The number 239

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ABSTRACT: We give some formulas related to the number 239.

I. Introduction

- The number 239 is a prime number
- 239 appears in Machin’s formula

$$\frac{\pi}{4} = 4\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right) \tag{1}$$

In this note we give some formulas related to 239.

Remark: $\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}$.

II. Some formulas related to 239

Entry 1.

$$239 = 3^5 - 2^2 \tag{2}$$

$$239 = 5\cdot 7^2 - 2\cdot 3 \tag{3}$$

$$239 = 5\cdot 7^2 - 2^2 - 2^1 \tag{4}$$

Entry 2.

$$239 = 2\cdot 5^3 - 2^3 - 3^1 \tag{5}$$

$$239 = 2\cdot 5^3 - 3^2 - 2^1 \tag{6}$$

Entry 3.

$$239 = 3\cdot 7\cdot 11 + 2^3 \tag{7}$$

$$239 = 2\cdot 7\cdot 17 + 1 \tag{8}$$

$$239 = 2^4\cdot 3\cdot 5 - 1 \tag{9}$$

Entry 4.

$$239 = 10^2 + 9^2 + 7^2 + 3^2 \tag{10}$$

$$239 = 12^2 + 11^2 - 5^2 - 1^2 \tag{11}$$

(12)

Entry 5.

$$239 = 17^2 - 7^2 - 1^2 \tag{13}$$

$$239 = 7^3 - 10^2 - 2^2 \tag{14}$$

$$239 = 5^3 + 4^3 + 7^2 + 1^2 \tag{15}$$

Entry 6.

$$239 = 3 \cdot 79 + 2 \tag{16}$$

$$239 = 2 \cdot 3 \cdot 5 \cdot 7 + 2^2 + 5^2 \tag{17}$$

$$239 = 5^3 + 3^4 + 2^5 + 1 \tag{18}$$

$$239 = 13^2 + 8^2 + 2 \cdot 3 \tag{19}$$

$$239 = 2^5 + 3^5 - 6^2 = 2^5 + 3^5 - 2^2 \cdot 3^2 \tag{20}$$

$$239 = 6^3 + 2^4 + 2^3 - 1 \tag{21}$$

$$239 = 15^2 + 15 - 1 \tag{22}$$

$$239 = 2^8 - 2^4 - 1 \tag{23}$$

$$239^2 = 2 \cdot 13^4 - 1 \tag{24}$$

$$239 = 29 \cdot 43 + 29^2 - 43^2 \tag{25}$$

$$239 = 14 \cdot 15 - 14^2 + 15^2 \tag{26}$$

Entry 7.

$$\frac{\pi}{4} = \tan^{-1}\left(\frac{275 - 24\sqrt{62}}{239}\right) + \tan^{-1}\left(\frac{18 + 12\sqrt{62}}{239}\right) \tag{27}$$

Entry 8.

$$\frac{\pi}{4} = \tan^{-1}\left(\frac{8 + 36\sqrt{3}}{239}\right) + \tan^{-1}\left(\frac{255 - 72\sqrt{3}}{239}\right) \tag{28}$$

Entry 9.

$$\pi = 6\sqrt{3} \sum_{n=0}^{\infty} \frac{(-1)^n}{239^{n+1}} \sum_{k=0}^{n} \binom{n}{k} 2^{2n-2k} \left(\frac{81}{10k+1} - \frac{27}{10k+3} + \frac{9}{10k+5} - \frac{3}{10k+7} + \frac{1}{10k+9}\right) \tag{29}$$

Entry 10.

$$\left|\left(\frac{2 \cdot 5 \cdot 13 \cdot 57 + \frac{1}{2}}{239}\right)^{1/3} - \pi\right| = 1.788\ldots \times 10^{-8} \tag{30}$$

$$\left|\frac{239}{76}\left(1 - 10^{-3}\right) - \pi\right| = 5.483\ldots \times 10^{-7} \tag{31}$$

Entry 11.

$$\pi = \frac{6\sqrt{53}}{\sqrt{239}} \sum_{n=0}^{\infty} \binom{2n}{n} \frac{2^{-4n}}{2n+1} 239^{-n} \sum_{k=0}^{n} \binom{2n+1}{2k} 3^{2n-2k} \cdot 53^{n-k} \tag{32}$$

Entry 12.

$$\pi = 2\sqrt{114241}\sum_{n=0}^{\infty}\binom{2n}{n}\frac{2^{-4n}}{2n+1}239^{-2n-1}\sum_{k=0}^{n}\binom{2n+1}{2k}114241^{n-k} \tag{33}$$

Entry 13.

$$\pi = 8\tan^{-1}\left(\frac{7}{17}\right) + 4\tan^{-1}\left(\frac{1}{239}\right) \tag{34}$$

Entry 14.

$$\pi = \frac{32}{239}\sum_{n=0}^{\infty}(-1)^n\,239^{-n}\cdot 17^n\left(24\,{}_4F_3\left(\begin{matrix}\frac{1}{4},\frac{1}{4},\frac{3}{4},-n\\ \frac{1}{2},1,\frac{5}{4}\end{matrix}\,\middle|\,\frac{16}{17}\right) + {}_4F_3\left(\begin{matrix}\frac{3}{4},\frac{3}{4},\frac{5}{4},-n\\ 1,\frac{3}{2},\frac{7}{4}\end{matrix}\,\middle|\,\frac{16}{17}\right)\right) \tag{35}$$

Remark: ${}_4F_3$ is the generalized hypergeometric function.

Entry 15.

$$\frac{\pi}{4} = \tan^{-1}\left(\frac{119}{239}\right) + \tan^{-1}\left(\frac{80}{239}\right) + \tan^{-1}\left(\frac{20}{239^2 - 40\cdot 239 + 20}\right) \tag{36}$$

Entry 16.

$$\pi = \frac{239}{76} - \frac{1}{76}\tan^{-1}\left(-i\,\frac{(\cos(1)+i\sin(1))^{239} - (\cos(1)-i\sin(1))^{239}}{(\cos(1)+i\sin(1))^{239} + (\cos(1)-i\sin(1))^{239}}\right) \tag{37}$$

where

$$-i\,\frac{(\cos(1)+i\sin(1))^{239} - (\cos(1)-i\sin(1))^{239}}{(\cos(1)+i\sin(1))^{239} + (\cos(1)-i\sin(1))^{239}} = 0.24361292896477\ldots,\quad i = \sqrt{-1} \tag{38}$$

III. References

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