

Formulas of the Fine-structure Constant and the Speed of Light in Atomic Units Based on ^{137}Ba in Terms of $137=56+81$

Gang Chen[†], Tianman Chen, Tianyi Chen

[†]Correspondence to: gang137.chen@connect.polyu.hk

Abstract

In our previous papers, we gave the formulas of the fine-structure constant and the speed of light in atomic units based on 2π -e formula and the natural end of the elements, i.e., the 112th element Cn^* , which was determined according to our previous theories. In this paper, based on the nuclide of ^{137}Ba with the proton number of 56, the neutron number of 81 and the total nucleon number of 137, we construct new formulas of the fine-structure constant and the speed of light in atomic units in terms of $137=56+81$ in which 56 is the most stable number in atomic nucleus according to our theories. By the way we also give these formulas based on $^{224}\text{Fr}^*$.

Keywords: the fine-structure constant, the speed of light in atomic units, ^{137}Ba , $^{224}\text{Fr}^*$, $137=56+81$, $137=224-87$.

1. Definitions of the Fine-structure Constant

The fine-structure constant (α) is a centennial mystery of physics. It was introduced by Arnold Sommerfeld in 1916, it has three definitions as follows.

$$\alpha = \frac{\lambda_e}{2\pi a_0}, \quad \alpha = \frac{2\pi r_e}{\lambda_e}, \quad \alpha = \frac{v_e}{c} = \frac{e^2}{4\pi\epsilon_0\hbar c}$$
$$\alpha = \frac{1}{137.035999\dots} \approx \frac{1}{137.036} \approx \frac{1}{137}$$

We suppose these three definitions correspond to α_1 , α_2 and α_c respectively.

2. Formulas of the Fine-structure Constant and the Speed of Light in Atomic Units Based on 2π -e Formula and the 112th element Cn^*

In our previous papers, we gave the formulas of the fine-structure constant and the speed of light in atomic units based on 2π -e formula and the natural end of the elements, i.e., the 112th element Cn^* , which was determined according to our theories

[1-14]. They are listed as follows.

$2\pi - e$ formula:

$$2\pi = \left(\frac{e}{e^{\gamma_c}}\right)^2 = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots$$

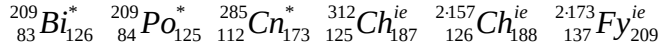
$$(2\pi)_{Chen-k} = \left(\frac{e}{e^{\gamma_{c-k}}}\right)^2 = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \dots \frac{e^2}{\left(\frac{k+1}{k}\right)^{2k+1}}$$

Formulas of the fine-structure constant:

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7(2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}}$$

$$= 1/137.03599903741537918851722952874$$

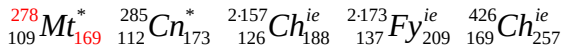
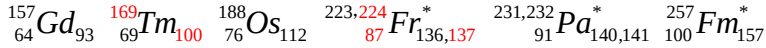
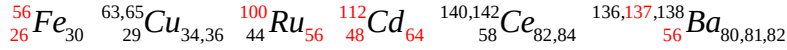
Relationships with nuclides:



$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13(2\pi)_{Chen-278}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}}$$

$$= 1/137.03599911187296275811920947793$$

Relationships with nuclides:



$$c_{au} = \frac{c}{v_e} = \frac{4\pi\epsilon_0\hbar c}{e^2} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1\alpha_2}}$$

$$= \sqrt{112(168 - \frac{1}{3} + \frac{1}{4 \cdot 141} - \frac{1}{14 \cdot 112(2 \cdot 173 + 1) + 13 + \frac{7}{72}})}$$

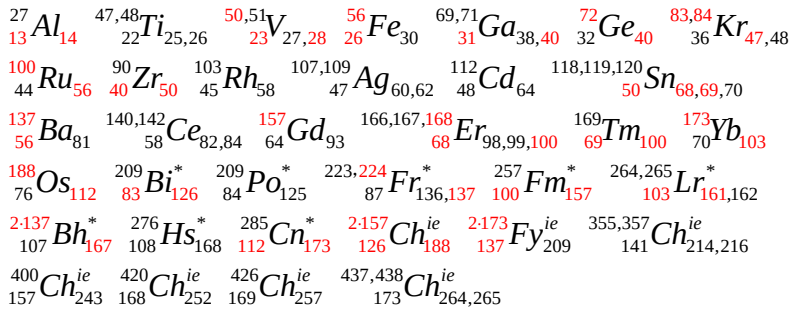
$$= 2 \sqrt{56(83 + \frac{157}{188} - (\frac{1}{8 \cdot 141} + \frac{1}{56^2(2 \cdot 173 + 1) + 26 + \frac{7}{36}}))}$$

$$= 137.035999074644171$$

$$c_{au} = \sqrt{112(168 - \frac{1}{3} + \frac{1}{4 \cdot 141} - \frac{1}{14 \cdot 112(2 \cdot 173 + 1) + 13 + \frac{7}{72 + \delta_c}})}$$

$$\begin{aligned}
&= 2 \sqrt{56(83 + \frac{157}{188} - (\frac{1}{8 \cdot 141} + \frac{1}{56^2(2 \cdot 173 + 1) + 26 + \frac{7}{36 + \delta_c / 2}}))} \\
&= 137.03599907464417096826121642708 \\
\delta_c &= \frac{1}{50 - \frac{1}{40 + \frac{1}{23 - \frac{1}{103 + \frac{7}{31}}}}}
\end{aligned}$$

Relationships with nuclides:



Among them, the last formula of c_{au} is somewhat new with δ_c for more precision.

3. Formulas of the Fine-structure Constant and the Speed of Light in Atomic Units Based on ¹³⁷Ba in Terms of 137=56+81

According to our theories [1], we supposed the fine-structure constant should be the functions of the proton number (Z), the neutron number (N) and the total nucleon number (A) of nuclides. So based on the nuclide of ¹³⁷Ba with Z=56, N=81 and A=137 which is the only nuclide with the total nucleon number of 137 among all primordial nuclides, we construct new formulas of the fine-structure constant and the speed of light in atomic units in terms of 137=56+81 as follows. It is worth noting that 56 is the most stable number in atomic nucleus according to our theories [15].

$$\begin{aligned}
\alpha_1 &= \frac{1}{56 + 81 + \frac{1}{28 - \frac{2}{9}} - \frac{1}{9 \cdot 5 \cdot 14 \cdot 17 \cdot 97 - \frac{8}{27}}} \\
&= \frac{1}{56 + 81 + \frac{1}{27 + \frac{7}{9}} - \frac{1}{4 \cdot 27(4 \cdot 5 \cdot 13 \cdot 37 - 1) + 18 - \frac{8}{27}}} \\
&= 1/137.035999037415379
\end{aligned}$$

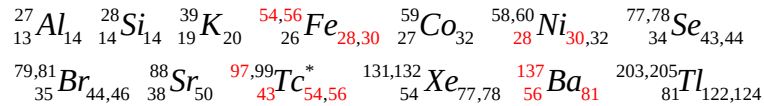
$$\alpha_1 = \frac{1}{56+81+\frac{1}{28-\frac{2}{9}}-\frac{1}{9\cdot 5\cdot 14\cdot 17\cdot 97-\frac{8}{27+\delta_1}}}$$

$$= \frac{1}{56+81+\frac{1}{27+\frac{7}{9}}-\frac{1}{4\cdot 27(4\cdot 5\cdot 13\cdot 37-1)+18-\frac{8}{27+\delta_1}}}$$

$$= 1/137.03599903741537918851722952874$$

$$\delta_1 = \frac{1}{56+\frac{1}{5-\frac{1}{43-\frac{19}{88-\frac{1}{9\cdot 13}}}}}$$

Relationships with nuclides:



$$\alpha_2 = \frac{1}{56+81+\frac{1}{28-\frac{2}{9}}-\frac{1}{28(4\cdot 9(4\cdot 9\cdot 31+1)+1)+1+\frac{1}{22}}}$$

$$= \frac{1}{56+81+\frac{1}{27+\frac{7}{9}}-\frac{1}{2\cdot 27\cdot 29(16\cdot 9\cdot 5-1)+10+1+\frac{1}{22}}}$$

$$= 1/137.035999111872963$$

$$\alpha_2 = \frac{1}{56+81+\frac{1}{28-\frac{2}{9}}-\frac{1}{28(4\cdot 9(4\cdot 9\cdot 31+1)+1)+1+\frac{1}{22-\delta_2}}}$$

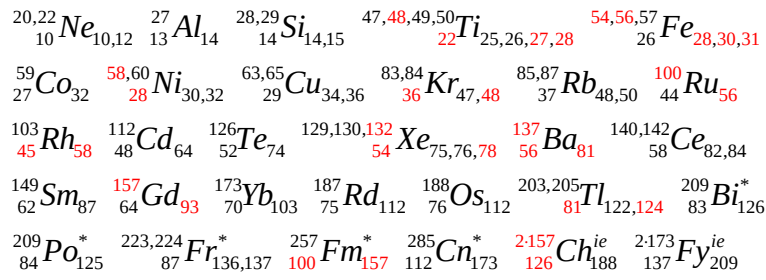
$$= \frac{1}{56+81+\frac{1}{27+\frac{7}{9}}-\frac{1}{2\cdot 27\cdot 29(16\cdot 9\cdot 5-1)+10+1+\frac{1}{22-\delta_2}}}$$

$$= 1/137.03599911187296275811920947793$$

$$\delta_2 = \frac{1}{13-\frac{5}{62-\frac{5}{132-\frac{2}{157-\frac{11}{39}}}}}$$

4

Relationships with nuclides:

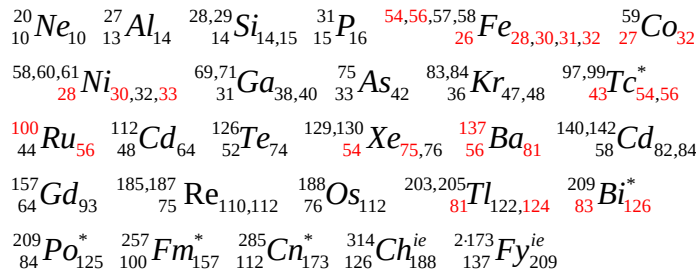


$$\begin{aligned}
 c_{au} &= 56 + 81 + \frac{1}{28 - \frac{2}{9}} - \frac{1}{3 \cdot 5 \cdot 28 \cdot 31 \cdot 83 + 5} + \frac{10}{27} \\
 &= 56 + 81 + \frac{1}{27 + \frac{7}{9}} - \frac{1}{25 \cdot 27(64 \cdot 25 + 1) - 10} + \frac{10}{27} \\
 &= 137.035999074644171
 \end{aligned}$$

$$\begin{aligned}
 c_{au} &= 56 + 81 + \frac{1}{28 - \frac{2}{9}} - \frac{1}{3 \cdot 5 \cdot 28 \cdot 31 \cdot 83 + 5} + \frac{10}{27 - \delta_c} \\
 &= 56 + 81 + \frac{1}{27 + \frac{7}{9}} - \frac{1}{25 \cdot 27(64 \cdot 25 + 1) - 10} + \frac{10}{27 - \delta_c} \\
 &= 137.03599907464417096826121642708
 \end{aligned}$$

$$\delta_c = \frac{1}{43 + \frac{1}{33 - \frac{1}{13 - \frac{1}{33 - \frac{1}{13 - \frac{19}{4 \cdot 13}}}}}$$

Relationships with nuclides:



Note: $28 = 56 / 2$, $81 = (3 / 2)54$, $27 = 54 / 2$,

$112 = 2 \times 56$, $224 = 4 \times 56$, $2 \times 2 \times 9 = 36$, $2 \times 7 \times 9 = 126$

In the above formulas of α_1 , α_2 and c_{au} , the characteristic factors 28 and 27 appear

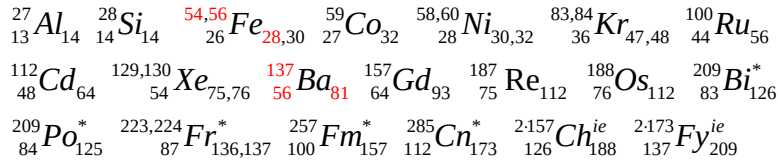
in each term of their polynomials, this demonstrates that α_1 , α_2 and c_{au} are functions of 56 and 81 which are the proton number and the neutron number of the nuclide ^{137}Ba .

The above formulas of α_1 , α_2 and c_{au} are complicated with many factors in the terms of the polynomials, however, the first three terms of the polynomials are the same and simple, so they could be simplified as follows.

$$\alpha = \frac{1}{56 + 81 + \frac{1}{28 - \frac{2}{9}}} = \frac{1}{56 + 81 + \frac{1}{27 + \frac{7}{9}}} = 1/137.036$$

$$c_{au} = \frac{1}{\alpha_c} = 56 + 81 + \frac{1}{28 - \frac{2}{9}} = 56 + 81 + \frac{1}{27 + \frac{7}{9}} = 137.036$$

Relationships with nuclides:



Note: $28 = 56 / 2$, $81 = (3 / 2)54$, $27 = 54 / 2$, $112 = 2 \times 56$,
 $224 = 4 \times 56$, $2 \times 2 \times 9 = 36$, $2 \times 7 \times 9 = 126$

These should be the most concise formulas of α and c_{au} but still meaningful.

4. Formulas of the Fine-structure Constant and the Speed of Light in Atomic Units Based on $^{224}\text{Fr}^*$ in Terms of $137=224-87$

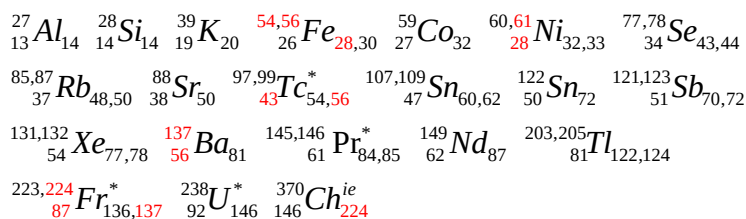
Based on the nuclide of $^{224}\text{Fr}^*$ with $Z=87$, $N=137$ and $A=224$, we construct new formulas of the fine-structure constant and the speed of light in atomic units in terms of $137=224-87$ as follows.

$$\alpha_1 = \frac{1}{224 - 87 + \frac{1}{28 - \frac{2}{9}} - \frac{1}{87(2 \cdot 7(2 \cdot 7 \cdot 61 - 1) - 1) - 3 - \frac{8}{27 + \delta_1}}}$$

$$= 1/137.03599903741537918851722952874$$

$$\delta_1 = \frac{1}{56 + \frac{1}{5 - \frac{1}{43 - \frac{19}{88 - \frac{1}{9 \cdot 13}}}}}$$

Relationships with nuclides:

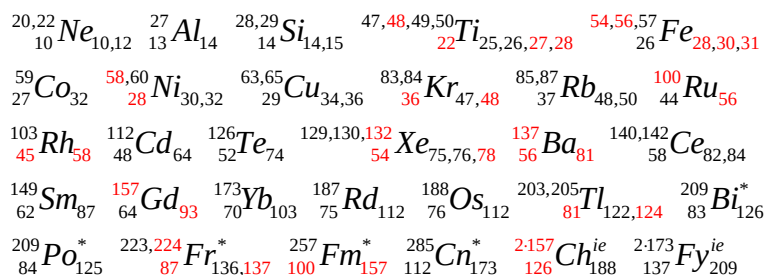


$$\alpha_2 = \frac{1}{224 - 87 + \frac{1}{28 - \frac{2}{9}} - \frac{1}{2 \cdot 9 \cdot 87(16 \cdot 9 \cdot 5 - 1) + 10 + 1 + \frac{1}{22 - \delta_2}}}$$

$$= 1/137.03599911187296275811920947793$$

$$\delta_2 = \frac{1}{13 - \frac{5}{62 - \frac{5}{132 - \frac{2}{157 - \frac{11}{39}}}}}$$

Relationships with nuclides:

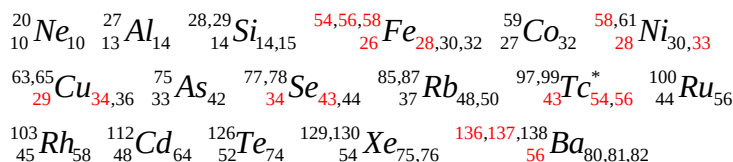


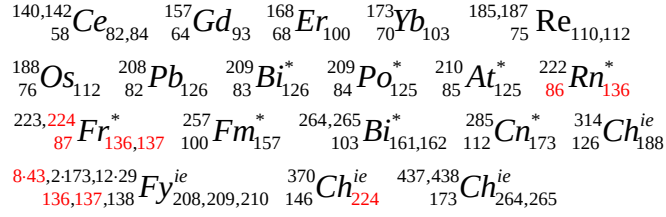
$$c_{au} = 224 - 87 + \frac{1}{28 - \frac{2}{9}} - \frac{1}{16 \cdot 17 \cdot 29 \cdot 137 + 9 + \frac{10}{27 - \delta_c}}$$

$$= 137.03599907464417096826121642708$$

$$\delta_c = \frac{1}{43 + \frac{2}{33 - \frac{1}{13 - \frac{1}{33 - \frac{1}{13 - \frac{19}{4 \cdot 13}}}}}$$

Relationships with nuclides:





It is worth noting that $224=4\times56=8\times28$ and $87=3\times29$. In the above formulas of α_1 , α_2 and c_{au} , the characteristic factors 224, 56, 28, 87 and 29 appear in each term of the polynomials, this demonstrates that α_1 , α_2 and c_{au} are functions of 224 and 87 which are the total nucleon number and the proton number of the nuclide $^{224}\text{Fr}^*$.

5. Some Supplement Formulas of the Fine-structure Constant and the Speed of Light in Atomic Units

The following are some supplement formulas of the fine-structure constant and the speed of light in atomic units based on ^{56}Ba in terms of $137=56+81$, especially having calculated values with 22 digits.

$$\begin{aligned}
\alpha_1 &= \frac{36}{7(2\pi)_{\text{Chen-112}}} \frac{1}{112 + \frac{1}{75^2}} \\
&= 1/137.0359990374153791885
\end{aligned}$$

$$\begin{aligned}
\alpha_2 &= \frac{13(2\pi)_{\text{Chen-278}}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} \\
&= 1/137.0359991118729627581
\end{aligned}$$

$$\begin{aligned}
\alpha_1 &= \frac{1}{56+81+\frac{1}{28-\frac{2}{9}}-\frac{1}{9\cdot5\cdot14\cdot17\cdot97-\frac{8}{27}+\frac{1}{28\cdot3\cdot61}}} \\
&= 1/137.0359990374153791885
\end{aligned}$$

$$\begin{aligned}
\alpha_1 &= \frac{1}{56+81+\frac{1}{28-\frac{2}{9}}-\frac{1}{9\cdot5\cdot14\cdot17\cdot97-\frac{8}{27+\frac{1}{56+\frac{1}{5}}}}} \\
&= 1/137.0359990374153791885
\end{aligned}$$

$$\alpha_2 = \frac{1}{56+81+\frac{1}{28-\frac{2}{9}}-\frac{1}{28(4\cdot 9(4\cdot 9\cdot 31+1)+1)+\frac{23}{22}+\frac{1}{5\cdot 14\cdot 89}}}$$

$$= 1/137.0359991118729627581$$

$$\alpha_2 = \frac{1}{56+81+\frac{1}{28-\frac{2}{9}}-\frac{1}{28(4\cdot 9(4\cdot 9\cdot 31+1)+1)+\frac{23}{22-\frac{7}{3(4\cdot 173-1)}}}}$$

$$= 1/137.0359991118729627581$$

$$c_{au} = \sqrt{112(168-\frac{1}{3}+\frac{1}{4\cdot 3\cdot 47}-\frac{1}{14\cdot 112(2\cdot 173+1)+13+\frac{7}{72+1/50}})}$$

$$= 2\sqrt{56(83+\frac{157}{188}-(\frac{1}{8\cdot 3\cdot 47}+\frac{1}{56^2(2\cdot 173+1)+26+\frac{7}{36+1/100}}))}$$

$$= 137.0359990746441709683$$

$$c_{au} = 56+81+\frac{1}{28-\frac{2}{9}}-\frac{1}{3\cdot 5\cdot 28\cdot 31\cdot 83+5+\frac{10}{27}+\frac{1}{56^2}}$$

$$= 137.0359990746441709683$$

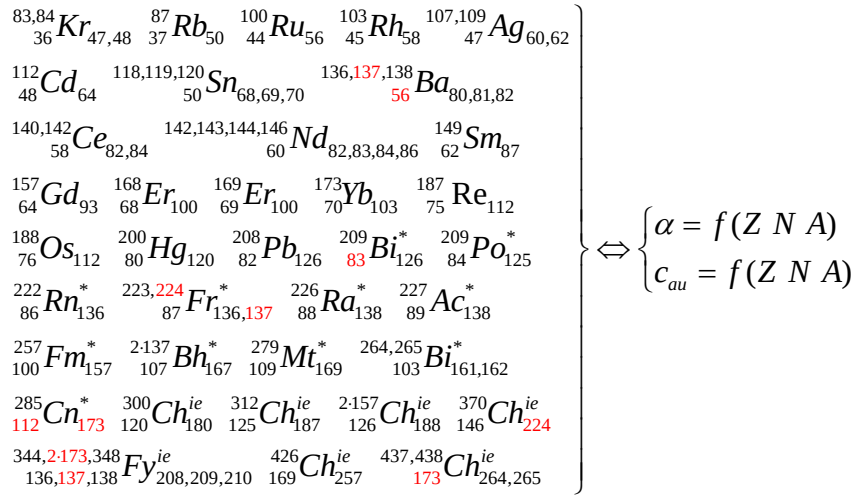
6. Discussion and Conclusion

In our previous papers, we constructed the formulas of the fine-structure constant and the speed of light in atomic units by transforming the natural end of elements, i.e., the 112th element Cn^* , to the Feynman ideal extended end of elements, i.e., the 137th element. In this paper, we construct them based on the 56th element's nuclide ^{137}Ba . The principle for these constructions is that α and c_{au} are supposed to be the functions of the proton number (Z), the neutron number (N) and the total nucleon number (A) of nuclides [1], and this can be illustrated as follows.

Concise:

$$\left. \begin{array}{l} {}^{136,137,138}_{56}\text{Ba}_{80,81,82} \quad {}^{209}_{83}\text{Bi}_{126}^* \quad {}^{209}_{84}\text{Po}_{125}^* \\ {}^{222}_{86}\text{Rn}_{136}^* \quad {}^{223,224}_{87}\text{Fr}_{136,137}^* \quad {}^{226}_{88}\text{Ra}_{138}^* \quad {}^{227}_{89}\text{Ac}_{138}^* \\ {}^{285}_{112}\text{Cn}_{173}^* \quad {}^{344,2\cdot 173,348}_{136,137,138}\text{Fy}_{208,209,210}^{ie} \end{array} \right\} \Leftrightarrow \begin{cases} \alpha = f(Z \ N \ A) \\ c_{au} = f(Z \ N \ A) \end{cases}$$

Detailed:



So the factors in these formulas are related to nuclides and hence the formulas possess physical meanings. The two kinds of formulas and the obtained values are consistent with each other, so we suppose that they should be reasonable and precise.

Referenc

1. E-preprint: vixra.org/abs/2002.0203
2. E-preprint: vixra.org/abs/2002.0020
3. E-preprint: vixra.org/abs/2012.0107
4. E-preprint: vixra.org/abs/2102.0162
5. E-preprint: vixra.org/abs/2103.0088
6. E-preprint: vixra.org/abs/2104.0053
7. E-preprint: vixra.org/abs/2106.0042
8. E-preprint: vixra.org/abs/2106.0151
9. E-preprint: vixra.org/abs/2308.0168
10. E-preprint: vixra.org/abs/2407.0038
11. E-preprint: vixra.org/abs/2409.0044
12. E-preprint: vixra.org/abs/2501.0003
13. E-preprint: vixra.org/abs/2501.0095
14. E-preprint: vixra.org/abs/2503.0172
15. E-preprint: vixra.org/abs/2312.0055