Sophie Germain primes concept expanded to ultimate numbers

Number Genetics and the 3 to 2 ratio

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Abstract: *Here is expanded the concept of Sophie Germain prime and safe prime to ultimate numbers. More, this mathematical mechanism is also applied to the set of all whole numbers (ℕ) which are differentiated into ultimate and non-ultimate numbers. Thus, a complete study of the set ℕ is undertaken. This global investigation, universally broadening the mechanical-mathematical concept of Sophie Germain, makes it possible to propose a genetics of numbers very similar to biological genetics. According to these new numeric genetic criteria, in their start top organization, geometric distribution of whole numbers in various closed matrices, is organized into perfect ratios to exact 3/2 or 1/1 value.*

Keywords: Sophie Germain primes, whole numbers, symmetry, set theory, ultimate numbers, number theory, Genetics.

1. Introduction

Until now, many studies and mathematical literature in number theory have invested the field of Sophie Germain numbers also called *"Sophie Germain primes"* [1]. These investigations uniquely just highlight these Sophie Germain primes associating with safe primes. Therefore, in these different works, the rest of the other whole numbers are not really studied (others primes, non-primes, etc.) As in other investigations in number theory, the same negligence is often done with for example the study of sequence of Pi decimals forgetting to also investigate the equally interesting sequence of decimals of 1/Pi [2].

Identically, it is still made the same scientific error to be interested only in the prime numbers sequence without, in parallel, also studying that of non primes. It is therefore proposed here to apply the mathematical concept of Sophie Germain both to prime numbers and to non-prime numbers inside the whole numbers set (ℕ).

Also, these mathematical investigations integrate the concept of ultimate numbers. This concept giving a variant definition of that of prime numbers and allowing to merge the exotic numbers 0 and 1 with the sequence of prime numbers. This innovative but still little publicized concept is so introduced before the development of that of Sophie Germain numbers (safe and Sophie Germain primes).

The simultaneous study of all the categories of whole numbers differentiated by these double notions of ultimity and safety makes it possible to introduce a genetic description of the ℕ set. We call this *"the number genetics"*.

2. Ultimity concept depiction

In preview papers *"The ultimate numbers and the 3/2 ratio. Just two primary sets of whole numbers"* [3] and *"New whole number classification and the 3 to 2 ratio"* [4], we have introduced concept of ultimates numbers and others derived numbers classified from that. These studies invest the whole numbers set (ℕ) and proposes a mathematical definition to integrate the number *zero* (0) and the number *one* (1) into the thus called prime numbers sequence. This enlarged set is called the set of ultimate numbers.

So, according to a new mathematical definition, whole numbers are divided into two sets, one of which is the merger of the sequence of prime numbers and numbers zero and one. Using a definition with inescapable characters and not subject to any ambiguity, we introduce here the concept of ultimity that a whole number can possess. Recalling the notion of prime number, we then introduce a compact and absolute definition splitting the set N into two primary sets. A development, taking the very first numbers as an example, clarifies this innovative concept which turns out to be of a strong simplicity.

2.1 Prime number definition

In mathematical literature the definition of primes looks like this:

> *"Prime numbers are numbers greater than 1. They only have two factors, 1 and the number itself. This means these numbers cannot be divided by any number other than 1 and the number itself without leaving a remainder."*

This is often supplemented by:

> *"Numbers that have more than 2 factors are known as composite numbers."*

These definitions supposed to describe and classify the whole numbers immediately present two great ambiguities since they are embarrassed by the two singular numbers that are zero (0) and one (1).

2.2 New definition approach

The conventional definition applied to define whether a number is prime or not does not specify the character of inferiority of the divisors. This seems obvious, trivial. However, with regard to the particular numbers that are zero and one, this notion has all its importance. Indeed, the number zero, the first of the whole numbers, has many divisors. But these dividers are all superior to it in value. Also, number one, second whole number, has not divisor inferior to it because it cannot be divided by zero, the one which is inferior to it.

This novel approach to the concept of divisibility of numbers which includes the notion of inferiority (and consequently the notion of superiority) of divisors allows the creation of two unique sets where all whole numbers can be referenced.

2.3 Absolute definitions

Considering the set of all whole numbers (ℕ), these are organized into two sets: ultimate numbers and non-ultimate numbers.

Ultimate number definition:

An ultimate number admits at most one divisor being inferior to it in value.

Non-ultimate number definition:

A non-ultimate number admits more than one divisor being inferior to it in value.

2.4 Conventional designations

As "primes" designates prime numbers, it is agree that appellation "ultimates" designates ultimate numbers. Also it is agree that appellation "non-ultimates" designates non-ultimate numbers. So, the concept introduced here is therefore called that of ultimity of whole numbers. Otherwise transcendent conventional appellations will be applied to these two different classes of whole number just defined, in particular with the development of Sophie Germain's number concept, which we are going to extend to all whole number.

2.5 Development

This new concept of ultimity, which may seem intriguing, is now explained in detail with, more concretely, the first of the whole numbers as representative examples, including the exotic numbers zero (0) and one (1).

2.5.1 Expended definitions

Let *n* be a whole number (belonging to ℕ), this one is ultimate if **at most one divisor being inferior to it in value** divides it.

Let *n* be a whole number (belonging to ℕ), this one is non-ultimate if **more than one divisor being inferior to it in value** divides it.

2.5.2 Development

Below are listed, to illustration of definition, some of the first ultimate or non-ultimate numbers defined above, especially particular numbers zero (0) and one (1).

- 0 is ultimate: although it admits an infinite number of divisors superior to it, **since it is the first whole number**, number 0 does not admit any divisor **being inferior to it**.

- 1 is ultimate: since the division by 0 has no defined result, number 1 does not admit any divisor (whole number) being less than it.

- 2 is ultimate: since the division by 0 has no defined result, number 2 admit just one divisor (1) being less than it.

- 4 is non-ultimate: number 4 admits 1 and 2 as divisors being less than it, so more than one divisor.

- 6 is non-ultimate: number 6 admits numbers 1, 2 and 3 as divisors being less than it, so more than one divisor.

- 7 is ultimate: since the division by 0 has no defined result, number 7 admit just one divisor (1) being less than it.

- 12 is non-ultimate: number 12 admits numbers 1, 2, 3, 4 and 6 as divisors being less than it, so more than one divisor.

Thus, by these previous definitions and demonstrations, the set of whole numbers (ℕ) is organized just into these two entities:

- the set of ultimate numbers, which is the fusion of the prime numbers sequence with the number 0 and number 1.
- the set of non-ultimate numbers identifying to the non-prime numbers sequence, free of the numbers 0 and 1.

2.5.3 Abbreviated definition

It is therefore possible to classify very clearly and unequivocally all whole numbers according to ultimity concept. Figure 1 summarizes the process of identifying any whole number that can only be either ultimate or non-ultimate.

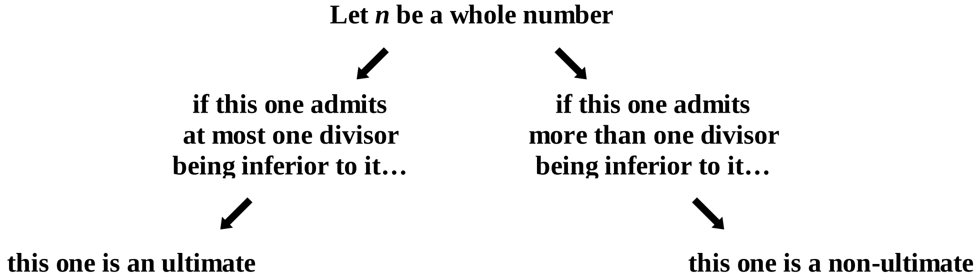


Figure 1: Process of identifying any whole number according to ultimity concept. See Figure 2 also.

This ultimity or non-ultimity identification mechanism is universal for all the sequence of whole numbers starting with the number zero.

2.5.4 The first ten ultimate numbers and the first ten non-ultimate numbers

Considering the previous double definition, the sequence of ultimate numbers is initialized by these ten numbers:

0 1 2 3 5 7 11 13 17 19

Considering the previous double definition, the sequence of non-ultimate numbers is initialized by these ten numbers:

4 6 8 9 10 12 14 15 16 18

We can make the first observation here, which is that the set of the first twenty wholes numbers (from 0 to 19) is made up of exactly ten ultimates and ten non-ultimates.

2.5.5 First twenty whole numbers classification

Figure 2 is a complete illustration summarizing the concept of ultimity applied to the set of all whole numbers with the first twenty numbers as example.

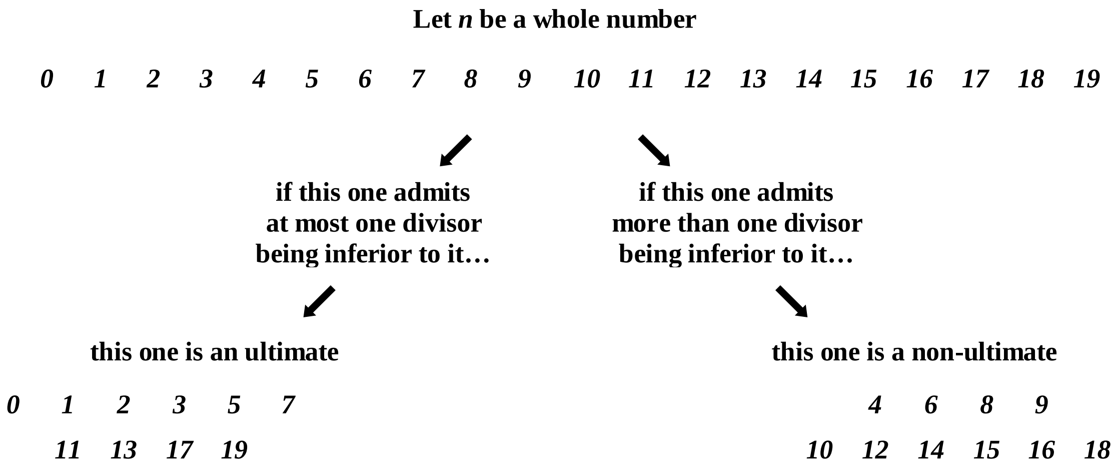


Figure 2: Clear and unequivocal classification of the first twenty whole numbers according to ultimity concept.

The illustration Figure 2, which uses the first twenty whole numbers, is not chosen by chance. We are very soon going to demonstrate a non-random organization of these twenty mathematics entities according to the concept of ultimity which has just been introduced. We will demonstrate later that these twenty digital entities are organized along the same path in our development of the concept of Sophie Germain's number.

3 The twenty fundamental numbers

It turns out that the tenth ultimate number is the number 19, a number* located in twentieth place in the sequence of the whole numbers. This peculiarity undeniably links the ultimate numbers and the decimal system. So the first twenty numbers (twice ten numbers) are organized into different 1/1 and 3/2 ratios according to their different attributes.

> ** In statements, when this is not specified, the term "number" always implies a "whole number". Also, It is agreed that the number zero (0) is well integrated into the set (ℕ) of whole numbers.*

3.1 The ultimate numbers and the decimal system

By the nature of the decimal system, as shown in Figure 3, the ten *digit numbers* (digits confused as numbers) are opposed to the first ten *non-digit numbers* by a ratio of 1/1. Also, there are exactly the same quantity of ultimates and non-ultimates among these twenty numbers, so ten entities in each category. In a double 3/2 value ratio, six ultimates versus four are among the ten digit numbers and six non-ultimates versus four are among the first ten non-digit numbers.

10 digit numbers	**← 1/1 ratio →**	10 non-digit numbers
6 ultimates	**← 3/2 ratio →**	**4 ultimates**
0 1 2 3 4 **5** 6 **7** 8 9		10 **11** 12 **13** 14 15 16 **17** 18 **19**
4 non-ultimates	**← 2/3 ratio →**	6 non-ultimates
↑ 3/2 ratio ↓		**↑ 2/3 ratio ↓**

Figure 3: Differentiation of the 20 fundamental numbers according to their digitality or non-digitality: the 10 digit numbers (digits confused as numbers) and the first 10 non-digit numbers.

As shown in Figure 4, it is also possible to describe this arithmetic phenomenon by crossing criteria. Thus, the first ten ultimates are opposed to the ten non-ultimates by a 1/1 value ratio. Also, there are exactly the same quantity of digit numbers and non-digit numbers among these twenty numbers. In a twice 3/2 ratio, six digits versus four are among the ten ultimates and six non-digits numbers versus four are among the first ten non-ultimates.

The first 10 ultimate numbers	**← 1/1 ratio →**	The first 10 non-ultimate numbers
6 digit numbers	**← 3/2 ratio →**	**4 digit numbers**
0 1 2 3 5 7 11 13 17 19		**4 6 8 9** 10 12 14 15 16 18
4 non-digit numbers	**← 2/3 ratio →**	6 non-digit numbers
↑ 3/2 ratio ↓		**↑ 2/3 ratio ↓**

Figure 4: Differentiation of the 20 fundamental numbers according to their ultimity or non-ultimity: 10 ultimates versus 10 non-ultimates.

3.2 The twenty fundamental numbers

Whole numbers sequence is therefore initialized by twenty numbers with symmetrically and asymmetrically complementary characteristics of reversible 1/1 and 3/2 ratios. Illustrated Figure 5, this transcendent entanglement of the first twenty numbers according to their ultimate or non-ultimate nature (ultimate numbers or non-ultimate numbers) and according to their digit or non-digit nature (digits or non-digit numbers) allows, by convention, to qualify them as "fundamental numbers" among the whole numbers set. Figure 5 describes the total entanglement of these twenty fundamental numbers.

	digitality		
ultimity	**0 1 2 3 5 7**	**4 6 8 9**	non-ultimity
	11 13 17 19	*10 12 14 15 16 18*	
	non-digitality		

Figure 5: Entanglement of the 20 fundamental numbers according to their ultimity or non-ultimity and their digitality or non-digitality.

Thus, the set of the first twenty whole numbers is simultaneously made up to a set of twenty entities including ten ultimate numbers and ten non-ultimate numbers and to a (same) set of twenty entities including ten digit numbers (10 digits) and ten non-digit numbers (not digits).

Also, each of these four entangled subsets of ten entities with their own properties opposing two by two in 1/1 value ratio is composed of two opposing subsets in 3/2 value ratio according to the mixed properties of its components. This set of the first twenty numbers is defined as the set of fundamental numbers among the whole numbers. So it is agreed that designation "fundamentals" designates these twenty fundamental numbers previously defined.

Particular attention to these first twenty numbers, qualified as fundamental, finds its justification in the numerous demonstrations of the development of the concept of Sophie Germain primes expended to the concept of ultimate numbers.

4. Sophie Germain ultimate numbers

Here is applied to ultimate numbers (u) the concept of Sophie Germain prime and of safe prime. As a reminder, if p and $2p + 1$ are both prime, then p is a prime number of Sophie Germain and $2p + 1$ is a safe prime number.

4.1 Concept of safe ultimate number

So we can agree that if u and $2u + 1$ are ultimate, then u is **an ultimate number of Sophie Germain** and $2u + 1$ is **a safe ultimate number**.

From this convention, it follows that the two particular numbers *zero* (0) and *one* (1), recognized as ultimate since the definition of this type of number (definition introduced in Chapter 2.1), are both ultimate numbers of Sophie Germain:

0 and [(2 × 0) + 1] are both ultimates → 0 is Sophie Germain ultimate and 1 is safe ultimate number.
1 and [(2 × 1) + 1] are both ultimates → 1 is Sophie Germain ultimate and 3 is safe ultimate number.

4.2 Concept of safe non-ultimate number

Since ultimate numbers concept (introduced above Chapter 2.1) allows you to split the set $\mathbb{N}$ in just two sub-sets, We propose here the ambitious and innovative idea to applying the Sophie Germain mechanism both to the ultimates and non-ultimates.

So we can extend this concept *of safety* to non-ultimate numbers ($\cancel{u}$) and agree that if $\cancel{u}$ and $2\cancel{u} + 1$ are non-ultimate, then $\cancel{u}$ is a non-ultimate number of Sophie Germain and $2\cancel{u} + 1$ **a safe non-ultimate number**.

4.3 Concept of fertile number

Thus, according to these new conventions and the ultimity degree of whole numbers these can only belong to one of four different types of numbers including two types of *Sophie Germain numbers* (which can be ultimates or non-ultimates) and two types of *no Sophie Germain numbers* (which can be ultimates or non-ultimates). By using numeric genetic language, We propose here to qualify these Sophie Germain numbers as *fertile* and these no Sophie Germain numbers as *sterile*. A whole number is therefore:

- either a fertile ultimate (u): $2u + 1 = u'$ → u is fertile ultimate, u' is safe* ultimate,
- or a sterile ultimate (u): $2u + 1 = \cancel{u}$ → u is sterile ultimate, $\cancel{u}$ is unsafe* non-ultimate,
- or a fertile non-ultimate ($\cancel{u}$): $2\cancel{u} + 1 = \cancel{u}'$ → $\cancel{u}$ is fertile non-ultimate, $\cancel{u}'$ is safe* non-ultimate,
- or a sterile non-ultimate ($\cancel{u}$): $2\cancel{u} + 1 = u$ → $\cancel{u}$ is sterile non-ultimate, u is unsafe* ultimate.

Therefore, it is agreed that designation "fertiles" designates fertile numbers (which can be ultimates or non-ultimates) and designation "steriles" designates sterile numbers (which can be ultimates or non-ultimates).

*In some time, introduced in Chapter 4.6, more appropriate terms that safe or unsafe and using genetics language will proposed to qualify these numbers.

So, to resume, by the Sophie Germain's expanded numeric-genetic mechanism:

- if an ultimate generate a ultimate → this one is a fertile ultimate.

- if an ultimate generate a non-ultimate → this one is a sterile ultimate.

- if an non-ultimate generate a non-ultimate → this one is a fertile non-ultimate.

- if an non-ultimate generate a ultimate → this one is a sterile non-ultimate.

Also, to the traditional mathematical formula $2x + 1$, we prefer this one:

the addition of the number x with the adjoining number greater than x

This of course has the same meaning, but is more explicit from a numeric genetics point of view: union of two entities. This concept of number genetics will be largely revealed in chapter 9. So, for the next demonstrations we will now use formula:

$$x + (x+1) \text{ or } u + (u+1)$$

4.4 Charting of number fertility concept

Chart in Figure 6 synthesizes, more clearly than a simple text, the mixed notions of fertility and sterility and ultimity and non-ultimity which are applied to whole numbers. We uses respectively *u* and ~~*u*~~ to designate ultimate and non-ultimates numbers.

One can therefore understand that the two numerical species that are the ultimates and the non-ultimates are subdivided into four sub-species that can be either fertile or sterile.

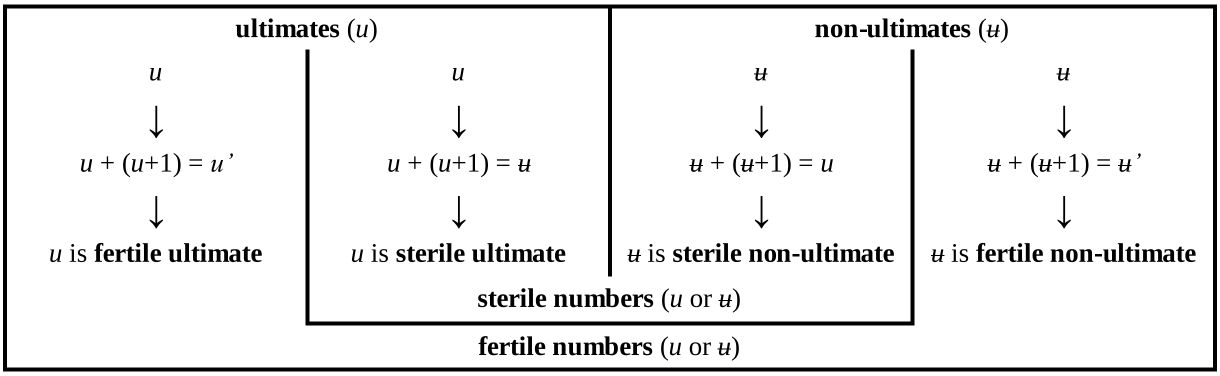


Figure 6: The two species and four subspecies of number.

Figure 7 illustrates this genetics of whole numbers with some representative numeric entities of each species and subspecies.

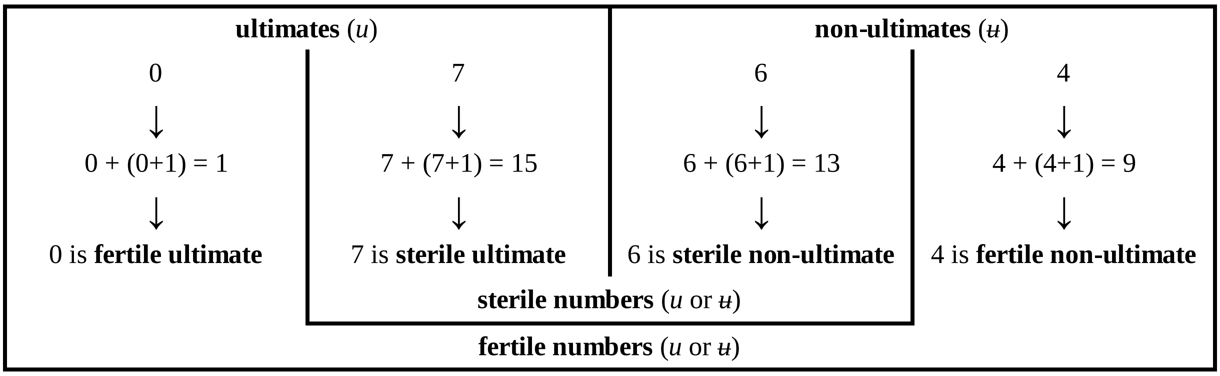


Figure 7: The two species and four subspecies of number.

4.4.1 The two species and four subspecies of number.

Chart in Figure 8 clarifies more simply this classification of numerical species and subspecies.

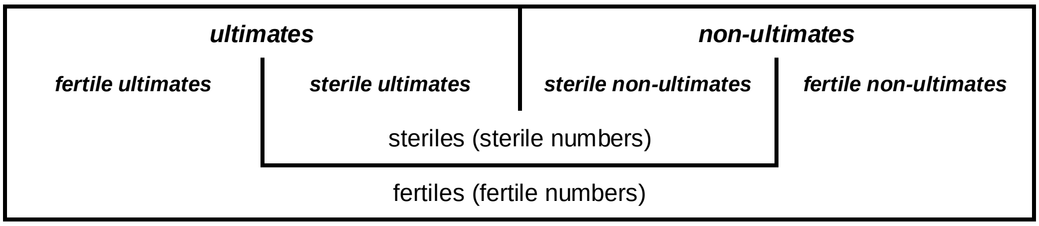


Figure 8: The two species of number and the four subspecies according to the ultimity concept and them fertility by Sophie Germain genetic concept expanded to ultimates and to non-ultimates.

Chart in Figures 9 takes up these genetic arrangements by classifying the first twenty numbers whose importance will soon be revealed.

species		subspecies
ultimates: 0-1-2-3-5-7-11-13-17-19	↗	fertile ultimates: 0-1-2-3-5-11
	↘	sterile ultimates: 7-13-17-19
non-ultimates: 4-6-8-9-10-12-14-15-16-18	↗	sterile non-ultimates: 6-8-9-14-15-18
	↘	fertile non-ultimates: 4-10-12-16

Figure 9: Genetic classification (typing and sub typing) of twenty fundamental numbers.

4.5 Fertility of twenty fundamental numbers

In Figure 10 are listed the twenty fundamental numbers (introduced in Chapter 3) and the twenty corresponding numbers that they generate by the arithmetic concept of Sophie Germain. We can also say: *"by the Sophie Germain's number genetic concept"*.

n →	**0**	**1**	**2**	**3**	4	**5**	6	**7**	8	9	10	**11**	12	**13**	14	15	16	**17**	18	**19**
	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘	↘
$2n + 1$ →	**1**	**3**	**5**	**7**	9	**11**	**13**	15	**17**	**19**	21	**23**	25	27	**29**	**31**	33	35	**37**	39

Figure 10: The twenty fundamental numbers and *them Sophie Germain's number genetic descent*.

In Figure 11, are classified the twenty fundamental numbers (so the first twenty whole numbers) in accordance with fertility concept that we have just introduced above. By this *number genetic* mechanism of Sophie Germain, it turns out that 10 of the 20 fundamental numbers are fertile and 10 others are sterile, so two sets of equal size.

Let *n* be a whole number

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

↙	↘
if *n* and (2*n* + 1) are both ultimate or both non-ultimate…	**if *n* and (2*n* + 1) are not both ultimate or not both non-ultimate…**
↙	↘
this one is a fertile number	**this one is a sterile number**

fertile		sterile
0 1 2 3 5 11	**← ultimates →**	***7 13 17 19***
4 10 12 16	**← non-ultimates →**	***6 8 9 14 15 18***

Figure 11: Clear and unequivocal classification of the first twenty whole numbers according to fertility concept. See Figure 8 also. See configuration similarity with Figure 2 Chapter 2.5 according to ultimity concept.

Regardless of considering here the twenty fundamental, this table illustrates the one way of identifying a fertile number or a sterile number. About these twenty fundamentals, of the 10 listed as fertiles, in an exact ratio of 3/2, 6 are ultimate numbers and 4 are non-ultimates. Also in a reverse configuration, of the 10 listed as steriles, in an exact ratio of 2/3, 4 are ultimate numbers and 6 are non-ultimates. Figure 11 is illustrated this transcendent arithmetic arrangement.

4.6 Number genetics concept

Thus, a first approach to the number genetics has just been introduced by the notions of fertility and sterility concerning the faculties of whole numbers to produce, according to the mechanics of Sophie Germain number, entities of the same species or of opposite species (ultimates or non-ultimates). We will now expand the number genetics by studying their *ancestry*.

4.6.1 Number ancestrality concept

After introducing the genetic notions of fertility and sterility of numbers from Sophie Germain number concept expanded to ultimates, we are now going to propose a complete genetic terminology to describe all whole numbers.

About primes, in Sophie Germain mathematics literature, one use safe or unsafe as terms. In order to enrich and finalize the concept of number genetics previously introduced, we therefore propose to speak of the ancestrality of ultimate and non-ultimate numbers and to use, according to their origin (*fx = (x-1)/2*), terms “pure”, “hybrid” or “orphan” to qualify them.

ancestral numbers:	⟵ ⟶ **number genetics mechanism:**	**descendant numbers:**
fertile ultimate (*u*)	**→ *u* + (*u*+1) = *u’* →** **← (*u’* -1)/2 = *u* ←**	**pure ultimate (*u’*)**
sterile non-ultimate (*ʉ*)	**→ *ʉ* + (*ʉ*+1) = *u’* →** **← (*u’* -1)/2 = *ʉ* ←**	**hybrid ultimate (*u’*)**
no ancestral number (x) ***x ≠ as whole number***	→ *x* + (*x*+1) = *u’* → ← (*u’* -1)/2 = *x* ←	**orphan ultimate (*u’*)**
fertile non-ultimate (*ʉ*)	**→ *ʉ* + (*ʉ*+1) = *ʉ’* →** **← (*ʉ’* -1)/2 = *ʉ* ←**	**pure non-ultimate (*ʉ’*)**
sterile ultimate (*u*)	**→ *u* + (*u*+1) = *ʉ’* →** **← (*ʉ’* -1)/2 = *u* ←**	**hybrid non-ultimate (*ʉ’*)**
no ancestral number (x) ***x ≠ as whole number***	*x* + (*x*+1) = *ʉ’* → ← (*ʉ’*-1)/2 = *x*	**orphan non-ultimate (*ʉ’*)**

Figure 12: Sophie Germain genetics mechanism of whole numbers applied to ultimate number concept.

Thus, as explained in Figure 12, any whole number can, depending on its origin (Sophie Germain numerical mechanism expended to all whole number), be either a pure number (as safe number of Sophie Germain), or a hybrid (safe) or even an orphan. Orphan numbers therefore have no ancestor which is a whole number. in fact, all the even numbers are therefore, by this numerical genetics, orphans.

Figure 13 explain the number genetics mechanism with some representative whole numbers as example.

ancestral numbers:	⟵ ⟶ **number genetics mechanism:**	**descendant numbers:**
fertile ultimate **1**	**→ 1 + (1+1) = 3 →** **← (3-1)/2 = 1 ←**	**pure ultimate** **3**
sterile non-ultimate **6**	**→ 6 + (6+1) = 13 →** **← (13 -1)/2 = 6 ←**	**hybrid ultimate** **13**
no ancestral number ***0,5 ≠ as whole number***	→ 0,5 + (0,5+1) = 2 → ← (2 -1)/2 = 0,5 ←	**orphan ultimate** **2**
fertile non-ultimate **4**	**→ 4 + (4+1) = 9 →** **← (9 -1)/2 = 4 ←**	**pure non-ultimate** **9**
sterile ultimate **7**	**→ 7 + (7+1) = 15 →** **← (15 -1)/2 = 7 ←**	**hybrid non-ultimate 15**
no ancestral number ***1,5 ≠ as whole number***	1,5 + (1,5+1) = 4 → ← (4-1)/2 = 1,5	**orphan non-ultimate** **4**

Figure 13: Sophie Germain genetic mechanism applied to ultimate numbers concept with some whole numbers as example. See Figure 12 also.

4.6.2 Complet number genetics concept

Thus, as well illustrated in Figure 14, it is therefore possible to propose a complete mechanism of number genetics. Thus, by applying and extending Sophie Germain number concept to the entire set ℕ, all whole number therefore have a complete and unique genetic scheme with ancestry and descent. This, resembling, a biological genetic organization.

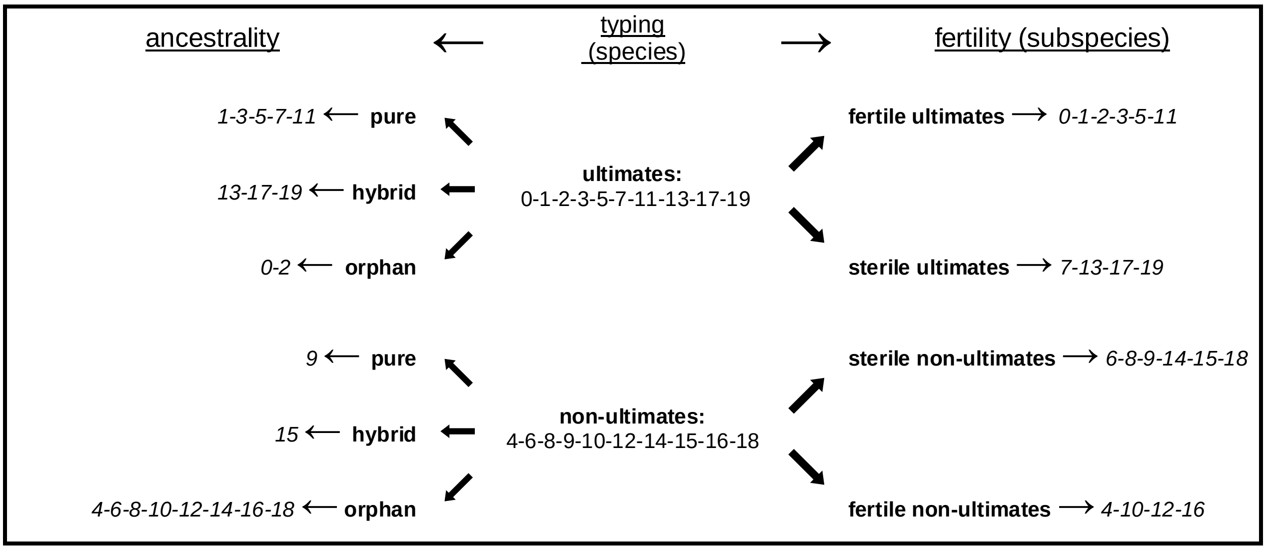


Figure 14: Numeric genetics of the twenty fundamental numbers: ancestrality, typing and fertility.

In complement to Figure 14, and in order to clarify the understanding of this new concept of number genetics, table in Figure 15 lists with many detail, the numbers genetics of the twenty fundamentals (first twenty whole numbers).

Number genetics:			**ancestral number (n-1)/2**		←**n**→		**descendant number n + (n+1)**
orphan	fertile	ultimate	*-0,5*	⟵	**0**	⟶	**1**
pure	fertile	ultimate	**0**	⟵	**1**	⟶	**3**
orphan	fertile	ultimate	*0,5*	⟵	**2**	⟶	**5**
pure	fertile	ultimate	**1**	⟵	**3**	⟶	**7**
orphan	fertile	non-ultimate	*1,5*	⟵	**4**	⟶	**9**
pure	fertile	ultimate	**2**	⟵	**5**	⟶	**11**
orphan	sterile	non-ultimate	*2,5*	⟵	**6**	⟶	**13**
pure	sterile	ultimate	**3**	⟵	**7**	⟶	**15**
orphan	sterile	non-ultimate	*3,5*	⟵	**8**	⟶	**17**
pure	sterile	non-ultimate	**4**	⟵	**9**	⟶	**19**
orphan	fertile	non-ultimate	*4,5*	⟵	**10**	⟶	**21**
pure	fertile	ultimate	**5**	⟵	**11**	⟶	**23**
orphan	fertile	non-ultimate	*5,5*	⟵	**12**	⟶	**25**
hybrid	sterile	ultimate	**6**	⟵	**13**	⟶	**27**
orphan	sterile	non-ultimate	*6,5*	⟵	**14**	⟶	**29**
hybrid	sterile	non-ultimate	**7**	⟵	**15**	⟶	**31**
orphan	fertile	non-ultimate	*7,5*	⟵	**16**	⟶	**33**
hybrid	sterile	ultimate	**8**	⟵	**17**	⟶	**35**
orphan	sterile	non-ultimate	*8,5*	⟵	**18**	⟶	**37**
hybrid	sterile	ultimate	**9**	⟵	**19**	⟶	**39**

Figure 15: Number genetics of the twenty fundamentals.

According to number genetics, there are therefore twelve possible combinations to define all whole numbers : 3 ancestral situations by 2 types of numbers (species) by 2 descendants possibilities. Among these twelve possibilities, the orphan-sterile-ultimate combination does not apply to any of the whole numbers. It therefore does not appear in this table listing the twenty fundamentals. Three other genetic configurations do not appear here for these first twenty numbers either but they are quickly found in the next whole numbers:

25 ⟶ pure, sterile non-ultimates
27 ⟶ hybrid, fertile non-ultimate
29 ⟶ hybrid, fertile ultimate

Thus, these three missing configurations for the twenty fundamentals, not probably appear by chance in three consecutive odd numbers: 25-27-29.

5. Highlighting of twenty fundamental numbers

5.1 The first ten fertile numbers and the first ten sterile numbers

By the Sophie Germain's expanded numeric-genetic mechanism previously introduced (Chapter 4.3), the sequence of fertile numbers is initialized by these ten numbers:

0 1 2 3 4 5 10 11 12 16

Again, also by this genetic numerical concept, the sequence of sterile numbers is initialized by these ten numbers:

6 7 8 9 13 14 15 17 18 19

Thus, just as the first 10 ultimates and the first 10 non-ultimates represent exactly the first twenty numbers, the first 10 fertiles and the first 10 steriles constitute the same set. In fact, it will turn out that this twice set of 10 entities of equal fertility and sterility, transcends a lot of other fundamental attributes qualifying the whole numbers.

Indeed, according to their digitality or non digitality, their ultimity or non-ultimity and indeed to their orphanity or non orphanity (pure or hybrid numbers), in always an exact distribution of 3/2 value ratio, these 10 fertile and 10 sterile numbers are differentiated. In this regard, Figure 16, which illustrates these transcendences, needs no comment.

The ten first few fertile numbers ↘

↙ **The ten first few sterile numbers**

6 digits	0 1 2 3 4 5	**← 3/2 ratio →**	6 7 8 9	**4 digits**
	↑ 3/2 ratio ↓		**↑ 2/3 ratio ↓**	
4 non-digits	10 11 12 16	**← 2/3 ratio →**	13 14 15 17 18 19	**6 non-digits**

6 ultimates	0 1 2 3 5 11	**← 3/2 ratio →**	7 13 17 19	**4 ultimates**
	↑ 3/2 ratio ↓		**↑ 2/3 ratio ↓**	
4 non-ultimates	4 10 12 16	**← 2/3 ratio →**	6 8 9 14 15 18	**6 non-ultimates**

6 orphans	0 2 4 10 12 16	**← 3/2 ratio →**	6 8 14 18	**4 orphans**
	↑ 3/2 ratio ↓		**↑ 2/3 ratio ↓**	
4 non-orphans	1 3 5 11	**← 2/3 ratio →**	7 9 13 15 17 19	**6 non-orphans**

Figure 16: Transcendent distribution of twin sets of first 10 fertile and sterile numbers in exact ratios of 3/2 value according to several numerical and genetic criteria.

5.2 Entanglement of the twenty fundamentals

Because the first twenty numbers therefore have singular numerical arrangements, we therefore qualify them as fundamental numbers among the whole numbers. Also, by convention, we now use the term "fundamentals" to refer to them. Figure 17 describes the total entanglement of these twenty fundamental numbers at the whole numbers sequence beginning.

Also, we strongly draw the reader's attention to this numerical entanglement singularity which occurs at the initial segment of set ℕ. This singularity does not appear in any way to fun numerology as the next investigations will confirm.

	digitality		
fertility	**0 1 2 3 4 5**	**6 7 8 9**	sterility
	10 11 12 16	*13 14 15 17 18 19*	
	non digitality		

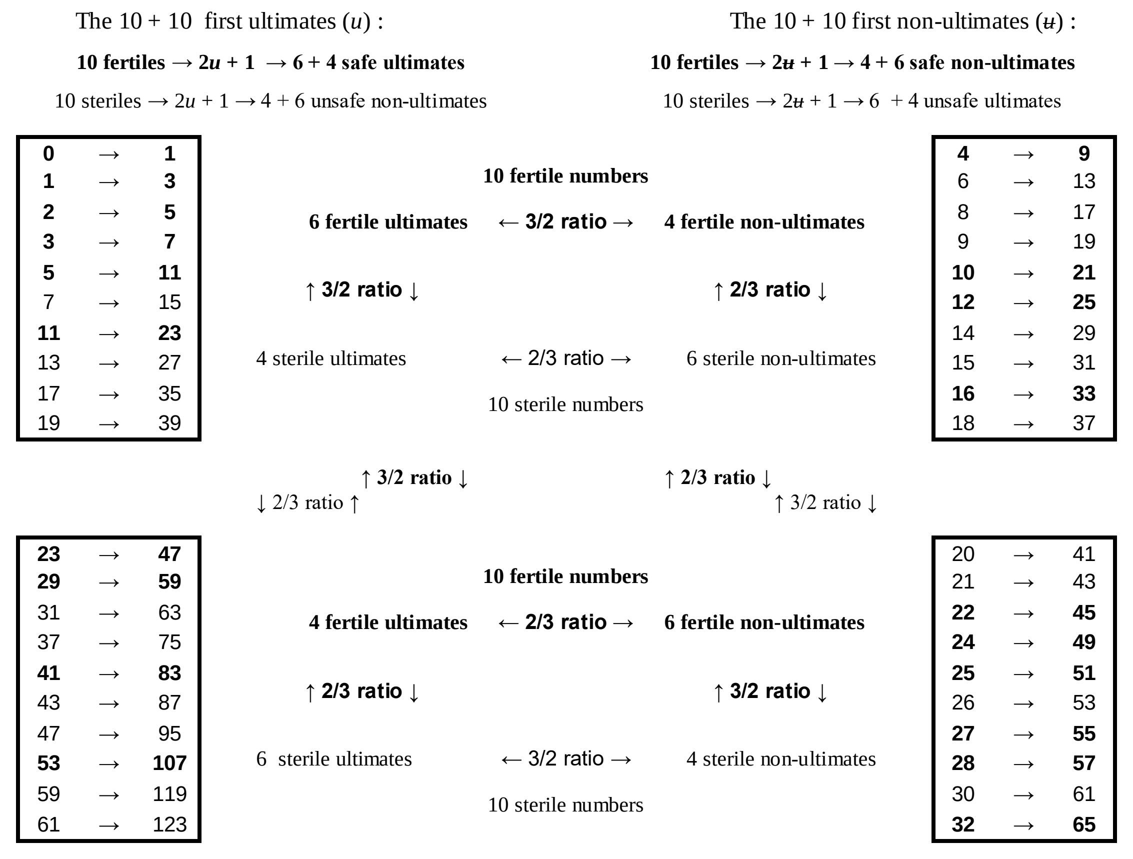
	ultimity		
fertility	**0 1 2 3 5 11**	**7 13 17 19**	sterility
	4 10 12 16	*6 8 9 14 15 18*	
	non ultimity		

	orphanity		
fertility	**0 2 4 10 12 16**	**6 8 14 18**	sterility
	1 3 5 11	*7 9 13 15 17 19*	
	non orphanity		

Figure 17: Triple entanglement of the twenty fundamental numbers according: 1) to their fertility or sterility and to their digitality or non-digitality, 2) to their fertility or sterility and to their ultimity or non ultimity, 3) to their fertility or sterility and to their orphanity or non orphanity.

5.3 Fertility entanglement

It turns out that among the twenty fundamental numbers (called *fundamentals*), which are simultaneously the first 20 numbers but also the first 10 ultimates and the first ten non-ultimates, are 50% of fertile numbers of which, in a ratio of 3/2 value, six fertile ultimates versus four fertile non-ultimates.



The 10 + 10 first ultimates (u) :

10 fertiles → 2u + 1 → 6 + 4 safe ultimates

10 steriles → 2u + 1 → 4 + 6 unsafe non-ultimates

The 10 + 10 first non-ultimates ($\cancel{u}$) :

10 fertiles → 2$\cancel{u}$ + 1 → 4 + 6 safe non-ultimates

10 steriles → 2$\cancel{u}$ + 1 → 6 + 4 unsafe ultimates

0	→	**1**
1	→	**3**
2	→	**5**
3	→	**7**
5	→	**11**
7	→	15
11	→	**23**
13	→	27
17	→	35
19	→	39

10 fertile numbers

6 fertile ultimates ← **3/2 ratio** → **4 fertile non-ultimates**

↑ **3/2 ratio** ↓ ↑ **2/3 ratio** ↓

4 sterile ultimates ← 2/3 ratio → 6 sterile non-ultimates

10 sterile numbers

4	→	**9**
6	→	13
8	→	17
9	→	19
10	→	**21**
12	→	**25**
14	→	29
15	→	31
16	→	**33**
18	→	37

↑ **3/2 ratio** ↓ ↑ **2/3 ratio** ↓

↓ 2/3 ratio ↑ ↑ 3/2 ratio ↓

23	→	**47**
29	→	**59**
31	→	63
37	→	75
41	→	**83**
43	→	87
47	→	95
53	→	**107**
59	→	119
61	→	123

10 fertile numbers

4 fertile ultimates ← **2/3 ratio** → **6 fertile non-ultimates**

↑ **2/3 ratio** ↓ ↑ **3/2 ratio** ↓

6 sterile ultimates ← 3/2 ratio → 4 sterile non-ultimates

10 sterile numbers

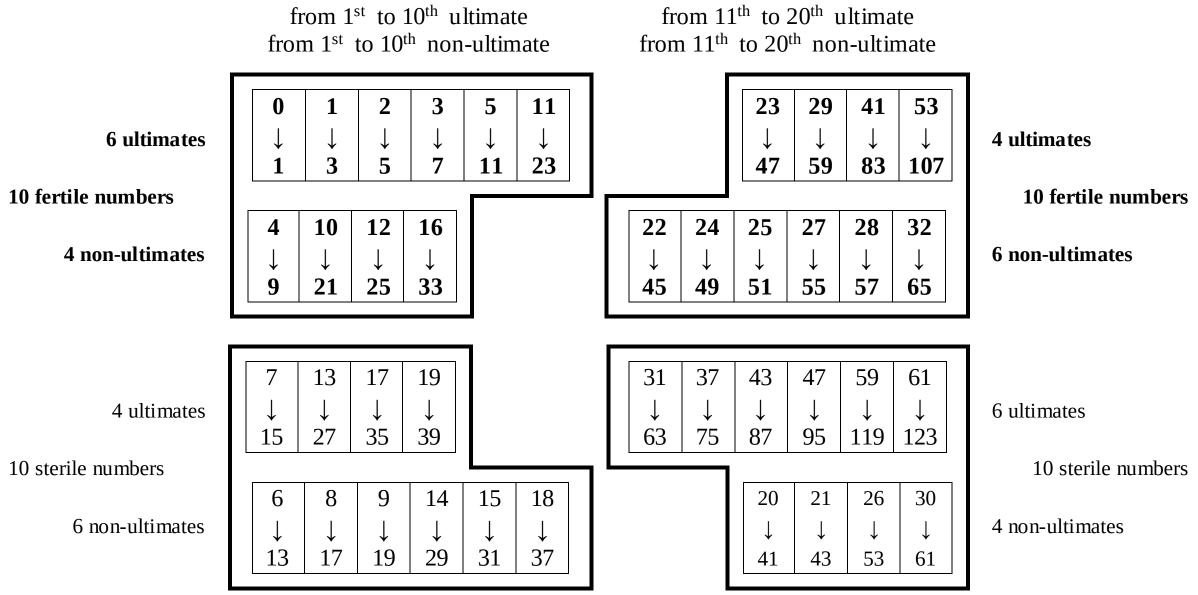
20	→	41
21	→	43
22	→	**45**
24	→	**49**
25	→	**51**
26	→	53
27	→	**55**
28	→	**57**
30	→	61
32	→	**65**

Figure 18: Fertility entanglement (from safety Sophie Germain concept) of twice 10 first ultimates and twice 10 first non-ultimates in a great many of 3/2 and reverse 2/3 ratios. See Figure 19 also.

As illustrated in the upper part of Figure 18, many transcendent ratios of 3/2 value (or reverse ratio to 2/3 value) thus operate according to the different natures of the entities constituting this set of twenty fundamental numbers. For example, and as cause consequence, there are 4 sterile ultimates versus 6 sterile non-ultimates among these twenty fundamentals.

Also (lower part of Figure 18), the group of twenty other numbers made up of the 10 ultimates and the 10 non-ultimates directly following of the entities of the previous group of the twenty fundamentals is organized in exactly inverse ratios according to the same natures considered: fertile or sterile ultimates, fertile or sterile non-ultimates. This is all the more remarkable since, differently from the first group of the twenty fundamentals made up of the first 20 numbers (from 0 to 19), this last group is not made up of the following twenty numbers (from 20 to 40) but just by the next 10 ultimates and the next 10 non-ultimates.

Thus, according to the concept of purity (safety of Sophie Germain) applied here as well to the ultimate numbers as to the non-ultimate numbers, we can observe a very strong entanglement between the four double groups of whole numbers made up of twice the first 10 ultimates and twice first 10 non-ultimates and, transversely, of twice the first 10 fertile or sterile ultimates and of twice first 10 fertile or sterile non-ultimates. Figure 19, which complements the previous one, illustrates this entanglement of the safety of the first 20 ultimates and first 20 non-ultimates from another angular mapping.



from 1st to 10th ultimate
from 1st to 10th non-ultimate

from 11th to 20th ultimate
from 11th to 20th non-ultimate

10 fertile numbers (from 1st to 10th)

6 ultimates

0	**1**	**2**	**3**	**5**	**11**
↓	↓	↓	↓	↓	↓
1	**3**	**5**	**7**	**11**	**23**

4 non-ultimates

4	**10**	**12**	**16**
↓	↓	↓	↓
9	**21**	**25**	**33**

10 fertile numbers (from 11th to 20th)

4 ultimates

23	**29**	**41**	**53**
↓	↓	↓	↓
47	**59**	**83**	**107**

6 non-ultimates

22	**24**	**25**	**27**	**28**	**32**
↓	↓	↓	↓	↓	↓
45	**49**	**51**	**55**	**57**	**65**

10 sterile numbers (from 1st to 10th)

4 ultimates

7	13	17	19
↓	↓	↓	↓
15	27	35	39

6 non-ultimates

6	8	9	14	15	18
↓	↓	↓	↓	↓	↓
13	17	19	29	31	37

10 sterile numbers (from 11th to 20th)

6 ultimates

31	37	43	47	59	61
↓	↓	↓	↓	↓	↓
63	75	87	95	119	123

4 non-ultimates

20	21	26	30
↓	↓	↓	↓
41	43	53	61

Figure 19: Fertility entanglement (from safety Sophie Germain concept) of twice first 10 ultimates and twice first 10 non-ultimates. See Figure 18 also.

5.4 Addition matrix of the first few ten fertile and few ten sterile numbers

the first ten sterile numbers

	+	**6**	**7**	**8**	**9**	**13**	**14**	**15**	**17**	**18**	**19**
	0	6	7	8	9	13	14	15	17	18	19
	1	7	8	9	**10**	14	15	**16**	18	19	20
	2	8	9	**10**	**11**	15	**16**	17	19	20	21
	3	9	**10**	**11**	**12**	**16**	17	18	20	21	**22**
the first ten fertile numbers	**4**	**10**	**11**	**12**	13	17	18	19	21	**22**	**23**
	5	**11**	**12**	13	14	18	19	20	**22**	**23**	**24**
	10	**16**	17	18	19	**23**	**24**	**25**	**27**	**28**	**29**
	11	17	18	19	20	**24**	**25**	26	**28**	**29**	30
	12	18	19	20	21	**25**	26	**27**	**29**	30	31
	16	**22**	**23**	**24**	**25**	**29**	30	31	33	**34**	35

x = 6 →	1x+2x+3x+4x =	60 steriles	→	6	12	18	24
↑ 3/2 ratio ↓		↑ 3/2 ratio ↓		↑3/2↓	↑3/2↓	↑3/2↓	↑3/2↓
x = 4 →	**1x+2x+3x+4x =**	**40 fertiles**	→	**4**	**8**	**12**	**16**
	3/2 ratio →	36/24 steriles		36			24
				↑ 3/2 ratio ↓			↑ 3/2 ratio ↓
	3/2 ratio →	**24/16 fertiles**		**24**			**16**

Figure 20: Cross additions table of the first ten fertile and first ten sterile numbers identified as twenty fundamentals.

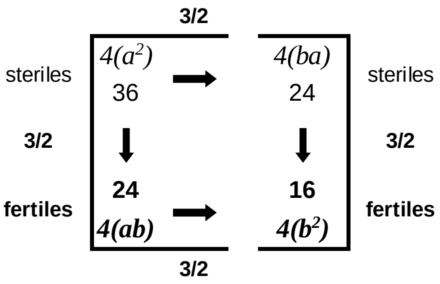
As we have demonstrated extensively previously, the first twenty whole numbers correspond to the first ten fertile and the first ten sterile. In Figure 20, a cross additions table of these first 10 fertile and these first 10 sterile is constructed.

As illustrated by the addition table in Figure 20, the results of these cross additions between these first ten fertile numbers and these first ten sterile numbers generate one hundred numbers of which, in a proportion of value ratio 3/2, 60 sterile numbers versus 40 fertile numbers.

Also, in a regular geometric progression of one then two then three and finally four columns, the proportion of sterile and fertile numbers always remains the same with 3*x* sterile versus 2*x* fertile.

5.4.1 Addition matrix and remarkable identity

This progressive geometric arrangement introduced in the addition table of 10 fertile and 10 sterile allows the creation of two sub-matrices of size 3*x* versus 2*x* entities, i.e. 60 numbers (1 + 2 + 3 columns) versus 40 (4 columns). As shown in Figure 21, within these two sub-matrices opposing each other in a ratio of 3/2 as value, fertile and sterile numbers also oppose each other in this ratio of 3/2 as values.



6	7	8	9	13	14	15	17	18	19
7	8	9	**10**	14	15	**16**	18	19	20
8	9	**10**	**11**	15	**16**	17	19	20	21
9	**10**	**11**	**12**	**16**	17	18	20	21	**22**
10	**11**	**12**	13	17	18	19	21	**22**	**23**
11	**12**	13	14	18	19	20	**22**	**23**	**24**
16	17	18	19	**23**	**24**	**25**	**27**	**28**	**29**
17	18	19	20	**24**	**25**	26	**28**	**29**	30
18	19	20	21	**25**	26	**27**	**29**	30	31
22	**23**	**24**	**25**	**29**	30	31	33	**34**	35

36 stériles
24 fertiles

24 stériles
16 fertiles

Figure 21: Remarkable identity revelled in le cross additions matrix of the first ten fertile and first ten sterile numbers. See Figure 20 also.

Thus, in these two sub-matrices, respectively, 36 entities oppose 24 others and then 24 entities oppose 16 others. The arithmetic arrangements of these four values (36→24→24→16) are some variants of the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* have 3 and 2 as respective value, primary values opposing in the 3/2 ratio. In some others chapters similar arrangements connected to this remarkable identity will be observed for other matrices of 5*x* entities depending on the nature of the numbers considered.

Already, just with these last two sub-matrices introduced, a singular geometric arrangement, Figure 22, generates the same distribution of numbers qualified as fertile or sterile in a variant of this remarkable identity.

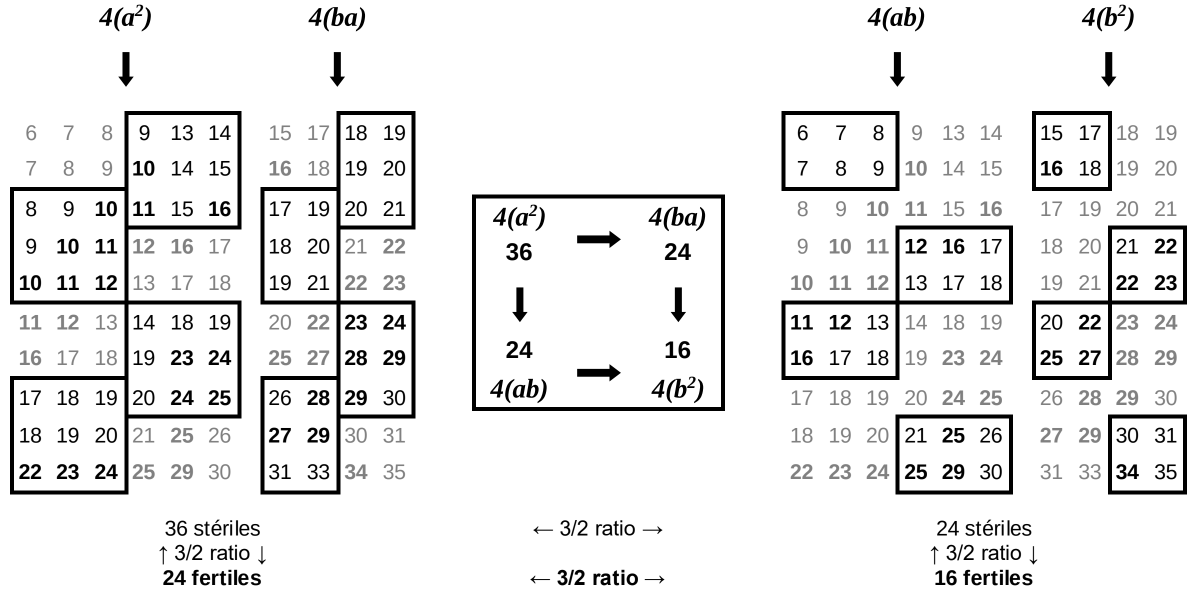


Figure 22: Remarkable identity revelled in the cross additions matrix of the first ten fertile and first ten sterile numbers. See Figures 21 and 23 also.

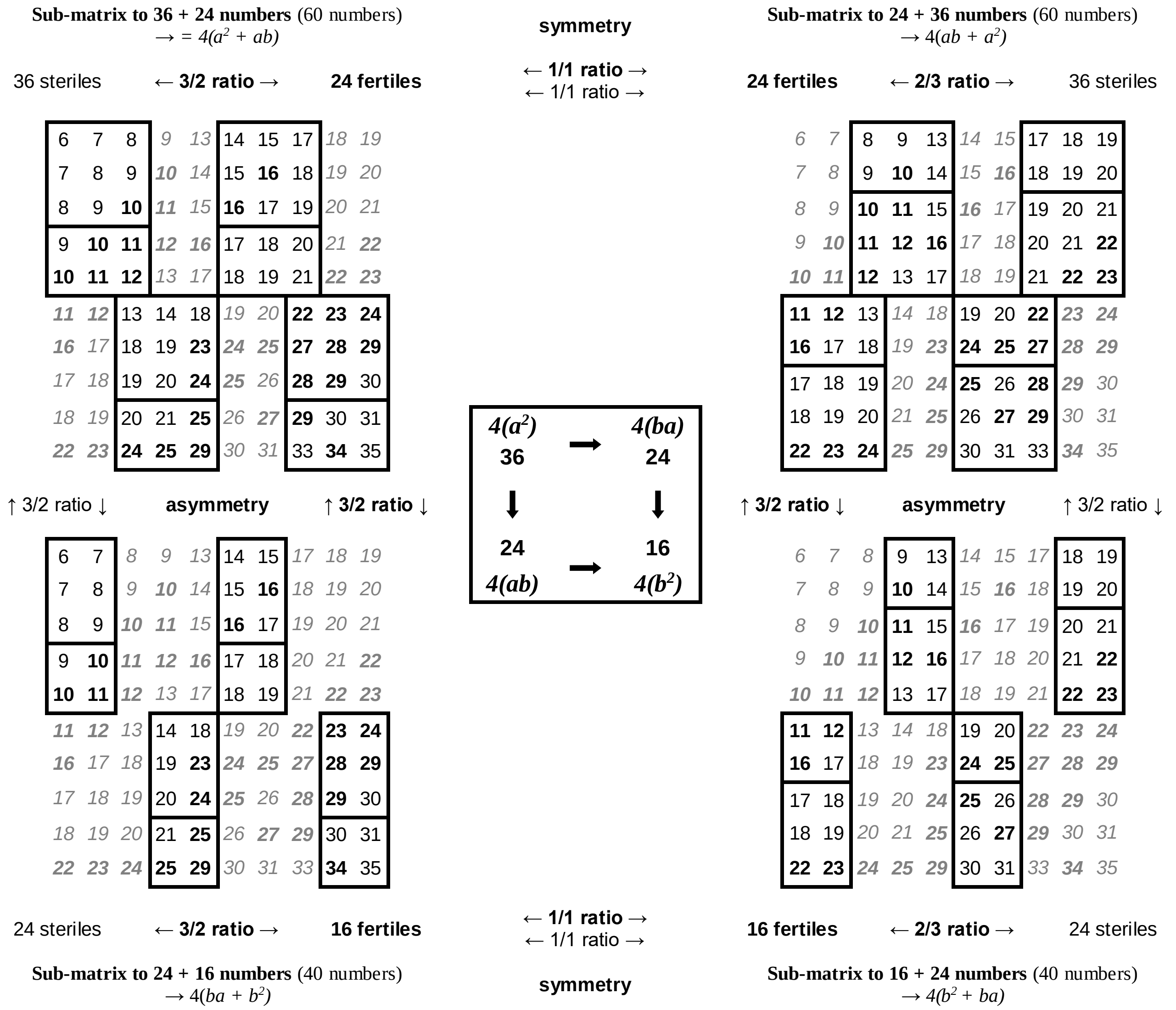
5.4.2 Addition matrix, symmetry and remarkable identity

Figure 23, from the cross additions table of the first ten fertile and first ten sterile, an other singular and similar geometric arrangement generates the same distribution of numbers we have qualified as fertile or sterile in a variant of the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$.

Here, the phenomenon manifests itself by considering two twice symmetrical sub-matrices of size 3*x* versus 2*x* entities. Also, these sub-matrices are assemblies of zones which have the size of the four components of this remarkable identity. So these sub-matrices are assemblies of zones of 9→6→6→4 numbers or $a^2 + ab + ba + b^2$ with a and b of 3 and 2 as respective value.

From the cross additions table of the first ten fertile and first ten sterile numbers identified as twenty fundamentals:

Two twice symmetrical semi alternating sub-matrices of 60 versus 40 numbers



Sub-matrix to 36 + 24 numbers (60 numbers)
→ = $4(a^2 + ab)$

symmetry

Sub-matrix to 24 + 36 numbers (60 numbers)
→ $4(ab + a^2)$

36 steriles **← 3/2 ratio →** **24 fertiles**

← 1/1 ratio →
← 1/1 ratio →

24 fertiles **← 2/3 ratio →** 36 steriles

6	7	8	*9*	*13*	14	15	17	*18*	*19*
7	8	9	***10***	*14*	15	**16**	18	*19*	*20*
8	9	**10**	***11***	*15*	**16**	17	19	*20*	*21*
9	**10**	**11**	***12***	***16***	17	18	20	*21*	***22***
10	**11**	**12**	*13*	*17*	18	19	21	***22***	***23***
11	***12***	13	14	18	*19*	*20*	**22**	**23**	**24**
16	*17*	18	19	**23**	***24***	***25***	**27**	**28**	**29**
17	*18*	19	20	**24**	***25***	*26*	**28**	**29**	30
18	*19*	20	21	**25**	*26*	***27***	**29**	30	31
22	***23***	**24**	**25**	**29**	*30*	*31*	33	**34**	35

6	*7*	8	9	13	*14*	*15*	17	18	19
7	*8*	9	**10**	14	*15*	***16***	18	19	20
8	*9*	**10**	**11**	15	***16***	*17*	19	20	21
9	***10***	**11**	**12**	**16**	*17*	*18*	20	21	**22**
10	***11***	**12**	13	17	*18*	*19*	21	**22**	**23**
11	**12**	13	*14*	*18*	19	20	**22**	***23***	***24***
16	17	18	*19*	***23***	**24**	**25**	**27**	***28***	***29***
17	18	19	*20*	***24***	**25**	26	**28**	***29***	*30*
18	19	20	*21*	***25***	26	**27**	**29**	*30*	*31*
22	**23**	**24**	***25***	***29***	30	31	33	***34***	*35*

$$\begin{tikzcd}
\begin{matrix}4(a^2)\\36\end{matrix} \arrow[r] \arrow[d] & \begin{matrix}4(ba)\\24\end{matrix} \arrow[d] \\
\begin{matrix}24\\4(ab)\end{matrix} \arrow[r] & \begin{matrix}16\\4(b^2)\end{matrix}
\end{tikzcd}$$

↑ 3/2 ratio ↓ **asymmetry** **↑ 3/2 ratio ↓**

↑ 3/2 ratio ↓ **asymmetry** ↑ 3/2 ratio ↓

6	7	*8*	*9*	*13*	14	15	*17*	*18*	*19*
7	8	*9*	***10***	*14*	15	**16**	*18*	*19*	*20*
8	9	***10***	***11***	*15*	**16**	17	*19*	*20*	*21*
9	**10**	***11***	***12***	***16***	17	18	*20*	*21*	***22***
10	**11**	***12***	*13*	*17*	18	19	*21*	***22***	***23***
11	***12***	*13*	14	18	*19*	*20*	***22***	**23**	**24**
16	*17*	*18*	19	**23**	***24***	***25***	***27***	**28**	**29**
17	*18*	*19*	20	**24**	***25***	*26*	***28***	**29**	30
18	*19*	*20*	21	**25**	*26*	***27***	***29***	30	31
22	***23***	***24***	**25**	**29**	*30*	*31*	*33*	**34**	35

6	*7*	*8*	9	13	*14*	*15*	*17*	18	19
7	*8*	*9*	**10**	14	*15*	***16***	*18*	19	20
8	*9*	***10***	**11**	15	***16***	*17*	*19*	20	21
9	***10***	***11***	**12**	**16**	*17*	*18*	*20*	21	**22**
10	***11***	***12***	13	17	*18*	*19*	*21*	**22**	**23**
11	**12**	*13*	*14*	*18*	19	20	***22***	***23***	***24***
16	17	*18*	*19*	***23***	**24**	**25**	***27***	***28***	***29***
17	18	*19*	*20*	***24***	**25**	26	***28***	***29***	*30*
18	19	*20*	*21*	***25***	26	**27**	***29***	*30*	*31*
22	**23**	***24***	***25***	***29***	30	31	*33*	***34***	*35*

24 steriles **← 3/2 ratio →** **16 fertiles**

← 1/1 ratio →
← 1/1 ratio →

16 fertiles **← 2/3 ratio →** 24 steriles

Sub-matrix to 24 + 16 numbers (40 numbers)
→ $4(ba + b^2)$

symmetry

Sub-matrix to 16 + 24 numbers (40 numbers)
→ $4(b^2 + ba)$

Figure 23: Remarkable identity revelled in the cross additions matrix of the first ten fertile and first ten sterile numbers. See Figures 21 and 22 also.

6. The twenty fundamentals matrix

Distribution of fertiles and steriles inside the matrix of the 20 fundamental numbers reveals, singular phenomena in their respective geometric arrangements. Table in Figure 24 illustrates just one of many more quirky interaction between the distribution of fertile and sterile numbers in this matrix to 4 rows of 5 columns.

From the horizontally matrix of the 20 fundamental numbers:

Super symmetrical sub-matrices to 6 + 4 entities

(10 numbers) → $3x + 2x = 5x$ (with $x = 2$)

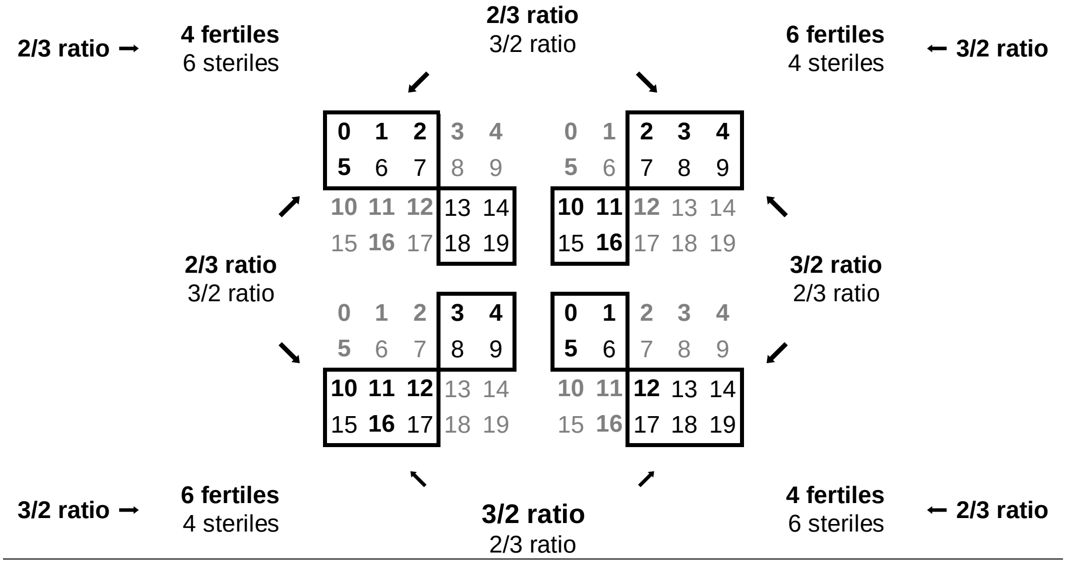
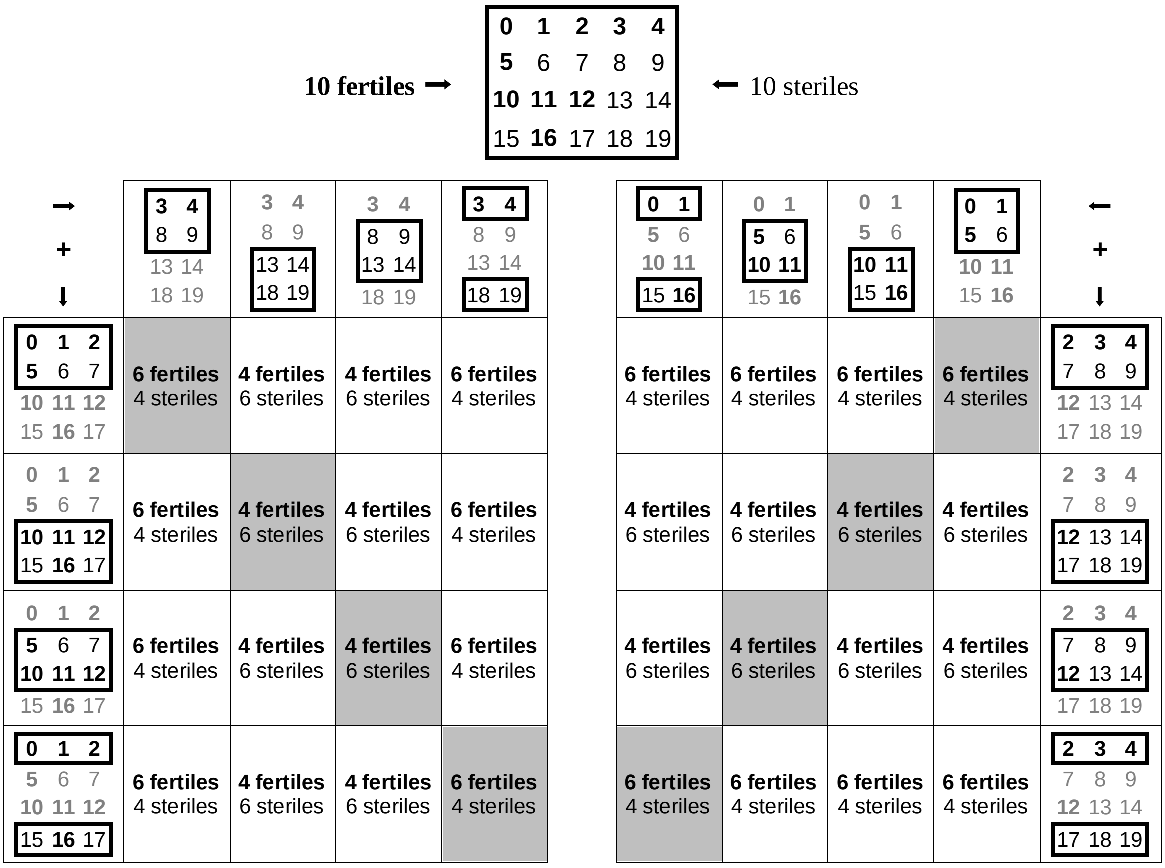


Figure 24: Fertiles and steriles distribution inside four super symmetrical sub-matrices to 6 + 4 entities (10 numbers) from the matrix of the 20 fundamental numbers. See Figure 25 also.

6.1 Twenty fundamentals matrix arrangement

Presentation of phenomena in the form of matrices which is very widely used in this paper has nothing to do with an exercise in numerology. These matrices all have an area equal to $5x$ entities and therefore have at least one side, if not both in some cases, of a dimension to $5x$.

The twenty fundamentals matrix



6 fertiles 4 steriles	4 fertiles 6 steriles	4 fertiles 6 steriles	6 fertiles 4 steriles		6 fertiles 4 steriles	6 fertiles 4 steriles	6 fertiles 4 steriles	6 fertiles 4 steriles
6 fertiles 4 steriles	4 fertiles 6 steriles	4 fertiles 6 steriles	6 fertiles 4 steriles		4 fertiles 6 steriles	4 fertiles 6 steriles	4 fertiles 6 steriles	4 fertiles 6 steriles
6 fertiles 4 steriles	4 fertiles 6 steriles	4 fertiles 6 steriles	6 fertiles 4 steriles		4 fertiles 6 steriles	4 fertiles 6 steriles	4 fertiles 6 steriles	4 fertiles 6 steriles
6 fertiles 4 steriles	4 fertiles 6 steriles	4 fertiles 6 steriles	6 fertiles 4 steriles		6 fertiles 4 steriles	6 fertiles 4 steriles	6 fertiles 4 steriles	6 fertiles 4 steriles

Figure 25: From the twenty fundamentals matrix: detailed fertiles versus steriles distribution in 3/2 ratio oppositions inside alls symmetrical sub areas associations to still $3x + 2\,x$ entities. Grey boxes correspond to crossed identical configurations. See Figure 24 also.

In fact, these matrices are therefore more precisely of dimension (3*x* + 2*x*) entities. It is for this reason that these matrices are very often "cut" into sub-areas of 3*x* and 2*x* numbers. Also, what is highlighted in the above Figure 25 and which invests in great detail the organization of the twenty fundamental numbers (which are simultaneously the first 10 fertile and the first 10 sterile numbers) is a revealing illustration that the arithmetic phenomena observed are in no way due to any chance but rather the proof of singular arrangements of the whole numbers according to their own but interactive attributes.

Thus, as presented in Figure 25, within this matrix of the twenty fundamentals defined above, it turns out that all the associations of sub-zones of 6 numbers with those of 4 numbers produce oppositions between the fertile numbers and sterile numbers in ratios of always 3/2 as values, or reversibly of 2/3 values.

These singular arrangements (Figure 25), can be arranged such as to form either symmetric or complementary sub-matrices as illustrated in the next three mapping Figures 26, 27 and 28. These formed sub-matrices, therefore oppose again in various ratios of value 3/2 according to the numerical fertility or sterility of the different entities of which they are made up.

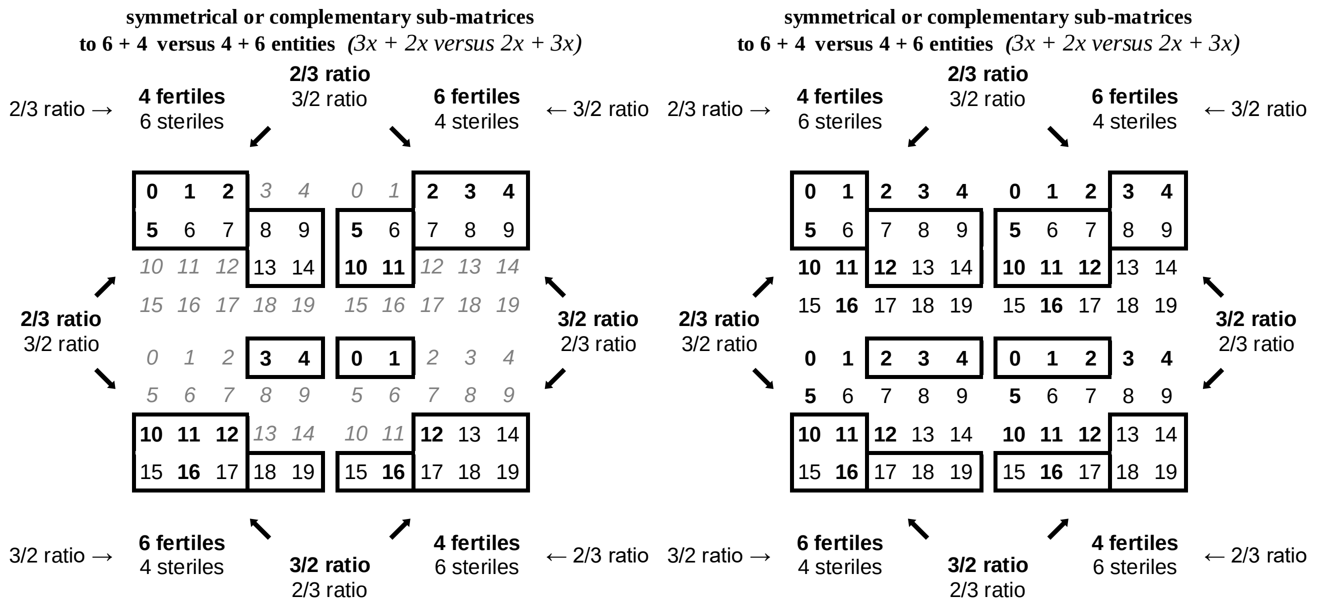


Figure 26: Fertiles and steriles distribution inside many symmetrical or complementary sub-matrices to 6 + 4 entities (10 numbers) from the matrix of the 20 fundamental numbers. See Figure 25 also.

Figure 27 illustrates others arrangements resulting from associations of the various areas of 3*x* and 2*x* entities described in Figure 25.

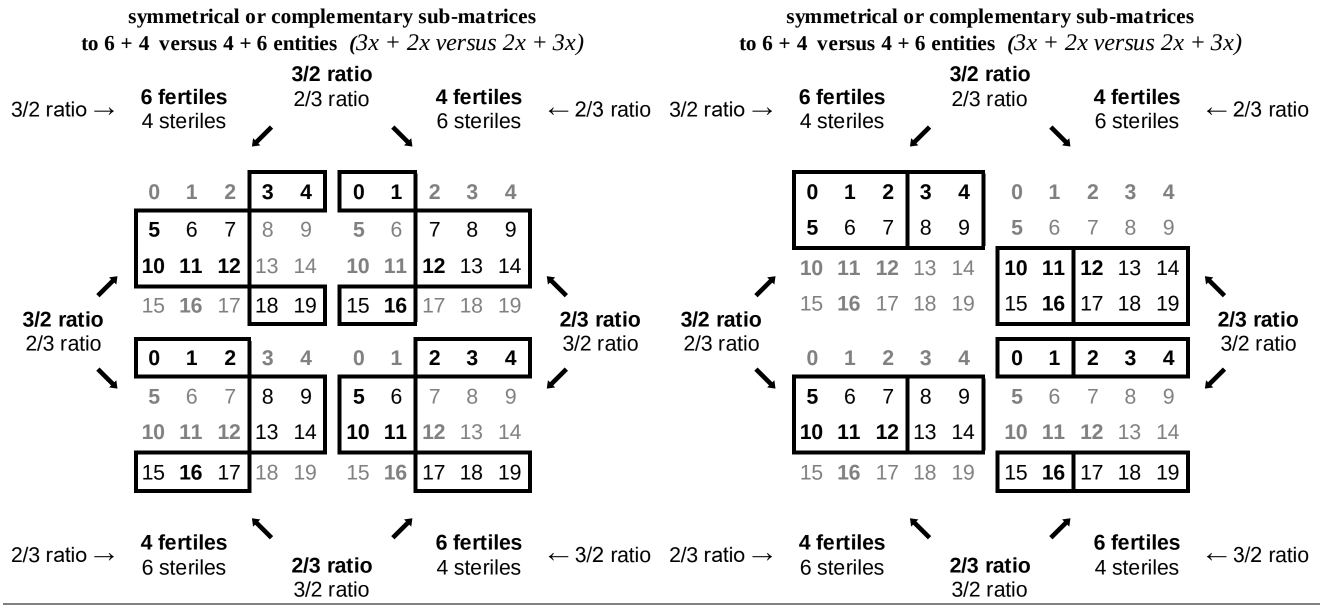


Figure 27: Fertiles and steriles distribution inside many symmetrical or complementary sub-matrices to 6 + 4 entities (10 numbers) from the matrix of the 20 fundamental numbers. See Figure 25 also.

To complet demonstrations of Figures 25, 26 and 27, Figure 28 illustrates others remaining arrangements resulting from associations of the various areas of 3*x* and 2*x* entities described in Figure 25.

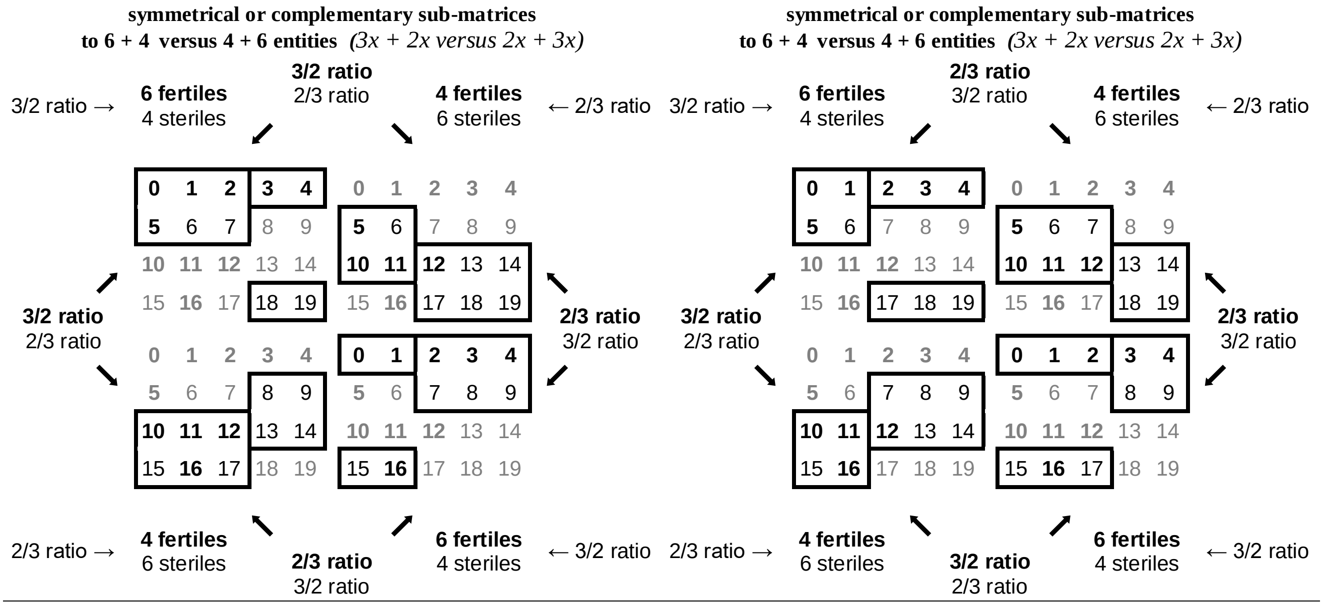


Figure 28: Fertiles and steriles distribution inside many symmetrical or complementary sub-matrices to 6 + 4 entities (10 numbers) from the matrix of the 20 fundamental numbers. See Figure 25 also.

6.2 Other symmetrical configurations

The configurations of 3*x* + 2*x* entities just presented in the previous tables are not the only ones to generate oppositions of values in 3/2 ratios of the quantities of fertile or sterile numbers inside the matrix of the twenty fundamentals.

As shown in Figure 29, zones more cut of only 3*x* and 2*x* entities but nevertheless with reciprocal relations of symmetry or complementarity still generate by their associations, oppositions of values in 3/2 ratio concerning the fertility or sterility of the numbers considered.

From the horizontally matrix of the 20 fundamental numbers:

Symmetrical or complementary sub-matrices to twice 3 + twice 2 entities (10 numbers) → $3x + 2x = 5x$ (with $x = 2$)

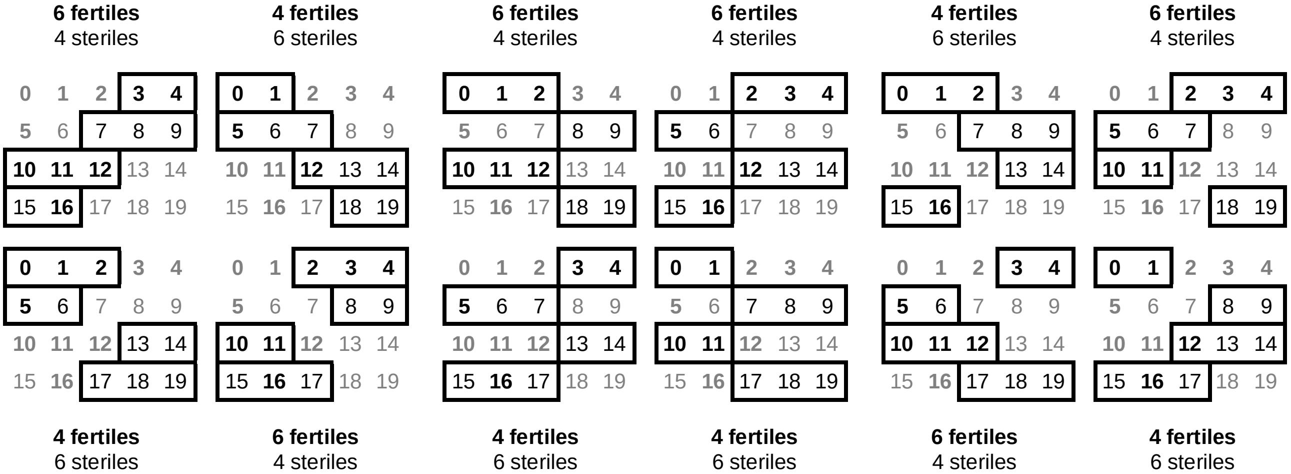


Figure 29: Fertiles and steriles distribution inside many symmetrical or complementary sub-matrices to 6 + 4 entities (**twice 3 + twice 2 entities**) from the matrix of the 20 fundamental numbers. See Figure 30 also.

6.2.1 Sub-matrices construction mechanism

The progressive graphic demonstration illustrated in Figure 30 shows how the different symmetrical zones of 3*x* + 2*x* entities which oppose each other in the proposed mappings in Figure 29 are constructed.

Thus (see Figures 29 and 30), two symmetrical linear zones of 3 entities are associated with two zones of 2 entities that are symmetrical to each other but anti-symmetrical to the zones of 3 entities. This therefore gives sets of 5*x* entities (10 numbers) where the fertiles and the steriles are always opposed in a ratio of 3/2 (or 2/3) as value with 6 fertiles versus 4 steriles or the opposite. And the complementary configuration is of course in reverse ratios.

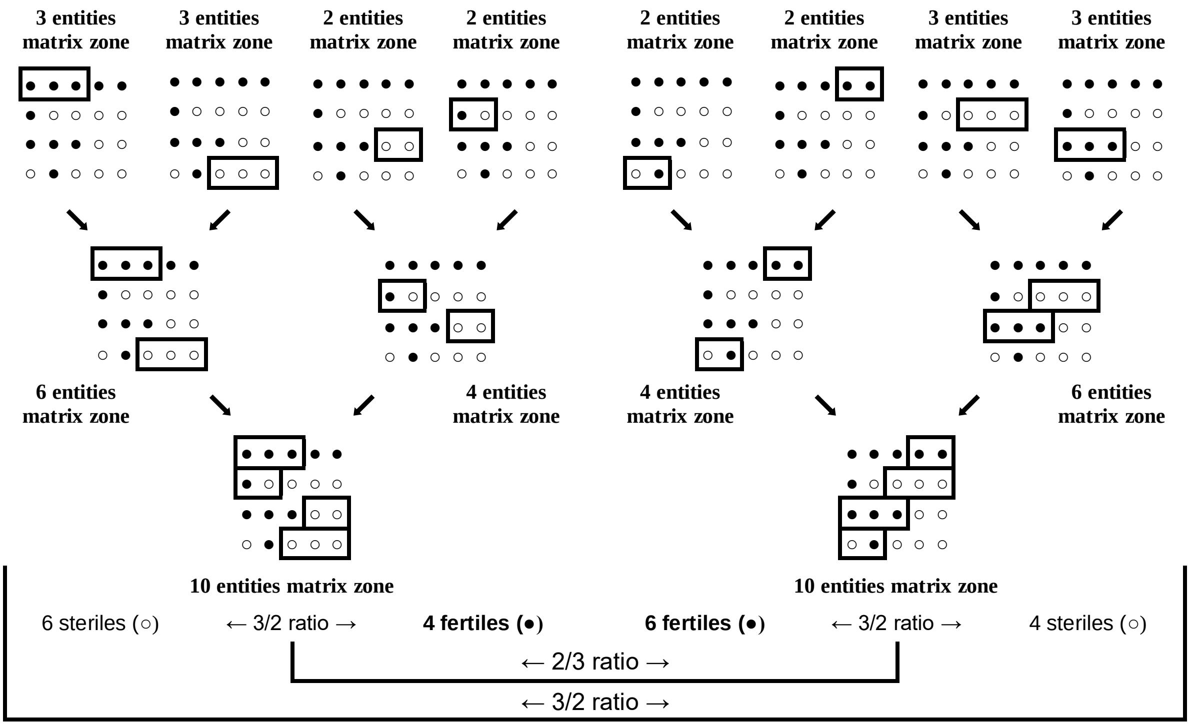


Figure 30: Detailed schematization of the construction of symmetrical and anti-symmetrical zones of 3*x* + 2*x* entities inside the matrix of the twenty fundamentals. (● **fertiles** ○ steriles)

Thus one can affirm that the concept of symmetry inside the matrix of the twenty fundamental numbers has a great importance according the relative distribution of the ten fertiles ones and the ten steriles ones of which it is made up.

6.3 Other symmetric configurations

Still inside twenty fundamentals matrix, many other symmetrical configurations but where the zones are even more cut out generate the same oppositions of fertiles and steriles in various ratios of value 3/2 (or 2/3). Figure 31 illustrates some singular examples of these arithmetical phenomena.

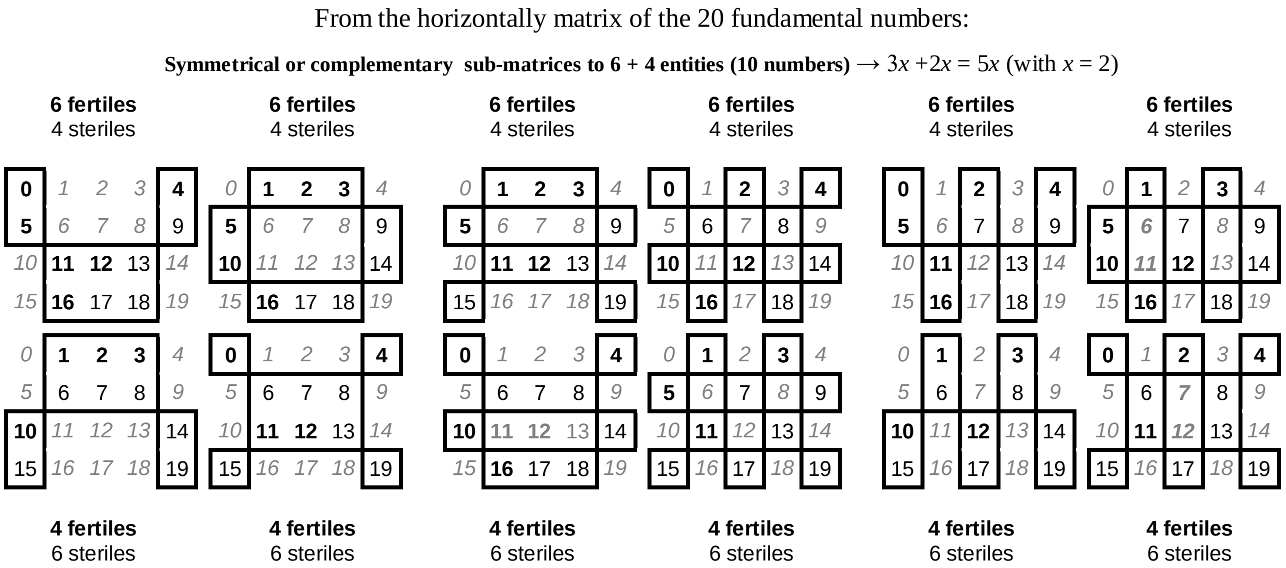


Figure 31: Fertiles and steriles distribution inside some others symmetrical or complementary sub-matrices to 6 + 4 entities (10 numbers) from the matrix of the 20 fundamental numbers.

7. Fertility of the first hundred numbers

The analysis of the matrix of the twenty fundamentals therefore reveals a non-random distribution of its components depending on whether they are fertile or sterile in nature according to the concept developed above in Chapters 4 and 5 introducing the genetics of numbers. We now extend similar investigations to the first hundred whole numbers.

7.1 Matrix of the first hundred numbers

Figure 32, is constructed a matrix of 5x by 5x entities with $x = 2$. So a geometric set of 10 by 10 numbers and we place there, in their order and line by line, the first hundred numbers.

50 fertiles

0	**1**	**2**	**3**	**4**	**5**	6	7	8	9
10	**11**	**12**	13	14	15	**16**	17	18	19
20	21	**22**	**23**	**24**	**25**	26	**27**	**28**	**29**
30	31	**32**	33	**34**	35	36	37	**38**	39
40	**41**	**42**	43	44	**45**	**46**	47	48	**49**
50	51	**52**	**53**	54	**55**	56	**57**	**58**	59
60	61	**62**	63	**64**	65	66	67	68	69
70	71	**72**	73	74	75	**76**	**77**	78	79
80	81	**82**	**83**	**84**	**85**	86	**87**	**88**	**89**
90	**91**	**92**	**93**	**94**	95	96	97	98	99

50 steriles

Figure 32: Matrix of the first 100 whole numbers. Matrix, which is made up of exactly **50 fertiles** and 50 steriles.

Just as the matrix of the twenty fundamentals contains the same quantity of fertiles and steriles, that of the first 100 numbers presents the same *genetic** particularity. There are therefore exactly 50 fertiles and 50 steriles in this matrix at 5x entities.

* It is of course agree that *genetic* or *genetics* terms are about number genetics in totality of this paper.

7.2 Alternating sub-matrices of the first hundred numbers

Creation of alternating sub-matrices within that of the first 100 numbers (from 0 to 99) reveals singular phenomena as shown in Figure 33. Opposite here, frontally and transversely, matrices of 20 entities and 30 entities (2x and 3x entities).

Matrix of the first 100 numbers:

50 fertile numbers ← **1/1 ratio** → 50 sterile numbers

25 fertiles ← **1/1 ratio** → 25 steriles ← **1/1 ratio** → ← 1/1 ratio → 25 steriles ← **1/1 ratio** → **25 fertiles**

0	**1**	**2**	**3**	**4**					
					15	**16**	17	18	19
20	21	**22**	**23**	**24**					
					35	36	37	**38**	39

← 0 to 39 →
(40 numbers)
20 fertiles
20 steriles
1/1 ratio

					5	6	7	8	9
10	**11**	**12**	13	14					
					25	26	**27**	**28**	**29**
30	31	**32**	33	**34**					

10 fertiles ← **1/1 ratio** → 10 steriles ← 1/1 ratio → ← **1/1 ratio** → 10 steriles ← **1/1 ratio** → **10 fertiles**

↑ **2/3 ratio** ↓ ↑ 2/3 ratio ↓ ↑ **2/3 ratio** ↓ ↑ 2/3 ratio ↓ ↑ 2/3 ratio ↓ ↑ **2/3 ratio** ↓

15 fertiles ← **1/1 ratio** → 15 steriles ← **1/1 ratio** → ← 1/1 ratio → 15 steriles ← **1/1 ratio** → **15 fertiles**

40	**41**	**42**	43	44					
					55	56	**57**	**58**	59
60	61	**62**	63	**64**					
					75	**76**	**77**	78	79
80	81	**82**	**83**	**84**					
					95	96	97	98	99

← 40 to 99 →
(60 numbers)
30 fertiles
30 steriles
1/1 ratio

					45	**46**	47	48	**49**
50	51	**52**	**53**	54					
					65	66	67	68	69
70	71	**72**	73	74					
					85	86	**87**	**88**	**89**
90	**91**	**92**	**93**	**94**					

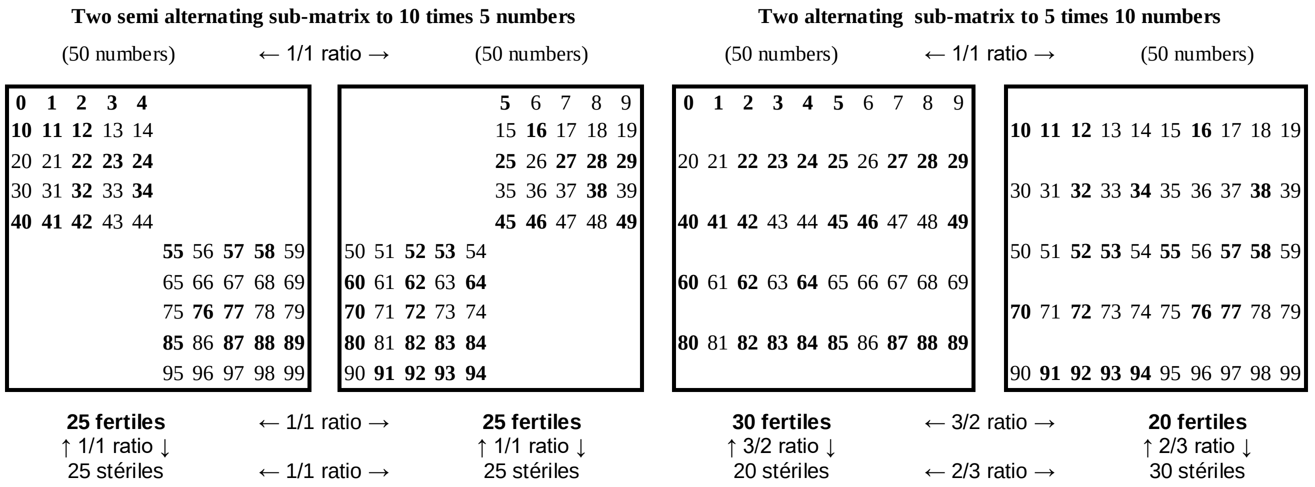
Figure 33: Distribution of the **50 fertile numbers** and the 50 sterile numbers in four alternated sub-matrices from the matrix of the first 100 numbers.

So, of the first 100 whole numbers, exactly 50 are fertile numbers and another 50 are sterile numbers. Also, isolation, in a 2/3 value ratio, of the first 40 numbers set and that of the next 60, shows the same split in groups with the same amount of entities. Finally, very sophisticated intricacies of these subsets of numbers appear in the interlacing of packets of always 5 consecutive entities. These intricacies generate numerous oppositions of the groups considered in various ratios of always 3/2 or 1/1 value.

Recall about numbers 0 and 1: All this remarkable arithmetic mechanics (like all the other demonstrations of this study of whole numbers) manifests itself only if it is well agreed that the numbers zero (0) and one (1) are merged with the set of prime numbers by creating a new set called *the set of ultimate numbers* as defined in the article introduction. Without this consideration, all of the demonstrations from this study about whole numbers would be destroyed.

7.3 Others alternating sub-matrices of the first hundred numbers

As shown in the left part of Figure 34, the association of the *north-west* and *south-east* quarters of the matrix of the first 100 numbers generates an equal distribution of fertile and barren with exactly 25 entities listed for each of these two number categories. The association of the opposite *north-east* and *south-west* zones therefore generates the same phenomena with also 25 fertiles and 25 steriles listed.



Two semi alternating sub-matrix to 10 times 5 numbers — **Two alternating sub-matrix to 5 times 10 numbers**

(50 numbers) ← 1/1 ratio → (50 numbers) — (50 numbers) ← 1/1 ratio → (50 numbers)

0	**1**	**2**	**3**	**4**					
10	**11**	**12**	13	14					
20	21	**22**	**23**	**24**					
30	31	**32**	33	**34**					
40	**41**	**42**	43	44					
					55	56	**57**	**58**	59
					65	66	67	68	69
					75	**76**	**77**	78	79
					85	86	**87**	**88**	**89**
					95	96	97	98	99

					5	6	7	8	9
					15	**16**	17	18	19
					25	26	**27**	**28**	**29**
					35	36	37	**38**	39
					45	**46**	47	48	**49**
50	51	**52**	**53**	54					
60	61	**62**	63	**64**					
70	71	**72**	73	74					
80	81	**82**	**83**	**84**					
90	**91**	**92**	**93**	**94**					

0	**1**	**2**	**3**	**4**	**5**	6	7	8	9
20	21	**22**	**23**	**24**	**25**	26	**27**	**28**	**29**
40	**41**	**42**	43	44	**45**	**46**	47	48	**49**
60	61	**62**	63	**64**	65	66	67	68	69
80	81	**82**	**83**	**84**	**85**	86	**87**	**88**	**89**

10	**11**	**12**	13	14	15	**16**	17	18	19
30	31	**32**	33	**34**	35	36	37	**38**	39
50	51	**52**	**53**	54	**55**	56	**57**	**58**	59
70	71	**72**	73	74	75	**76**	**77**	78	79
90	**91**	**92**	**93**	**94**	95	96	97	98	99

25 fertiles	← 1/1 ratio →	**25 fertiles**		**30 fertiles**	← 3/2 ratio →	**20 fertiles**
↑ 1/1 ratio ↓		↑ 1/1 ratio ↓		↑ 3/2 ratio ↓		↑ 2/3 ratio ↓
25 stériles	← 1/1 ratio →	25 stériles		20 stériles	← 2/3 ratio →	30 stériles

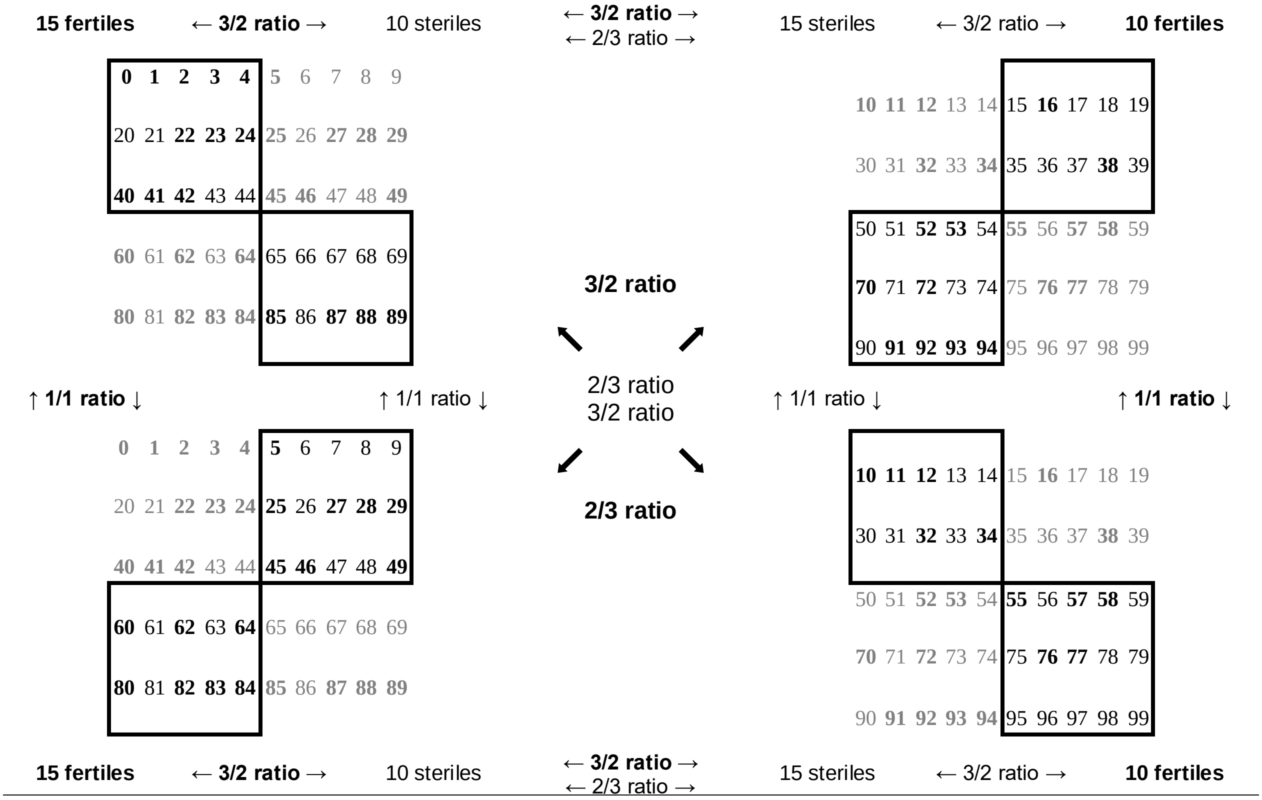
Figure 34: Distribution of the **50 fertile numbers** and the 50 sterile numbers in perfect 1/1 or 3/2 ratios inside four alternate sub-matrices from the matrix of the first 100 numbers.

Transversely to this, the association of 5 times 10 numbers regularly alternated from 10 to 10 entities as described in the right part of Figure 34 generates an opposition of the fertiles and the steriles in crossed ratios of value 3/2 and 2/3. Thus, in one and the other of these configurations, there are respectively 30 fertiles versus 20 steriles and 20 fertiles versus 30 steriles.

7.3.1 Mixed alternating sub-matrices of the first hundred numbers

The mixing of the two types of geometric configurations presented in Figure 34 generates singular arithmetic phenomena within the matrix of the first 100 whole numbers. So, in four semi alternating sub-matrices to 5 times 5 numbers as depicted Figure 35, fertiles and steriles continue to oppose each other in various exact ratios having 1/1 or 3/2 as value

From the matrix of the first 100 numbers : **Four semi alternating sub-matrices to 5 times 5 numbers**



0	**1**	**2**	**3**	**4**	**5**	6	7	8	9
20	21	**22**	**23**	**24**	**25**	26	**27**	**28**	**29**
40	**41**	**42**	43	44	**45**	**46**	47	48	**49**
60	61	**62**	63	**64**	65	66	67	68	69
80	81	**82**	**83**	**84**	**85**	86	**87**	**88**	**89**

10	**11**	**12**	13	14	15	**16**	17	18	19
30	31	**32**	33	**34**	35	36	37	**38**	39
50	51	**52**	**53**	54	**55**	56	**57**	**58**	59
70	71	**72**	73	74	75	**76**	**77**	78	79
90	**91**	**92**	**93**	**94**	95	96	97	98	99

0	**1**	**2**	**3**	**4**	**5**	6	7	8	9
20	21	**22**	**23**	**24**	**25**	26	**27**	**28**	**29**
40	**41**	**42**	43	44	**45**	**46**	47	48	**49**
60	61	**62**	63	**64**	65	66	67	68	69
80	81	**82**	**83**	**84**	**85**	86	**87**	**88**	**89**

10	**11**	**12**	13	14	15	**16**	17	18	19
30	31	**32**	33	**34**	35	36	37	**38**	39
50	51	**52**	**53**	54	**55**	56	**57**	**58**	59
70	71	**72**	73	74	75	**76**	**77**	78	79
90	**91**	**92**	**93**	**94**	95	96	97	98	99

Figure 35: Distribution of the **50 fertile numbers** and the 50 sterile numbers in perfect 1/1 or 3/2 ratios inside four alternate sub-matrices from the matrix of the first 100 numbers. See Figure 34 also.

7.4 Progressive alternating sub matrices of the first hundred numbers

Figure 36, from the matrix of the first hundred numbers, two sub-matrices of 60 and 40 entities are constructed. This is done by isolating sets of one, two, three and four rows of 10 consecutive numbers and then alternately associating two by two these four sets into two larger ones.

Sub-matrices to 1*x* + 2*x* + 3*x* + 4*x* numbers → *x* = 10

2*x* + 4*x* numbers → 6*x* numbers → *x* = 10 **← 3/2 →** **1*x* + 3*x* numbers → 4*x* numbers → *x* = 10**

10 numbers →	**0**	**1**	**2**	**3**	**4**	**5**	6	7	8	9	**0**	**1**	**2**	**3**	**4**	**5**	6	7	8	9	**← 10 numbers**
20 numbers →	**10**	**11**	**12**	13	14	15	**16**	17	18	19	**10**	**11**	**12**	13	14	15	**16**	17	18	19	**← 20 numbers**
	20	21	**22**	**23**	**24**	**25**	26	**27**	**28**	**29**	20	21	**22**	**23**	**24**	**25**	26	**27**	**28**	**29**	
30 numbers →	30	31	**32**	33	**34**	35	36	37	**38**	39	30	31	**32**	33	**34**	35	36	37	**38**	39	**← 30 numbers**
	40	**41**	**42**	43	44	**45**	**46**	47	48	**49**	**40**	**41**	**42**	43	44	**45**	**46**	47	48	**49**	
	50	51	**52**	**53**	54	**55**	56	**57**	**58**	59	50	51	**52**	**53**	54	**55**	56	**57**	**58**	59	
40 numbers →	**60**	61	**62**	63	**64**	65	66	67	68	69	**60**	61	**62**	63	**64**	65	66	67	68	69	**← 40 numbers**
	70	71	**72**	73	74	75	**76**	**77**	78	79	**70**	71	**72**	73	74	75	**76**	**77**	78	79	
	80	81	**82**	**83**	**84**	**85**	86	**87**	**88**	**89**	**80**	81	**82**	**83**	**84**	**85**	86	**87**	**88**	**89**	
	90	**91**	**92**	**93**	**94**	95	96	97	98	99	90	**91**	**92**	**93**	**94**	95	96	97	98	99	

60 numbers →	**30 fertiles** **↑ 1/1 ratio ↓** 30 steriles	**← 3/2 →** ← 3/2 →	**20 fertiles** **↑ 1/1 ratio ↓** 20 steriles	**← 40 numbers**

Figure 36: Distribution of the **50 fertile numbers** and the 50 sterile numbers in perfect 1/1 or 3/2 ratios inside two alternated sub-matrices from the matrix of the first 100 numbers. See Figure 33 also.

In these two new sets of 3/2 as respective side, which are sub-matrices of the original matrix of the first hundred whole numbers, again, the numbers called fertile and those called sterile are distributed in perfect equal quantity.

Also, within these two sub-matrices, their splitting by alternating half-rows of five numbers as illustrated in Figure 37 generates oppositions of the fertiles and the steriles in transcendent ratios of value 3/2 or 2/3.

2*x* + 4*x* numbers = 6*x* numbers → *x* = 5 **2*x* + 4*x* numbers = 6*x* numbers → *x* = 5**

18 fertiles **← 3/2 ratio →** 12 steriles 18 steriles ← 3/2 ratio → **12 fertiles**

					15	**16**	17	18	19		**10**	**11**	**12**	13	14					
					25	26	**27**	**28**	**29**		20	21	**22**	**23**	**24**					
60	61	**62**	63	**64**						**← 3/2 ratio →** ← 2/3 ratio →						65	66	67	68	69
70	71	**72**	73	74												75	**76**	**77**	78	79
80	81	**82**	**83**	**84**												**85**	86	**87**	**88**	**89**
90	**91**	**92**	**93**	**94**												95	96	97	98	99
↑ 3/2 ratio ↓									↑ 3/2 ratio ↓		↑ 3/2 ratio ↓									**↑ 3/2 ratio ↓**
0	**1**	**2**	**3**	**4**												**5**	6	7	8	9
					35	36	37	**38**	39	**← 3/2 ratio →** ← 2/3 ratio →	30	31	**32**	33	**34**					
					45	**46**	47	48	**49**		**40**	**41**	**42**	43	44					
					55	56	**57**	**58**	59		50	51	**52**	**53**	54					

12 fertiles **← 3/2 ratio →** 8 steriles 12 steriles ← 3/2 ratio → **8 fertiles**

1*x* + 3*x* numbers = 4*x* numbers → *x* = 5 **1*x* + 3*x* numbers = 4*x* numbers → *x* = 5**

Figure 37: Distribution of the **50 fertile numbers** and the 50 sterile numbers in transcendent 3/2 ratios inside four alternated sub-matrices from the matrix of the first 100 numbers. See Figure 36 also.

Thus, this matrix of 100 entities generates in a way and in reverse, sets of 18 fertiles versus 12 sterile numbers and of 12 versus 8 fertile or sterile numbers. Like in some previous demonstrations, the arithmetic arrangements of these four values (18→12→12→8) are some variants of the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where *a* and *b* have 3 and 2 as respective value, primary values opposing in the 3/2 ratio.

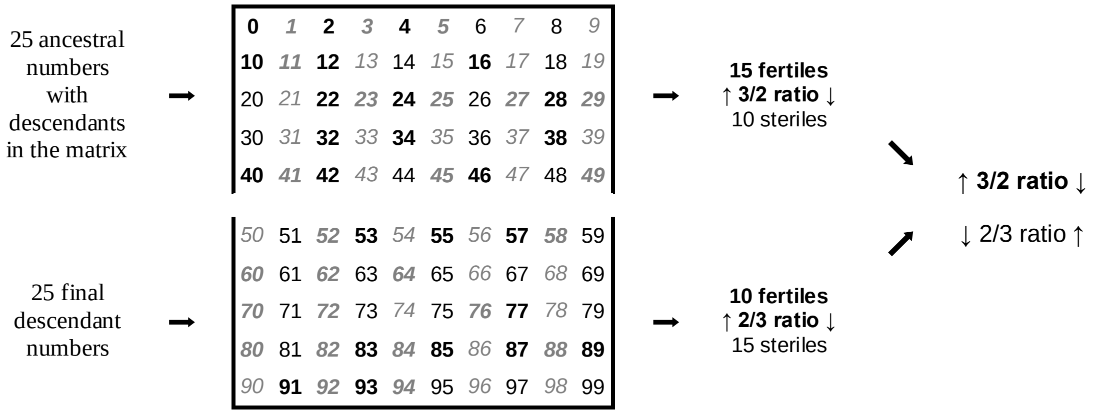
7.5 Matrix of the first hundred numbers and number lineage

Inside matrix of the first hundred numbers, twenty-five numbers have descendants. These descendants are 50 in number, including 25 final descendants. Figure 38 illustrates this.

prime ascendants		→ genetic number lineage →				
0	1	3	7	15	31	**63**
2	5	11	23	47	**95**	
4	9	19	39	**79**		
6	13	27	**55**			
8	17	35	**71**			
10	21	43	**87**			
12	25	**51**				
14	29	**59**				
16	33	**67**				
18	37	**75**				
20	41	**83**				
22	45	**91**				
24	49	**99**				
26	**53**					
28	**57**					
30	**61**					
32	**65**					
34	**69**					
36	**73**					
38	**77**					
40	**81**					
42	**85**					
44	**89**					
46	**93**					
48	**97**					

Figure 38: Lineage of the 25 prime ascendants inside the matrix of the first 100 whole numbers.

As shown in Figure 39, it turns out that among the first 25 ascending numbers (which are therefore the first 25 even numbers), 15 fertile numbers are opposed to 10 sterile numbers. It also appears that, among the 25 final descendants (which correspond to the last 25 odd numbers of the matrix), 10 fertile ones are opposed to 15 sterile ones. Thus, the reciprocal distribution of fertile numbers and sterile numbers in these geometric arrangements is not random but is organized in transcendent ratios of 3/2 as value. Thus, we count exactly 25 fertile and 25 sterile numbers in this configuration.



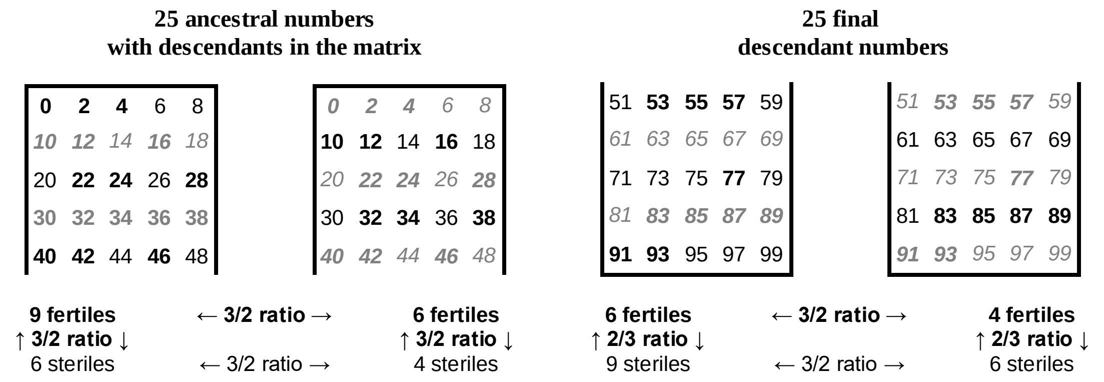
0	***1***	**2**	***3***	**4**	***5***	6	*7*	8	*9*
10	***11***	**12**	*13*	14	*15*	**16**	*17*	18	*19*
20	*21*	**22**	***23***	**24**	***25***	26	***27***	**28**	***29***
30	*31*	**32**	*33*	**34**	*35*	36	*37*	**38**	*39*
40	***41***	**42**	*43*	44	***45***	**46**	*47*	48	***49***

50	51	***52***	**53**	*54*	**55**	*56*	**57**	***58***	59
60	61	***62***	63	***64***	65	*66*	67	*68*	69
70	71	***72***	73	*74*	75	***76***	**77**	*78*	79
80	81	***82***	**83**	***84***	**85**	*86*	**87**	***88***	**89**
90	**91**	***92***	**93**	***94***	95	*96*	97	*98*	99

Figure 39: transcendent distribution of fertile and sterile numbers among the first 25 ascending and final 25 descending of the first 100 numbers matrix.

Also, in more detail, considering in isolation the two sub-matrices of the 25 prime ancestral and the 25 final descendant, it appears that, by alternation of lines as illustrated in Figure 40, sets of fertile numbers and of sterile numbers continue to oppose each other in a transcendent ratio of 3/2 as value.

Once again, this is in connection with the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$, with oppositions of 9 versus 6 entities or 6 versus 4 entities.



25 ancestral numbers with descendants in the matrix

0	**2**	**4**	6	8
10	***12***	*14*	***16***	*18*
20	**22**	**24**	26	**28**
30	**32**	**34**	**36**	**38**
40	**42**	44	**46**	48

0	***2***	***4***	*6*	*8*
10	**12**	14	**16**	18
20	***22***	***24***	*26*	***28***
30	**32**	**34**	36	**38**
40	***42***	*44*	***46***	*48*

9 fertiles **↑ 3/2 ratio ↓** 6 steriles

← **3/2 ratio** →

← 3/2 ratio →

6 fertiles **↑ 3/2 ratio ↓** 4 steriles

25 final descendant numbers

51	**53**	**55**	**57**	59
61	*63*	*65*	*67*	*69*
71	73	75	**77**	79
81	***83***	***85***	***87***	***89***
91	**93**	95	97	99

51	***53***	***55***	***57***	*59*
61	63	65	67	69
71	*73*	*75*	***77***	*79*
81	**83**	**85**	**87**	**89**
91	***93***	*95*	*97*	*99*

6 fertiles **↑ 2/3 ratio ↓** 9 steriles

← **3/2 ratio** →

← 3/2 ratio →

4 fertiles **↑ 2/3 ratio ↓** 6 steriles

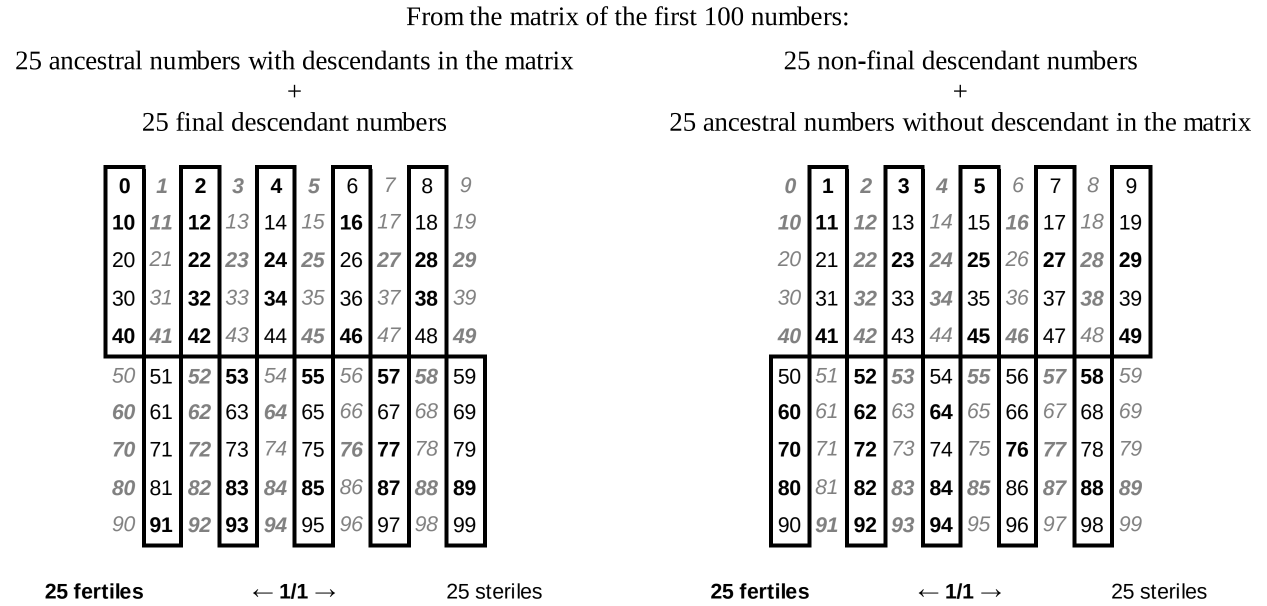
Figure 40: Transcendent distribution of fertile and sterile numbers among the first 25 ascending and final 25 descending of the first 100 numbers matrix.

7.5.1 Vertical and horizontal sub-matrices

As shown in Figure 32 of the introduction to this chapter, the matrix of the first hundred whole numbers is made up of 50 fertile and 50 sterile numbers. To conclude, this matrix can be split into two sub-matrices of symmetric geometric construction:

- Sub matrix to 25 ancestral numbers with descendants in the matrix + 25 final descendant numbers,
- Sub matrix to 25 non-final descendant numbers + 25 ancestral numbers without descendant in the matrix.

As show in Figure 41, in these two symmetric sub-matrices constructed from a criterion of lineage of numbers, fertiles and steriles are in exact equal quantities.



From the matrix of the first 100 numbers:

25 ancestral numbers with descendants in the matrix
+
25 final descendant numbers

0	***1***	**2**	***3***	**4**	***5***	6	*7*	8	*9*
10	***11***	**12**	*13*	14	*15*	**16**	*17*	18	*19*
20	*21*	**22**	***23***	**24**	***25***	26	***27***	**28**	***29***
30	*31*	**32**	*33*	**34**	*35*	36	*37*	**38**	*39*
40	***41***	**42**	*43*	44	***45***	**46**	*47*	48	***49***
50	51	***52***	**53**	*54*	**55**	*56*	**57**	***58***	59
60	61	***62***	63	***64***	65	*66*	67	*68*	69
70	71	***72***	73	*74*	75	***76***	**77**	*78*	79
80	81	***82***	**83**	***84***	**85**	*86*	**87**	***88***	**89**
90	**91**	***92***	**93**	***94***	95	*96*	97	*98*	99

25 fertiles ← **1/1** → 25 steriles

25 non-final descendant numbers
+
25 ancestral numbers without descendant in the matrix

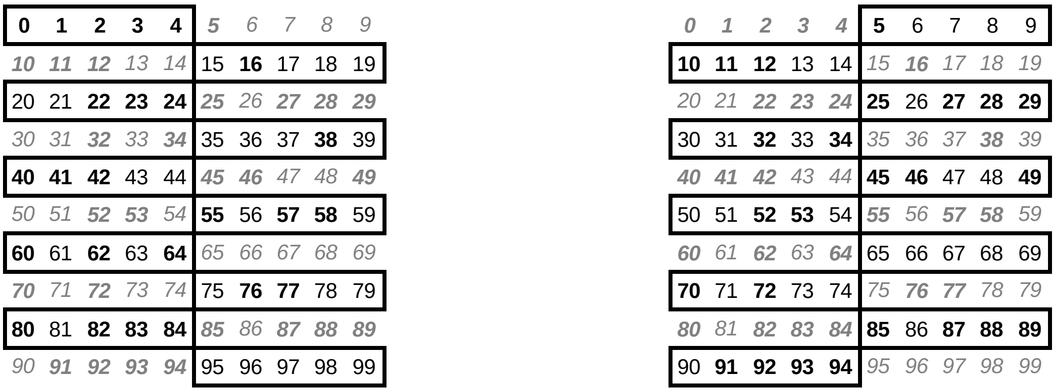
0	**1**	***2***	**3**	***4***	**5**	*6*	7	*8*	9
10	**11**	***12***	13	*14*	15	***16***	17	*18*	19
20	21	***22***	**23**	***24***	**25**	*26*	**27**	***28***	**29**
30	31	***32***	33	***34***	35	*36*	37	***38***	39
40	**41**	***42***	43	*44*	**45**	***46***	47	*48*	**49**
50	*51*	**52**	***53***	54	***55***	56	***57***	**58**	*59*
60	*61*	**62**	*63*	**64**	*65*	66	*67*	68	*69*
70	*71*	**72**	*73*	74	*75*	**76**	***77***	78	*79*
80	*81*	**82**	***83***	**84**	***85***	86	***87***	**88**	***89***
90	***91***	**92**	***93***	**94**	*95*	96	*97*	98	*99*

25 fertiles ← **1/1** → 25 steriles

Figure 41: The two symmetric sub-matrices constructed from a criterion of lineage of the numbers inside the first hundred numbers matrix.

Also, as a reminder of the configuration introduced in Figure 33, in two sub-matrices of 10 alternating semi-lines, the fertile and the sterile are in same exact proportion as shown in Figure 42. Thus, both in the vertical sub-matrices (Figure 41) and in the horizontal ones (Figure 42), the whole numbers that we call fertile and those that we call sterile are in exactly the same proportions.

From the matrix of the first 100 numbers:



0	**1**	**2**	**3**	**4**	***5***	*6*	*7*	*8*	*9*
10	***11***	***12***	*13*	*14*	15	**16**	17	18	19
20	21	**22**	**23**	**24**	***25***	*26*	***27***	***28***	***29***
30	*31*	***32***	*33*	***34***	35	36	37	**38**	39
40	**41**	**42**	43	44	***45***	***46***	*47*	*48*	***49***
50	*51*	***52***	***53***	*54*	**55**	56	**57**	**58**	59
60	61	**62**	63	**64**	*65*	*66*	*67*	*68*	*69*
70	*71*	***72***	*73*	*74*	75	**76**	**77**	78	79
80	81	**82**	**83**	**84**	***85***	*86*	***87***	***88***	***89***
90	***91***	***92***	***93***	***94***	95	96	97	98	99

0	***1***	***2***	***3***	***4***	**5**	6	7	8	9
10	**11**	**12**	13	14	*15*	***16***	*17*	*18*	*19*
20	*21*	***22***	***23***	***24***	**25**	26	**27**	**28**	**29**
30	31	**32**	33	**34**	*35*	*36*	*37*	***38***	*39*
40	***41***	***42***	*43*	*44*	**45**	**46**	47	48	**49**
50	51	**52**	**53**	54	***55***	*56*	***57***	***58***	*59*
60	*61*	***62***	*63*	***64***	65	66	67	68	69
70	71	**72**	73	74	*75*	***76***	***77***	*78*	*79*
80	*81*	***82***	***83***	***84***	**85**	86	**87**	**88**	**89**
90	**91**	**92**	**93**	**94**	*95*	*96*	*97*	*98*	*99*

25 fertiles ← **1/1** → 25 steriles **25 fertiles** ← **1/1** → 25 steriles

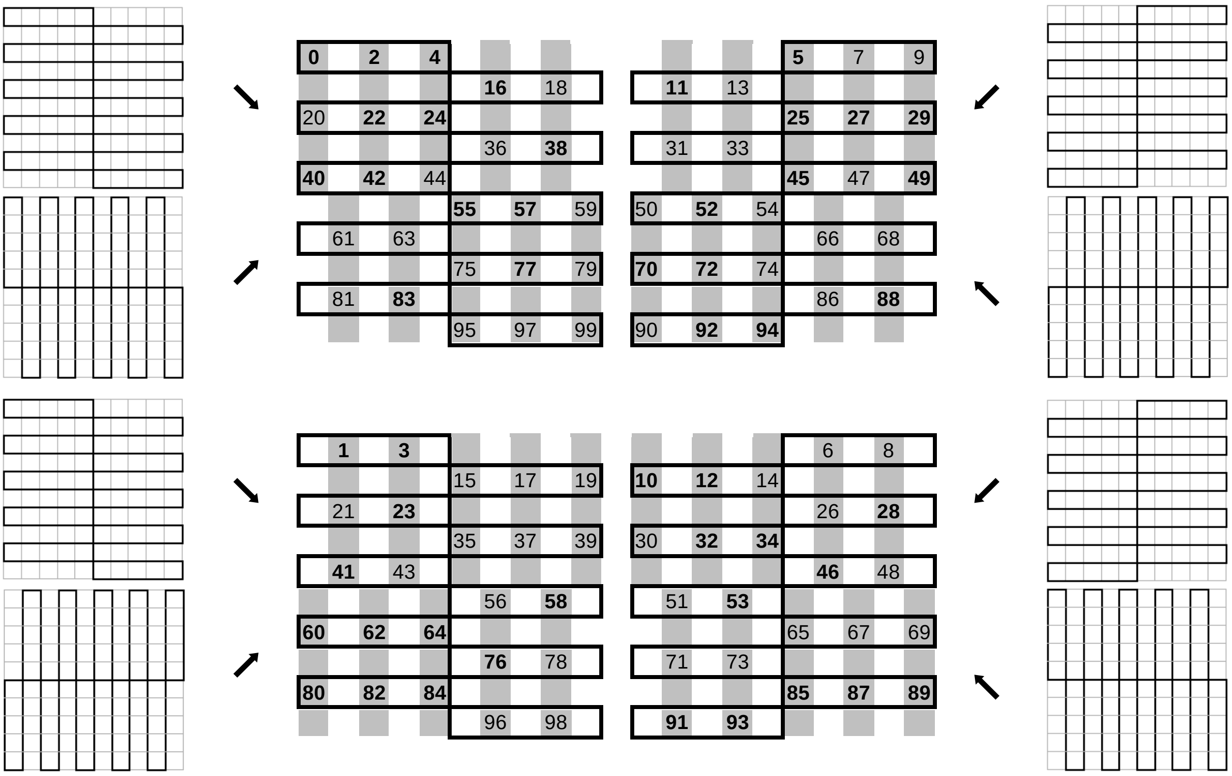
Figure 42: The two symmetric sub-matrices of 10 alternating semi lines inside the first hundred numbers matrix. See Figure 33 also and Figure 41 to comparison.

7.6 Crossed sub-matrices of the first hundred numbers matrix

At the end of this study of the matrix of the first hundred numbers, it appears undeniably that the distribution of numbers qualified as "fertile" and numbers qualified as "sterile" is absolutely not random within this closed matrix of 10 times 10 entities.

Indeed, as the illustration in Figure 43 clearly demonstrates, the crossings of the sub-matrices of 10 semi-alternating horizontal rows and the sub-matrices of 10 semi-alternating vertical rows generate sets of fertile and sterile entities of exact sizes of twice 13 or twice 12 numbers.

13 fertiles ← **1/1** → 13 steriles ← **1/1** → ← 1/1 → **13 fertiles** ← **1/1** → 13 steriles



0		**2**		**4**					
						16		18	
20		**22**		**24**					
						36		**38**	
40		**42**		44					
					55		**57**		59
	61		63						
					75		**77**		79
	81		**83**						
					95		97		99

					5		7		9
	11		13						
					25		**27**		**29**
	31		33						
					45		47		**49**
50		**52**		54					
						66		68	
70		**72**		74					
						86		**88**	
90		**92**		**94**					

	1		**3**						
					15		17		19
	21		**23**						
					35		37		39
	41		43						
						56		**58**	
60		**62**		**64**					
						76		78	
80		**82**		**84**					
						96		98	

						6		8	
10		**12**		14					
						26		**28**	
30		**32**		**34**					
						46		48	
	51		**53**						
					65		67		69
	71		73						
					85		**87**		**89**
	91		**93**						

12 fertiles ← **1/1** → 12 steriles ← **1/1** → ← 1/1 → **12 fertiles** ← **1/1** → 12 steriles

Figure 43: Crossed symmetric sub-matrices of 10 alternating semi lines or semi columns inside the first hundred numbers matrix. See Figures 41 and 42 also.

8. Matrix of the first fifty fertile numbers

We have just demonstrated that among the matrix of the first 100 numbers there are just 50% of numbers qualified as fertile according to our previously introduced definitions. We now study a matrix of these first 50 fertile numbers (so 5*x* numbers → *x* = 10) which turns out, according to our previously introduced definitions, to be made up of 15 pure numbers (so 5*x* numbers → *x* = 3).

first 100 numbers matrix

50 fertiles

50 steriles

0	**1**	**2**	**3**	**4**	**5**	6	7	8	9
10	**11**	**12**	13	14	15	**16**	17	18	19
20	21	**22**	**23**	**24**	**25**	26	**27**	**28**	**29**
30	31	**32**	33	**34**	35	36	37	**38**	39
40	**41**	**42**	43	44	**45**	**46**	47	48	**49**
50	51	**52**	**53**	54	**55**	56	**57**	**58**	59
60	61	**62**	63	**64**	65	66	67	68	69
70	71	**72**	73	74	75	**76**	**77**	78	79
80	81	**82**	**83**	**84**	**85**	86	**87**	**88**	**89**
90	**91**	**92**	**93**	**94**	95	96	97	98	99

➡

first 50 fertiles matrix

0	**1**	2	**3**	4
5	10	**11**	12	16
22	**23**	24	**25**	27
28	29	32	34	38
40	41	42	**45**	46
49	52	53	**55**	**57**
58	60	62	64	70
72	76	**77**	80	82
83	84	**85**	87	88
89	**91**	92	**93**	94

15
pure and fertile
whole numbers

35 non-pure fertiles

Figure 44: From the first 100 numbers matrix, construction of the first 50 fertiles matrix with inside, 15 pure numbers.

8.1 Pure and fertile numbers

Thus, Figure 44 we construct a matrix of 5 columns by 10 rows where are arranged, in their order of magnitude, the first 50 fertile numbers. In this new matrix are identified 15 pure numbers and 35 non-pure ones. We have previously explained, in the introductory Chapter 2 of these concepts, that the "genetic" notions of fertility and purity can be applied to both types (two species) of numbers. Figure 45 lists the first 15 simultaneously pure and fertile numbers, showing their numerical ancestry and descent.

6 pure fertile ultimates					← **3/2 ratio** →	**9 pure fertile non-ultimates**			
ancestral number		⟵n⟶		**descendant number**	**ancestral number**		⟵n⟶		**descendant number**
(n-1)/2				**n + (n+1)**	**(n-1)/2**				**n + (n+1)**
0	←	**1**	→	3	12	←	**25**	→	51
1	←	**3**	→	7	22	←	**45**	→	91
2	←	**5**	→	11	24	←	**49**	→	99
5	←	**11**	→	23	27	←	**55**	→	111
11	←	**23**	→	47	28	←	**57**	→	115
41	←	**83**	→	167	38	←	**77**	→	155
					42	←	**85**	→	171
					45	←	**91**	→	183
					46	←	**93**	→	187

Figure 45: The first 15 pure and fertiles numbers: 6 ultimates versus 9 non-ultimates. See Figure 44 also.

Thus, according to these concepts of numerical genetics, a number *n* is simultaneously pure and fertile if *n*, the ascendant of *n* and its descendent are all three either ultimate numbers or all three non-ultimate numbers. Figure 45 shows that among the first few 15 simultaneously pure and fertile numbers, in an exact ratio of 3/2 as value, six are ultimates and 9 are non-ultimates.

8.2 Mirror symmetry inside the first fifty fertile numbers

Inside the matrix of the first 50 fertile numbers configured in 10 lines of 5 rows, pure and non-pure numbers are distributed very singularly according to symmetrical mirror configurations. Indeed, in each pair of symmetrically opposite numbers, there is always the same quantity of pure and non-pure fertile numbers as in the pair of numbers configured in mirror symmetry.

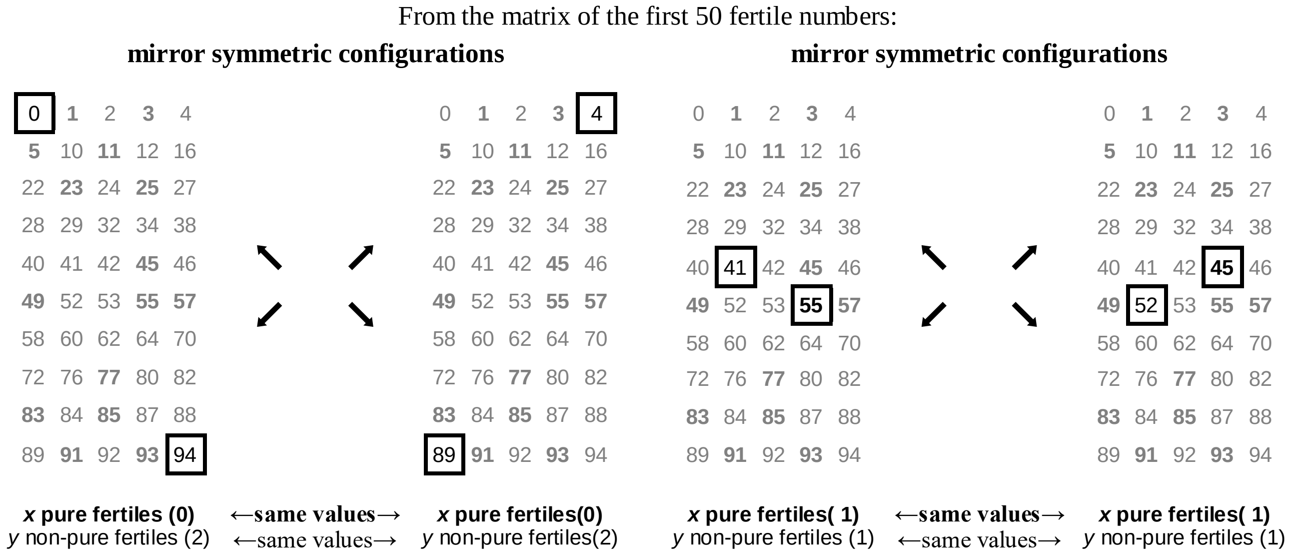


Figure 46: identical quantification of pure and non-pure numbers in symmetric mirror configurations of the matrix of the first 50 fertile numbers. See Figures 44 and 47 also.

Figure 45 describes this singular phenomenon in detail with two examples of two pairs of numbers arranged in mirror symmetry. Figure 46 lists all the mirror configurations where this phenomenon operates, that is, for two times ten even numbers.

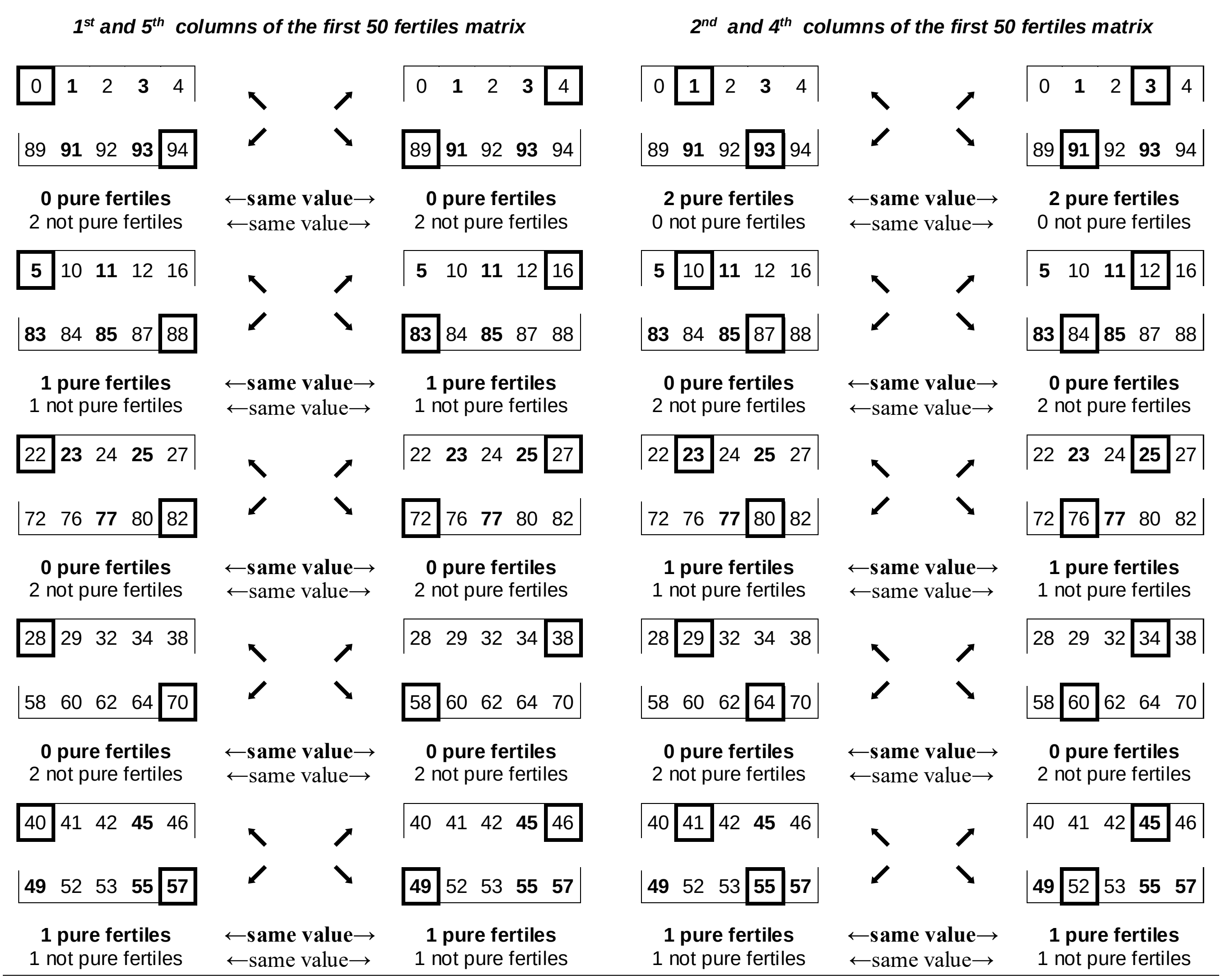


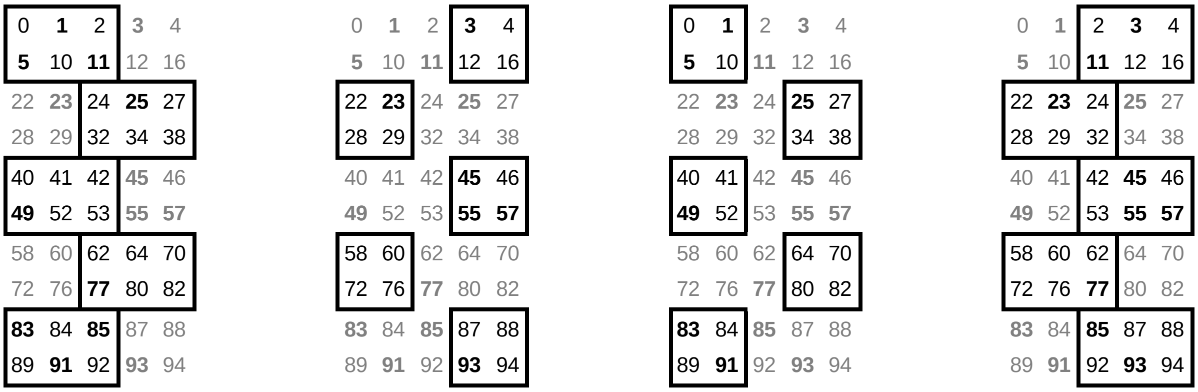
Figure 47: Identical quantification of pure and non-pure numbers in symmetric mirror configurations of the matrix of the first 50 fertile numbers. See Figures 44 and 46 also.

8.3 Mirror symmetry and 3/2 ratio

In the matrix of the first 50 fertile numbers (5 columns by 10 rows) many sub-matrices of configuration 30 versus 20 numbers are made up of 3*x* versus 2*x* pure numbers and consequently of 3*x* non-pure versus 2*x* non-pure. Also, and following the previous singular phenomenon introduced, the different categories of whole numbers are identically distributed in the mirror symmetry configurations, as illustrated in detail in Figure 47.

From the matrix of the first 50 fertile numbers:

30 versus 20 fertiles sub-matrices **20 versus 30 fertiles sub-matrices**



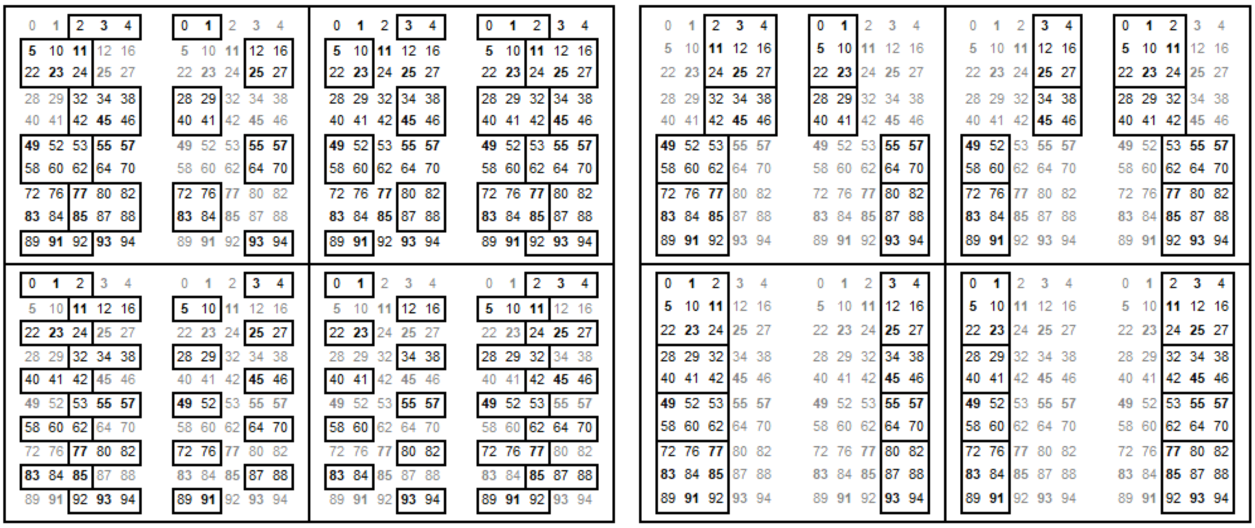
9 pure fertiles ← **3/2** → **6 pure fertiles**
21 not pure fertiles ← 3/2 → 14 not pure fertiles

6 pure fertiles ← **2/3** → **9 pure fertiles**
14 not pure fertiles ← 2/3 → 21 not pure fertiles

Figure 47: Symmetric mirror configurations of 3*x* versus 2*x* entities inside the matrix of the first 50 fertiles numbers.
See Figure 44 also.

The following Figure 48 shows all the other configurations where this double phenomenon manifests itself: mirror symmetry and 3/2 ratio.

9 pure 21 not pure	**6 pure** 14 not pure	**6 pure** 14 not pure	**9 pure** 21 not pure	**9 pure** 21 not pure	**6 pure** 14 not pure	**6 pure** 14 not pure	**9 pure** 21 not pure



9 pure 21 not pure	**6 pure** 14 not pure	**6 pure** 14 not pure	**9 pure** 21 not pure	**9 pure** 21 not pure	**6 pure** 14 not pure	**6 pure** 14 not pure	**9 pure** 21 not pure

Figure 48: Many symmetric mirror configurations of 3*x* versus 2*x* entities inside the matrix of the first 50 fertiles numbers.

9. The numerical genetics of the whole numbers

We will now finalize in great detail the introduction of the concept of number genetics by describing it as a pure system of biology applying to whole numbers.

9.1 Whole number emanations

The algebraic mechanics of Sophie Germain making it possible to respectively qualify the entities "x" and "$2x + 1$" of Sophie Germain prime and prime pure (and whose concept we have expanded to fertile number and pure number) compares two mathematics entities one of which ($2x + 1$) consists of a part (1 and more precisely + 1) which does not identify with x.

We are now going to propose that this second entity which is "$2x + 1$" is in fact totally derived from the first entity "x".

In mathematical literature, in number theory, there is always the debate whether or not to integrate the exotic number zero (0) into the set $\mathbb{N}$. As we specified in the paper introduction, we have decided to include this number in the set of whole numbers.

This integration has an important consequence regarding the attributes of the different whole numbers. Indeed, as the first number of the set N has 0 for value, this value is different from the value of its rank which is equal to 1. Thus, for any x, x being any whole number, the value of its rank is equal to $x + 1$. We can therefore already say that any x has two attributes: its value, which we call V(x) and the value of its rank which we call R(x). We propose to consider these two different attributes (these two values) as emanations of x.

Thus, we suggest that the classical algebraic notation "$2x + 1$" used to define the Germain's descent of x be replaced by V(x) + R(x). We also propose to call G(x) the result of this *fusion* of the two entities V(x) and (Rx). We therefore have the following mechanism of numerical genetics: the descent of x is equal to the sum of the value of x and the value of the rank of x. We thus write "G(x) = V(x) + R(x)" and consider that these three values are only emanations of x. We consider these values as real emanations of number x since no other mathematical entity not depending on x composes them. This, in contrast to the algebraic operation "f(x) = $2x + 1$" where the numeric entity "1" is foreign to it.

9.2 V(x), R(x) and G(x) emanations

We have logically proposed the names V(x) and R(x) in reference to the Value of x and the Rank of x, two emanations of x. For this third emanation, we propose the letter G in reference to "generation of x" but also in reference (and homage) to Sophie Germain, initiator of the concept of Germain prime and safe prime; concept that we expand in this paper. Table in figure 49 visually explains these concepts of emanations and their different new conventional writing.

entity (*emanation*)	➡	***Value of x***	***Rank of x***	***Descent of x***
notation	➡	***V(x)***	***R(x)***	***G(x)***
formula	➡	***V(x) = x***	***R(x) = V(x) + 1***	***G(x) = V(x) + R(x)***
final value (standard value)	➡	*x*	*x + 1*	*2x + 1*

Figure 49: Numeric emanations of entity x as integrate parts of x.(x being any whole number). Whole number 3 as illustrated example. See Figure 50 also.

9.3 Enriched number genetics mechanism

Thus we have just demonstrated that the entity generated from x and of the old formulation "$2x + 1$" is in fact a pure emanation from x. The illustrations in Figure 50 clearly describe how x first generates two primary entities (pure attributes of x) then the secondary entity G(x), final emanation of x. Whole number 3 is used here as example to make this new arithmetic concept better understand

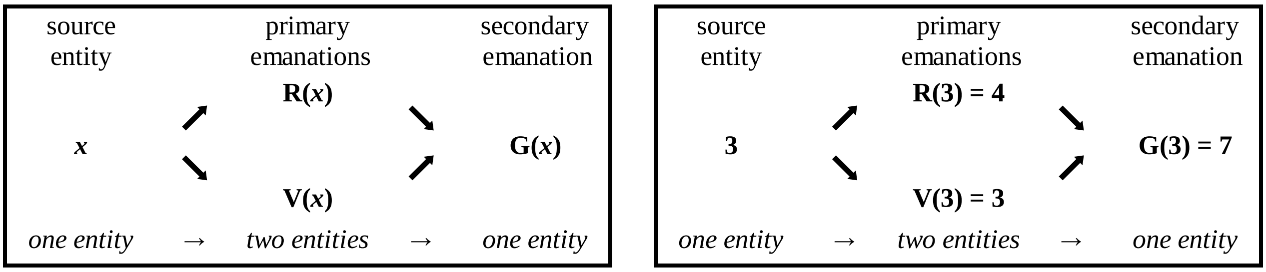


Figure 50: Numeric emanations of entity x as integrate parts of x.(x being any whole number). Whole number 3 as illustrated example. See Figure 49 also.

9.3.1 Enriched number writing

From this enriched numeric genetics mechanism, we logically propose now a enriched number writing. Thus we can write any whole number in this expanded form:

$$^{R(x)}x_{G(x)}$$

Table in Figure 51 lists this wider writing of the first ten numbers.

Rank of x → ***R(x)*** ***= x + 1***	1	2	3	4	5	6	7	8	9	10
Value of x → ***V(x)*** ***= x***	0	1	2	3	4	5	6	7	8	9
Descent of x → ***G(x) = V(x) + R(x) = 2 x + 1***	1	3	5	7	9	11	13	15	17	19
wider writing of x	$^{1}0_{1}$	$^{2}1_{3}$	$^{3}2_{5}$	$^{4}3_{7}$	$^{5}4_{9}$	$^{6}5_{11}$	$^{7}6_{13}$	$^{8}7_{15}$	$^{9}8_{17}$	$^{10}9_{19}$

Figure 51: Wider genetics writing of the first ten whole numbers.

9.4 Numeric genetics identifying with a biological process

Since we talk a lot in this article about number genetics, especially in development of the famous one Sophie Germain's concept, we allow ourselves to also introduce a closer link with the field of biology.

We therefore introduce the idea that the arithmetic mechanism producing G(*x*) from *x* is purely as a biological process of cellular reproduction. Thus, *x* (any whole number) really divides like a biological cell into two daughter cells to generate V(*x*) and R(*x*). Then these two cells merge biologically to form a new cell which is none other than G(*x*).

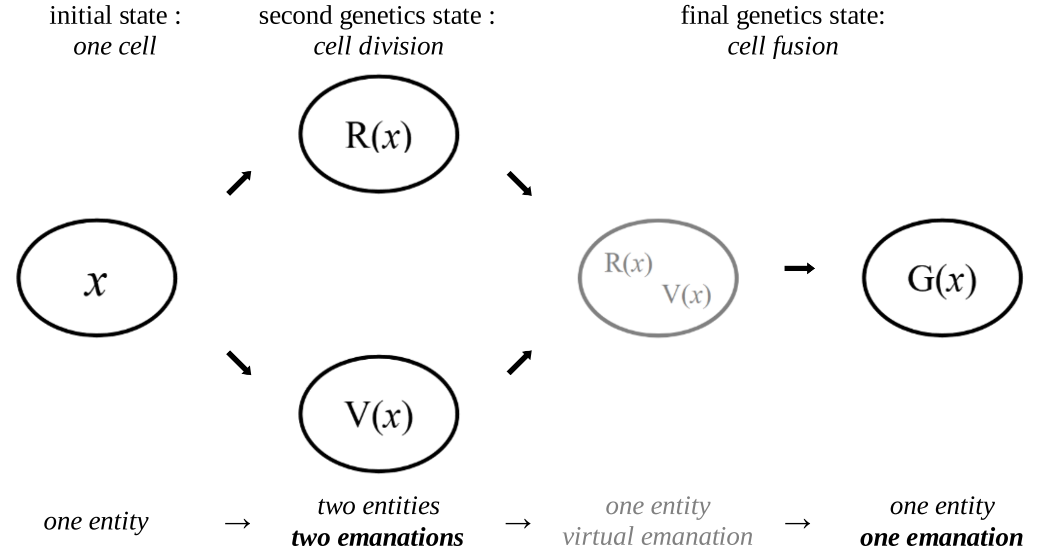


Figure 52: Arithmetic Sophie Germain's concept as a biological process of cellular reproduction.
See Figure 50 also.

Figure 52 illustrates this number biology with the creation of an intermediate virtual cell forming before G(*x*). Mathematical mechanics is of course timeless and this entity is therefore just to be considered as ghost in the formation of G(*x*). Figure 53 completes this illustration with whole number 3 as an example: numbers 4 and 7 are pure emanations of number 3.

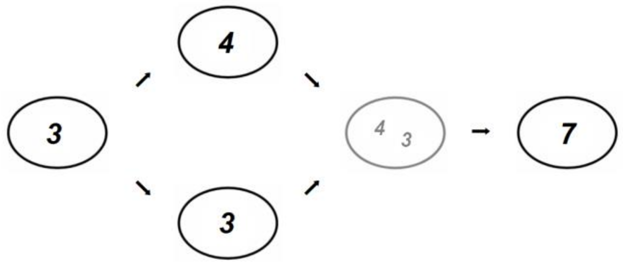


Figure 53: Arithmetic Sophie Germain's concept as a biological process of cellular reproduction. Whole number 3 as illustration example. See Figure 50 also.

9.5 Enriched numeric genetics: hyper fertility and ultra purity

In Chapters 4 and 5, the notions of fertility and purity of whole numbers were introduced. This by comparing the numerical species of any *x* with its ascendant and its descendant. The enriched numeric genetics mechanism can now allow us to introduce the notions of ***hyper fertility*** and of ***ultra purity*** to these numbers.

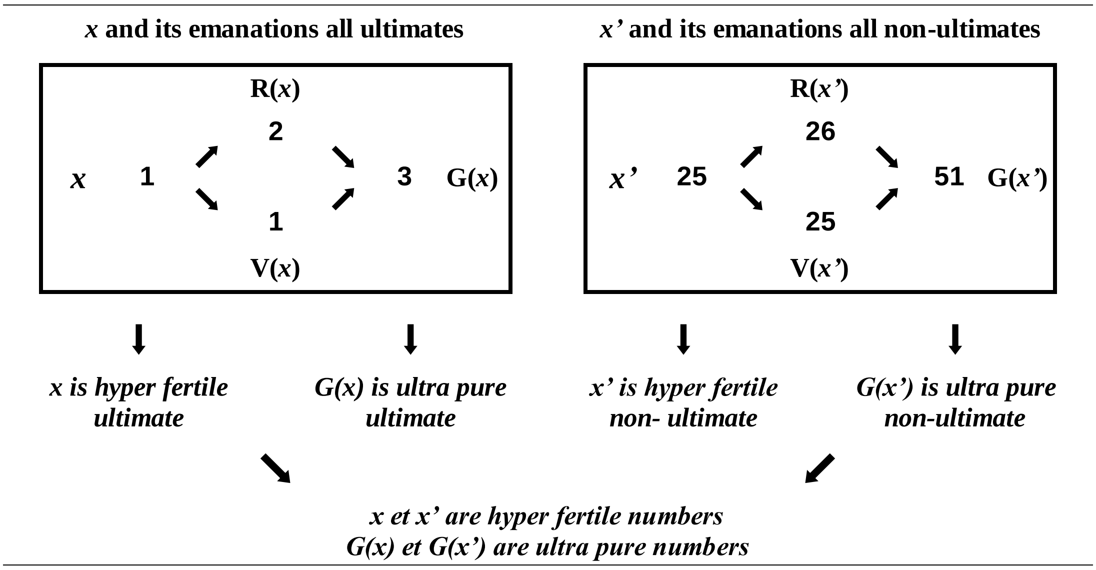


Figure 54: Description of the concepts of hyper fertility and ultra purity of whole numbers. Here 1 and 25 as examples of emanation source.

The illustration Figure 54 clearly describes how the entities *x* and G(*x*) rank respectively in hyper fertile and ultra pure numbers.

We therefore propose the following new definitions. These ones use entities of enriched numeric genetics already above introduced: V(*x*), R(*x*) and G(*x*).

Hyper fertile ultimate and ultra pure ultimate definition:

- It is agreed that if *x* (confused with V(*x*)), R(*x*) and G(*x*) are all ultimate numbers, then *x* is a hyper fertile ultimate and G(*x*) is an ultra pure ultimate.

Hyper fertile non-ultimate and ultra pure non-ultimate definition:

- It is agreed that if *x* (confused with V(*x*)), R(*x*) and G(*x*) are all non-ultimate numbers, then *x* is a hyper fertile non-ultimate and G(*x*) is an ultra pure non-ultimate.

Hyper fertile number and ultra pure number definition:

- It is agreed that if *x* (confused with V(*x*)), R(*x*) and G(*x*) are either all ultimates or, in opposite, all non-ultimates, then *x* is a hyper fertile number and G(*x*) is an ultra pure number.

This third definition is so an universal definition for any whole numbers according with hyper fertility and ultra purity concepts.

10. Hyper fertility and ultra purity arrangements

Hyper fertility and ultra purity concepts just introduced highlight singular arrangements in the closed matrix of the first 100 numbers. The existence of these geomatrix phenomena strongly accredits the idea of granting veracity to these mathematical concepts proposed as an extension of Sophie Germain's numbers concept.

10.1 First 100 numbers matrix to first 50 fertiles matrix

In Chapter 7 it was introduced that the matrix of the first 100 whole numbers (from 0 to 99) consists of exactly 50 fertiles and 50 steriles. We draw attention again to the fact that these values are equal to 5*x* entities, and more precisely to 3*x* + 2*x* entities.

first 100 numbers matrix

50 fertiles

50 steriles

0	**1**	**2**	**3**	**4**	**5**	6	7	8	9
10	**11**	**12**	13	14	15	**16**	17	18	19
20	21	**22**	**23**	**24**	**25**	26	**27**	**28**	**29**
30	31	**32**	33	**34**	35	36	37	**38**	39
40	**41**	**42**	43	44	**45**	**46**	47	48	**49**
50	51	**52**	**53**	54	**55**	56	**57**	**58**	59
60	61	**62**	63	**64**	65	66	67	68	69
70	71	**72**	73	74	75	**76**	**77**	78	79
80	81	**82**	**83**	**84**	**85**	86	**87**	**88**	**89**
90	**91**	**92**	**93**	**94**	95	96	97	98	99

➡

first 50 fertiles matrix

$^{1}0_{1}$	**$^{2}1_{3}$**	**$^{3}2_{5}$**	$^{4}3_{7}$	$^{5}4_{9}$
$^{6}5_{11}$	$^{11}10_{21}$	$^{12}11_{23}$	$^{13}12_{25}$	$^{17}16_{33}$
$^{23}22_{45}$	$^{24}23_{47}$	**$^{25}24_{49}$**	**$^{26}25_{51}$**	**$^{28}27_{55}$**
$^{29}28_{57}$	$^{30}29_{59}$	**$^{33}32_{65}$**	**$^{35}34_{69}$**	**$^{39}38_{77}$**
$^{41}40_{81}$	$^{42}41_{83}$	$^{43}42_{85}$	**$^{46}45_{91}$**	$^{47}46_{93}$
$^{50}49_{99}$	$^{53}52_{105}$	$^{54}53_{107}$	**$^{56}55_{111}$**	**$^{58}57_{115}$**
$^{59}58_{117}$	$^{61}60_{121}$	**$^{63}62_{125}$**	**$^{65}64_{129}$**	$^{71}70_{141}$
$^{73}72_{145}$	**$^{77}76_{153}$**	**$^{78}77_{155}$**	**$^{81}80_{161}$**	$^{83}82_{165}$
$^{84}83_{167}$	**$^{85}84_{169}$**	**$^{86}85_{171}$**	**$^{88}87_{175}$**	$^{89}88_{177}$
$^{90}89_{179}$	**$^{92}91_{183}$**	**$^{93}92_{185}$**	**$^{94}93_{187}$**	**$^{95}94_{189}$**

25 hyper fertiles

25 fertiles

Figure 55: Highlighting the 25 hyper fertiles in the sub matrix of the first 50 fertiles. Sub matrix derived from that of the first 100 numbers which is made up of 50 fertile and 50 sterile numbers. See Figures 54 and 56 also.

As it is possible to discover in the illustration Figure 55, the matrix of the first 50 fertiles (sub-matrix of that of the first 100 numbers) is itself composed of an equal quantity of hyper fertiles and (simple) fertiles , i.e. 25 entities from each number category. In this table, numbers are presented in their wider writing: $^{R(x)}x_{G(x)}$.

10.2 First 50 fertiles matrix

The respective distributions of these two categories of numbers (hyper or simple fertile) are not random but are organized according to the zones of this sub-matrix in different interactive ratios of value 3/2 or 2/3.

first 50 fertiles matrix
25 hyper fertiles - 25 fertiles

10 hyper fertiles
10 fertiles

0	**1**	**2**	3	4
5	10	11	12	16
22	*23*	***24***	***25***	***27***
72	***76***	***77***	***80***	*82*
83	**84**	**85**	**87**	88
89	**91**	**92**	**93**	**94**

↓ **2/3 ratio** ↑

↑ 2/3 ratio ↓

22	23	**24**	**25**	**27**
28	29	**32**	**34**	**38**
40	41	42	**45**	46
49	52	53	**55**	**57**
58	60	**62**	**64**	70
72	**76**	**77**	**80**	82

15 hyper fertiles
15 fertiles

0	**1**	**2**	3	4
5	10	11	12	16
22	23	**24**	**25**	**27**
28	29	**32**	**34**	**38**
40	41	42	**45**	46
49	52	53	**55**	**57**
58	60	**62**	**64**	70
72	**76**	**77**	**80**	82
83	**84**	**85**	**87**	88
89	**91**	**92**	**93**	**94**

0	**1**	**2**	3	4
5	10	11	12	16
22	23	**24**	**25**	**27**
28	29	**32**	**34**	**38**
40	41	42	**45**	46

10 hyper fertiles
15 fertiles

↓ **2/3 ratio** ↑

↑ 3/2 ratio ↓

49	52	53	**55**	**57**
58	60	**62**	**64**	70
72	**76**	**77**	**80**	82
83	**84**	**85**	**87**	88
89	**91**	**92**	**93**	**94**

15 hyper fertiles
10 fertiles

Figure 56: Distribution of 25 hyper fertiles and 25 (simple) fertiles in the first 50 fertiles matrix and in some sub matrices derived from its.

Thus, as Figure 56 demonstrates, 10 hyper fertiles are opposed to 15 fertiles in the first half (25 entities) of the matrix of the first 50 fertiles whereas an inverse ratio appears in the second half of this one (25 entities following). We therefore note that since, in these two areas of 25 entities, the respective quantities of hyper fertile and simple fertile cannot be split into equal parts, they are divided into ratios of value 3/2 and 2/3.

Conversely, in areas of 20 or 30 entities, which can be divided into equal parts, the respective quantities of hyper fertiles and fertiles are equally distributed with 10 against 10 entities or 15 against 15 entities, these two groups of opposing values in a final ratio of 3/2 value. Difficult to consider here these distributions of numbers as just a coincidence.

10.2.1 First 50 fertiles matrix anatomy

Inside the two halves of the matrix of the first 50 fertiles (from the 1st to the 25th and from the 2nd to the 50th fertile numbers), the respective distribution of the hyper fertiles and (simple) fertiles is organized as the algebraic entities of the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where a and b respectively equal to 2 and 3. This, by opposing matrix fractions consisting alternately of 2 versus 3 consecutive numbers as shown in Figure 57.

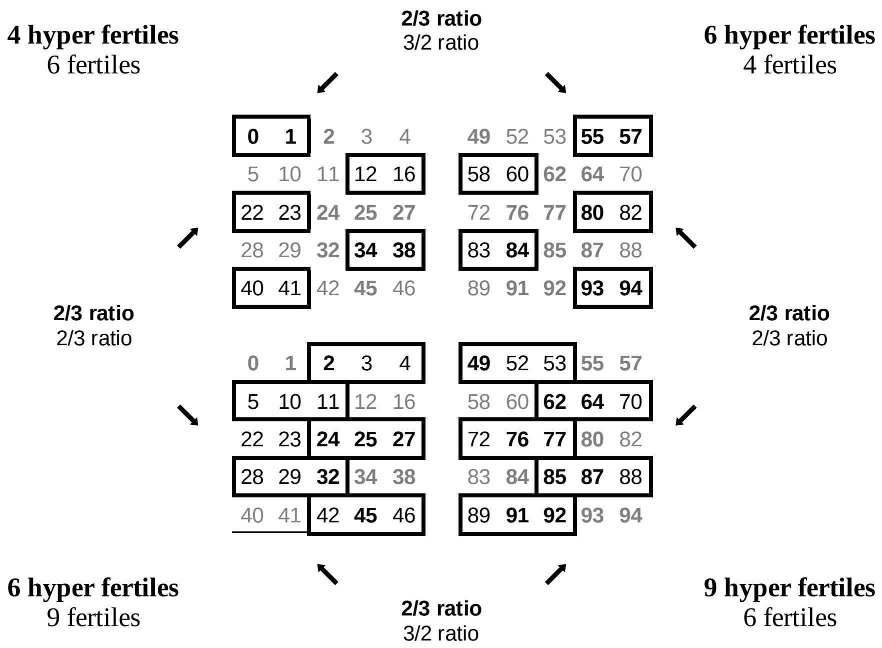
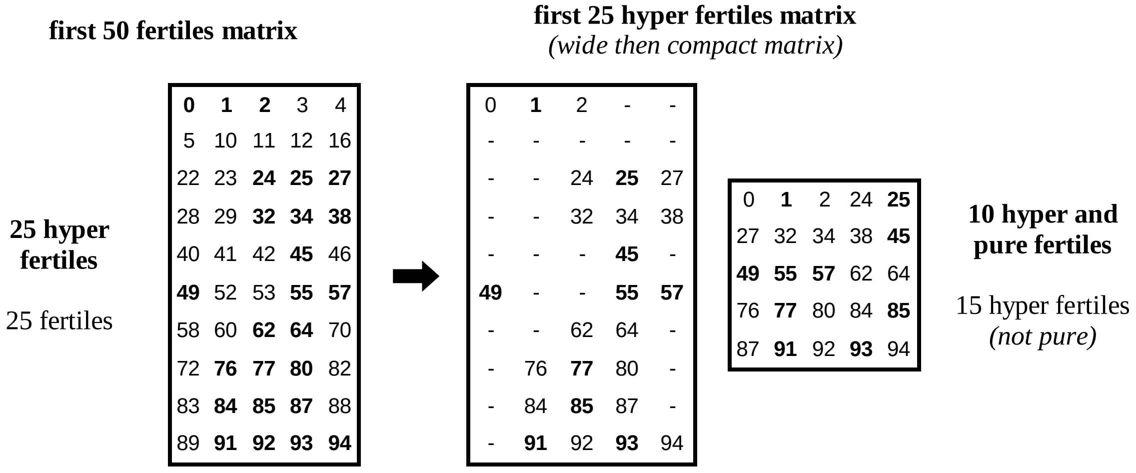


Figure 57: Distribution of hyper and simple fertile numbers inside complementary fractions of the first 50 fertiles matrix. This according with the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$

Indeed, according to their hyper fertility or simple fertility, in these fractions of the matrix, the numbers are distributed and opposed in 4, 6 or 9 entities. Value representing the algebraic entities of the previously defined remarkable identity.

10.3 First 50 fertiles matrix to first 25 hyper fertiles matrix

Figure 58, since the matrix of the first 50 fertiles contains 50%, i.e. 25 hyper fertiles, it is then possible to introduce a new matrix once again to 5*x* entities: the matrix of the first 25 hyper fertile numbers.



0	**1**	**2**	3	4
5	10	11	12	16
22	23	**24**	**25**	**27**
28	29	**32**	**34**	**38**
40	41	42	**45**	46
49	52	53	**55**	**57**
58	60	**62**	**64**	70
72	**76**	**77**	**80**	82
83	**84**	**85**	**87**	88
89	**91**	**92**	**93**	**94**

0	**1**	2	-	-
-	-	-	-	-
-	-	24	**25**	27
-	-	32	34	38
-	-	-	**45**	-
49	-	-	**55**	**57**
-	-	62	64	-
-	76	**77**	80	-
-	84	**85**	87	-
-	**91**	92	**93**	94

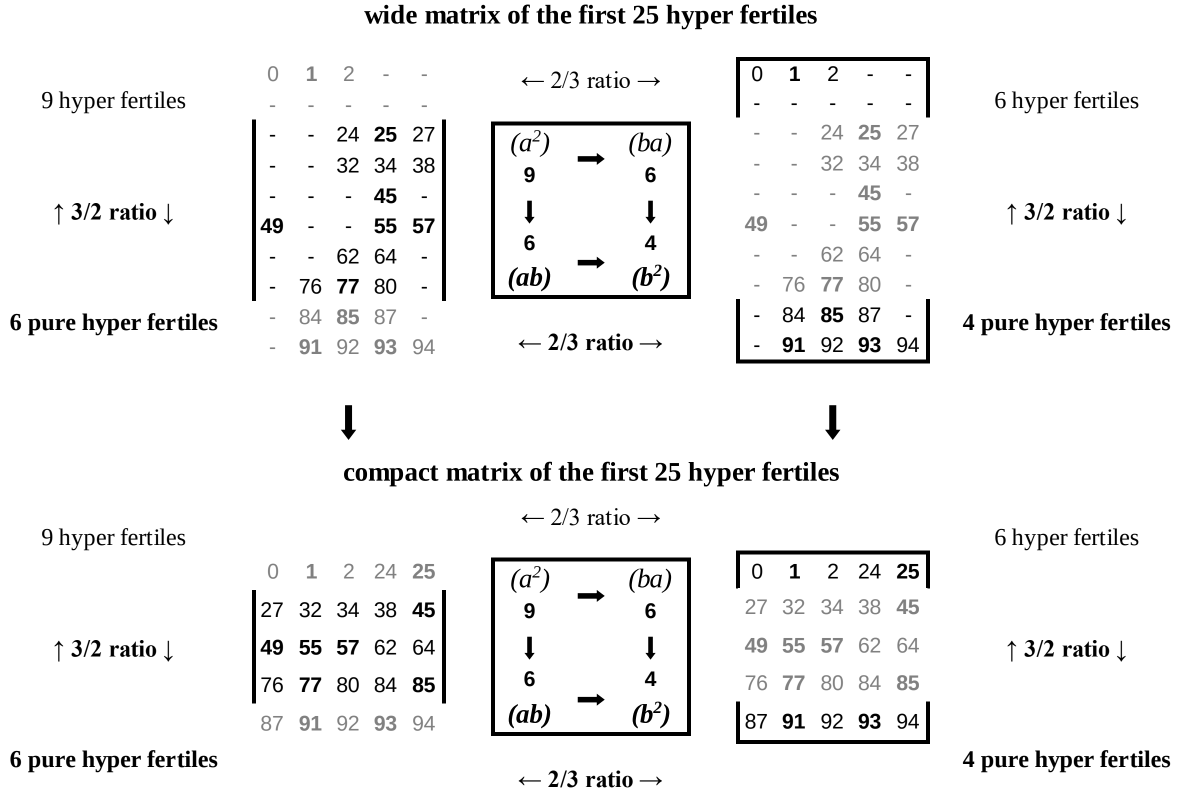
0	**1**	2	24	**25**
27	32	34	38	**45**
49	**55**	**57**	62	64
76	**77**	80	84	**85**
87	**91**	92	**93**	94

Figure 58: Creation of the first 25 hyper fertiles matrix (wide then compact) from the first 50 fertiles matrix.

This matrix of 25 entities itself contains two groups of numbers differentiated according to their purity or non-purity. Also, according to this criterion of purity, a concept of number genetics, these two sets also contain 5*x* and 5*x'* entities: respectively, and in a 2/3 ratio, 10 pure hyper fertiles and 15 non-pure hyper fertiles numbers.

10.3.1 Wide then compact matrix of the first 25 hyper fertiles

We introduce this matrix in two geo chart shapes: a wide version keeping the same space as the source matrix of the first 50 fertiles then a compact version of just 25 space boxes.



0	**1**	2	-	-
-	-	-	-	-
-	-	24	**25**	27
-	-	32	34	38
-	-	-	**45**	-
49	-	-	**55**	**57**
-	-	62	64	-
-	76	**77**	80	-
-	84	**85**	87	-
-	**91**	92	**93**	94

0	**1**	2	-	-
-	-	-	-	-
-	-	24	**25**	27
-	-	32	34	38
-	-	-	**45**	-
49	-	-	**55**	**57**
-	-	62	64	-
-	76	**77**	80	-
-	84	**85**	87	-
-	**91**	92	**93**	94

0	**1**	2	24	**25**
27	32	34	38	**45**
49	**55**	**57**	62	64
76	**77**	80	84	**85**
87	**91**	92	**93**	94

0	**1**	2	24	**25**
27	32	34	38	**45**
49	**55**	**57**	62	64
76	**77**	80	84	**85**
87	**91**	92	**93**	94

Figure 59: Wide and compact matrix of the first 25 hyper fertiles: same arrangements in connections with remarkable identity and according with hyper fertility and purity concepts.

Figure 59 shows that, despite two different geo-distributions, identical arithmetic phenomena operate both in the compact and expanded matrix of the first 25 hyper fertiles.

In these different configurations, whose dimensions of the zones of entities are in a ratio of value 3/2, the pure and non-pure numbers (all hyper fertile) are always opposed in many transcendent 3/2 ratios. This by organizing itself according to the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where a and b respectively equal to 3 and 2.

10.3.2 First 25 hyper fertiles matrix and remarkable identity

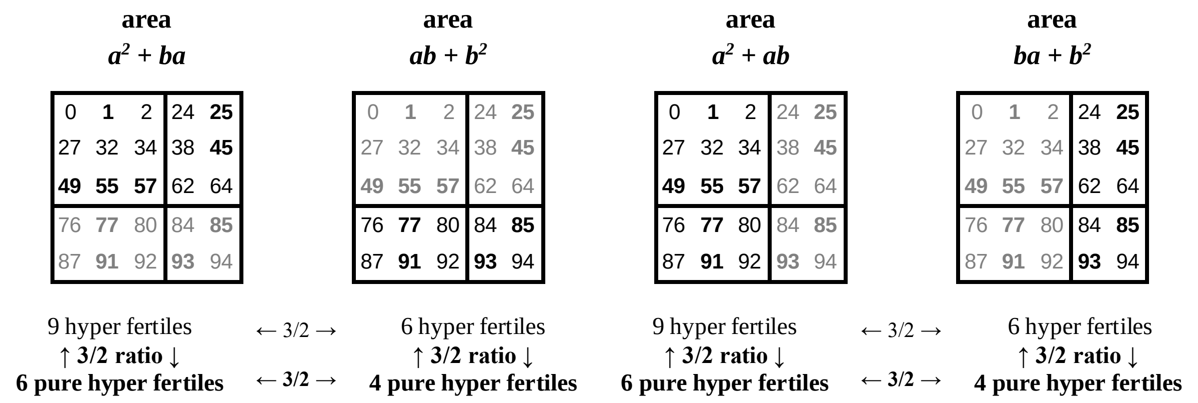
The matrix of the first 25 hyper fertiles is intimately linked to the remarkable identity a lot previously introduced. As shown in Figure 60, this matrix of 25 entities can be split into four geo zones identifying with the four components of this remarkable identity.

a^2	0	**1**	2	24	**25**	*ba*
	27	32	34	38	**45**	
	49	**55**	**57**	62	64	
ab	76	**77**	80	84	**85**	b^2
	87	**91**	92	**93**	94	

Figure 60: First 25 hyper fertiles matrix as a geo expression of the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$

Also, depicted Figure 61, it is possible to oppose two by two these charting zones in a ratio of value 3/2:

$$(a^2 + ab)/(ba + b^2) = 3/2 \text{ or } (a^2 + ba)/(ab + b^2) = 3/2$$



area $a^2 + ba$				
0	**1**	2	24	**25**
27	32	34	38	**45**
49	**55**	**57**	62	64
76	**77**	80	84	**85**
87	**91**	92	**93**	94

area $ab + b^2$				
0	**1**	2	24	**25**
27	32	34	38	**45**
49	**55**	**57**	62	64
76	**77**	80	84	**85**
87	**91**	92	**93**	94

area $a^2 + ab$				
0	**1**	2	24	**25**
27	32	34	38	**45**
49	**55**	**57**	62	64
76	**77**	80	84	**85**
87	**91**	92	**93**	94

area $ba + b^2$				
0	**1**	2	24	**25**
27	32	34	38	**45**
49	**55**	**57**	62	64
76	**77**	80	84	**85**
87	**91**	92	**93**	94

9 hyper fertiles ← 3/2 → 6 hyper fertiles
↑ **3/2 ratio** ↓ ↑ **3/2 ratio** ↓
6 pure hyper fertiles ← 3/2 → 4 pure hyper fertiles

9 hyper fertiles ← 3/2 → 6 hyper fertiles
↑ **3/2 ratio** ↓ ↑ **3/2 ratio** ↓
6 pure hyper fertiles ← 3/2 → 4 pure hyper fertiles

Figure 61: Transcendent distribution of pure and non-pure hyper fertiles in the matrix of first 25 hyper fertiles.

So, in this 25 entities matrix, with *a* and *b* respectively equal to 3 and 2, in zone of $a^2 + ba$ entities, a^2 non-pure hyper fertiles (therefore 9) and *ba* pure hyper fertiles (therefore 6) oppose in 3/2 ratio. In transcendence, same phenomena operate in zone of $ab + b^2$ entities where 6 non-pure hyper fertiles and 4 pure hyper fertiles oppose. Even more, the reversibility of zones *ab* and *ba* also produces the same phenomena as Figure 61 clearly explains.

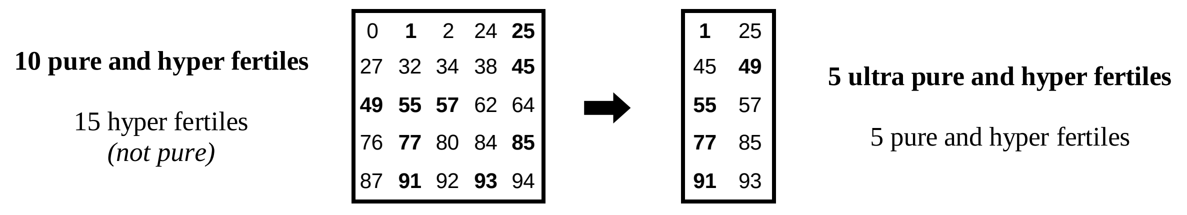
10.4 First 25 hyper fertiles matrix to first 10 pure hyper fertiles matrix

In an ever-deeper number genetics investigation, from the matrix of first 25 hyper fertiles, we propose now to create the first 10 pure and hyper fertiles matrix.

10.4.1 First 10 pure hyper fertiles matrix

first 25 hyper fertiles matrix

first 10 pure and hyper fertiles matrix



10 pure and hyper fertiles

15 hyper fertiles
(not pure)

0	**1**	2	24	**25**
27	32	34	38	**45**
49	**55**	**57**	62	64
76	**77**	80	84	**85**
87	**91**	92	**93**	94

1	25
45	**49**
55	57
77	85
91	93

5 ultra pure and hyper fertiles

5 pure and hyper fertiles

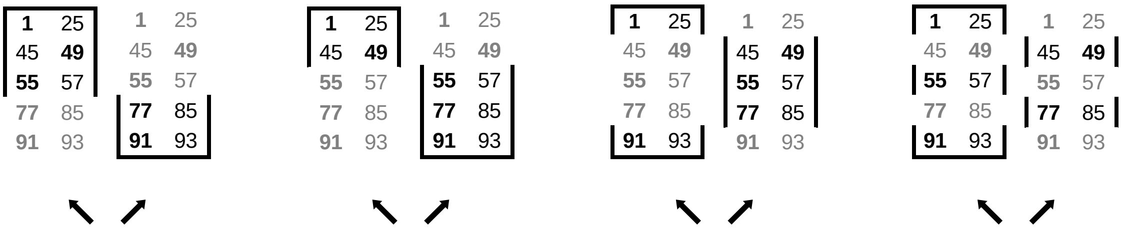
Figure 62: Creation of the first 10 pure and hyper fertiles matrix from the first 25 hyper fertiles matrix.

Inside this concentrated matrix, and according with ultra purity concept, each pair of two consecutive numbers is always made up of two different entities, one of which is ultra pure, the other simply pure.

As illustrated Figure 63, this phenomenon generates sub areas of 6 versus 4 entities with always the same equal distribution of ultra or pure hyper fertiles numbers, so 3 versus 2 (and reverse).

first 10 pure and hyper fertiles matrix:

5 ultra pure and hyper fertiles and 5 (simple) pure and hyper fertiles



1	25	1	25
45	**49**	45	49
55	57	55	57
77	85	**77**	85
91	93	**91**	93

1	25	1	25
45	**49**	45	49
55	57	**55**	57
77	85	**77**	85
91	93	**91**	93

1	25	1	25
45	49	45	**49**
55	57	**55**	57
77	85	**77**	85
91	93	91	93

1	25	1	25
45	49	45	**49**
55	57	55	57
77	85	**77**	85
91	93	91	93

3/2 ratio
3/2 ratio

2/3 ratio
2/3 ratio

3/2 ratio
3/2 ratio

2/3 ratio
2/3 ratio

Figure 63: First 10 pure and hyper fertiles matrix with equal distribution of ultra pure and (simple) pure hyper fertiles numbers in some opposite areas to four versus 6 entities.

10.4.2 Ultimate criteria of enriched number genetics.

Thus, from the closed matrix of the first 100 whole numbers, according to increasingly ultimate criteria of number genetics, from matrices to concentrated matrices, we have therefore defined a set of five numbers with absolute criteria of ultra purity and hyper fertility. These are the numbers *1-49-55-77-91*. Figure 64 explains this concept of enriched number genetics with number 1, this one being the first simultaneously a hyper fertile and ultra pure number, so with two ultimate criteria of number genetics.

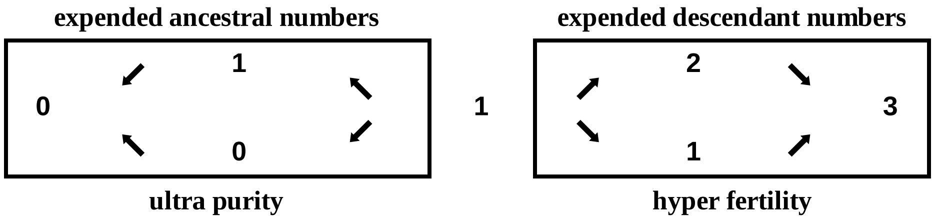
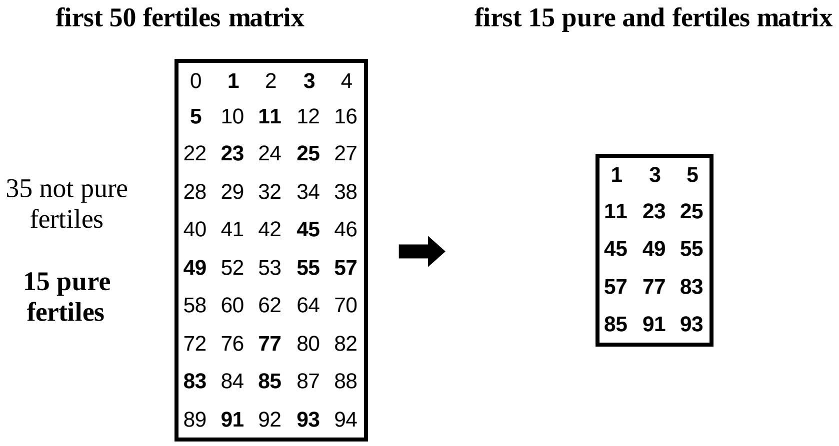


Figure 64: Enriched number genetics of the whole number 1 (one) being the first simultaneously a hyper fertile and ultra pure number.

10.5 First 50 fertiles matrix to first 15 pure and fertiles matrix

As illustrated Figure 65, from the first 50 fertiles matrix, another matrix path is possible to isolate the first 15 numbers to the ultimate criteria of number genetics.



0	**1**	2	**3**	4
5	10	**11**	12	16
22	**23**	24	**25**	27
28	29	32	34	38
40	41	42	**45**	46
49	52	53	**55**	**57**
58	60	62	64	70
72	76	**77**	80	82
83	84	**85**	87	88
89	**91**	92	**93**	94

1	**3**	**5**
11	**23**	**25**
45	**49**	**55**
57	**77**	**83**
85	**91**	**93**

Figure 65: From the 50 fertiles matrix, creation of the first 15 pure and fertiles matrix.

Chapter 8 has been highlighted the first 15 pure and fertile numbers. Their grouping into a matrix of 15 entities also makes it possible to reveal different groups of numbers of still 5*x* entities. This according to increasingly ultimate criteria of number genetics.

Thus, as Figure 66 explains, inside this new matrix, can we differentiate 10 hyper fertile and 5 ultra pure numbers. Also, these two categories of numbers differ from other numbers and according to their own criteria (hyper fertile or ultra pure) by transcendent ratios of value 3/2 in matrix areas of 3*x* and 2*x* entities (9 versus 6 number areas).

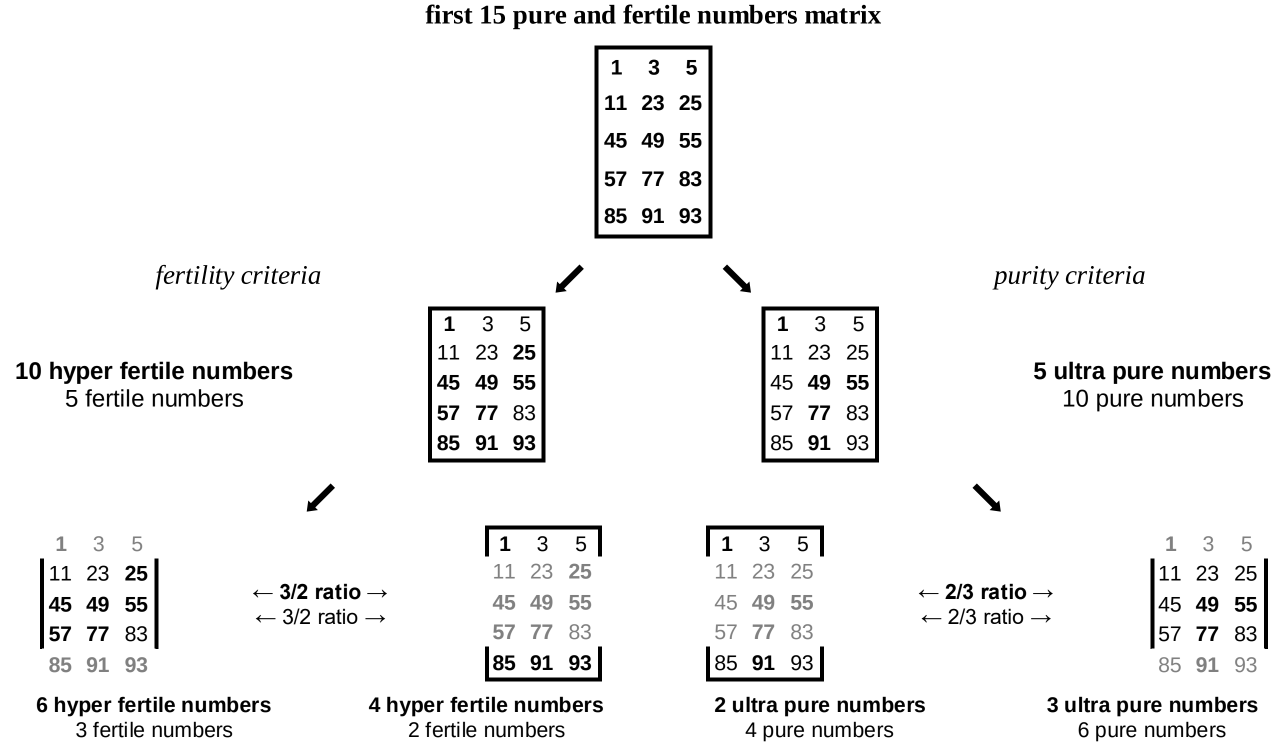


Figure 66: From the first 15 pure and fertiles matrix, differentiation of 10 hyper fertile and 5 ultra pure numbers. Distribution in 3/2 ratios of these in sub matrices at 9 or 6 entities.

In fact as illustrated Figure 67, this 15 numbers matrix is composed of three sets of numbers with strict and evolutionary criteria in accordance with the concept of the number genetics (and enriched genetics):

- 5 just pure and fertile numbers,
- 5 just pure and hyper fertile numbers,
- 5 ultra pure and hyper fertile numbers.

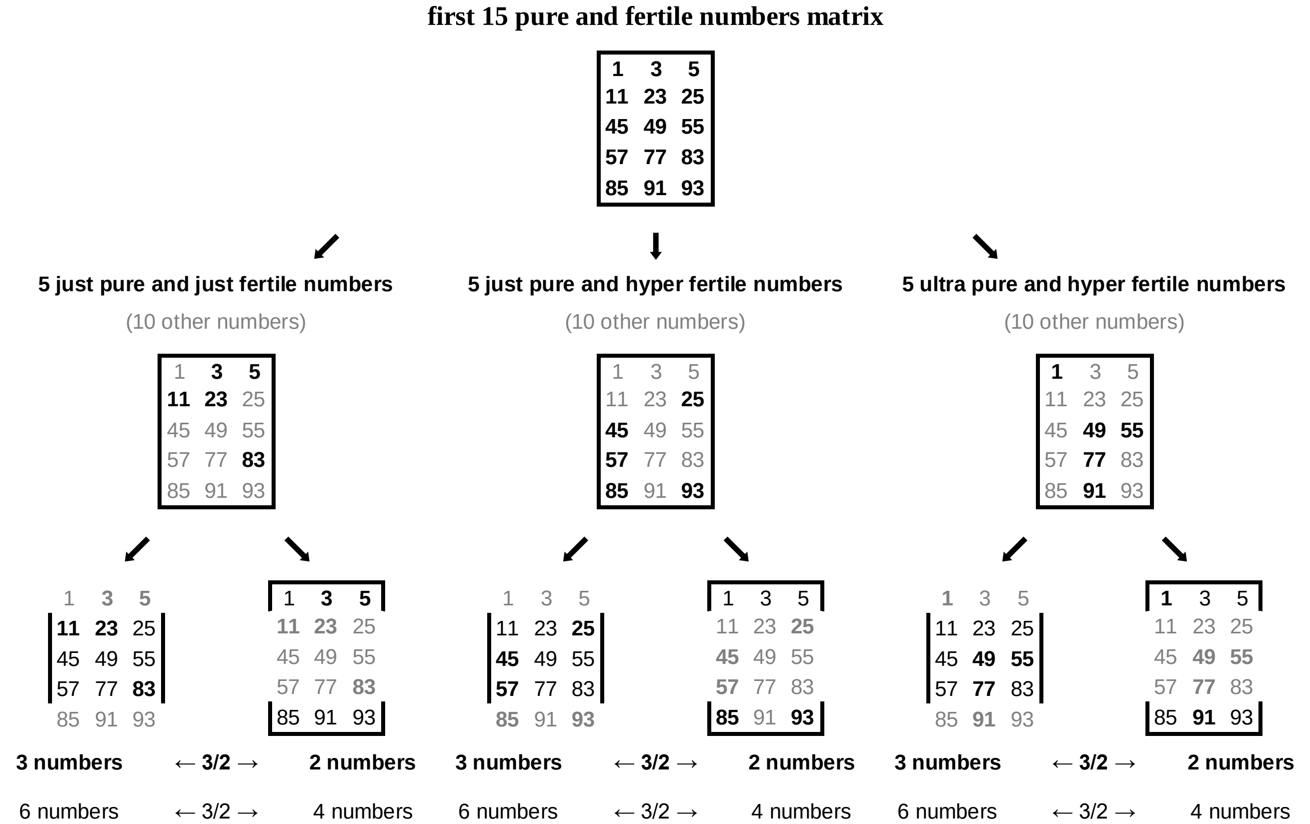


Figure 67: From the first 15 pure and fertiles matrix, differentiation of three sets of five numbers with progressive criteria of number genetics. Distribution in 3/2 ratios of these in sub matrices at 9 or 6 entities.

So, according to gradated criteria of number genetics, as Figure 67 demonstrates its, inside the matrix of first 15 pure and fertile numbers, three entities sets stand out clearly. First is there a set of five just pure and just fertile numbers: *3-5-11-23-83*. Then a set of five just pure and hyper fertile numbers: *25-45-57-85-93*. Finally a third set of five ultra pure and hyper fertile numbers: *1-49-55-77-91*.

Also, Figure 67, these three sets of numbers are all divided into ratios of value 3/2 in same matrix areas of 3*x* and 2*x* entities (9 versus 6 number areas).

Finally, the last five entities set is therefore constituted with the first five numbers, among the first hundred whole numbers, to cumulate the ultimate criteria of number genetics: ultra purity and hyper fertility. Figure 68 illustrates their enriched numeric genetics with detailed anatomy.

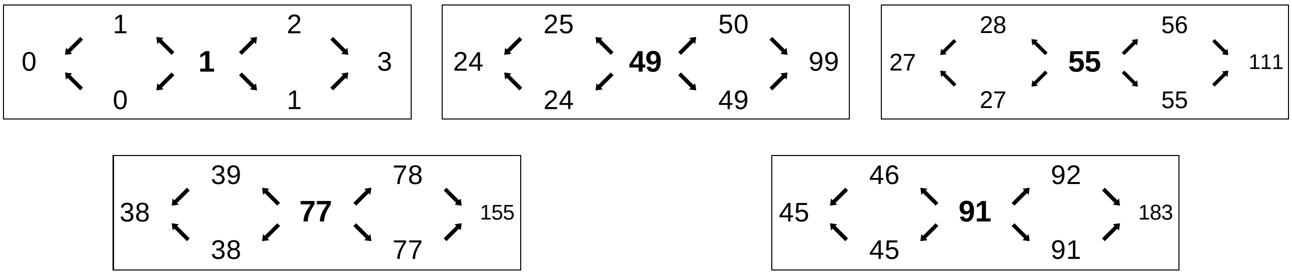
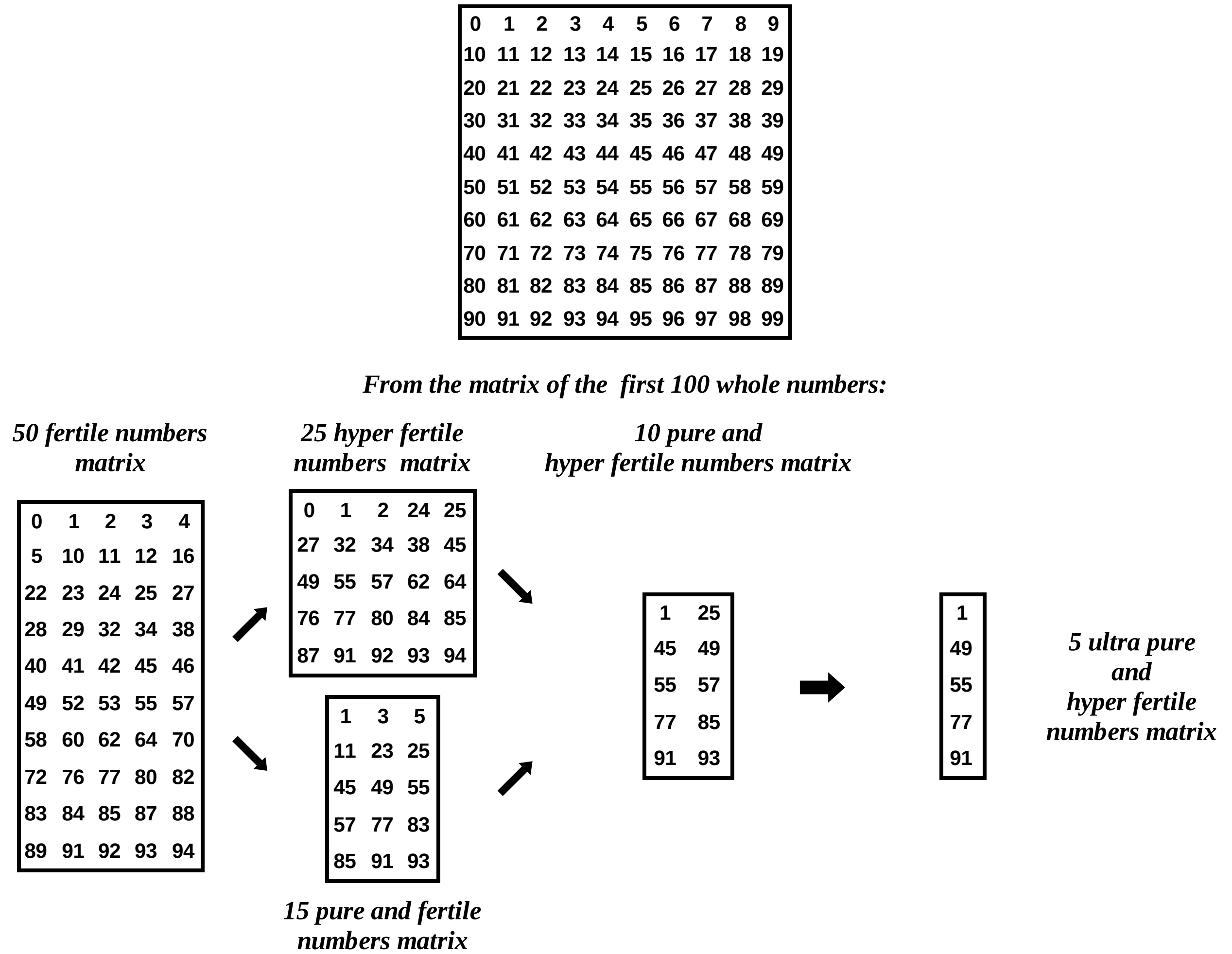


Figure 68: The first five ultra pure and hyper fertile numbers and their enriched numeric genetics. See Figure 64 also.

10.6 Number enriched genetics summary of first hundred numbers set

Here is a summary of how the set of the first 100 whole numbers differ from each other in relation to the concept of number genetics. As Figure 69 clearly illustrates, and as this has been progressively presented previously, the first 100 numbers are always differentiated into sets and subsets which are made up of 5*x* entities. This is a fundamental point to consider.



0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

From the matrix of the first 100 whole numbers:

50 fertile numbers matrix

0	1	2	3	4
5	10	11	12	16
22	23	24	25	27
28	29	32	34	38
40	41	42	45	46
49	52	53	55	57
58	60	62	64	70
72	76	77	80	82
83	84	85	87	88
89	91	92	93	94

25 hyper fertile numbers matrix

0	1	2	24	25
27	32	34	38	45
49	55	57	62	64
76	77	80	84	85
87	91	92	93	94

1	3	5
11	23	25
45	49	55
57	77	83
85	91	93

15 pure and fertile numbers matrix

10 pure and hyper fertile numbers matrix

1	25
45	49
55	57
77	85
91	93

1
49
55
77
91

5 ultra pure and hyper fertile numbers matrix

Figure 69: according to number genetics, organization of the first 100 whole numbers in concentrated matrices at always 5*x* entities as value.

Therefore, matrix of first 100 numbers is composed with 50 fertile versus 50 sterile numbers. Matrix of first 50 fertile numbers is composed with 25 hyper fertile versus 25 (basic) fertile numbers bat also with 15 pure fertile numbers versus 35 non-pure fertile numbers.

Matrix of first 25 hyper fertile numbers is composed with 10 pure versus 15 non-pure numbers. Also in the 15 pure and fertile numbers matrix, 10 numbers are hyper fertile.

Finally, set of first 10 pure hyper fertile numbers is composed with 5 pure (only pure) and hyper fertile numbers versus 5 simultaneous ultra pure and hyper fertile numbers. This five ultra pure and hyper fertile whole numbers inside the set of the first hundred are:

1-49-55-77-91

Here is the final synthesis of the organization of the set of the first hundred numbers in relation to the concept of genetics of numbers that we propose as an extension of Sophie Germain's number concept :

100		*50*		*25*		*10*		*5*
whole numbers	→	*fertiles*	→	*hyper fertiles*	→	*pure hyper fertiles*	→	*ultra pure hyper fertiles*

As we have demonstrated above in different demonstrations, there are two different paths of exploration depending on whether we consider the genetics notions of fertility or purity of numbers. So we can also present this other similar organization:

100		*50*		*15*		*10*		*5*
whole numbers	→	*fertiles*	→	*pure fertiles*	→	*pure hyper fertiles*	→	*ultra pure hyper fertiles*

So, from the matrix of the first 100 whole numbers, all these different number sets are distributed or/and opposed in various configurations of 5x entities. Also, we have amply demonstrated that within all these different sets of 5x entities as side, according to the considerations of genetics of numbers, these various entities can be opposed in various ratios of 3/2 as value.

11 Discussions and conclusions

In the previous paper *"The ultimate numbers and the 3/2 ratio. Just two primary sets of whole numbers"* [3], we introduced the concept of ultimity or non-ultimity of whole numbers:

An ultimate number admits at most one divisor being inferior to it in value,

A non-ultimate number admits more than one divisor being inferior to it in value.

The twin definition of ultimate and non-ultimate numbers proposed here makes it possible to properly divide the set of whole numbers ($\mathbb{N}$),starting with the number 0 (zero), into only two sets of numbers with well-defined and absolute characteristics: a number is either ultimate or non-ultimate. Also, the fact of specifying the numerically lower nature of a divisor to any envisaged number effectively allows that there is no difference in status between the ultimate numbers zero (0) and one (1) and any other number described as ultimate.

Thus, the set of whole numbers ($\mathbb{N}$) is organized just into these two entities:

- the set of ultimate numbers, which is the fusion of the prime numbers sequence with the number 0 and number 1.

- the set of non-ultimate numbers identifying to the non-prime numbers sequence, free of the numbers 0 and 1.

"Safe and Sophie Germain primes" is a concept that only applies to the set of prime numbers. The subjects of Mathematics studies where this concept is exploited therefore do not apply to other whole numbers including the exotic numbers 0 (zero) and 1 (one). On the contrary, here we have expanded this concept to the two unique sets of numbers that we define as constituting the set $\mathbb{N}$: ultimate numbers and non-ultimate numbers.

So we defined as category of whole numbers *"Safe and Sophie Germain ultimates"* but also *"Safe and Sophie Germain non-ultimates"* and we have studied with the same consideration these two *"species"* of number. By analogy with biology and genetics, we replaced the terms of *Save* and *Sophie Germain prime* with *pure number* and *fertile number*. We have done this both with numbers we call ultimate and with those we call non-ultimate. Also in order to complete this genetic nomenclature, we also qualify the other numbers as *non-pure* and *sterile*. This always in consideration of the two species of number that we have defined as ultimate or non-ultimate. Finally, allowing the classification of any number, we introduced the term *orphan* which, by the arithmetic nature of Sophie Germain's concept, applies to any even number which therefore has no "ancestor".

Combining Sophie Germain's arithmetic mechanism with a process of cell reproduction biology, we suggest that the classical algebraic notation "$2x + 1$" used to define the Germain's descent of x be replaced by $V(x) + R(x)$. We denote by $V(x)$ the value of x and by $R(x)$ the value of its rank in the sequence of whole numbers starting with the number 0. Thus, for any whole number, $R(x)$ is equal to $x + 1$.

We also propose to call $G(x)$ the result of this *fusion* of the two entities $V(x)$ and (Rx). We therefore have the following mechanism of numerical genetics: the descent of x is equal to the sum of the value of x and the value of the rank of x. We thus write "$G(x) = V(x) + R(x)$" and consider that these three values are only emanations of x. We consider these values as real emanations of number x since no other mathematical entity not depending on x composes them. This, in contrast to the algebraic operation "$f(x) = 2x + 1$" where the numeric entity "1" is foreign to it.

We have logically proposed the names $V(x)$ and $R(x)$ in reference to the Value of x and the Rank of x, two emanations of x. For this third emanation, we propose the letter G in reference to "generation of x" but also in reference (and homage) to Sophie Germain, initiator of the concept of Germain prime and safe prime; concept that we expand in this paper.

We therefore introduce the idea that the arithmetic mechanism producing $G(x)$ from x is purely as a biological process of cellular reproduction. Thus, x (any whole number) really divides like a biological cell into two daughter cells to generate $V(x)$ and $R(x)$. Then these two cells merge biologically to form a new cell which is none other than $G(x)$.

Considering this mechanism of cellular reproduction, we have enriched this concept of genetics of numbers by considering new notions of *"hyper fertility"* and *"ultra purity"*. So, if all the ancestral components of a number are of its same species, then it is called *ultra pure*. Also, if all its descendant components are of its same species, then we call it *hyper fertile*.

Using all this genetics of numbers that we have built since the concept of safe and Sophie Germain primes, we have demonstrated that the different genetic families of whole number are closely related in closed matrices of side $5x$. This is manifested in starting closed matrices of the set $\mathbb{N}$. In these matrices, globally and/or transversally, the different varieties of numbers are organized very much in opposition of ratios of exact value 3/2.

Also, many of the phenomena presented, in addition to involving this arithmetic ratio of 3/2, revolve around the remarkable identity $(a + b)^2 = a^2 + 2ab + b^2$ where a and b have the values 3 and 2. This generates many entanglement in the arithmetic arrangements operating between the different entities considered and therefore strengthens their credibility by the dimensional amplification of these arithmetic phenomena.

To conclude, from the various arithmetic demonstrations introduced in this investigation of the whole number set including number 0 (zero), we legitimately propose a twin new mathematical concept classifying all whole numbers just into two primary sets of entities with distinct and well defined properties:

- the ultimates, admitting at most one divisor being inferior to them in value,
- the non-ultimates, admitting more than one divisor being inferior to them in value.

Also, we propose to enrich and extend notions of Sophie Germain numbers (prime, safe) simultaneously with this double new concept of ultimity and non-ultimity of the whole numbers.

Finally, we also propose to consider Sophie Germain's arithmetic concept as a pure biological process of cellular reproduction and to introduce the notion of genetics of numbers in order to invest, using the extension of Sophie Germain's concept, the entire set $\mathbb{N}$, the set of prime numbers starting with the number 0 (zero).

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Number genetics glossary

This glossary list many propositions introduced in this paper about new concept of ultimate numbers and development of concept of Sophie Germain primes expanded to ultimate and non-ultimate numbers.

Ultimate number:

→ An ultimate number admits **at most one divisor** being inferior to it in value

Non-ultimate number:

→ A non-ultimate number admits **more than one divisor** being inferior to it in value

Fertile number:

→ If x and $(2x + 1)$ are either both ultimates or both non-ultimates, then x is **a fertile number**

Sterile number:

→ If x and $(2x + 1)$ are not either both ultimates nor both non-ultimates, then x is a **sterile number**

Pure number:

→ If x and $(2x + 1)$ are either both ultimates or both non-ultimates, then $(2x + 1)$ is **a pure number**

Hybrid number:

→ If x and $(2x + 1)$ are not either both ultimates nor both non-ultimates, then $(2x + 1)$ is a **hybrid number**

Orphan number:

→ If $(x - 1)/2$ is not a whole number, then x is **an orphan number.** → **Any even whole number** is an orphan number

Hyper fertile number:

→ If x, $(x + 1)$ and $(2x + 1)$ are either both ultimates or both non-ultimates, then x is **a hyper fertile number**

Ultra pure number:

→ If x, $(x + 1)$ and $(2x + 1)$ are either both ultimates or both non-ultimates, then $(2x + 1)$ is **a ultra pure number**