# Lorentz effects on observed distance and lookback as a function of cosmic redshift

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**Abstract**: The Universe is generally considered to be about 14 billion years old. There was an 'inflationary period', in which it underwent instant expansion, then abruptly slowed down to a more gentle rate which persists to the present day. This motion picture of the observable Universe derives from a 'Hubble parameter' $H$: A star's radial recession rate $v$, divided by its 'proper distance' $d$; $H = v/d$. Both observed $v$ and observed $d_{obs}$ from starlight are supposed to increase with cosmic redshift $z$, and $v$ does, but for $z > 1$, $d_{obs}$ drops off. The current correction for this is to take the expansion of space over time into account, via integration. Integration is also used to calculate 'lookback' $(t_0 - t)$. Herein, the author shows that both $d$ and $(t_0 - t)$ are calculable without integration, via the Lorentz factor $\gamma$: $d = d_{obs}\gamma$ and $(t_0 - t) = d_{obs}\gamma^2/c$. This method is proper: Time does not elapse at the speed of light $c$, so space doesn't expand for light. The Universe's $(t_0 - t)$ is found this way as at least 170 billion years old. We also examine the slowdown of time in an increasingly dense Universe. Time came to a halt at a certain density.

**Technical Abstract**: When we treat the observable Universe as a flat perfect fluid, special relativity's constant speed of light ($c$) gives photon travel distance ($d$) and time ($t_\lambda$) from source to receptor as invariant references. For any ΛCDM Hubble parameter $H = v/d_\Lambda$, there is an instant $v/c = \beta$, found from the star's observed cosmic redshift $z$. A Lorentz frame $\gamma_\beta$ is defined, with domain $\beta$. A star's $d_\Lambda$ connects to $d$ as the instant product $d = d_\Lambda\gamma_\beta$. A 'Lorentz wall' $W_L = c/H$ is defined, giving a 'Lorentz sphere' in which all Universal energy density $\varepsilon$ resides. Rest mass's 'Lorentz distance' $d_L = W_L\beta$ cannot reach or cross $W_L$. Lorentz $d_L$ and ΛCDM $d_\Lambda$ are numerically identical. Source lookback $(t_0 - t)$ is proposed as the instant product $t_\lambda\gamma_\beta$. This gives $(t_0 - t) > 170$ billion years for the observable Universe. A general frame $\gamma_\varepsilon$ is defined, with domain ε. A 'redshift of singularity' $z_s$ is found. Density $\varepsilon$ has an upper boundary at $z_s$. Dilation in $\gamma_\varepsilon$ constitutes the physical basis of inflation.

**Key words**: Cosmology; proper distance; lookback; special relativity; cosmic inflation.

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## 1 INTRODUCTION

A star's Hubble parameter $H$ derives from two calculations: its observed redshift $z$ and observed distance $d_{obs}$. Much progress has been made since Vesto Slipher's first observation of $H$ [1]. The efforts of many astronomers over the last century, now including the vast trove of data from DESI [2][3], give potential $H$ measurements numbering in the millions. These are aggregated to produce simple models of $H$ as a function of $z$. Since 1998 [4][5], the standard model has been ΛCDM, presently under fire from DESI. This paper delves neither into the controversy surrounding the tension between ΛCDM and DESI's findings [6], nor the 'Hubble tension' [7]. We will use ΛCDM only as a proxy for observations to date. We can deduce a fundamental principle of stellar observation through ΛCDM without resort to extensive comparison with large datasets. This fundamental principle is the Lorentz effect. Lorentz contraction is built into observed $H$. Its effect on $d_{obs}$ is a core property, and will apply to any more accurate distance-ladder model that may arise.

We first explore the concept of an 'instant flat Universe', Euclidean at scale and frozen in time. We then connect its instant behavior to the observed Universe, which looks back in time. The Lorentz effect plays a central role in both. Most of the present paper's content is well known and found in introductory texts, e.g. [8]-[11], to which the reader is referred.

## 2 REFERENCE FRAMES IN A FLAT PERFECT FLUID

Under the Cosmological Principle, at any point in time $t$, the instant Universe at very large scale is treated by the texts as a perfect fluid. This fluid is also considered perfectly 'flat' over all $t$, with Euclidean geometry in both Cartesian $x, y, z$ and radial $d, \theta, \phi$ comoving coordinates. Radial $d$ is called *proper distance*. In the texts, $d$ is found by temporal adjustment to observed distance $d_{obs}$. We express $d$ with Lorentz adjustment instead.

A flat perfect fluid has four reference frames:

1) The receptor or *rest frame* is where we sit, here and now.
2) The *source frame* $\gamma_\beta$ is transposable with the rest frame. We can sit here, or we can sit there, and slow-moving source mass around us follows Newton's laws. Fast-moving source mass is observably affected in the rest frame, by special relativity. Both observed distance increase, and observed time increase, between source and receptor diminish to zero in the receptor's rest frame as source recession approaches the speed of light. When such a fast-moving source is next to us, it just disappears from view, but if it's a star a long way off, its $\Delta d/d$ hardly changes at all. The star is still about the same distance away from one day to the next.
3) The *light frame*. In the light frame, there is no such thing as time. Light moves instantly in its frame from one coordinate $d, \theta, \phi$ to another, irrespective of $d$. Time has meaning

only for rest mass. Proper $d$ is found with the light frame, and is unaffected by any rest mass behavior before or after photon emission.

4) The *density frame*, termed $\gamma_\varepsilon$, expresses general time dilation. It's developed in section 5.

## 3 LORENTZ EFFECTS IN THE INSTANT UNIVERSE

Under special relativity, proper distance $d$ and photon travel time $t_\lambda$ between source and receptor are collinear:

$$c = \frac{d}{t_\lambda} \tag{1}$$

Where $c$ is the speed of light. A scalar Hubble parameter $H$ is applied to the instant coordinates:

$$H = \frac{v}{d_L} \tag{2}$$

Where $v$ is the radial recession rate between two points. The *Lorentz distance* $d_L$ depends on $v$. When $v \ll c$, the Lorentz effect is negligible; $d_L = d$. As $v \to c$, Lorentz contraction 'shortens' $d_L$:

$$d_L = d\sqrt{1 - \frac{v^2}{c^2}} = d\sqrt{1 - \beta^2} = \frac{d}{\gamma_\beta} \tag{3}$$

Where $\beta = v/c$, and $\gamma_\beta$ is the Lorentz factor:

$$\gamma_\beta = \frac{1}{\sqrt{1 - \beta^2}} \tag{4}$$

The '$\beta$' subscript in $\gamma_\beta$ distinguishes it from $\gamma_\varepsilon$, discussed in section 5. We will say $\gamma_\beta = \gamma$ until then.

When $\beta = 0.001$, $v = 300$ km s$^{-1}$, and $d_L = 0.9999995d$. Proper $d$ and $d_L$ are effectively the same. We partition the $d$ line into proper segments $d_{seg}$, each with $\beta_{seg} = 0.001$, and assign a point as the rest frame. Without Lorentz effects, the $d_{seg}$'s add to give a serial $d_L$:

$$d_L = \sum d_{seg} \tag{5}$$

With Lorentz effects, $d_L$ is given by:

$$d_L = \sum \frac{d_{seg}}{\gamma_{seg}} \tag{6}$$

Where $\gamma_{seg}$ is the instant $\gamma$ at one boundary point of the $d_{seg}$. A distant $d_{seg}$ gets 'shorter': $'d_{seg}' \to 0$. Photon traverse time $t_{\lambda_{seg}}$ is collinear with proper $d_{seg}$ and does not change, so added speed $v_{seg} \to 0$ in the rest frame.

When you move the rest frame anywhere along the $d$ line, distant $'d_{seg}'$s on one side get closer. They stretch out to their proper value: $d_L \to d$. The $d_{seg}$'s on the other side of the rest frame grow more distant, and 'shorter'.

Serial $d_L$ approaches a *Lorentz wall* $W_L$ (Figure 1) as the $'d_{seg}'$s $\sum v_{seg} \to c$:

$$W_L = \frac{c}{H} \tag{7}$$

The number of proper $d_{seg}$'s required to reach $W_L$ approaches infinity: $\sum d_{seg} \to \infty$.

A more tractable approach to Eq. (6) is linear in $\beta$:

$$d_L = W_L \beta \tag{8}$$

From Eqs. (3) and (8), proper $d$ is thus:

$$d = d_L \gamma = W_L \beta \gamma = \frac{c\beta\gamma}{H} \tag{9}$$

Figure 1. The Lorentz wall $\mathbf{W_L}$
($\gamma \to \infty$ as $\beta \to 1$)



Table 1 gives some instant $d$ values as $\beta \to 1$ for three different $H$'s. As $v \to c$, $d_L \to W_L$ for each $H$. Proper $d$, however, increases dramatically. This is most pronounced at lower $H$ (and $z$). For today's cosmic redshift $z = 0$, with our chosen $H_0 = 73$ km s$^{-1}$ Mpc$^{-1}$, the last entry in Table 1 ($\beta = 0.9999999999$) gives $d = 947{,}000$ gigalight-years (Glyr). At $z = 1089$, the time of last scattering

[13], $W_L$ was much less. The corresponding $d$ is 41 Glyr. Photons from a light source receding that fast, that long ago, would take 41 billion years to see. This 'source' is the present-day cosmic microwave background (CMB); there were no stars back then. These same CMB photons would have a modal ten-kilometer wavelength by the time they reach our successors.

| TABLE 1: Instant Lorentz and proper distances vs. $\beta$, for selected $z$ and $\Lambda$CDM $H$ values* | | | | | | |
|---|---|---|---|---|---|---|
| Cosmic redshift: | $z = 0$ | | $z = 10$ | | $z = 1089$ | |
| $\Lambda$CDM $H$, Gyr$^{-1}$: | 0.07465 | | 1.518 | | 1,716.0 | |
| $W_L$, Glyr: | 13.395814 | | 0.658797 | | 0.000583 | |
| | | | | | | |
| $\beta$ | $d_L$, Glyr | $d$, Glyr | $d_L$, Glyr | $d$, Glyr | $d_L$, Glyr | $d$, Glyr |
| 0.01 | 0.134 | 0.134 | 0.00659 | 0.00659 | 0.000006 | 0.000006 |
| 0.1 | 1.340 | 1.35 | 0.0659 | 0.0662 | 0.000058 | 0.000059 |
| 0.5 | 6.698 | 7.73 | 0.329 | 0.380 | 0.000291 | 0.000336 |
| 0.9 | 12.06 | 27.7 | 0.593 | 1.36 | 0.000524 | 0.00120 |
| 0.99 | 13.26 | 94.0 | 0.652 | 4.62 | 0.000577 | 0.00409 |
| 0.999 | 13.38 | 299 | 0.658 | 14.7 | 0.000582 | 0.0130 |
| 0.9999 | 13.394 | 947 | 0.6587 | 46.6 | 0.000582 | 0.0412 |
| 0.99999 | 13.3957 | 2,995 | 0.658790 | 147 | 0.000583 | 0.130 |
| 0.999999 | 13.3958 | 9,472 | 0.658796 | 466 | 0.000583 | 0.412 |
| 0.9999999 | 13.395812 | 29,954 | 0.658797 | 1,473 | 0.000583 | 1.30 |
| 0.99999999 | 13.395813 | 94,723 | 0.658797 | 4,658 | 0.000583 | 4.12 |
| 0.999999999 | 13.395814 | 299,540 | 0.658797 | 14,731 | 0.000583 | 13.0 |
| 0.9999999999 | 13.395814 | 947,227 | 0.658797 | 46,584 | 0.000583 | 41.2 |
| * $\Lambda$CDM parameters: $H_0 = 73.00$ km s$^{-1}$Mpc$^{-1}$ [7]; enhanced precision. | | | | | | |
| Others from [12]: $\Omega_{(b+c)_0} = 0.3091$; $\Omega_{\lambda_0} = 0.000091 \pm 0.000005$; $\Omega_{\Lambda_0} = 0.6908$. | | | | | | |

When we use $\beta$ as the linear domain, the Lorentz wall $W_L$ becomes the radius of a *Lorentz sphere* which expands at the speed of light, and contains the entire Universe's kinetic energy and rest mass. Its infinite energy lies almost entirely within a thin outermost $\beta$ shell $[W_L - d_L]$. Almost all Euclidean space is compressed into this shell. The sphere's coordinates $\beta, \theta, \phi$ constitute dimensionless *Lorentz space* having physical meaning via $H$ and $c$. Coordinates $\beta, \theta, \phi$ are only transposable with the sphere's center or rest frame, and not each other. Any $\beta > 1$ in Lorentz space is unphysical.

The Lorentz sphere's finite radius $W_L$ means that we can talk about the Universe having a finite 'size'. It was much 'smaller' long ago than today. However, if perfectly flat, the Universe is unbound in $d$, and has always been infinitely large in Euclidean space. It's just been getting less dense. In the texts, today's wall $W_{L_0} = c/H_0$ is called the 'Hubble distance'. The term 'horizon' is also used.

The *Planck* mission's CMB-derived $H_0$ is an instant Universe expression, extrapolated forward from $z = 1089$ to $z = 0$ [12]. In *Planck*'s treatment, baryons at $z = 1089$ were moving away from each other, not from us.

## 4 LORENTZ EFFECTS IN THE OBSERVED STELLAR UNIVERSE

We now examine the stellar Universe and its distance-ladder approach. This approach is fundamentally different from *Planck*'s instant Universe. While an instant 'universe's rest mass at any one $z$ follows ΛCDM along its isotemporal $d$, the Lorentz effect on stellar observation gives substantially lower $H$ values at higher $z$'s than ΛCDM alone. A much older Universe results. The Lorentz effect wasn't obvious enough until JWST's deployment in 2021: At $z \leq 0.3$, $\gamma \leq 1.03$, so the instant and observed 'universes' more-or-less converge for many of the stars we see.

In section 4.1, we connect $d$ to ΛCDM's calculated distance $d_\Lambda$ ($\equiv d_{obs}$). The 'Λ' subscript in $d_\Lambda$ refers to ΛCDM *in toto*. In section 4.2, we calculate lookback $(t_0 - t)$ via $t_\lambda$. By inclusion of Lorentz effects, we will see how high-redshift luminous objects can appear both closer and older than predicted by ΛCDM alone.

### 4.1 Proper distance *vs.* cosmic redshift

A star's observed redshift $z$ gives its $\beta$:

$$\beta = \frac{(z+1)^2-1}{(z+1)^2+1} \tag{10}$$

Combining Eqs. (4) and (10) gives $\gamma(z)$:

$$\gamma = \sqrt{\frac{1}{2}z^2 + z + 1} \tag{11}$$

The 'minimum flat-Universe' ΛCDM model is:

$$H = \frac{v}{d_\Lambda} = \frac{\beta c}{d_\Lambda} = H_0\sqrt{\Omega_{\lambda_0}(z+1)^4 + \Omega_{(b+c)_0}(z+1)^3 + \Omega_{\Lambda_0}} \tag{12}$$

The $\Omega$ and $H_0$ terms are common and found in the texts. Their values are given in Table 1. The remaining $d_\Lambda$ term is an aggregate of individual distances $d_{obs}$ derived from measurement of luminosity and/or angular size of the source. Recession rate $v$ is, in general, more precisely found than is $d_\Lambda$ for each measurement.

| TABLE 2. ΛCDM distance, Lorentz distance, tension, tension error, and proper distance. | | | | | |
|---|---|---|---|---|---|
| $z$ | $d_\Lambda$ | $d_L$ | $d_\Lambda/d_L$ | $(d_\Lambda - d_L)/d_L$ | $d(= d_L\gamma)$ |
| | Glyr | Glyr | | | Glyr |
| 0.01 | 0.132671 | 0.132668 | 1.000021 | 2.053388090E-05 | 0.1327 |
| 0.1 | 1.212392 | 1.212367 | 1.000021 | 2.053388090E-05 | 1.2179 |
| 0.2 | 2.182471 | 2.182426 | 1.000021 | 2.053388090E-05 | 2.2188 |

| | | | | | |
|---|---|---|---|---|---|
| 0.5 | 3.912135 | 3.912054 | 1.000021 | 2.053388090E-05 | 4.2381 |
| 0.6 | 4.195630 | 4.195544 | 1.000021 | 2.053388090E-05 | 4.6675 |
| 0.7 | 4.377932 | 4.377842 | 1.000021 | 2.053388090E-05 | 5.0088 |
| 0.8 | 4.480904 | 4.480812 | 1.000021 | 2.053388090E-05 | 5.2774 |
| 0.9 | 4.522642 | 4.522550 | 1.000021 | 2.053388090E-05 | 5.4866 |
| 1 | 4.517814 | 4.517721 | 1.000021 | 2.053388090E-05 | 5.6472 |
| 1.1 | 4.478121 | 4.478029 | 1.000021 | 2.053388090E-05 | 5.7681 |
| 1.2 | 4.412790 | 4.412699 | 1.000021 | 2.053388090E-05 | 5.8569 |
| 1.3 | 4.329025 | 4.328936 | 1.000021 | 2.053388090E-05 | 5.9193 |
| 1.4 | 4.232408 | 4.232321 | 1.000021 | 2.053388090E-05 | 5.9605 |
| 1.5 | 4.127233 | 4.127148 | 1.000021 | 2.053388090E-05 | 5.9844 |
| **1.6** | **4.016779** | **4.016697** | **1.000021** | **2.053388090E-05** | **5.9941** |
| 1.7 | 3.903536 | 3.903456 | 1.000021 | 2.053388090E-05 | 5.9925 |
| 1.8 | 3.789375 | 3.789298 | 1.000021 | 2.053388090E-05 | 5.9817 |
| 1.9 | 3.675692 | 3.675617 | 1.000021 | 2.053388090E-05 | 5.9634 |
| 2 | 3.563511 | 3.563438 | 1.000021 | 2.053388090E-05 | 5.9391 |
| 3 | 2.610761 | 2.610707 | 1.000021 | 2.053388090E-05 | 5.5478 |
| 4 | 1.970316 | 1.970275 | 1.000021 | 2.053388090E-05 | 5.1227 |
| 5 | 1.541486 | 1.541454 | 1.000021 | 2.053388090E-05 | 4.7528 |
| 6 | 1.243610 | 1.243585 | 1.000021 | 2.053388090E-05 | 4.4414 |
| 7 | 1.028614 | 1.028593 | 1.000021 | 2.053388090E-05 | 4.1787 |
| 8 | 0.868139 | 0.868121 | 1.000021 | 2.053388090E-05 | 3.9548 |
| 9 | 0.744915 | 0.744900 | 1.000021 | 2.053388090E-05 | 3.7617 |
| 10 | 0.648010 | 0.647997 | 1.000021 | 2.053388090E-05 | 3.5934 |

Table 2 gives selected results using Eqs. (9) – (12).

We can see that the $\Lambda$CDM-calculated $d_\Lambda$ and Lorentz $d_L$ distances are identical to 20 ppm for any instant $z$. The miniscule error is itself constant out to nine decimal places. Such precise and accurate identification suggests that what we see through a telescope is Lorentz-contracted. To get the proper distance, we can use $d_\Lambda$ instead of $d_L$ in Eq. (9). Combining Eqs. (9) - (12) gives:

$$d = d_\Lambda \gamma = d_\Lambda \sqrt{\frac{1}{2}z^2 + z + 1} \qquad (13)$$

Equation (13) returns a surprising result: Proper distance between source and receptor reached a maximum at $z = 1.6$, when $d = 6$ Glyr. Luminous bodies at $z > 1.6$ were *actually closer* to us than they were at $z = 1.6$. They got even more close as $z$ rises further. This explains why high-$z$ stars and galaxies can appear brighter and larger, e.g. [14], than $\Lambda$CDM's prediction. See [15] for a summary (and alternate interpretation) of these observations. Received photon flux and observed angular distance are reduced (or enhanced) by an increase (or decrease) in proper distance upon emission. Stars and galaxies get larger and brighter, or smaller and fainter, depending on whether $\partial d / \partial z$ is negative or positive. However, intervening dust (e.g. [16]-[20]), and/or general relativistic lensing in overdense regions of the Universe [21] can affect observed size.

## 4.2 Lookback *vs.* cosmic redshift

Lookback $(t_0 - t)$ is elapsed cosmic time between source $(-t)$ and receptor $(t_0)$. In this subsection, author endeavors to show that lookback has monotonic variance with redshift. Stars at $z > 1.6$ may have grown ever closer to us, but they were also ever older.

Lookback approaches $t_\lambda$ only if the source is receding at $\beta << 1$. When recession is fast enough, elapsed time at the receptor (us) drops in our rest frame, by $\gamma^{-1}$. Elapsed time at the source is given by Eq. (14):

$$(t_0 - t) = t_\lambda \gamma \tag{14}$$

Some readers may benefit from the following description. A source just sitting there can emit light pulses at a rate one per second. If the source moves away at $\beta = 0.998611$, the pulse rate we see drops sixtyfold, to one per minute. It doesn't matter how far away the source was, is, or will be. As long as it recedes at that same $\beta$, the pulse rate will be one per minute. In the source's rest frame, we're the ones who have slowed down. We know that our light pulses are one second apart, so we have to speed things up on the other end to get an accurate calculation of elapsed source time since emission. Equation (14) does this. Cosmic time comoves at about the same rate in either frame. If the source is another Earth mass, then time dilation due to general relativistic $\gamma_\varepsilon$ is identical in both frames, and their elapsed cosmic times converge within the flat perfect fluid.

For a source with constant $\beta$, Eq. (14) applies to any proper $d$. The author believes that Eq. (14) remains an accurate description of elapsed time at the source with variant $\beta$ as well. Once that photon is emitted, any later change in the source's $\beta$ doesn't change the photon's $t_\lambda = d/c$. Change in $\beta$ would affect the source's age *after* emission, but we are only interested in its instant age. Author contends that lookback time to the source can be expressed as an instant product, and Eq. (14) applies to any instant $z$. In the texts and literature, line integrations $\iint H/H_0\, \partial z \partial d$ and $\iint H/H_0\, \partial z \partial t$ are used to adjust for the purported effect of the expansion of space on $d$ and $(t_0 - t)$ respectively. Space does not expand in the light frame, nor does $d$. When $\gamma$ is factored into $t_\lambda$, numeric $\iint H/H_0\, \partial z \partial t$ simply returns the same $(t_0 - t)$ as the instant Eq. (14). Line integration may still be required to estimate present-day source distance $d_0$, and *is* required for the effect of $\gamma_\varepsilon$ on elapsed cosmic time in the very early Universe (section 5).

Combining Eqs. (1), (13), and (14) gives:

$$(t_0 - t) = \frac{d_A \gamma^2}{c} = \frac{d_A}{c}\left(\frac{1}{2}z^2 + z + 1\right) \tag{15}$$

Figure 2 shows the range of Eqs. (7), (8), (9), and (15) for $z = 0 \rightarrow 20$. At $z = 5$, Eq. (15) gives a ΛCDM 'universe' 14.6 Gyr older than today. At $z = 20$, it's 27.5 Gyr. At $z = 1089$, those photons from the CMB reached us after only 0.317 Gyr, but baryonic rest mass back then was ≈173 Gyr older than that. The 173 Gyr value uses $H_0 = 73$ km s$^{-1}$ Mpc$^{-1}$ [7]. Using $H_0 = 68$ instead [12], Eq. (15) gives $(t_0 - t)_{1089} = 186$ Gyr. Baryons at that time can be consistently described as

having been isotropic, un-ionized, and monatomic [9]. Equation (15) allows plenty of time afterward for galaxy evolution.



Figure 2. Lorentz wall, observed and proper distances, and lookback vs. redshift
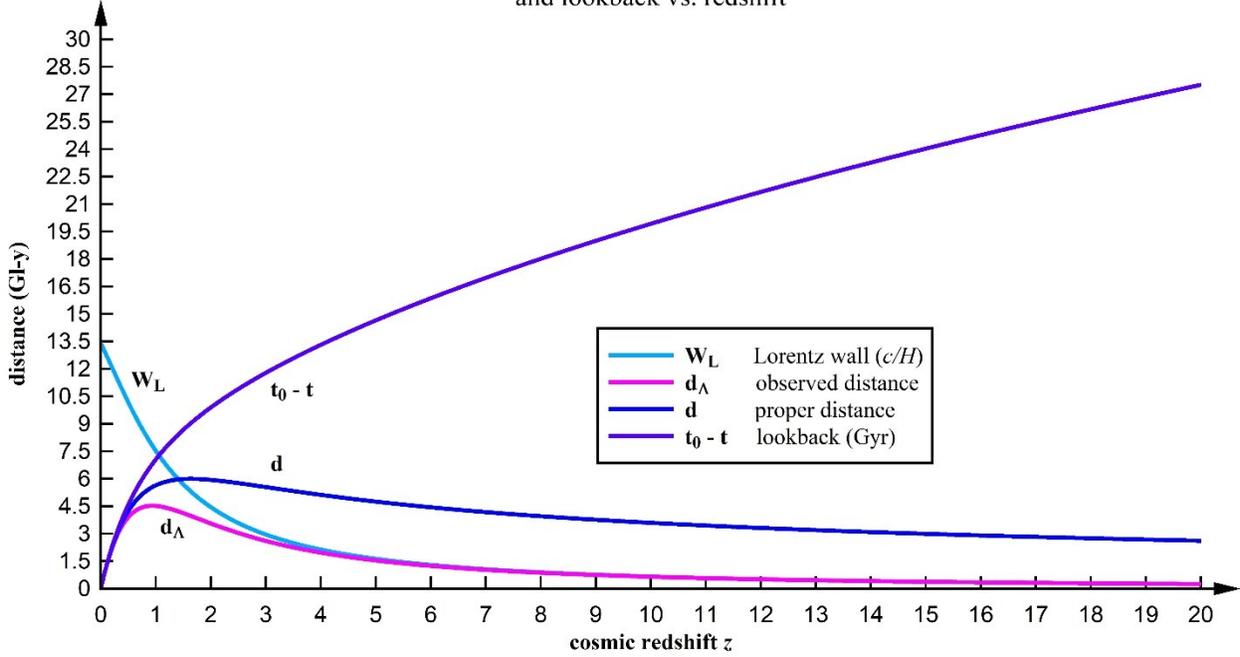
When Eq. (15) is multiplied by *c*, we get the *static distance*, which would be the proper distance in a noncomoving or static 'universe'. The Universe comoves, so static distance is unphysical.

## 5 THE DENSITY FRAME OF REFERENCE IN A FLAT PERFECT FLUID

The density frame $\gamma_\varepsilon$ is governed by energy density $\varepsilon$. One expression of $\gamma_\varepsilon$ is for a spherical mass $M$ in an otherwise empty 'universe':

$$\frac{d_{t_2}}{d_{t_1}} = \frac{\gamma_{\varepsilon_2}}{\gamma_{\varepsilon_1}} = \frac{\left(1 - \frac{2GM}{r_2 c^2}\right)^{\frac{1}{2}}}{\left(1 - \frac{2GM}{r_1 c^2}\right)^{\frac{1}{2}}} \tag{20}$$

Where $d_{t_2}/d_{t_1}$ is the relative progress of time (i.e., dilation), $r$ is proper distance $d$ from the sphere's center, and $G$ is the gravitational constant. For example, using Earth mass $M_E = 5.972 * 10^{24}$ kg and radius $r_E = 6378$ km, $\gamma_{\varepsilon_E} = 0.999999999305$.

For any $M$, when $r_1 \to r_2$, $\gamma_{\varepsilon_1} \to \gamma_{\varepsilon_2}$, so if we disperse a dense $M$ out into a sphere having a satellite's (circular) radius, gravitational time dilation of its rest mass is identical to the satellite's.

Newton's shell theorem allows us to use an 'empty universe' model for a flat perfect fluid. A sphere of $M$ could be anywhere or everywhere in the fluid.

We introduce the scale factor $a$:

$$a = \frac{1}{z+1} = \frac{r_2}{r_1} \tag{21}$$

Inserting Eq. (21) into Eq. (20) and rearranging gives:

$$\frac{d_t}{d_{t_0}} = \frac{\gamma_\varepsilon}{\gamma_{\varepsilon_0}} = \frac{\left(1 - \frac{2GM(z+1)}{r_1 c^2}\right)^{\frac{1}{2}}}{\left(1 - \frac{2GM}{r_1 c^2}\right)^{\frac{1}{2}}} \tag{22}$$

Where $r_1$ is the radius of arbitrary mass $M$ today $(t_0)$.



Figure 3. The 'density wall' of time
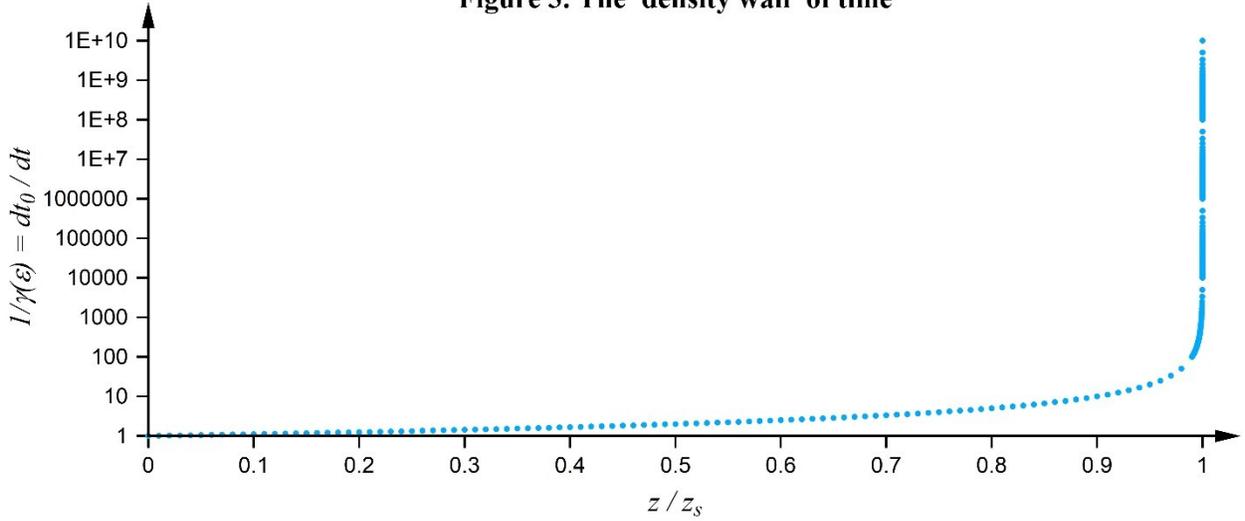


Figure 4. The physical basis of inflation
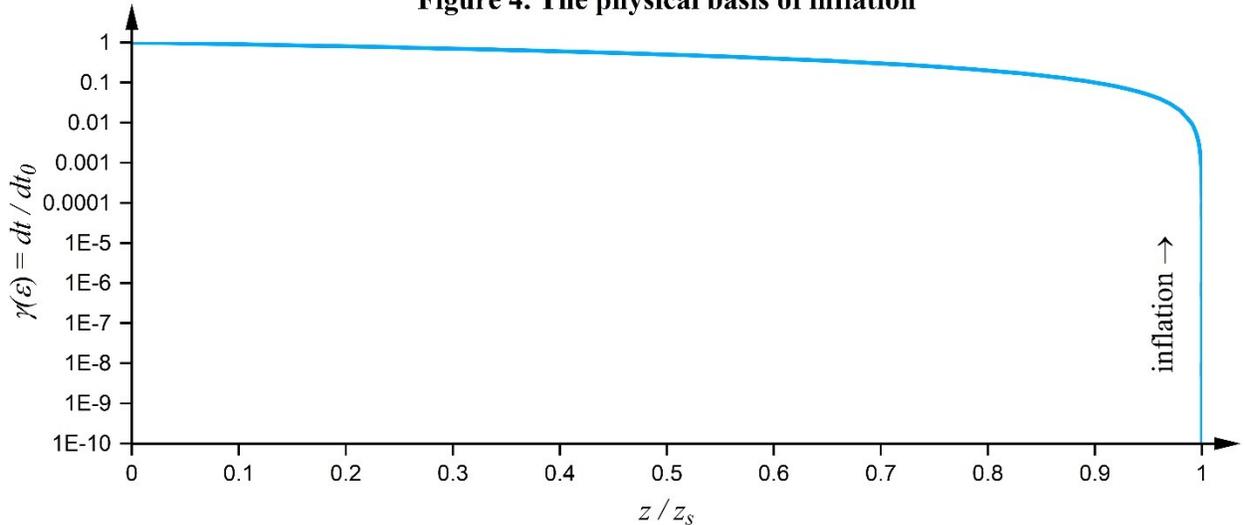
Equation (22) does not appear to account for radial contraction of $d$ in FLRW (Friedmann-

Lemaître-Robertson-Walker) space due to $\gamma_\varepsilon$. Author suggests *ad hoc* that squaring Eq. (22) is appropriate:

$$\frac{d_t}{d_{t_0}} = \left(\frac{\gamma_\varepsilon}{\gamma_{\varepsilon_0}}\right)^n = \frac{1 - \frac{2GM(z+1)}{r_1 c^2}}{1 - \frac{2GM}{r_1 c^2}} \quad \text{(for n = 2)} \tag{23}$$

Equations (22) and (23) have the same upper boundary $z_s \geq z$. This $z_s$ is the *redshift of singularity*. At $z_s$, time came to a halt, as first alluded to by Georges Lemaître [9][22]. This finite $z_s$ means that 'singularity' has a finite value of $\varepsilon$. One might suppose that time still elapses at $z_s$, but that would translate into uncountable trillions of years in our density frame. At $z > z_s$, the arrow of time reverses. This reversal appears to be inconsistent with the second law of thermodynamics.

For a rest-mass-only 'universe', $z_s$ is given by Eq. (24):

$$z_s = \frac{r_1 c^2}{2GM} - 1 \tag{24}$$

Using Earth's $r_E$ and $M_E$, $z_s = 7.2 * 10^8$. For $M = M_E / 10^{10}$, $z_s = 7.2 * 10^{18}$.

Figure 3 gives the reader a general idea of how much faster time progresses in our density frame $\gamma_{\varepsilon_0}$ as $z \to z_s$. The redshift of singularity $z_s$ gives a 'density wall' of infinite $t$, analogous to $W_L$'s infinite $d$, but with $1/\gamma_\varepsilon$ as the range instead of $\gamma_\beta$. When Figure 3's range is inverted $(1/\gamma_\varepsilon \to \gamma_\varepsilon)$, cosmic time $\gamma_\varepsilon / \gamma_{\varepsilon_0}$ plunges to zero. Figure 4 graphically illustrates this, the physical basis of inflation [23].

Equations (23) and (24) only apply to a 'universe' containing rest mass, at rest, exclusively. Instant $\varepsilon$ is less simple. Fortunately, $\gamma_\varepsilon$ has minimal effect in the observable Universe [21], so the instant Eqs. (13) and (15) still apply. In the early Universe, however, $\gamma_\varepsilon$ was significant as $\varepsilon$ became large:

$$\varepsilon = \left(\sum \sqrt{M^2 c^4 + p^2 c^2} + \sum hc/\lambda\right)/V \tag{25}$$

Term $p$ is momentum, $h$ is Planck's constant, $\lambda$ is photon wavelength, and $V$ is proper volume. 'Dark energy' $\Lambda$ is excluded from Eq. (25), as it is negligible for all $z > 1089 \to z_s$. Equation (15) suggests that the Universe is at least 170 Gyr old. Just how much older is an open question, expressed by Eq. (26):

$$(t_0 - t) = \int_0^{z_s} \iiint_0^{z_s} \frac{\partial\left[(H)(\gamma_\beta)(\gamma_\varepsilon)dt\right]}{H_0 \partial z^4} \partial z^4 \tag{26}$$

## 6 OTHER COMMENTS

The present paper connects observable numbers only to $[x, y, z, t]$ metrics. It does not say why the Universe expands in the first place. In another paper [24], the present author shows $H$ as arising from *entropic pressure*. Entropic pressure in this 'GCDM' model is a thermodynamic function of unaccreted $M$ and $p$. We sit at the 'center' of what is essentially an exploded gas bomb with initial radius $W_L$. Mass and momentum are more simply described now than at singularity, when today's stable baryon, electron, neutrino, and non-neutrino or 'dark' rest masses are believed to have comprised a quark-gluon (-electron-neutrino-dark-photon-Higgs) plasma, a long-studied phase of matter-energy [25][26] whose particles' relative movement was close to the speed of light. Author's position is that these particles' near-singular kinetic equilibria and other properties are still imperfectly described, in particular with respect to the entropy of formation of free neutrinos. This neutrino entropy must be found if they can be conclusively excluded as a candidate for cold dark matter.

## 7 CONCLUSION

The present paper gives proper or 'lookback' distance $(d)$ and time $(t_0 - t)$ as instant products in the observable Universe. Both are calculable without resort to integration along $z$. This paper's findings do not obviate a model's dependence of $H$ on aggregate $d_{obs}$. They do, however, provide needed corrections for estimates of $d$ and $(t_0 - t)$. These corrected estimates will provide a better understanding of the Universe around us.

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LEGENDS

TABLE 1: Instant Lorentz and proper distances vs. $\beta$, for selected $z$ and $\Lambda$CDM $H$ values.

TABLE 2.  $\Lambda$CDM distance, Lorentz distance, tension, tension error, and proper distance.

Figure 1. The Lorentz wall $W_L$.

Figure 2. Lorentz wall, observed and proper distances, and lookback vs. cosmic redshift.

Figure 3. The 'density wall' of time.

Figure 4. The physical basis of inflation.