

Magic of Natural integers

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In this article, I studied the behaviour of natural integers, after performing operations on them. The results I have obtained are intriguing and I cannot explain them.

I share this original work with young researchers for deepening and improvement.

Key words: Integers, operations, sequences.

Magic of Natural integers

Introduction: this work consists, of making a product of two integers and using the behaviour of this product, when we multiply it in turn with another integer.

I notice that some of the properties are valid only when one of the factors is 17.

Example 1: Consider the following product: $17 \cdot 23 = 391$

By performing the following operations we obtain:

(I)

$$\begin{aligned} 391 \cdot 53 &= 20723 \rightarrow 207 = 9 \cdot 23 \\ 391 \cdot 153 &= 59823 \rightarrow 598 = 26 \cdot 23 \\ 391 \cdot 253 &= 98923 \rightarrow 989 = 43 \cdot 23 \\ 391 \cdot 353 &= 138023 \rightarrow 1380 = 60 \cdot 23 \\ 391 \cdot 453 &= 177123 \rightarrow 1771 = 77 \cdot 23 \\ 391 \cdot 553 &= 216223 \rightarrow 2162 = 94 \cdot 23 \\ 391 \cdot 653 &= 255323 \rightarrow 2553 = 111 \cdot 23 \\ &\vdots \\ &.etc \end{aligned}$$

(II)

$$\begin{aligned} 391 \cdot 87 &= 34017 \rightarrow 340 = 20 \cdot 17 \\ 391 \cdot 187 &= 73117 \rightarrow 731 = 43 \cdot 17 \\ 391 \cdot 287 &= 112217 \rightarrow 1122 = 66 \cdot 17 \\ 391 \cdot 387 &= 151317 \rightarrow 1513 = 89 \cdot 17 \\ 391 \cdot 487 &= 190417 \rightarrow 1904 = 112 \cdot 17 \\ 391 \cdot 587 &= 229517 \rightarrow 2295 = 135 \cdot 17 \\ 391 \cdot 687 &= 268617 \rightarrow 2686 = 158 \cdot 17 \\ &\vdots \\ &.etc \end{aligned}$$

In the equalities (I), we have the coefficients of (23)

$$\begin{array}{l} 9 \cup 26 \cup 43 \cup 60 \cup 77 \cup 94 \cup 111 \dots etc \\ .17 \cup 17 \cup 17 \cup 17 \cup 17 \cup 17 \end{array} \quad (A).$$

Arithmetic sequence of reason 17

In the equalities (II), we have the coefficients of (17)

$$\begin{array}{l} 20 \cup 43 \cup 66 \cup 89 \cup 112 \cup 135 \cup 158 \dots etc \\ \dots 23 \cup 23 \cup 23 \cup 23 \dots 23 \cup 23 \end{array} \quad (B).$$

Arithmetic sequence of reason 23

If we take the terms of the sequence (A) , and the third term(66) of the sequence (B), making the sum of their square:

$$\begin{array}{l}
 66^2 + 9^2 = 261 \cdot 17 \\
 66^2 + 26^2 = 296 \cdot 17 \\
 66^2 + 43^2 = 365 \cdot 17 \\
 66^2 + 60^2 = 468 \cdot 17 \\
 66^2 + 77^2 = 605 \cdot 17 \\
 66^2 + 94^2 = 776 \cdot 17 \\
 66^2 + 111^2 = 981 \cdot 17 \\
 . \\
 .etc
 \end{array}
 \begin{array}{l}
 261 \cup 296 \cup 365 \cup 468 \cup 605 \cup 776 \cup 981....etc \\
35 \cup69 \cup103 \cup137 \cup171 \cup205 \\
34 \cup34 \cup34 \cup34 \cup34 \\
 34 = 2 \cdot 17
 \end{array}$$

We notice that

$$\begin{array}{lll}
 261 + 9 = 270 & 365 + 43 = 408 & 605 + 77 = 682 \\
 296 - 26 = 270 & 468 - 60 = 408 & 776 - 94 = 682.....etc \\
 \\
 408.....270.....682 \\
138.....274 \\
136 = 8 \cdot 17
 \end{array}$$

Note: why the sum of the squares of the equalities (I) is valid only with the equality 3 of (II) ???

Example 2: Consider the following product: $17 \cdot 29 = 493$

(III)	(IV)
$493 \cdot 53 = 26129 \rightarrow 261 = 9 \cdot 29$	$493 \cdot 69 = 34017 \rightarrow 340 = 20 \cdot 17$
$493 \cdot 153 = 75429 \rightarrow 754 = 26 \cdot 29$	$493 \cdot 169 = 83317 \rightarrow 833 = 49 \cdot 17$
$493 \cdot 253 = 124729 \rightarrow 1247 = 43 \cdot 29$	$493 \cdot 269 = 132617 \rightarrow 1326 = 78 \cdot 17$
$493 \cdot 353 = 174029 \rightarrow 1740 = 60 \cdot 29$	$493 \cdot 469 = 181917 \rightarrow 1819 = 107 \cdot 17$
$493 \cdot 453 = 223329 \rightarrow 2233 = 77 \cdot 29$	$493 \cdot 569 = 231217 \rightarrow 2312 = 136 \cdot 17$
$493 \cdot 553 = 272629 \rightarrow 2726 = 94 \cdot 29$	$493 \cdot 669 = 280517 \rightarrow 2805 = 165 \cdot 17$
$493 \cdot 653 = 321929 \rightarrow 3219 = 111 \cdot 29$	.
.	.etc
.etc	

$$\begin{array}{l}
 9 \cup 26 \cup 43 \cup 60 \cup 77 \cup 94 \cup 111....etc \\
 .17 \cup 17 \cup 17 \cup 17 \cup 17 \cup 17
 \end{array}
 \quad (C)$$

$$20_49_78_106_135_164_193....etc \quad (D)$$

$$...29_29_29_29_29_29_29$$

We notice that:

If we take the terms of the sequence (C) , and the second term(49) of the sequence (D), making the sum of their square:

$$49^2 + 9^2 = 146 * 17$$

$$49^2 + 26^2 = 181 * 17$$

$$49^2 + 43^2 = 250 * 17$$

$$49^2 + 60^2 = 353 * 17$$

$$49^2 + 77^2 = 490 * 17$$

$$49^2 + 94^2 = 661 * 17$$

$$146_181_250_353_490_661.....etc$$

$$....35_69_103_137_171$$

$$.....34_34_34_34$$

$$34 = 2 * 17$$

$$146 + 9 = 155 \quad 250 + 43 = 293 \quad 490 + 77 = 567$$

$$181 - 26 = 155 \quad 353 - 60 = 293 \quad 661 - 94 = 567$$

$$155....293.....567$$

$$.....138.....274$$

$$.....136 = 8 * 17$$

Example 3: Consider the following product $17 * 59 = 1003$

(V)

$$1003 * 53 = 53159 \rightarrow 531 = 9 * 59$$

$$1003 * 153 = 153459 \rightarrow 1534 = 26 * 59$$

$$1003 * 253 = 253759 \rightarrow 2537 = 43 * 59$$

$$1003 * 353 = 354059 \rightarrow 3540 = 60 * 59$$

$$1003 * 453 = 454359 \rightarrow 4543 = 77 * 59$$

$$1003 * 553 = 554659 \rightarrow 5546 = 94 * 59$$

(VI)

$$1003 * 39 = 39117 \rightarrow 391 = 23 * 17$$

$$1003 * 139 = 139417 \rightarrow 1394 = 82 * 17$$

$$1003 * 239 = 239717 \rightarrow 2397 = 141 * 17$$

$$1003 * 339 = 340017 \rightarrow 3400 = 200 * 17$$

$$1003 * 439 = 440317 \rightarrow 4403 = 259 * 17$$

$$1003 * 539 = 540617 \rightarrow 5406 = 318 * 17$$

$$.$$

$$1003 * 839 = 841517 \rightarrow 8415 = 495 * 17$$

$$9_26_43_60_77_94_111....etc \quad (E)$$

$$.17_17_17_17_17_17$$

23_82_141_200_259_318_377_436_495....etc (F)
 ..59_..59_..59_..59_..59_..59_..59_..59_..59

We notice that:

If we take the terms of the sequence (E) , and the ninth term(495) of the sequence (F), making the sum of their square:

$$\begin{array}{l}
 495^2 + 9^2 = 14418 * 17 \\
 495^2 + 26^2 = 14453 * 17 \\
 495^2 + 43^2 = 14522 * 17 \\
 495^2 + 60^2 = 14625 * 17 \\
 495^2 + 77^2 = 14762 * 17 \\
 495^2 + 94^2 = 14933 * 17
 \end{array}
 \begin{array}{l}
 14418_14453_14522_14625_14762_14933 \\
35.....69.....103.....137.....171 \\
34.....34.....34.....34
 \end{array}$$

14418 + 9 = 14427	14522 + 43 = 14565	14762 + 77 = 14839
14453 - 26 = 14427	14625 - 60 = 14565	14933 - 94 = 14839

$$\begin{array}{l}
 14427.....14565.....14839 \\
138.....276 \\
136 = 8 * 17
 \end{array}$$

Example 4: Consider the following product: $17 * 109 = 1853$

(VII)

(VIII)

$1853 * 89 = 164917 \rightarrow 1649 = 97 * 17$	$1853 * 353 = 654109 \rightarrow 654 = 6 * 109$
$1853 * 189 = 350217 \rightarrow 3502 = 206 * 17$	$1853 * 1353 = 2507109 \rightarrow 2507 = 23 * 109$
$1853 * 289 = 535517 \rightarrow 5355 = 315 * 17$	$1853 * 2353 = 4360109 \rightarrow 4360 = 40 * 109$
$1853 * 389 = 720817 \rightarrow 7208 = 424 * 17$	$1853 * 3353 = 6213109 \rightarrow 6213 = 57 * 109$
$1853 * 489 = 906117 \rightarrow 9061 = 533 * 17$	$1853 * 4353 = 8066109 \rightarrow 8066 = 74 * 109$
$1853 * 589 = 1091417 \rightarrow 10914 = 642 * 17$	$1853 * 5353 = 9919109 \rightarrow 9919 = 91 * 109$
.	.
.etc	.etc

97_206_315_424_533_642_751_860....etc (G)
 .109_109_109_109_109_109_109

$$6_23_40_57_74_91_108....etc \quad (H)$$

$$.17_17_17_17_17_17$$

We notice that:

-If we take the terms of the sequence (G) , and the third term (40) of the sequence (H), making the sum of their square

-And we take the terms of the sequence (H), and the eight term (860) of the sequence (G), making the sum of their square, we obtain:

(E)	(F)
$40^2 + 97^2 = 101 * 109$	$860^2 + 6^2 = 43508 * 17$
$40^2 + 206^2 = 404 * 109$	$860^2 + 23^2 = 43537 * 17$
$40^2 + 315^2 = 925 * 109$	$860^2 + 40^2 = 43600 * 17$
$40^2 + 424^2 = 1664 * 109$	$860^2 + 57^2 = 43697 * 17$
$40^2 + 533^2 = 2621 * 109$	$860^2 + 74^2 = 43828 * 17$
$40^2 + 642^2 = 3796 * 109$	$860^2 + 91^2 = 43993 * 17$
101_404_925_1664_2621_3796	43508_43537_43600_43697_43828_43993
...303..521..739....957....1175	29.....63.....97.....131.....165
.....218...218....218....218 $\rightarrow 2 * 109$	34.....34.....34.....34 $\rightarrow 34 = 2 * 17$

Example 5: Consider the following product $43 * 47 = 2021$

$2021 * 7 = 14147 \rightarrow 141 = 3 * 47$	$2021 * 83 = 167743 \rightarrow 1677 = 39 * 43$
$2021 * 107 = 216247 \rightarrow 2162 = 46 * 47$	$2021 * 183 = 369843 \rightarrow 3698 = 86 * 43$
$2021 * 207 = 418347 \rightarrow 4183 = 89 * 47$	$2021 * 283 = 571943 \rightarrow 5719 = 133 * 43$
$2021 * 307 = 620447 \rightarrow 6204 = 132 * 47$	$2021 * 383 = 774043 \rightarrow 7740 = 180 * 43$
$2021 * 407 = 822547 \rightarrow 8225 = 175 * 47$	$2021 * 483 = 976143 \rightarrow 9761 = 227 * 43$
$2021 * 507 = 1024647 \rightarrow 10246 = 218 * 47$	$2021 * 583 = 1178243 \rightarrow 11782 = 274 * 43$
.	.
.etc	.etc

$$3_46_89_132_175_218_261....etc....(I)$$

$$.43_43_43_43_43_43$$

$$39_86_133_180_227_274_321....etc.....(J)$$

$$...47_47_47_47_47_47$$

We notice that: in this case, I cannot find a sequence or there is the sum of squares. Is it valid just when the product of the two numbers, one of them is 17.