Lucas Number 123 Demystified: Mathematics of a Divine Number

Dedicated to Pope Leo XIY, who once studied mathematics

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Abstract

We show the importance of *Lucas* number 123, symbolized by others for divine order or trinity, as enigmatic number for life, physics and the cosmos. This number is related to fundamental constants of nature and to powers of the golden mean that governs phase transitions from particle to cosmic scale. It is also related to the *Higgs* boson and to the Great Pyramid. Our number theoretical approach may help to understand unsolved problems in physics and supports applications.

Keywords: Number Theory, Lucas Numbers, Fibonacci Numbers, Golden Mean, Phase Transitions, Sommerfeld's Structure Constant, Gyromagnetic Factor of the Electron, Higgs Boson, Dirac's Large Number, Great Pyramid.

Introduction

This is only a first short essay about the 'angel' number **123**, a symbol for balance, human creativity, divine order or trinity of God the Father, the Son and the Holy Spirit. The author thinks that important information about this number is not well known. Here this enigmatic number is interpreted mathematically as beautiful golden relation, connected with the golden mean and its fifth power that is vice versa governed by phase transitions from particle up to cosmic scale [1]. The golden ratio is the most irrational number with the simplest infinite continued fraction representation at all and a very adaptable number-theoretical chameleon. Special attention is paid to the reciprocity property of the golden ratio as effective pre-calculator of natures creativness.We use the definition

$$\varphi = \frac{\sqrt{5}-1}{2} = \frac{1}{1+\frac{1}{1+\frac{1}{1+\cdots}}} = 0.6180339887\ldots \tag{1}$$

However, the golden ratio is frequently used by others as the reciprocal of this value

$$\Phi = \varphi^{-1} = 1 + \varphi = \frac{\sqrt{5}+1}{2} = 1.6180339887\ldots \tag{2}$$

There is another nice continued fraction approach to represent φ^{-1}

$$r = \sqrt{1+\sqrt{1+\sqrt{1+\cdots}}} = \sqrt{1+r} \tag{3}$$

giving the quadratic equation $r^2 - r - 1 = 0$ with the solution

$$r_1 = \varphi^{-1} = 1.6180339887\ \ldots \tag{4}$$

The next important number, the fifth power of the golden ratio, shows also a simple infinitely continued fraction representation

$$\varphi^5 = \frac{\sqrt{125}-11}{2} \tag{5a}$$

$$\varphi^{-5} = \frac{\sqrt{125}+11}{2} \tag{5b}$$

$$\varphi^5 = 0.0901699\ \ldots = \frac{1}{11+\frac{1}{11+\frac{1}{11+\cdots.}}} \tag{6}$$

This fundamental number not only governs phase transitions from particle up to cosmic scale [1] [2], but also determines the mass constituents of the universe [3] [4]. Its connection to the Great Pyramid in combination with π is also remarkable [5] [6] [7].

Lucas Number 123

Lucas numbers L_n are represented by the number sequence that begins with n = 0 [8]

$$L_n = \{2,1,3,4,7,11,18,29,47,76,\mathbf{123},199,\ldots\} \tag{7}$$

The numbers can be generated by the formula

$$L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n = \varphi^{-n} + (-\varphi)^n \tag{8}$$

Now we summarize some beautiful relations for *Lucas* number 123 [8]. We write down a relation with reciprocal terms (for comparison see relation (5b))

$$\sqrt{123 - \frac{1}{123}} = 11 + \varphi^5 = \varphi^{-5} \tag{9a}$$

$$\frac{1}{\sqrt{123-\frac{1}{123}}} = \varphi^5 \tag{9b}$$

$$\sqrt[5]{123 - \frac{1}{123}} = 2.618033991 = \Phi^2 = \frac{1}{\varphi^2} \tag{10}$$

$$\sqrt[10]{123 - \frac{1}{123}} = 1.618033989 = \varphi + 1 = \frac{1}{\varphi} \tag{11}$$

and $$123 \approx \frac{720}{\sqrt{5}} \varphi^2 = 122.99068 = \frac{2}{\sqrt{5}} \cdot 137.50776 \tag{12}$$

where $360° \cdot \varphi^2 = 137.507764°$ is known as the golden angle.
We formulate a quadratic equation in order to verify relation (9a)

$$x^2 - (11 + \varphi^5)^2 x - 1 = 0 \tag{13}$$

The solutions are $$x_{1,2} = \frac{(11+\varphi^5)^2}{2} \pm \sqrt{\left[\frac{(11+\varphi^5)^2}{2}\right]^2 + 1} \tag{14}$$

$$x_1 = \frac{122.991869 \ldots}{2} + \frac{123.008129 \ldots}{2} = 123.00000 \ldots \tag{15}$$

$$x_2 = -\frac{1}{x_1} = -\frac{1}{123} \tag{16}$$

The infinitely continued fraction of number 123 leads to the golden reciprocity relation that can be compared to the second sum term of relation (13)

$$123.008129543 \ldots = \frac{1}{123.008129543 \ldots} \tag{17}$$

Another representation of φ^5used a series representation of the inverse of number 123. We yield by summing only over uneven numbers $n = 1,3,5, \ldots$ [3] [9]

$$\left(\sum_{n=1,3,5\ldots}^{\infty} \left(\frac{1}{123}\right)^n\right)^{1/2} = \frac{1}{\sqrt{123 - \frac{1}{123}}} = \varphi^5 \tag{18a}$$

In practice, in order to generate a fairly good approximation of φ^5, only the first two terms of the sum are needed

$$\sqrt{\frac{1}{123} + \left(\frac{1}{123}\right)^3} = 0.09016994 \ldots \tag{18b}$$

For all positive real numbers *m* we get

$$\left(\sum_{n=1,3,5\ldots}^{\infty} m^{-n}\right)^{-1} = m - \frac{1}{m} \tag{19}$$

This connection to golden mean mathematics, dominating the life as well as the entire universe, is the very meaning of *Lucas* number 123. The concept of reciprocity is an important tool of nature. An excerpt of this short contribution can be found in reference [3].
Further insight into the mathematics of number 123 can be gained by considering the quartic polynomial equation and its selected golden mean solutions given in **Table 1** [10].

$$x^4 - (n-2)x^2 + 1 = 0 \tag{20}$$

We find for $n = 123 = 11^2 + 2$ the solution

$$x_1 = 10.999624310 \approx 11 \tag{21}$$

Furthermore, we can generate *Sommerfeld*'s reciprocal structure constant α^{-1}[11] as fundamental number of rotational movement and precession by using number 123

$$\frac{7}{2\pi} \cdot 123 = 137.032406 \tag{22}$$

$$\frac{123 \cdot \sqrt{5} - 1}{2} = 137.01818 \tag{23}$$

The last relation can be tentatively considered as an extension of the basis relation (1) for φ.

An approximation between number 123 and the galactic velocity β_g of *Guynn* [12] can be formulated using *Fibonacci* numbers and the second power of φ

$$123 \approx \frac{5 \cdot \varphi^2}{21 \cdot \beta_g} = 123.00140 \ldots \tag{24}$$

The elegant reciprocity relation between β_g and *Sommerfeld*'s structure constant α is added [13]

$$\frac{\alpha}{\pi} = \pi \cdot \beta_g \tag{25}$$

Also an approximation for the anomalous part of the gyromagnetic factor of the electron is deduced from relation (22)

$$\Delta g_e \approx \frac{2}{7 \cdot 123} - \frac{1}{2}\left(\frac{2}{7 \cdot 123}\right)^2 = 0.00232018 \tag{26}$$

$$\Delta g_e \approx \frac{6}{13}\frac{\varphi}{123 - \frac{1}{123}} = \frac{6}{13}\varphi^{11} = 0.00231923 \tag{27}$$

whereas our φ-based approximation is more accurate [10]

$$\Delta g_e = \frac{\varphi^6}{24} - \frac{1}{2}\left(\frac{\varphi^6}{24}\right)^2 - \frac{1}{3}\left(\frac{\varphi^6}{24}\right)^3 = 0.002319304 \ldots \tag{28}$$

However, *Guynn* derived the exact value for g_e by avoiding any *QED* construct and verified the extraordinary precise experimental value [12].

Furthermore, number 123 can be related to our newly introduces α_1 angle that is an expression of creative doubling [14]

$$\alpha_1 = arccos\left(\frac{\sqrt[3]{2}}{2}\right) = 50.9527898 \tag{29}$$

We yield

$$123 \approx \alpha_1 \cdot (1+\sqrt{2}) \tag{30}$$

Relation (30) would apply exactly for $\alpha_1 = 50.948268°$ ....

Last but not least we relate *Lucas* number 123 to the circumsphere radius of an icosahedron

$$\sqrt{\frac{123}{34}} = 1.902011 \approx \sqrt{3+\varphi} = 1.902113 \tag{31}$$

where 34 is again a *Fibonacci* number [15]. One can decompose number 123 into *Fibonacci* numbers $123 = 13 + 2 \cdot 55$ and proceed with

$$\frac{123}{34} = \frac{13}{34} + 2\frac{55}{34} \approx \varphi^2 + 2\varphi^{-1} = 3 + \varphi \tag{32}$$

Number 123 divided by *Fibonacci* number 55 yields

$$\frac{123}{55} = 2.23636363 \approx \sqrt{5} = 2.236067978 \tag{33}$$

In **Table 1** we find also number 171 as last listed one, which is a coefficient of the icosahedron equation. Like number 123 we find the golden expression

$$\sqrt{171 - \frac{1}{171}} = 13.0764732\ldots = \frac{1}{0.0764732\ldots} \tag{34}$$

and $$\sqrt[3]{\frac{171}{123}} \approx \frac{\sqrt{5}}{2} \quad \text{respectively} \quad \sqrt[3]{\frac{171}{123}} - \frac{1}{2} \approx \varphi \tag{35}$$

However, the quotient of another coefficient of the icosahedron equation and the golden angle is intriguing

$$\frac{494}{360° \cdot \varphi^2 - 14} = \frac{19}{\alpha_1 \cdot \varphi^2 - \frac{7}{13}} = 3.9997488 \approx 4 \tag{36}$$

where all coefficients of the icosahedron equation are multiples of prime 19: $494 = 26 \cdot 19$, $228 = 12 \cdot 19$, $171 = 9 \cdot 19$.

Number 123 and 494 are quite simply connected by the golden mean. In this way coefficients of the icosahedron equation [16] can be replaced by number 123 [10]

$$123 \cdot \frac{\varphi}{2} = 38.009090 \approx \frac{494}{13} = 38 \tag{37}$$

The icosahedron equation maps for instance the positions of the face centers of an icosahedron with unit in-radius projected onto a complex plane where *z* is the coordinates

[16]. Instead of following *Klein*'s quintic icosahedral solution, the substitution of the complex variable $z^5 \rightarrow x$ formally leads to a quartic polynomial [3]

$$H(z,1) = z^{20} - 228z^{15} + 494z^{10} + 228z^5 + 1 \tag{38a}$$

$$H(x,1) = x^4 - 228x^3 + 494x^2 + 228x + 1 \tag{38b}$$

If we replace the coefficients according to relation (34), we obtain the quartic polynomial as approximation by using number 123

$$H(x,1) = x^4 - 3\varphi \cdot 123 \cdot x^3 + \frac{13}{2}\varphi \cdot x^2 + 3\varphi \cdot 123 \cdot x + 1 \tag{38c}$$

The reader is frequently confronted with *Fibonacci* number 13, which obviously plays an important role besides φ respectively φ^5 or number 123 when assessing bio-coding and related storage and processing of information. Number 123 is connected with the icosahedron as important structural entity of life and with a helix composed of 13 filaments. We report a further approximation for number 123 using *Fibonacci* number 13

$$123 \approx \frac{4}{9\varphi}\left(13 + \frac{1}{13}\right)^2 = 122.9748 \tag{39}$$

Considering that the golden mean is the most irrational number, it is intriguing that the sixth root of the golden mean gives a palindromic relation consisting of only two numbers. With the number string one can again generate approximately number 123.

$$\sqrt[6]{\varphi} = 0.922929922929 \ldots \approx \frac{12}{13} \tag{40}$$

$$92.29 \cdot \frac{4}{3} = 123.05333 \ldots \tag{41}$$

It remains to connect number 123 with the circle constant π. We obtain very good approximations [3]

$$\frac{1}{\sqrt{123 - \frac{1}{123}}} \approx \frac{5}{2}\left(\sqrt{\frac{6}{5\pi}} - \frac{6}{5\pi}\right) - \frac{1}{2} = 0.0901671 \tag{42}$$

$$\frac{1}{\sqrt{123 - \frac{1}{123}}} \approx \sqrt{2\pi} \cdot \pi(\pi(\pi + 1) - 13) = 0.090169666 \tag{43}$$

$$\sqrt[3]{123} = 4.97318 \ldots \approx \pi \cdot (\pi + 1) \cdot \varphi^2 = 4.9698 \ldots \tag{44}$$

As a final remarkable result we connect the 'spherical energy center' of the Great Pyramid at *Giza* with the pyramid geometry and properties via the large and the small of the trinity number 123 and the circle constant, giving the exact, but nevertheless enigmatic formula [5] [6]

$$\frac{V_{sph}}{V_{pyr}} = \pi \cdot \varphi^5 = \frac{\pi}{\sqrt{123 - \frac{1}{123}}} \tag{45}$$

where V_{sph} is the in-sphere volume of the pyramid and V_{pyr} the volume of the pyramid itself. The same relation holds for the surfaces.

Number 123 is frequently involved in number-theoretical considerations of modern physics and seems to be omnipresent. We derive quite simple relations between this number and *Dirac*'s large number *DLN*, the number N_{sd} for small distances, *Sommerfeld*'s reciprocal structure constant α^{-1} and the mass m_{Hi} of the *Higgs* boson. We obtain the following golden mean based interrelations by using again our angle α_1 [3] [17] [18] [19]

$$\sqrt[20]{\frac{10^{43}}{\pi}} = \sqrt[20]{DLN} \approx \frac{\alpha_1}{\varphi^2} = 133.3959128 \tag{46}$$

$$\sqrt{\alpha^{-1} \cdot N_{sd}} = 133.3959128 \tag{47}$$

$$\alpha^{-1} \approx N_{sd}(1 + \varphi^6) \tag{48}$$

$$N_{sd} = 129.85250805 \approx 123 \cdot (1 + \varphi^6) \tag{49}$$

$$\alpha^{-1} \approx 123 \cdot (1 + \varphi^6)^2 \tag{50}$$

$$\sqrt{\alpha^{-1} \cdot N_{sd}} \approx 123 \cdot \sqrt{(1 + \varphi^6)^3} = 133.4237 \ldots \tag{51}$$

$$m_{Hi} = \frac{\alpha_1}{\varphi^2}\left(m_p + m_e\right) = 125.23\ GeV/c^2 \tag{52}$$

$$m_{Hi} \approx 123 \cdot \sqrt{(1 + \varphi^6)^3} \cdot \left(m_p + m_e\right) \tag{53}$$

Relation (47) is given by *Kosinov* [20] as the geometric mean between the reciprocal *Sommerfeld* constant α^{-1} and the number for small distances $N_{sd} = 129.85250805$.

Resume:

The golden mean and its fifth power as well as helical structures and icosahedral entities belong together, and number 123 is connected with all that. Such intimate interweaving determines the observed beautiful diversity of life and cosmos. For instance, brain waves of living creatures are golden mean clocked [21]. However, the connection of number 123 with its inverse must be considered not opposed, but rather interconnected as expressed by the Yin and the Yang in Chinese philosophy. A deeper insight into fractal geometric interrelations may be obtained when applying a curvilinear coordinate frame and the elegant *Clifford* algebra [22] [23]. But keep it simple!

Conclusion

The divine *Lucas* number 123 was demystified without losing its meaning and intrinsic beauty. Beauty and simplicity are sisters. It remains a golden number that is related to fundamental constants of nature and therefore sustainably influences life and the entire cosmos. People may henceforth consider this mathematical interpretation, beyond pure scientific importance, as helpful for esoteric or religious insights. It seems that number 123 is once again enhanced and shines brightly.

Conflicts of Interest

The author declares no conflict of interests regarding the publication of this paper.

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Table 1. Selected Solutions for the Quartic Polynomial Equation
$$\boldsymbol{x^4 - (n-2)x^2 + 1 = 0}$$

n		$\sqrt{n} = x_1 + x_1^{-1}$	notation	x_1	notation	x_1^{-1}
4		2	φ^0	1	φ^0	1
5		$\sqrt{5}$	φ^{-1}	1.6180339887	φ	0.6180339887
8		$2 \cdot \sqrt{2}$	${\delta_s}^{-1}$	2.4142135623	δ_s	0.414213562
9		3	φ^{-2}	2.6180339887	φ^2	0.381966011
18	$4^2 + 2$	$3 \cdot \sqrt{2}$		3.9921490369		
20		$2 \cdot \sqrt{5}$	φ^{-3}	4.2360679774	φ^3	0.236067977
27	$5^2 + 2$			4.9959919730		
32		$4 \cdot \sqrt{2}$		5.4741784358		0.1826758136
38	$6^2 + 2$			5.9976829489		
49		7	φ^{-4}	6.8541019662	φ^4	0.1458980337
51	$7^2 + 2$			6.9985415144		
66	$8^2 + 2$			7.9990231393		
83	$9^2 + 2$			8.9993139982		
102	$10^2 + 2$			9.9994999374		
123	$11^2 + 2$			10.999624310		
125		$5 \cdot \sqrt{5}$	φ^{-5}	11.090169943	φ^5	0.090169943
146	$12^2 + 2$			11.999710630		
171	$13^2 + 2$			12.999772406		