Turkish-English Dictionary and The Graphical Law

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Abstract

We study A Turkish-English Dictionary by H. C. Hony. We draw the natural logarithm of the number of head root words, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised. We find that the head root words underlie a magnetisation curve. The magnetisation curve i.e. the graph of the reduced magnetisation vs the reduced temperature is the exact Onsager solution of the two dimensional Ising model in the the absence of external magnetic field.

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letter	A	B	C	Ç	D	E	F	G	H	İ	I	J	K	L	M
number	1374	1188	432	591	991	609	636	704	1200	1091	113	10	2215	313	2850
letter	N	O	Ö	P	R	S	Ş	T	U	Ü	V	Y	Z		
number	559	252	179	545	387	1697	512	1944	256	160	413	900	463		

TABLE I. Turkish head root words: the first(third) row represents letters of the Turkish alphabet,[1], in the serial order.
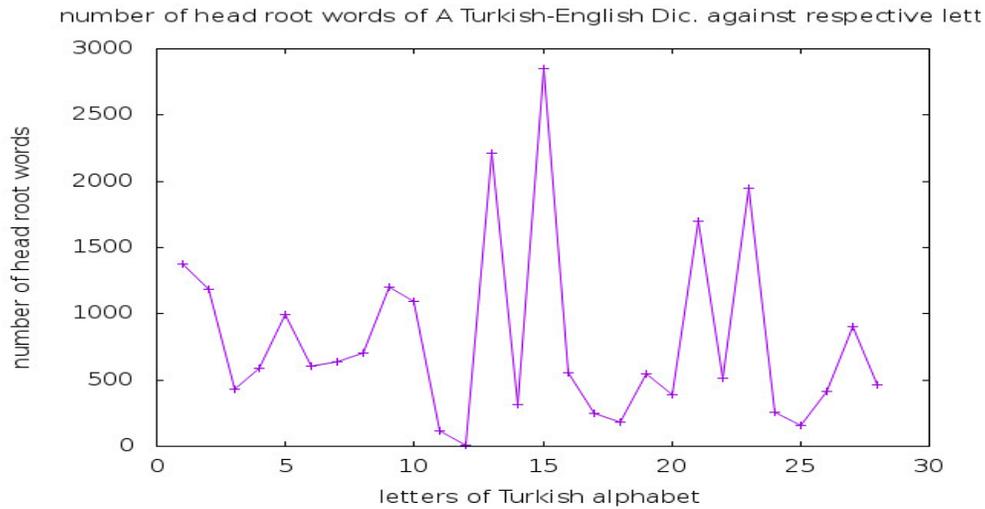

FIG. 1. The vertical axis is number of the head root words of A Turkish-English Dictionary, [1]. The horizontal axis is the letters of the Turkish alphabet. Letters are represented by the sequence number in the alphabet as it appears in the dictionary, [1].

I. INTRODUCTION

In this paper, we turn to A Turkish-English Dictionary by H.C.Hony, [1]. We go through the head root words. We count all the head root words of the dictionary,[1], one by one from the beginning to the end. The result is the table, tableI. To visualise we plot the number of head root words against the respective letters in the dictionary sequence, [1] , in the adjoining figure, fig.1.

Next we look for the graphical law. We have started considering magnetic field pattern in [2], in the languages we converse with. We have studied there, a set of natural languages, [2], and have found existence of a magnetisation curve under each language. We have termed this phenomenon as the Graphical Law. Then, we moved on to investigate, [3], into dictionaries

of five disciplines of knowledge and found the existence of a curve of magnetisation under each discipline. This was followed by finding of the graphical law in the references from [4] to [104].

The planning of the paper is as follows. In the next section, we describe the Graphical Law analysis of the head root words of A Turkish-English Dictionary, [1]. In the section III, we give an introduction to the standard curves of magnetisation of Ising model. The section IV is Acknowledgment. The last section is Bibliography.

II.   THE GRAPHICAL LAW ANALYSIS

For the purpose of exploring graphical law, we assort the letters according to the number of head root words, in the descending order, denoted by f and the respective rank, [114], denoted by k. k is a positive integer starting from one. Moreover, the minimum non-zero number of head root words is ten. Hence we attach a limiting head root word number one. The limiting rank is maximum rank plus one, here it is twenty nine. As a result both $\frac{lnf}{lnf_{max}}$ and $\frac{lnk}{lnk_{lim}}$ varies from zero to one. Then we tabulate in the adjoining table,II, and plot $\frac{lnf}{lnf_{max}}$ against $\frac{lnk}{lnk_{lim}}$ in the figure fig.2. We then ignore the letter with the highest number of head root words, tabulate in the adjoining table,II,and redo the plot, normalising the lnfs with lnf_{n-max}, and starting from $k = 2$ in the figure fig.3. Normalising the lnfs with lnf_{2n-max}, we tabulate in the adjoining table,II, and starting from $k = 3$ we draw in the figure fig.4. Normalising the lnfs with lnf_{3n-max} we record in the adjoining table,II, and plot starting from $k = 4$ in the figure fig.5. In this way we obtain figures up to the figure fig.10.

k	lnk	lnk/lnk_{lim}	f	lnf	lnf/lnf_{max}	lnf/lnf_{n-max}	lnf/lnf_{2n-max}	lnf/lnf_{3n-max}	lnf/lnf_{4n-max}	lnf/lnf_{5n-max}	lnf/lnf_{6n-max}	lnf/lnf_{7n-max}	$lnf/lnf_{10n-max}$
1	0	0	2850	7.955	1	Blank	Blank	Blank	Blank	Blank	Blank	Blank	Blank
2	0.69	0.205	2215	7.703	0.968	1	Blank	Blank	Blank	Blank	Blank	Blank	Blank
3	1.10	0.326	1944	7.573	0.952	0.983	1	Blank	Blank	Blank	Blank	Blank	Blank
4	1.39	0.412	1697	7.437	0.935	0.965	0.982	1	Blank	Blank	Blank	Blank	Blank
5	1.61	0.478	1374	7.225	0.908	0.938	0.954	0.971	1	Blank	Blank	Blank	Blank
6	1.79	0.531	1200	7.090	0.891	0.920	0.936	0.953	0.981	1	Blank	Blank	Blank
7	1.95	0.579	1188	7.080	0.890	0.919	0.935	0.952	0.980	0.999	1	Blank	Blank
8	2.08	0.617	1091	6.995	0.879	0.908	0.924	0.941	0.968	0.987	0.988	1	Blank
9	2.20	0.653	991	6.899	0.867	0.896	0.911	0.928	0.955	0.973	0.974	0.986	Blank
10	2.30	0.682	900	6.802	0.855	0.883	0.898	0.915	0.941	0.959	0.961	0.972	Blank
11	2.40	0.712	704	6.557	0.824	0.851	0.866	0.882	0.908	0.925	0.926	0.937	1
12	2.48	0.736	636	6.455	0.811	0.838	0.852	0.868	0.893	0.910	0.912	0.923	0.984
13	2.56	0.760	609	6.412	0.806	0.832	0.847	0.862	0.887	0.904	0.906	0.917	0.978
14	2.64	0.783	591	6.382	0.802	0.829	0.843	0.858	0.883	0.900	0.901	0.912	0.973
15	2.71	0.804	559	6.326	0.795	0.821	0.835	0.851	0.876	0.892	0.894	0.904	0.965
16	2.77	0.822	545	6.301	0.792	0.818	0.832	0.847	0.872	0.889	0.890	0.901	0.961
17	2.83	0.840	512	6.238	0.784	0.810	0.824	0.839	0.863	0.880	0.881	0.892	0.951
18	2.89	0.858	463	6.138	0.772	0.797	0.811	0.825	0.850	0.866	0.867	0.877	0.936
19	2.94	0.872	432	6.068	0.763	0.788	0.801	0.816	0.840	0.856	0.857	0.867	0.925
20	3.00	0.890	413	6.023	0.757	0.782	0.795	0.810	0.834	0.850	0.851	0.861	0.919
21	3.04	0.902	387	5.958	0.749	0.773	0.787	0.801	0.825	0.840	0.842	0.852	0.909
22	3.09	0.917	313	5.746	0.722	0.746	0.759	0.773	0.795	0.810	0.812	0.821	0.876
23	3.14	0.932	256	5.545	0.697	0.720	0.732	0.746	0.767	0.782	0.783	0.793	0.846
24	3.18	0.944	252	5.529	0.695	0.718	0.730	0.743	0.765	0.780	0.781	0.790	0.843
25	3.22	0.955	179	5.187	0.652	0.673	0.685	0.697	0.718	0.732	0.733	0.742	0.791
26	3.26	0.967	160	5.075	0.638	0.659	0.670	0.682	0.702	0.716	0.717	0.726	0.774
27	3.30	0.979	113	4.727	0.594	0.614	0.624	0.636	0.654	0.667	0.668	0.676	0.721
28	3.33	0.988	10	2.303	0.290	0.299	0.304	0.310	0.319	0.325	0.325	0.329	0.351
29	3.37	1	1	0	0	0	0	0	0	0	0	0	0

TABLE II. Turkish head root words: ranking, natural logarithm, normalisations

Turkish head root words and BP(4,beta H=0.02)and Onsager solution
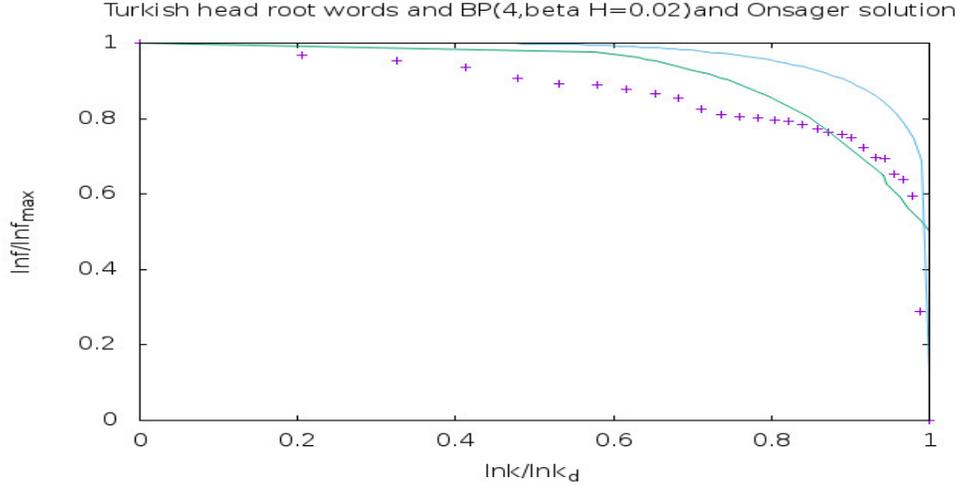
FIG. 2. Vertical axis is $\frac{lnf}{lnf_{max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the head root words of the Turkish language with the fit curve being the Bethe-Peierls curve of the Ising Model with four nearest neighbours, in the presence of little external magnetic field, m=0.01 or, $\beta H = 0.02$. The uppermost curve is the Onsager solution.
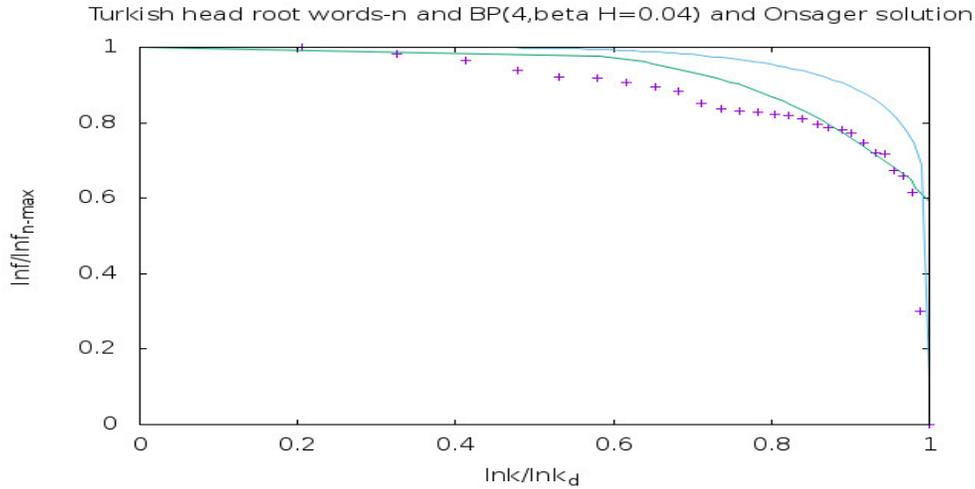

Turkish head root words-n and BP(4,beta H=0.04) and Onsager solution

FIG. 3. Vertical axis is $\frac{lnf}{lnf_{n-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the head root words of the Turkish language with the fit curve being the Bethe-Peierls curve of the Ising Model with four nearest neighbours, in the presence of little external magnetic field, m=0.02 or, $\beta H = 0.04$. The uppermost curve is the Onsager solution.
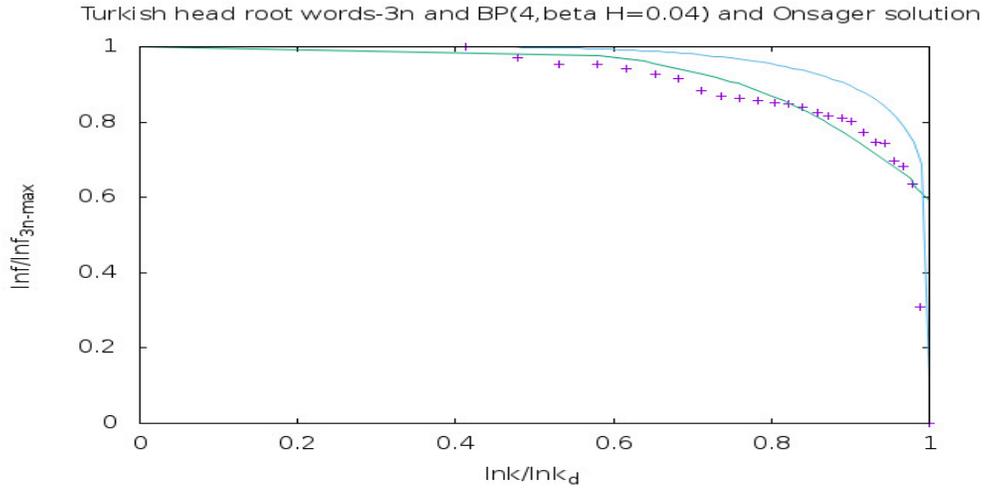
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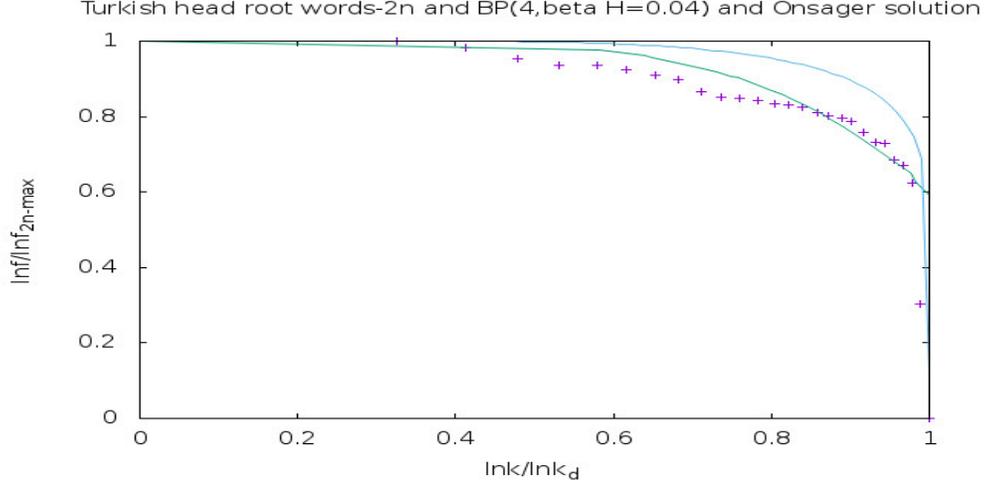
FIG. 4. Vertical axis is $\frac{lnf}{lnf_{2n-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the head root words of the Turkish language with the fit curve being the Bethe-Peierls curve of the Ising Model with four nearest neighbours, in the presence of little external magnetic field, m=0.02 or, $\beta H = 0.04$. The uppermost curve is the Onsager solution.
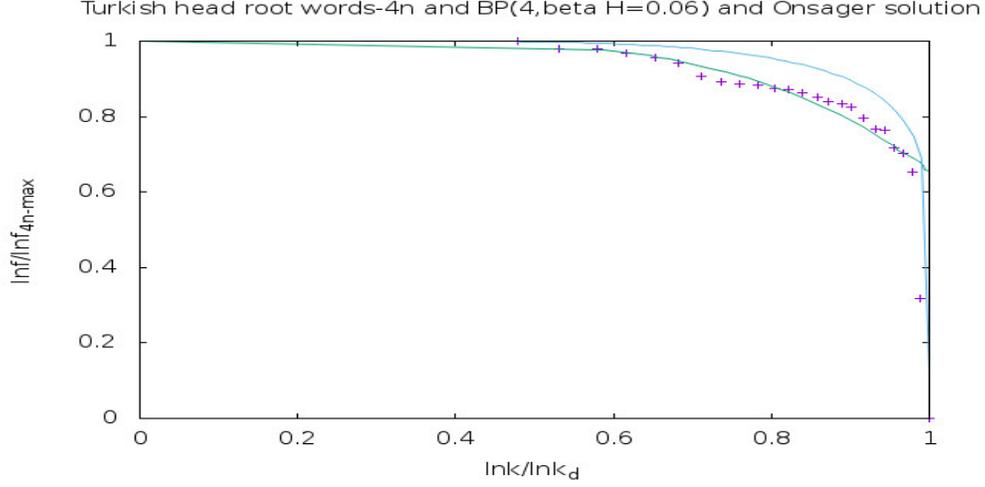

FIG. 5. Vertical axis is $\frac{lnf}{lnf_{3n-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the head root words of the Turkish language with the fit curve being the Bethe-Peierls curve of the Ising Model with four nearest neighbours, in the presence of little external magnetic field, m=0.02 or, $\beta H = 0.04$. The uppermost curve is the Onsager solution.

FIG. 6. Vertical axis is $\frac{lnf}{lnf_{4n-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the head root words of the Turkish language with the fit curve being the Bethe-Peierls curve of the Ising Model with four nearest neighbours, in the presence of little external magnetic field, m=0.03 or, $\beta H = 0.06$. The uppermost curve is the Onsager solution.
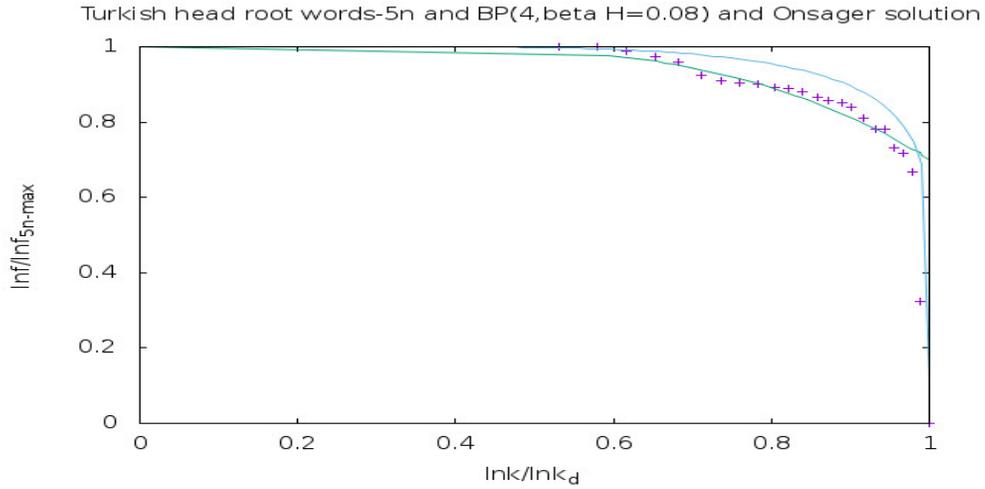

FIG. 7. Vertical axis is $\frac{lnf}{lnf_{5n-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the head root words of the Turkish language with the fit curve being the Bethe-Peierls curve of the Ising Model with four nearest neighbours, in the presence of little external magnetic field, m=0.04 or, $\beta H = 0.08$. The uppermost curve is the Onsager solution.
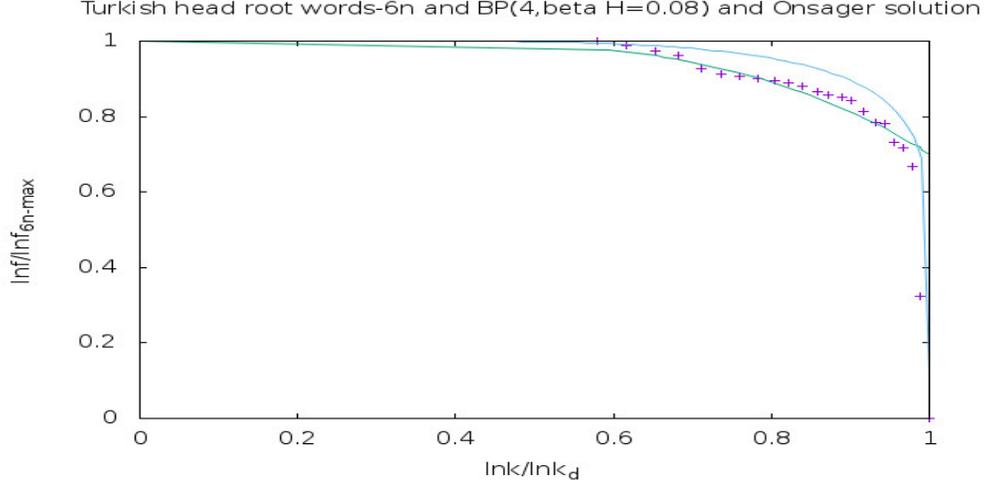
Turkish head root words-6n and BP(4,beta H=0.08) and Onsager solution

FIG. 8. Vertical axis is $\frac{lnf}{lnf_{6n-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the head root words of the Turkish language with the fit curve being the Bethe-Peierls curve of the Ising Model with four nearest neighbours, in the presence of little external magnetic field, m=0.04 or, $\beta H = 0.08$. The uppermost curve is the Onsager solution.

Turkish head root words-7n and BP(4,beta H=0.08) and Onsager solution

FIG. 9. Vertical axis is $\frac{lnf}{lnf_{7n-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the head root words of the Turkish language with the fit curve being the Bethe-Peierls curve of the Ising Model with four nearest neighbours, in the presence of little external magnetic field, m=0.04 or, $\beta H = 0.08$. The uppermost curve is the Onsager solution.
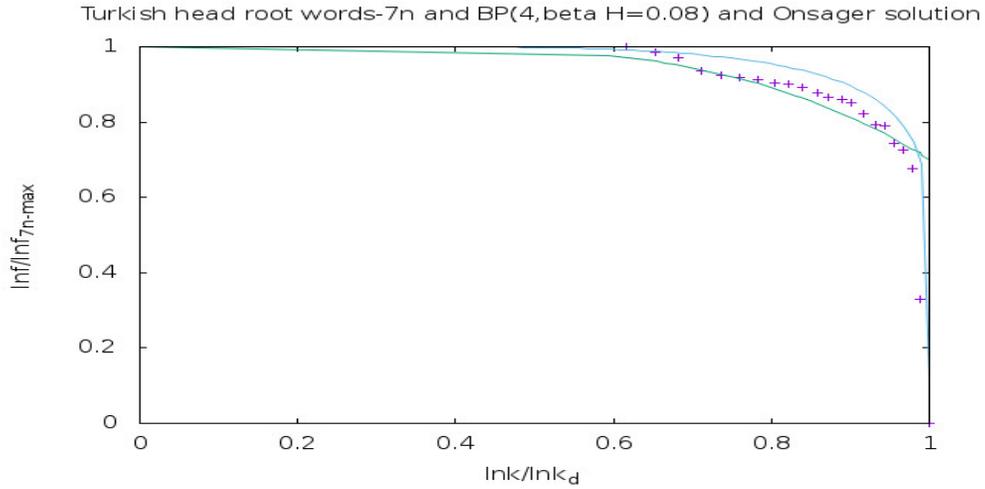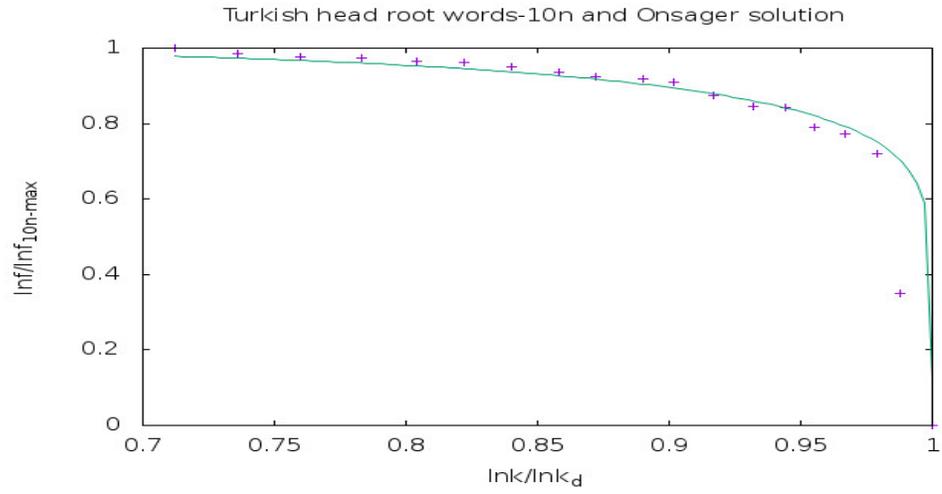
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FIG. 10. Vertical axis is $\frac{lnf}{lnf_{10n-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the head root words of the Turkish language with the fit curve being the Onsager solution of the Ising Model.

A. conclusion

From the figures (fig.2-fig.10), we observe that there is a curve of magnetisation, behind the head root words of Turkish language,[1]. The magnetisation curve i.e. the graph of the reduced magnetisation vs the reduced temperature is the exact Onsager solution of the two dimensional Ising model in the the absence of external magnetic field Moreover, the associated correspondence is,

$$\frac{ln f}{ln f_{10n-max}} \longleftrightarrow \frac{M}{M_{max}},$$
$$ln k \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [115].

III. APENDIX: MAGNETISATION

A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like para magnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N}\Sigma_i\sigma_i$, where σ_i is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where N_+ is the number of up spins, N_- is the number of down spins. $L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1 + L)$ and $N_- = \frac{N}{2}(1 - L)$. Magnetisation or, net magnetic moment , M is $\mu\Sigma_i\sigma_i$ or, $\mu(N_+ - N_-)$ or, μNL, $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$.

$\frac{M}{M_{max}}$ is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[105], for the lattice of spins, setting μ to one, is $-\epsilon\Sigma_{n.n}\sigma_i\sigma_j - H\Sigma_i\sigma_i$, where n.n refers to nearest neighbour pairs.

The difference $\triangle E$ of energy if we flip an up spin to down spin is, [106], $2\epsilon\gamma\bar{\sigma} + 2H$, where γ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_-}{N_+}$ equals $exp(-\frac{\triangle E}{k_BT})$, [107]. In the Bragg-Williams approximation,[108], $\bar{\sigma} = L$, considered in the thermal average sense. Consequently,

$$ln\frac{1+L}{1-L} = 2\frac{\gamma\epsilon L + H}{k_BT} = 2\frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2\frac{L + c}{\frac{T}{T_c}} \tag{1}$$

where, $c = \frac{H}{\gamma\epsilon}$, $T_c = \gamma\epsilon/k_B$, [109]. $\frac{T}{T_c}$ is referred to as reduced temperature.

Plot of L vs $\frac{T}{T_c}$ or, reduced magentisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [106]. W. L. Bragg was a professor of Hans Bethe. Rudolf Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudolf Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field

In the approximation scheme which is improvement over the Bragg-Williams, [105],[106],[107],[108],[109], due to Bethe-Peierls, [110], reduced magnetisation varies with reduced temperature, for γ neighbours, in absence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \tag{2}$$

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google
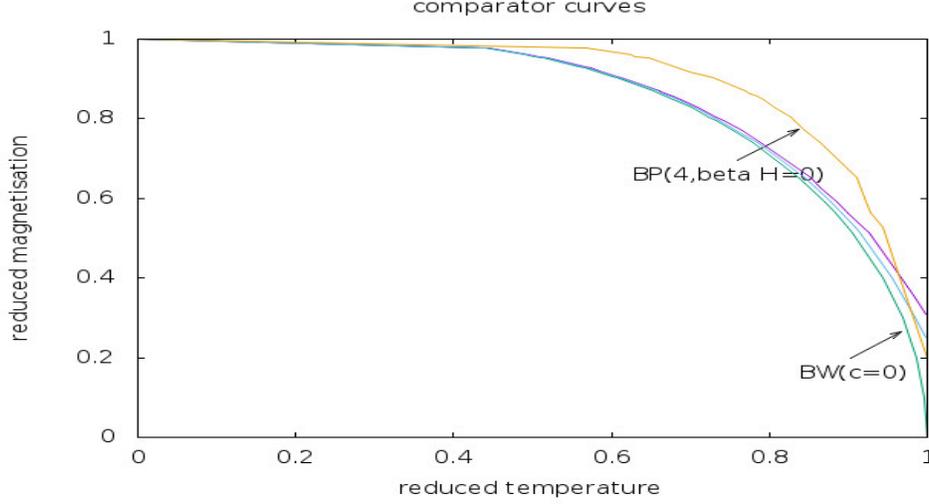
FIG. 11. Reduced magnetisation vs reduced temperature curves, for the Bragg-Williams approximation, in the absence (BW(c=0)) and in the presence (BW(c=0.005), BW(c=0.01)) of magnetic field, $c = 0$, $c = \frac{H}{\gamma\epsilon} = 0.005$, $c = \frac{H}{\gamma\epsilon} = 0.01$, outwards; and in the Bethe-Peierls approximation, BP(4,βH=0), in the absence of magnetic field, for four nearest neighbours (outer in the top).

search "reduced magnetisation vs reduced temperature curve". In the following, we describe data s generated from the equation(1) and the equation(2) in the table, III, and curves of magnetisation plotted on the basis of those data s. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.11. Empty spaces in the table, III, mean corresponding point pairs were not used for plotting a line.

C. Bethe-peierls approximation in presence of four nearest neighbours, in the presence of external magnetic field

In the Bethe-Peierls approximation scheme , [110], reduced magnetisation varies with reduced temperature, for γ neighbours, in presence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}\,factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}\,factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c} ; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{3}$$

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BW(c=0)	BW(c=0.005)	BW(c=0.01)	BP(4,$\beta H = 0$)	reduced magnetisation
0	0	0	0	1
0.435	0.437	0.439	0.563	0.978
0.439	0.441	0.443	0.568	0.977
0.491	0.493	0.495	0.624	0.961
0.501	0.504	0.507	0.630	0.957
0.514	0.517	0.519	0.648	0.952
0.559	0.562	0.565	0.654	0.931
0.566	0.569	0.573	0.7	0.927
0.584	0.587	0.590	0.7	0.917
0.601	0.604	0.607	0.722	0.907
0.607	0.610	0.613	0.729	0.903
0.653	0.658	0.661	0.770	0.869
0.659	0.663	0.666	0.773	0.865
0.669	0.674	0.678	0.784	0.856
0.679	0.684	0.688	0.792	0.847
0.701	0.705	0.709	0.807	0.828
0.723	0.728	0.732	0.828	0.805
0.732	0.736	0.743	0.832	0.796
0.753	0.758	0.766	0.845	0.772
0.779	0.784	0.788	0.864	0.740
0.838	0.844	0.853	0.911	0.651
0.850	0.858	0.864	0.911	0.628
0.870	0.877	0.885	0.923	0.592
0.883	0.891	0.899	0.928	0.564
0.899	0.908	0.918		0.527
0.905	0.914	0.926	0.941	0.513
0.944	0.956	0.968	0.965	0.400
		0.985		0.350
		0.998		0.310
0.969	0.985		0.965	0.300
	0.998			0.250
0.987			1	0.200
0.997			1	0.100
1			1	0

TABLE III. Datas for Reduced temperature[for the Bragg-Williams approximation, in the absence (BW(c=0)) and in the presence (BW(c=0.005), BW(c=0.01)) of magnetic field, $c = 0$, $c = \frac{H}{\gamma\epsilon} = 0.005$, $c = \frac{H}{\gamma\epsilon} = 0.01$ respectively and in the Bethe-Peierls approximation, BP(4,βH=0), in the absence of magnetic field, for four nearest neighbours] vs reduced magnetisation. Reduced temperature is drawn along the x-axis and Reduced magnetisation is drawn along the y-axis. In gnuplot the command is plot ".dat" using 1:2 with line; 1 standing for x-axis and 2 standing for y-axis datas.

Derivation of this formula ala [110] is given in the appendix of [7].

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For four neighbours,

$$\frac{0.693}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{4}$$

In the following, we describe datas in the table, IV, generated from the equation(4) and curves of magnetisation plotted on the basis of those datas. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.06$. calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.05$. calculated from the equation(4). BP(m=0.02) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.04$. calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.02$. calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.01$. calculated from the equation(4). The data set is used to plot fig.**??**. Empty spaces in the table, IV, mean corresponding point pairs were not used for plotting a line.

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BP(m=0.03)	BP(m=0.025)	BP(m=0.02)	BP(m=0.01)	BP(m=0.005)	reduced magnetisation
0	0	0	0	0	1
0.583	0.580	0.577	0.572	0.569	0.978
0.587	0.584	0.581	0.575	0.572	0.977
0.647	0.643	0.639	0.632	0.628	0.961
0.657	0.653	0.649	0.641	0.637	0.957
0.671	0.667		0.654	0.650	0.952
	0.716			0.696	0.931
0.723	0.718	0.713	0.702	0.697	0.927
0.743	0.737	0.731	0.720	0.714	0.917
0.762	0.756	0.749	0.737	0.731	0.907
0.770	0.764	0.757	0.745	0.738	0.903
0.816	0.808	0.800	0.785	0.778	0.869
0.821	0.813	0.805	0.789	0.782	0.865
0.832	0.823	0.815	0.799	0.791	0.856
0.841	0.833	0.824	0.807	0.799	0.847
0.863	0.853	0.844	0.826	0.817	0.828
0.887	0.876	0.866	0.846	0.836	0.805
0.895	0.884	0.873	0.852	0.842	0.796
0.916	0.904	0.892	0.869	0.858	0.772
0.940	0.926	0.914	0.888	0.876	0.740
	0.929			0.877	0.735
	0.936			0.883	0.730
	0.944			0.889	0.720
	0.945				0.710
	0.955			0.897	0.700
	0.963			0.903	0.690
	0.973			0.910	0.680
				0.909	0.670
	0.993			0.925	0.650
		0.976	0.942		0.651
	1.00				0.640
		0.983	0.946	0.928	0.628
		1.00	0.963	0.943	0.592
			0.972	0.951	0.564
			0.990	0.967	0.527
				0.964	0.513
			1.00		0.500
				1.00	0.400
					0.300
					0.200
					0.100
					0

TABLE IV. Bethe-Peierls approx. in presence of little external magnetic fields
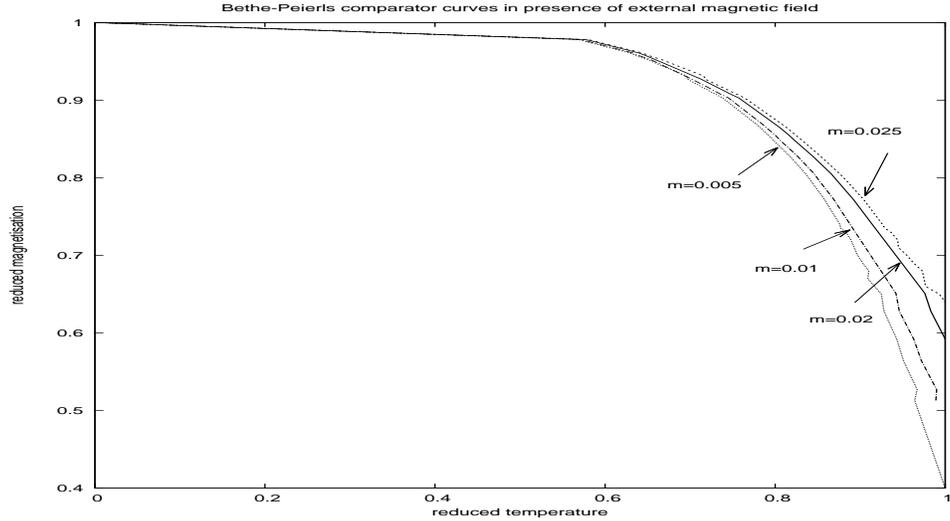

FIG. 12. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

D. Onsager solution

At a temperature T, below a certain temperature called phase transition temperature, T_c, for the two dimensional Ising model in absence of external magnetic field i.e. for H equal to zero, the exact, unapproximated, Onsager solution gives reduced magnetisation as a function of reduced temperature as, [111], [112], [113], [110],

$$\frac{M}{M_{max}} = [1 - (sinh\frac{0.8813736}{\frac{T}{T_c}})^{-4}]^{1/8}.$$

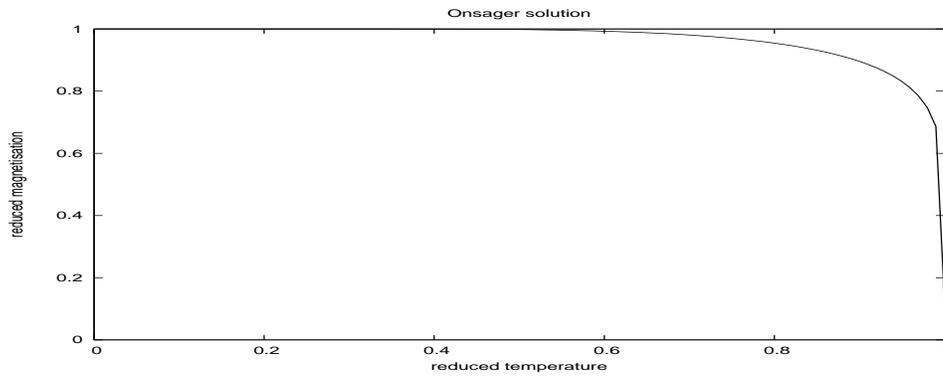Graphically, the Onsager solution appears as in fig.13.

FIG. 13. Reduced magnetisation vs reduced temperature curves for exact solution of two dimensional Ising model, due to Onsager, in absence of external magnetic field

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IV. ACKNOWLEDGMENT

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