The game show aka Monty Hall problem

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abstract

The game show problem aka Monty Hall problem [1], originated when Craig Whitaker posed a question of a winning strategy for a 3 door game show to Marilyn Savant who wrote articles for Parade magazine.
Her 1990 response was to switch doors when given the option. [2] [3]
The debate of probability of success as 2/3 vs 1/2 has continued until today.
This paper reveals errors in her response.

Whitaker's question

Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say #1, and the host, who knows what's behind the doors, opens another door, say #3, which has a goat. He says to you, "Do you want to pick door #2?" Is it to your advantage to switch your choice of doors?

Marilyn Savant's 1990 response [3]

Yes; you should switch. The first door has a 1/3 chance of winning, but the second door has a 2/3 chance. Here's a good way to visualize what happened. Suppose there are a million doors, and you pick door #1. Then the host, who knows what's behind the doors and will always avoid the one with the prize, opens them all except door #777,777. You'd switch to that door pretty fast, wouldn't you?
The benefits of switching are readily proven by playing through the six games that exhaust all the possibilities. For the first three games, you choose #1 and "switch" each time, for the second three games, you choose #1 and "stay" each time, and the host always opens a loser. Here are the results.. For the first three games, you choose #1 and "switch" each time, for the second three games, you choose #1 and "stay" each time, and the host always opens a loser. Here are the results.

	DOOR 1	DOOR 2	DOOR 3	RESULT
GAME 1	AUTO	GOAT	GOAT	Switch and you lose.
GAME 2	GOAT	AUTO	GOAT	Switch and you win.
GAME 3	GOAT	GOAT	AUTO	Switch and you win.
GAME 4	AUTO	GOAT	GOAT	Stay and you win.
GAME 5	GOAT	AUTO	GOAT	Stay and you lose.
GAME 6	GOAT	GOAT	AUTO	Stay and you lose.

fig.1

probability

Probability is a substitute for lack of knowledge, in an abstract mathematical form, and is not intended to be taken literally. It is an average result of a large number of events. The measure of success is the ratio (event of interest)/(all possible events). For the game show the ratio is (guess a car door)/(all possible guesses). The prizes are in fixed locations for the duration of a game. The doors never contain partial prizes.

Marilyn Savant was a victim of 'gambler's fallacy'.[4]

"The fallacy leads to the incorrect notion that previous failures will create an increased probability of success on subsequent attempts."

game rules

The game rules as proposed by Whitaker.

rule 1. the host cannot open the door from the players 1st guess.

rule 2. the host cannot open a door containing a car.

rule 3. the host must offer the player a 2nd guess.

Initial conditions: the player does not know the location of the car, thus they can only make a random guess.

The host knows the location of the car, thus their choice is not random, acknowledged by Savant:

"So let's look at it again, remembering that the original answer defines certain conditions, the most significant of which is that the host always opens a losing door on purpose. (There's no way he can always open a losing door by chance!)"

Player always guesses door 1.

g=goat, c=car, red is host choice to open and remove from play

Savant's game

e	1	2	3	r
1	g	g	c	c
2	g	c	g	c
3	c	g	g	g/2
3	c	g	g	g/2

fig.2

When the car is behind door 1 (game/event 3) Savant considers the host opening (door 2 or door 3) in the same game. If all games have the same duration, game 3 results in 1 goat instead of 2 goats due to host's choices. Savant did not consider the logical difference of (door 2 or door 3) and (door 2 and door 3). If all 3 games occur with equal frequency, the switch column 'r' contains 1 less g, which results in a bias of 1/3 for stay vs 2/3 for switch. If the host opened door 1 to verify the player's guess, it would show the player win 1 game if staying and 2 games if switching, but that is not Whitaker's game.

errors

Her example of a million doors is total nonsense.
The set D of 1M doors contains 1 car, with probability=1. The probability of 1 is by convention distributed over all doors with a value of 1/M per door. That does not mean each door contains 1/M of a car. It means if the player guessed a different door for 1M games, they would win 1 game and lose M-1 games. The player's guesses and the host's choices do not alter the contents of the doors. More doors results in a prolonged and boring game.

e	1	2	3	r
1	g	g	c	c
2	g	c	g	c
3	c	g	g	g
4	c	g	g	g

fig.3

The 2 goats exist simultaneously, thus require separate identities, such as g1 and g2. Savant does not distinguish the 2 goats as individual prizes, but fails to explain how 1 goat can be in 2 different locations at the same time.
There is no violation of game rules if the host opens (door 2 and door 3) in separate games.
Even with a generic goat 'g', her list of 3 games is missing a 4th game.
When the car is behind door 1, the host can open door 2 or door 3, but not both. That would end the game, and deny the player a 2nd guess.
An additional game is required to open the 2nd door.
The games should have the same format: player 1st guess, host opens 1 door, player 2nd guess, host verifies win or lose of players 2nd guess.

Comparing door 1 with door r (remaining closed door), the player wins 2 games for stay or switch. There is no advantage.

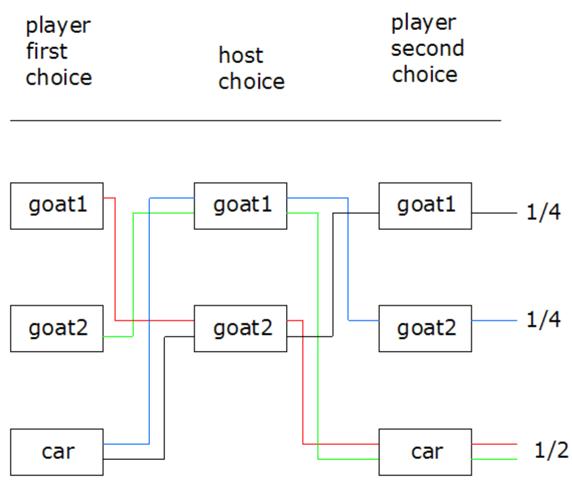

fig.4

The success ratio (guess a car door)/(all possible guesses) relates to guesses, NOT locations, as in fig.4. Each colored path is 1 of 4 possible games, with the player switching. The 1st column is the stay prizes. A comparison shows no advantage.

conclusion

The errors are due to Whitaker's choice of 2 goats vs 2 distinct prizes such as a goat and a dog as secondary prizes and Savant's lack of understanding simple probabilities.
Using the measure of success from 'probability',
the player doesn't win with their 1st guess with a 1 in 3 chance, since the host does not open that door to verify win or lose. When the player gets their 2nd guess, there are not 3 ways to choose 1 of 2 things. The host has benefited the player by increasing their chance from 1/3 to 1/2.
The host then verifies a win or lose by opening that door.
The answer to Whitaker's question is no.

reference

[1} The American Statistician, August 1975, Vol. 29, No. 3
[2] game show problem, Wikipedia Sep 2024

[3] Marilyn vos Savant,
https://web.archive.org/web/20130121183432/http://marilynvossavant.com
[4] Wikipedia, Gambler's fallacy, Apr 2025