The Periodic variation of Baryon masses as a function of their Magnetic Moments: Quantum interference in the femtometer scale of Hadrons .

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Abstract: The objective of this paper is to give full consideration to a revealing result which comes straight from *tabulated* data for all baryons of the octet and decuplet when these data are duly analyzed theoretically, namely: *The masses of spin ½ baryons, in proton mass-units, are simple periodic functions of their magnetic moments, in nuclear magneton-units.* As discussed here, this reveals the (overlooked) dynamic influence of quantum interference of closed currents inside the Particles, as observed in conventional superconducting rings confining magnetic flux.

keywords: Quantum interference, Regularization techniques, Casimir effect.

1) **Introduction, and discussion of the theory in previous work**.

In recent years the author has carried out investigations on the possible application of electrodynamics in the theoretical analysis of Particles properties, like in the determination of their masses, and their relation to magnetic flux confinement inside the Particle. This led to the association of the origin of Particles to a topological transition(to a current loop-state) starting from plane waves in an environment at 10^{13} K (possibly Big Bang conditions) [1,2]. In view of the amount of data available, the analysis has been concentrated on the baryons of the octet and decuplet.

Quantitative agreement between models and data has been achieved adopting a relativistic circular ring of currents model for the baryons[1], in which is included the concept (introduced by Asim Barut in the 1970s [3]) that a proton-state(with mass m_p)surrounded by a cloud of mesons, neutrinos, and electrons is taken as a fundamental element present in all baryons. It is inevitable to notice the potential similarity of this problem to the one of determining the dynamic properties of currents flowing around a conventional superconductor ring, which is confirmed by this investigation.

The ring model is explicitly developed to represent spin ½ particles, since elements of such quasi-proton can circulate in two opposite directions in the ring plane. Making reference to details available in previous publications[1], a Dirac equation is written for the ring-shaped distribution of charge that models a Particle. The propagation of momentum around the loop (of perimeter L) takes place by local elemental displacements, like in a vibrating string. Bohr-Sommerfeld quantum conditions impose a continuity of phase around the ring that introduces an infinite number of vibrating modes (indexed as k), given by the momenta $p_k = 2\pi\hbar k/L$. The fundamental state is obtained by summing up over these modes. This sum diverges, but the converging part of the solution can be isolated by applying a Regularization ("Reg" below) procedure that extracts the diverging parts, which are associated to the surrounding infinite environment[1]. The Dirac equation and its solution include a magnetic gauge field A , which introduces an amount of magnetic flux ϕ arrested inside the ring. The flux $\phi = A\,L$ is defined in numbers n of magnetic flux quanta $\phi_0 = hc/e$. The solutions of the Dirac hamiltonian provide energies which are associated with the rest energies Mc^2 of the baryons, through the expression(s $\rightarrow$ -1)[1]:

$$M\,c^2 = U_0 + \mathrm{Reg} \sum_k c\{(p_k + e\phi/Lc)^2 + m_p{}^2 c^2\}^{-S/2} \qquad (1)$$

for each baryon of mass M. Here U_0 is the parent state energy of the environment the loops originate from, and is obtained by comparing theory with the mass data for the baryons in the Tables. Equation (1) can be rewritten in dimensionless form. Here the dimensionless parameters adopted are $m' = m_p/m_0$, where $m_0 = 2\pi\hbar/cL$, and $u_0 = U_0/m_p c^2$:

$$M(n)/m_p = u_0 + (1/m')\mathrm{Reg} \sum_k \{(k+n)^2 + m'^2\}^{-s/2} \qquad (2)$$

The second term on the right of (2) corresponds to an internal correlation energy that turns the ring states energetically favorable as compared to the parent state, so that a condensation into ring form takes place. This would be the picture of ring-like Particles formation from a Parent state. The Regularization of the second term in (2) results in(s→ -1, and k + are the positive integers):

$$\mathrm{Reg} \sum_k \{(k+n)^2 + m'^2\}^{-s/2} =$$

$$= \frac{2\sqrt{\pi}}{\Gamma\left(\frac{1-s}{2}\right)} \left(\frac{\Gamma\left(-\frac{s}{2}\right)}{2m'^{-s}} + 2\pi^{-\frac{s}{2}} \sum_{k+} \left(\frac{k}{m'}\right)^{\frac{-s}{2}} K_{\frac{s}{2}}(2\pi m' k) \cos(2\pi k n + \delta) \right) \quad (3)$$

which must be multiplied by $(\pi^{\frac{2s-1}{2}} / \Gamma\left(\frac{s}{2}\right))\, \Gamma\left(\frac{1-s}{2}\right)$ and inserted in (2) to give the masses (see ref. 1 for details of the Regularization method). One immediately realizes that the $M(n)$ are periodic functions of n, where we have added a phase δ to allow breaks in phase continuity (see below). The presence of this phase mimics effects of topological changes in the currents path, as discussed later. Such phase may vary along the list of baryons. Therefore. it is predicted from (3) that the mass of spin ½ baryons should be periodical in n, provided no other effects modify this parameter. It must be pointed out that the Modified Bessel function K in (3) decays very fast with the argument and thus the only term of the sum that actually contributes is k= 1.

2) The inclusion of J=3/2 particles data in the analysis.

The data adopted in this analysis comes from Table 3.2 of ref[4] and is presented in Tables 1 and 2 (in the end of the paper).

Only the octet particles are spin J= ½ particles. One needs to obtain values of mass for the decuplet (J=3/2) particles in a spin ½ state, so that they can be compared to the octet data. Another issue to be resolved is the infuence of magnetic properties on mass. These properties are represented by n and μ, the magnetic moment

Firstly, in our previous publications the parameter n is defined for spin ½ particles through the equation[1]

$$n = (2c^2\alpha/e^3)\ \mu m. \quad (4)$$

which completely defines n from tabulated data, and where α is the fine-structure constant. In particular, one notices that m on the right side is proportional to the ratio n/μ, which contains the magnetic part of the mass in the form of a ratio. In principle, n and μ should be proportional to each other, since flux inside a loop of radius R is given by $2\pi\mu/R$. The radius should not change with spin, and thus magnetic effects on mass given by the n/μ ratio should not change for excited spin states like J= 3/2(that is, depending on spin, n and μ might have different values but would keep their proportionality ratio, and thus net magnetic effects essentially remain constant under spin variation). This is consitent with the expressions of mass obtained for both baryons and mesons by Corben[5] (J proportional to M^2 + constant), which are valid for particles with completely different magnetic properties. In addition, spin can be defined as a global property of Mass [6,7], which supplements its usual formal definition for puntual particles. It is thus consistent with Corben’s work that the increase in mass of the decuplet particles as compared with the octet particles can be attributed to the greater spin and not to the net changes in magnetic contribution in (4).

Values for n are relevant to this analysis. To obtain suitable values for n in the decuplet from eq.(4) and spin ½ we then proceeded as follows, imposing consistency with Corben´s work. Firstly we recognize from inspection of the tabulated magnetic moments that each of the Δ(including the nucleons N), Σ, and Ξ families of particles has in the decuplet 3/2 spin state a particle with a magnetic moment which is very close in absolute value to the one of a particle of same family in the octet ½ spin state. This indicates that hypothetical ½ spin state decuplet baryons will have about the same mass as the octet baryon particles of same family, and that the decuplet spin 3/2 magnetic moment may be attached to the octet particle provided n/μ remains unchanged, for consistency with Corben´s analysis. Therefore, from this reasoning one concludes that n is obtained for this state by inserting in (4) the octet particle mass and the respective tabulated spin 3/2 magnetic moment since this would keep the ratio n/μ unaltered by spin change.

This is clear for the Δ, Σ, and Ξ families of particles, but the decuplet Ω is a special case. It does not have a tabulated octet spin ½ version for comparison. Therefore, firstly mass for the hypothetical spin ½ state of the decuplet baryon Ω is obtained by subtracting from the spin 3/2 mass the overall averaged mass excess of all decuplet particles over the octet masses, which is 244 MeV/c^2 . Then, the value of n is determined as for the other particles.

The masses for the ½ spin versions of the decuplet Particles are called *transformed masses* m_t and shown in Table 2. Then, as said, values of n are obtained from eq (4) adopting the transformed masses to simulate spin ½ for decuplet particles, alongside the available values of μ for the decuplet, which we assume as good since values of n are chosen to keep the same n/μ ratios under spin change.

The Tables show that the obtained ratios n/μ are close to unit in dimensionless units for many Particles, which is the expected result for a circulating ring of current. However, the Tables also show there are "jumps" in the values of n towards integer values, something typical of the presence of weak-links in superconducting closed currents paths(see, for instance, [8]). Such weak-links are not considered in the perfect-ring circuit model calculations adopted, which resulted in the simple cosinusoidal expressions (1)-(3) with $\delta = 0$. For this reason we adopt the dimensionless μ in the analysis below, which might be considered the unperturbed version of the parameter n (cf. eq. 6 of ref.[6], taking $s = 1$), and adequate for comparison with the simple predictions of eq. (1)-(3). By proceeding this way the following plot reveals a relevant new result.

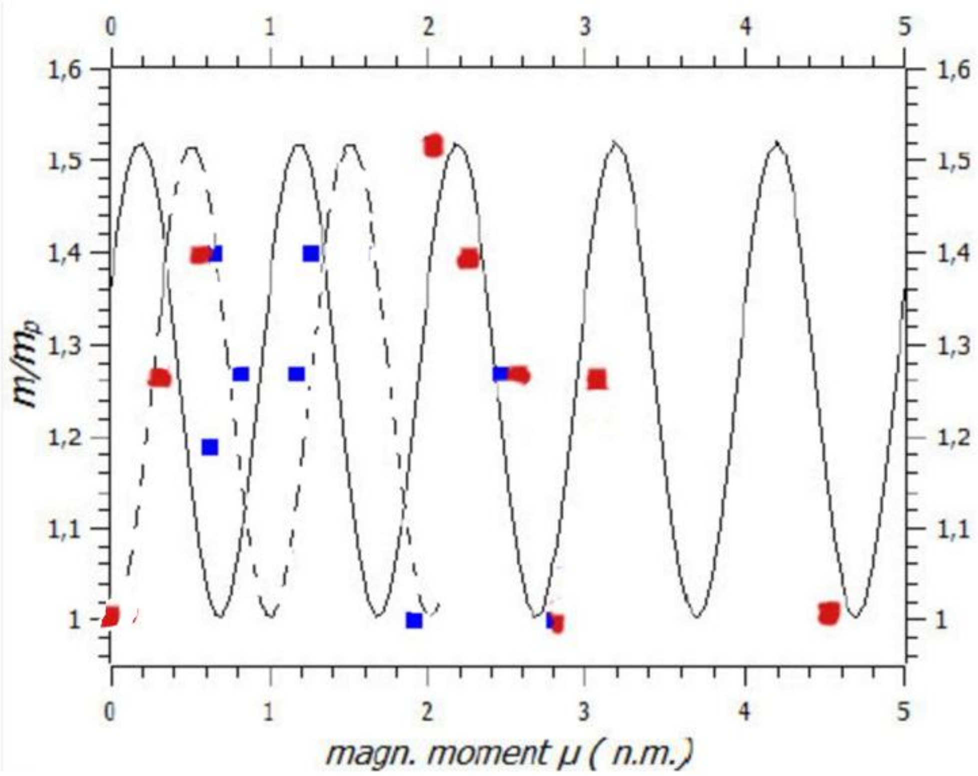

Figure 1: Plot of the ratio m/m_p from Tables 1(blue points) and 2 (red points, m_t adopted for Table 2 data) using the magnetic moments in the Tables in place of *n* in eq. (2-3). The phase-displaced cosine curves are the plot of eqs.(2)- (3) with μ in place of *n* (cf. ref. [6]). One notices the good fits obtained if two curves with δ=0 and 2π/3 are adopted in the fits, one (dashed) below and the other(solid) above μ=2.

Analysis and Conclusion.

Figure 1 clearly displays the periodic behavior of eqs. (2)-(3) in the fit of the data. The dashed line is a perfect cosine(δ =0) and fits data up to μ= 2. It is particularly interesting to note that calculations in [4], Table 3.2, predict the null magnetic moment of the Δ^0 particle, which perfectly agrees with the dashed δ=0 curve close to M/m_p =1(μ=0 red point on the Figure). This would be the only particle with null magnetic moment predicted by the present model, and the papers agree on this.

For magnetic moments μ between about 2 and 5 magnetons, the (solid) fit curve is a phase-displaced cosine, with δ=2π/3 approximately. At $\mu \approx 2$ there is a jump of the magnetic flux from eq.(4)(cf. n and Ω data), with essentially no increase in magnetic moment, which appears as a jump in mass in the Figure. In a rather detailed heuristic analysis, Post [9] associated this kind of effect to a closed winding path of the current superposed to the ring path. This would allow an increase of the confined magnetic flux with little effect on the magnetic moment, like in a trefoil-like current path [10]. Such change in the topology of currents might thus be estabilized by a decrease in the overall energy of the system despite a higher flux content.

In both curves in Fig. 1 the dimensionless parameters are $m' = 0.36$ and $u_0 = 2.5$ ($m' = m_p/m_0$, where $m_0 = 2\pi\hbar/cL$, and $u_0 = U_0/m_pc^2$.), which slightly correct numbers obtained in previous publications(a ½ factor is included here in the k=0 additive tern in (3) as compared to the expression in ref. [1]). This u_0 parameter corresponds to U_0= 2340 MeV, which is consistent with the peak position in the plot of cosmic rays protons flux analyzed in previous work[2].

The good results in this analysis reinforce Barut's proposal that the differences between these particles are not related to differences between inner point-like constituents, quite independently adding up their individual effects as point sources. Following Barut, an individual proton state and rest mass dominates, with mass modulated by interference of circulating clouds of charge, which manifest in terms of a magnetic moment of essentially orbital type (cf. [3], p. 127).

The periodic dependence of energy of currents (here determining the rest masses of baryons) as a function of trapped flux in a ring or tube-shaped superconductor was initially analyzed theoretically by Byers and Yang[11](cf. their Theorem 1), as a consequence of quantum interference of currents circulating around the trapped flux. The present work shows that existing tabulated data can be organized to display a similar periodic effect happening inside baryons. Both treatments have in common the imposition of gauge invariance and continuity of wavefunctions around the magnetic flux confined inside the structure.

In conclusion, this paper divulges the following (revealing) results, immediately obtainable from tabulated data: *the masses of spin ½ baryons, in proton mass-units, are simple periodic functions of their magnetic moments, in nuclear magneton-units*. Phase differences seem attributable to changes in the topology of currents, which would include closed winding paths as flux increases [9, 10].

Results are in accordance with predictions by Byers and Yang[11] for the periodic behavior of the energy of currents in multiply-connected superconductors trapping magnetic flux.

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Table 1: Baryon octet data(magnetic moments μ from ref. [4]). According to eq.(4) (gaussian units): n= 1.16x10^{47} μm. Theoretical values of μ in [4] are indicated. Other values are experimental

	abs μ (n.m.)	μ(erg/G) x 10^{23}	m(Mev/c^2)	m(g) x 10^{24}	n eq.(4)
p	2.79	1.41	939	1.67	2.73
n	1.91	0.965	939	1.67	1.9
Σ^+	2.46	1.24	1189	2.12	3
Σ^0	0.82(theor.)	0.414	1192	2.12	1
Σ^-	1.16	0.586	1197	2.12	1.5
Ξ^0	1.25	0.631	1314	2.34	1.7
Ξ^-	0.65	0.328	1321	2.34	0.9
Λ	0.61	0.308	1116	1.98	0.7

Table 2: Baryon decuplet data <u>in a spin ½ state</u> (magn. moments μ from ref. [4]). According to eq.(4): n= 1.16x10^{47} $\mu\, m_t$ (gaussian units). See text for the definition of m_t.

	abs μ (n.m.)	μ(erg/G) x 10^{23}	m_t(Mev/c^2) (**see text**)	m_t(g) x 10^{24}	n
Δ^{++}	4.52	2.28	939	1.67	4.4
Δ^{+}, Δ^{-} Δ^{0}	2.81,2.81 0(theor= t)	1.42	939	1.67 1.67	2.8 , 2.8
Σ^{+}	3.09 (t)	1.56	1189	2.12	3.8
Σ^{0}	0.27 (t)	0.136	1192	2.12	0.33
Σ^{-}	2.54 (t)	1.28	1197	2.12	3,1
Ξ^{0}	0.55 (t)	0.28	1314	2.34	0.76
Ξ^{-}	2.25 (t)	1.14	1321	2.34	3,1
Ω^{-}	2.02	1.02	1428	2.54	3