

Monty Hall game simplified

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abstract

The game show problem aka Monty Hall problem [1], originated when Craig Whitaker posed a question of a winning strategy for a 3 door game show to Marilyn Savant who wrote articles for Parade magazine.

Her 1990 response was to switch doors when given the option. [2] [3]

The debate of probability of success as $2/3$ vs $1/2$ has continued until today.

This paper reveals errors in her response.

Whitaker's question

Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say #1, and the host, who knows what's behind the doors, opens another door, say #3, which has a goat. He says to you, "Do you want to pick door #2?" Is it to your advantage to switch your choice of doors?

Marilyn Savant's 1990 response [3]

Yes; you should switch. The first door has a $1/3$ chance of winning, but the second door has a $2/3$ chance.

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Here are the results.

	DOOR 1	DOOR 2	DOOR 3	RESULT
GAME 1	AUTO	GOAT	GOAT	Switch and you lose.
GAME 2	GOAT	AUTO	GOAT	Switch and you win.
GAME 3	GOAT	GOAT	AUTO	Switch and you win.
GAME 4	AUTO	GOAT	GOAT	Stay and you win.
GAME 5	GOAT	AUTO	GOAT	Stay and you lose.
GAME 6	GOAT	GOAT	AUTO	Stay and you lose.

fig.1

Comparison

Let there be 3 distinct prizes, car 'c' and 2 goats 'g1 and g2'.

For the purpose of the game, the goats are always behind different doors, thus are identified with a generic 'g'.

Player guesses door 1 each game.

Host cannot open player 1st guess.
 Host cannot open door with a car.
 Host must offer the player a 2nd guess.
 Door 1 is stay results, door r is switch results.

e	1	2	3	r
1	g	g	c	c
2	g	c	g	c
3	c	g	g	g
4	c	g	g	g

fig.2

e	1	2	3	r
1	g	g	c	c
2	g	c	g	c
3	c/2	g/2	g/2	g/2
3	c/2	g/2	g/2	g/2

fig.3

In fig.2 all games have the same format: player 1st guess, host opens 1 door and eliminates it from play (red), player 2nd guess, host opens door from player 2nd guess to verify win or lose the car. Comparing column 1 (stay) to column r (remaining closed door) shows no advantage to switch. Door r eliminates the need to play the games twice. For each game the player's 2nd guess is always 1 of 2 doors, door 1 and door r. The 1st guess is never verified. The host's action transforms a 3-door game to a 2-door game, thus the 1 of 3 probability does not apply. The statistics for a 2-door game are the same as for a coin toss, 1/2 with no bias, but the entertainment value is greater if the player is involved.

How did Marilyn Savant conclude a different outcome?

She did not distinguish goat 1 from goat 2 either, but considered classes of cars and goats. When the car is behind the player's 1st guess, the host can open (door 2 and door 3), but not in the same game (revealing the car location). Thus an additional game is required for the 2nd door. In fig.3, she assumed the host opened (door 2 or door 3) by switching between the 2 doors equally for half of game 3. This effectively introduces a bias with 1 less win for stay and 1 more win for switch. Comparing column 1 to column r shows an apparent advantage to switch from 1/3 to 2/3 win ratio.

conclusion

The errors are due to Savant's lack of understanding simple probabilities. The apparent advantage is based on an inconsistent and distorted interpretation of the game.

Using the measure of success as (door with car)/(all possible guesses), the player doesn't win with their 1st guess with a 1 in 3 chance, since the host does not open that door to verify win or lose. When the player gets their 2nd guess, there are not 3 ways to choose 1 of 2 things. The host has benefited the player by increasing their chance from $1/3$ to $1/2$.

The host then verifies a win or lose by opening the door from the 2nd guess. Since the player never knows the location of the car until after their 2nd guess, there is no basis to define a strategy, thus the player can only make a random guess.

The answer to Whitaker's question is no.

reference

[1] The American Statistician, August 1975, Vol. 29, No. 3

[2] game show problem, Wikipedia Sep 2024

[3] Marilyn vos Savant,

<https://web.archive.org/web/20130121183432/http://marilynvossavant.com>