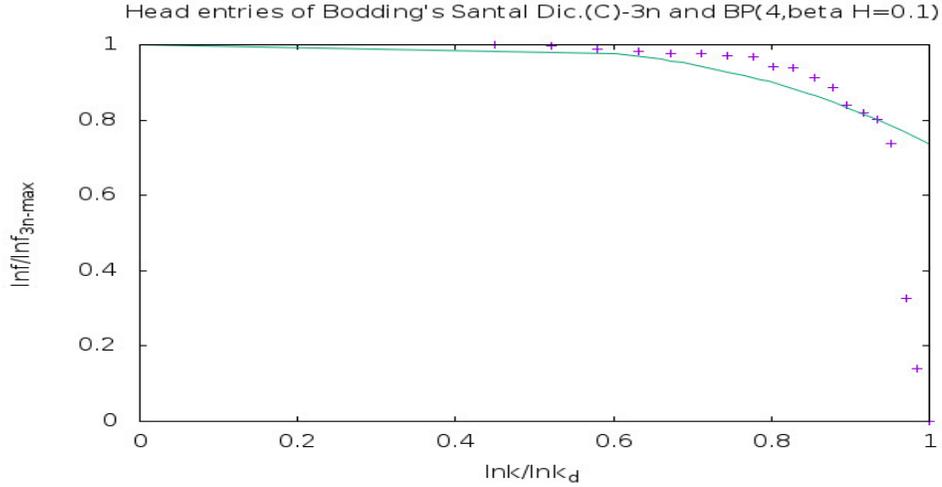


FIG. 14. The vertical axis is  $\frac{\ln f}{\ln f_{3n-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the head entries of the Santal Dictionary of P. O. Bodding in the alphabet scheme of Campbell, with the fit curve being the Bethe-Peierls curve,  $BP(4, \beta H = 0.1)$ , in the presence of four nearest neighbours and little external magnetic field,  $m = 0.05$  or,  $\beta H = 0.1$ .

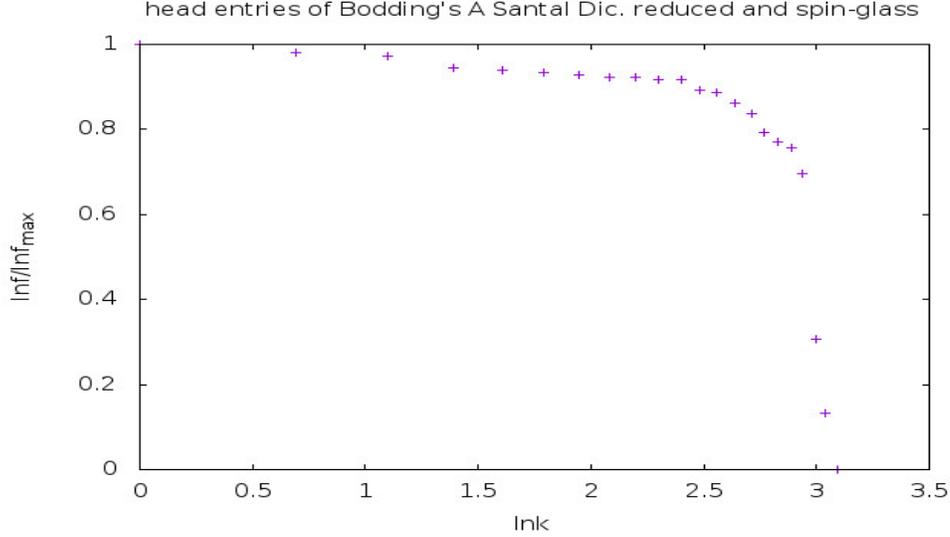


FIG. 15. The vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and the horizontal axis is  $\ln k$ . The + points represent the head entries of A Santal Dictionary of P. O. Bodding,[2], in the alphabet scheme of Campbell,[1].

### A. tentative conclusion

Matching of the plots in the figures fig.(11-14), with comparator curves i.e. the magnetisation curves of the Ising Model in various approximations, are with dispersions and dispersions do not reduce over higher orders of normalisations.

To explore for possible existence of spin-glass transition, in the presence of little external magnetic field,  $\frac{\ln f}{\ln f_{max}}$  is drawn against  $\ln k$  in the figure fig.15.

From the figure fig.15, we notice that the Santali head entries of A Santal Dictionary of P. O. Bodding,[2], in the alphabet scheme of Campbell,[1], is very well suited to be described to imbibe a magnetisation curve associated with spin-glass transition, in the presence of little external magnetic field.

A	ᱠ	B	Bh	C	ᱚ	Ch	D	Dh	ᱛ	ᱜ	E	ᱥ	G	Gh	H	I	J	Jh	K	ᱛ	Kh	L
1129	468	2639	895	1750	0	502	1410	766	675	424	228	322	1653	414	2180	327	1465	572	2798	0	1315	2149
M	N	Nh	ᱠ	ᱡ	ᱢ	O	ᱣ	P	ᱤ	Ph	R	ᱥ	S	T	ᱦ	Th	ᱧ	ᱨ	U	V	W	Y
1655	839	68	162	0	0	310	426	1765	0	594	1297	0	2506	1093	0	295	724	478	619	0	3	13

TABLE VII. Head entries of A Santal Dictionary by P. O. Bodding: the odd rows represent letters of the Santali alphabet ala P. O. Bodding, [2], in the serial order, the even rows are the respective number of head entries of A Santal Dictionary by P. O. Bodding, [2].

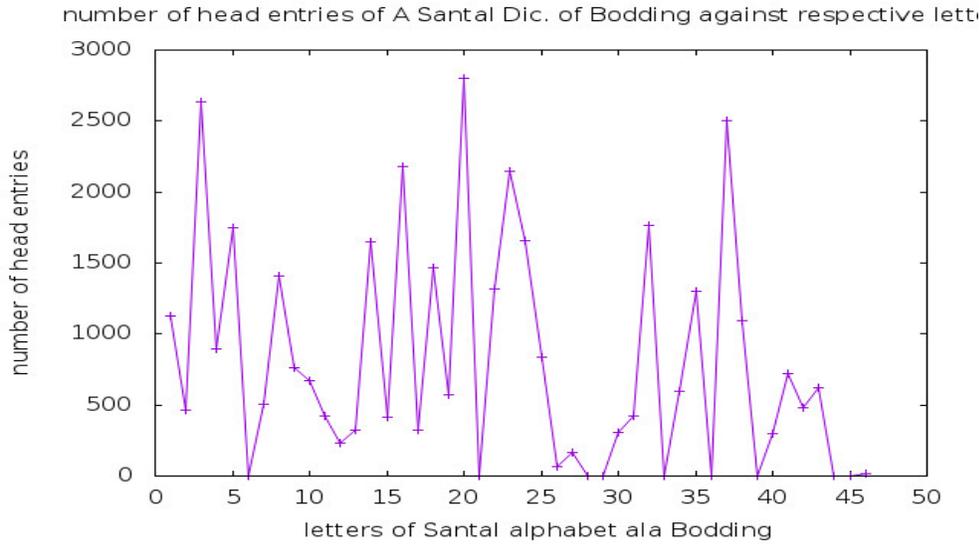


FIG. 16. Vertical axis is number of head entries of A Santal Dictionary by P. O. Bodding,[2]. Horizontal axis is the letters of the Santali alphabet ala P. O. Bodding, [2]. Letters are represented by the sequence number in the alphabet.

## V. A SANTAL DICTIONARY BY P. O. BODDING IN THE ALPHABET SCHEME OF P. O. BODDING: THE GRAPHICAL LAW ANALYSIS

Highest number of head entries, two thousand seven hundred ninety eight, starts with the letter K followed by head entries numbering two thousand six hundred thirty nine beginning with B, two thousand five hundred six with the letter S etc. To visualise we plot the number of head entries against the respective letters in the figure fig.16.

For the purpose of exploring graphical law, we assort the letters according to the number of head entries, in the descending order, denoted by  $f$  and the respective rank, [124], denoted

by  $k$ .  $k$  is a positive integer starting from one. Moreover, the minimum non-zero number of head entries is three. We attach a limiting rank,  $k_{lim}$ , and a limiting number of head entries. The limiting rank is maximum rank plus one, here it is thirty nine and the limiting number of head entries is one. As a result both  $\frac{lnf}{lnf_{max}}$  and  $\frac{lnk}{lnk_{lim}}$  varies from zero to one. Then we tabulate in the adjoining table,VIII, and plot  $\frac{lnf}{lnf_{max}}$  against  $\frac{lnk}{lnk_{lim}}$  in the figure fig.17. We then ignore the letter with the highest of head entries, tabulate in the adjoining table,VIII,and redo the plot, normalising the  $lnfs$  with  $lnf_{n-max}$ , and starting from  $k = 2$  in the figure fig.18. Normalising the  $lnfs$  with  $lnf_{2n-max}$ , we tabulate in the adjoining table,VIII, and starting from  $k = 3$  we draw in the figure fig.19. Normalising the  $lnfs$  with  $lnf_{3n-max}$  we record in the adjoining table,VIII, and plot starting from  $k = 4$  in the figure fig.20. In this way we obtain up to the figure fig.22.

k	lnk	lnk/ $\ln k_{lim}$	f	lnf	lnf/ $\ln f_{max}$	lnf/ $\ln f_{n-max}$	lnf/ $\ln f_{2n-max}$	lnf/ $\ln f_{3n-max}$	lnf/ $\ln f_{4n-max}$	lnf/ $\ln f_{5n-max}$
1	0	0	2798	7.937	1	Blank	Blank	Blank	Blank	Blank
2	0.69	0.189	2639	7.878	0.993	1	Blank	Blank	Blank	Blank
3	1.10	0.301	2506	7.826	0.986	0.993	1	Blank	Blank	Blank
4	1.39	0.380	2180	7.687	0.969	0.976	0.982	1	Blank	Blank
5	1.61	0.440	2149	7.673	0.967	0.974	0.980	0.998	1	Blank
6	1.79	0.489	1765	7.476	0.942	0.949	0.955	0.973	0.974	1
7	1.95	0.533	1750	7.467	0.941	0.948	0.954	0.971	0.973	0.999
8	2.08	0.568	1655	7.412	0.934	0.941	0.947	0.964	0.966	0.991
9	2.20	0.601	1653	7.410	0.934	0.941	0.947	0.964	0.966	0.991
10	2.30	0.628	1465	7.290	0.918	0.925	0.932	0.948	0.950	0.975
11	2.40	0.656	1410	7.251	0.914	0.920	0.927	0.943	0.945	0.970
12	2.48	0.678	1315	7.182	0.905	0.912	0.918	0.934	0.936	0.961
13	2.56	0.699	1297	7.168	0.903	0.910	0.916	0.932	0.934	0.959
14	2.64	0.721	1129	7.029	0.886	0.892	0.898	0.914	0.916	0.940
15	2.71	0.740	1093	6.997	0.882	0.888	0.894	0.910	0.912	0.936
16	2.77	0.757	895	6.797	0.856	0.863	0.869	0.884	0.886	0.909
17	2.83	0.773	839	6.732	0.848	0.855	0.860	0.876	0.877	0.900
18	2.89	0.790	766	6.641	0.837	0.843	0.849	0.864	0.866	0.888
19	2.94	0.803	724	6.585	0.830	0.836	0.841	0.857	0.858	0.881
20	3.00	0.820	675	6.515	0.821	0.827	0.832	0.848	0.849	0.871
21	3.04	0.831	619	6.428	0.810	0.816	0.821	0.836	0.838	0.860
22	3.09	0.844	594	6.387	0.805	0.811	0.816	0.831	0.832	0.854
23	3.14	0.858	572	6.349	0.800	0.806	0.811	0.826	0.827	0.849
24	3.18	0.869	502	6.219	0.784	0.789	0.795	0.809	0.811	0.832
25	3.22	0.880	478	6.170	0.777	0.783	0.788	0.803	0.804	0.825
26	3.26	0.891	468	6.148	0.775	0.780	0.786	0.800	0.801	0.822
27	3.30	0.902	426	6.054	0.763	0.768	0.774	0.788	0.789	0.810
28	3.33	0.910	424	6.050	0.762	0.768	0.773	0.787	0.788	0.809
29	3.37	0.921	414	6.026	0.759	0.765	0.770	0.784	0.785	0.806
30	3.40	0.929	327	5.790	0.729	0.735	0.740	0.753	0.755	0.774
31	3.43	0.937	322	5.775	0.728	0.733	0.738	0.751	0.753	0.772
32	3.47	0.948	310	5.737	0.723	0.728	0.733	0.746	0.748	0.767
33	3.50	0.956	295	5.687	0.717	0.722	0.727	0.740	0.741	0.761
34	3.53	0.964	228	5.429	0.684	0.689	0.694	0.706	0.708	0.726
35	3.56	0.973	162	5.088	0.641	0.646	0.650	0.662	0.663	0.681
36	3.58	0.978	68	4.220	0.532	0.536	0.539	0.549	0.550	0.564
37	3.61	0.986	13	2.565	0.323	0.326	0.328	0.334	0.334	0.343
38	3.64	0.995	3	1.099	0.138	0.140	0.140	0.143	0.143	0.147
39	3.66	1	1	0	0	0	0	0	0	0

TABLE VIII. Head entries of A Santal Dictionary by P. O. Bodding,[2](ala Bodding scheme of alphabet,[2]): ranking,natural logarithm, normalisations

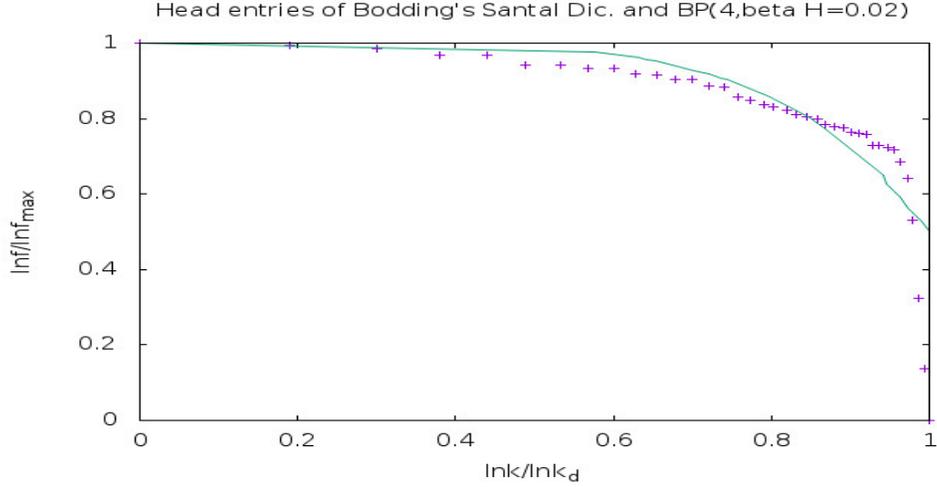


FIG. 17. The vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the head entries of the Santal Dictionary of P. O. Boddling with the fit curve being the Bethe-Peierls curve,  $BP(4, \beta H = 0.02)$ , in the presence of four nearest neighbours and little external magnetic field,  $m = 0.01$  or,  $\beta H = 0.02$ .

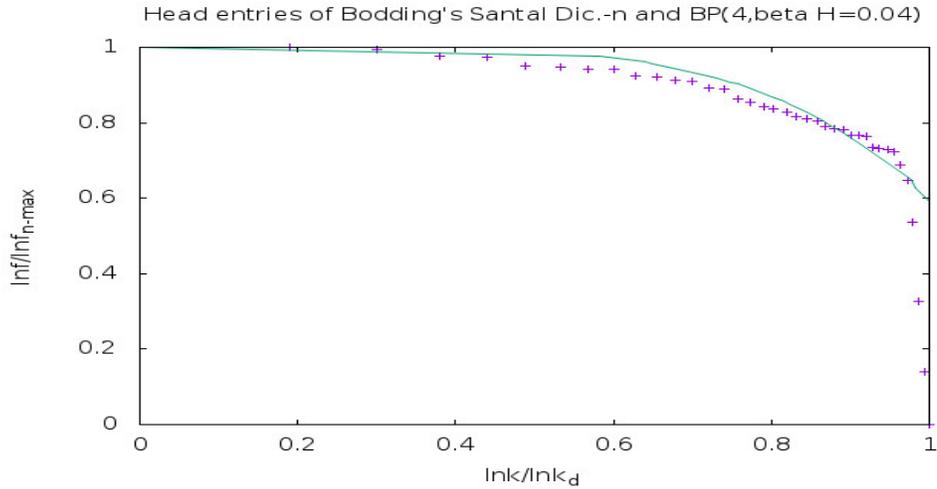


FIG. 18. The vertical axis is  $\frac{\ln f}{\ln f_{n-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the head entries of the Santal Dictionary of P. O. Boddling with the fit curve being the Bethe-Peierls curve,  $BP(4, \beta H = 0.04)$ , in the presence of four nearest neighbours and little external magnetic field,  $m = 0.02$  or,  $\beta H = 0.04$ .

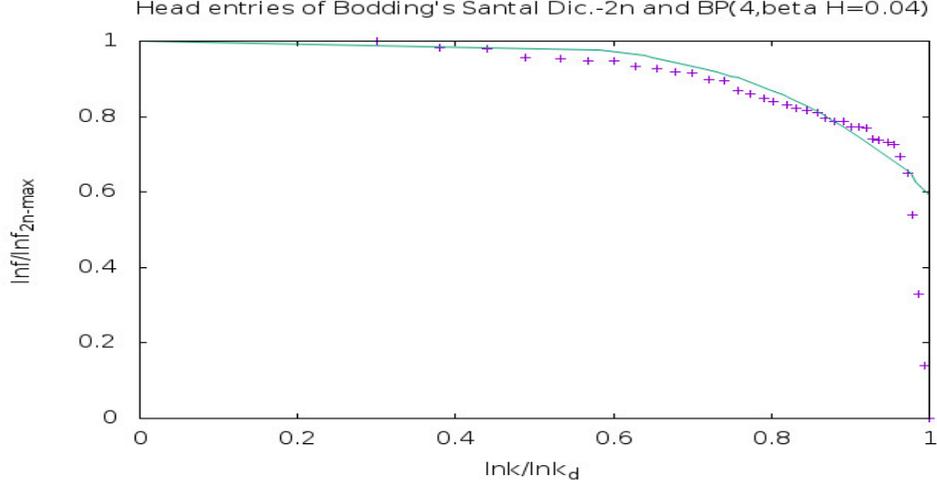


FIG. 19. The vertical axis is  $\frac{\ln f}{\ln f_{2n-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the head entries of the Santal Dictionary of P. O. Bodding with the fit curve being the Bethe-Peierls curve, BP(4, $\beta H = 0.04$ ), in the presence of four nearest neighbours and little external magnetic field,  $m = 0.02$  or,  $\beta H = 0.04$ .

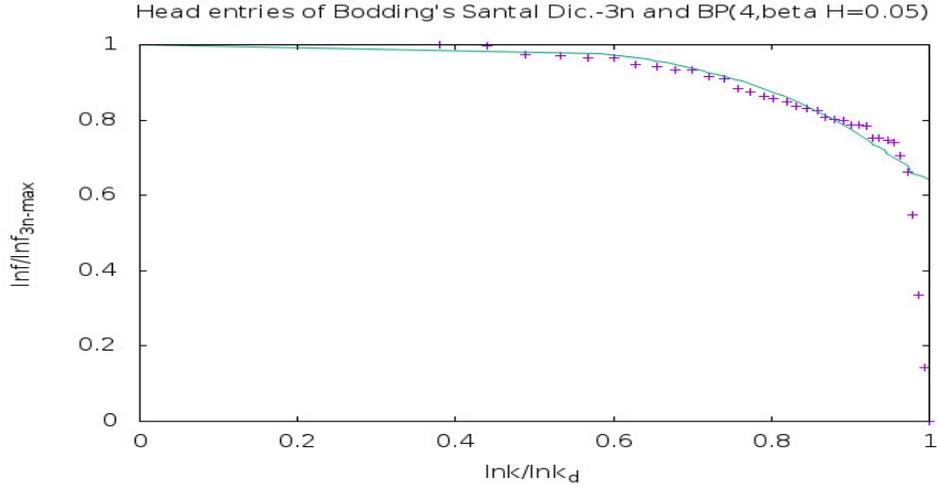


FIG. 20. The vertical axis is  $\frac{\ln f}{\ln f_{3n-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the head entries of the Santal Dictionary of P. O. Bodding with the fit curve being the Bethe-Peierls curve, BP(4, $\beta H = 0.05$ ), in the presence of four nearest neighbours and little external magnetic field,  $m = 0.025$  or,  $\beta H = 0.05$ .

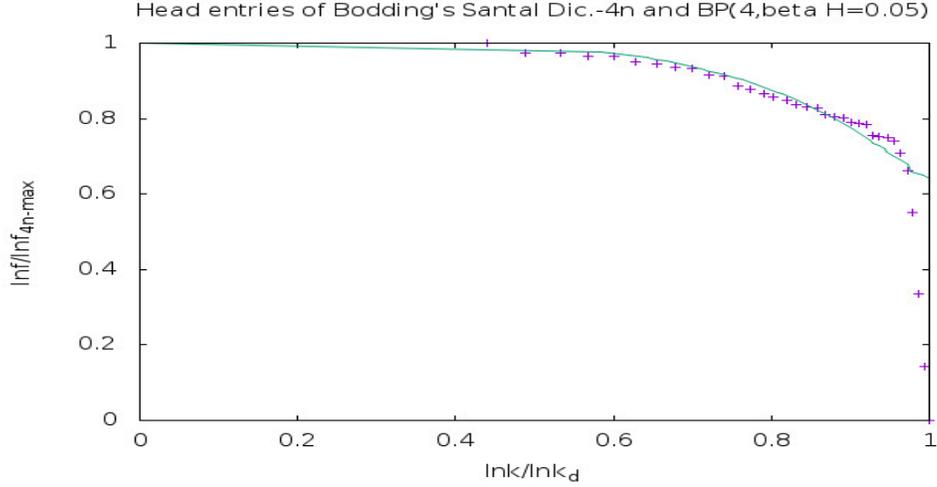


FIG. 21. The vertical axis is  $\frac{\ln f}{\ln f_{4n-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the head entries of the Santal Dictionary of P. O. Bodding with the fit curve being the Bethe-Peierls curve,  $BP(4, \beta H = 0.05)$ , in the presence of four nearest neighbours and little external magnetic field,  $m = 0.025$  or,  $\beta H = 0.05$ .

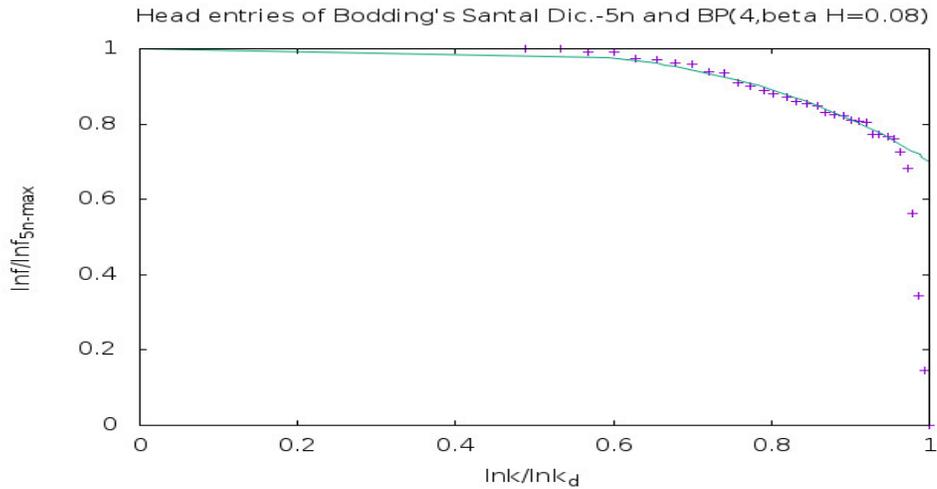


FIG. 22. The vertical axis is  $\frac{\ln f}{\ln f_{5n-max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the head entries of the Santal Dictionary of P. O. Bodding with the fit curve being the Bethe-Peierls curve,  $BP(4, \beta H = 0.08)$ , in the presence of four nearest neighbours and little external magnetic field,  $m = 0.04$  or,  $\beta H = 0.08$ .

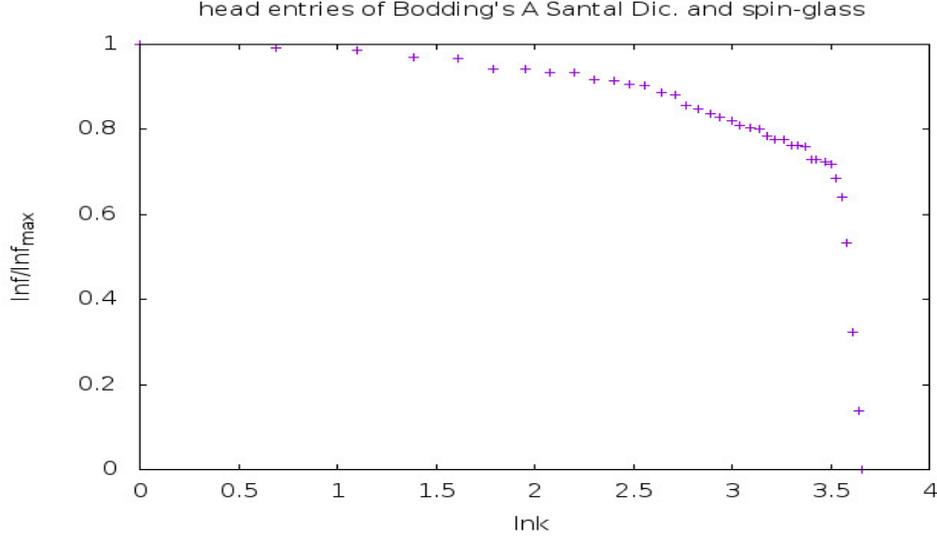


FIG. 23. The vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and the horizontal axis is  $\ln k$ . The + points represent the head entries of A Santal Dictionary of P. O. Bodding, in the alphabet scheme of P. O. Bodding.

#### A. tentative conclusion

From the figures (fig.17-fig.22), we observe that there appears to be a curve of magnetisation, behind the head entries of the Santal Dictionary of P. O. Bodding. This is the magnetisation curve, BP(4,  $\beta H = 0.08$ ), of the Ising model, in the presence of four nearest neighbours and little external magnetic field,  $m = 0.04$  or,  $\beta H = 0.08$ , modulo the almost vertically rising right hand part. Moreover, the associated correspondence is,

$$\frac{\ln f}{\ln f_{5n-max}} \longleftrightarrow \frac{M}{M_{max}}, \quad \ln k \longleftrightarrow T.$$

$k$  corresponds to temperature in an exponential scale, [125]. To explore for possible existence of spin-glass transition, in the presence of little external magnetic field,  $\frac{\ln f}{\ln f_{max}}$  is drawn against  $\ln k$  in the figure fig.23. From the figure fig.23, we notice that the Santali head entries of A Santal Dictionary of P. O. Bodding,[2], in the alphabet scheme of P. O. Bodding,[2], is well suited to be described to imbibe a magnetisation curve associated with spin-glass transition, in the presence of little external magnetic field.

## VI. CONCLUSION

From the figures (fig.9, fig.15, fig.23) respectively, we observe that the points-lines have smoothed transitions, [110]. Above the transition point(s), the lines are almost horizontal and below the transition point(s), points-lines rise like the branch of a rectangular hyperbola. Hence, Santali, [[1],[2]], is well-suited to be described by a Spin-Glass magnetisation curve, [95], in the presence of little external magnetic field. Moreover, the associated correspondence is (referred to Fig.15),

$$\frac{\ln f}{\ln f_{max}} \longleftrightarrow \frac{M}{M_{max}},$$
$$\ln k \longleftrightarrow T.$$

$k$  corresponds to temperature in an exponential scale, [125].

## VII. ACKNOWLEDGMENT

The author would like to thank the State Central Library, West Bengal, Calcutta, to allow us to use the Santal Dictionaries,[[1],[2]], in its premises. We have used gnuplot for plotting the figures in this paper.

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