

Fermat's Last Theorem

Fermat's Last Theorem: There are no natural numbers (1, 2, 3, ...) a , b , and c such that $a^n + b^n = c^n$, in which n is a natural number greater than 2.

Proof

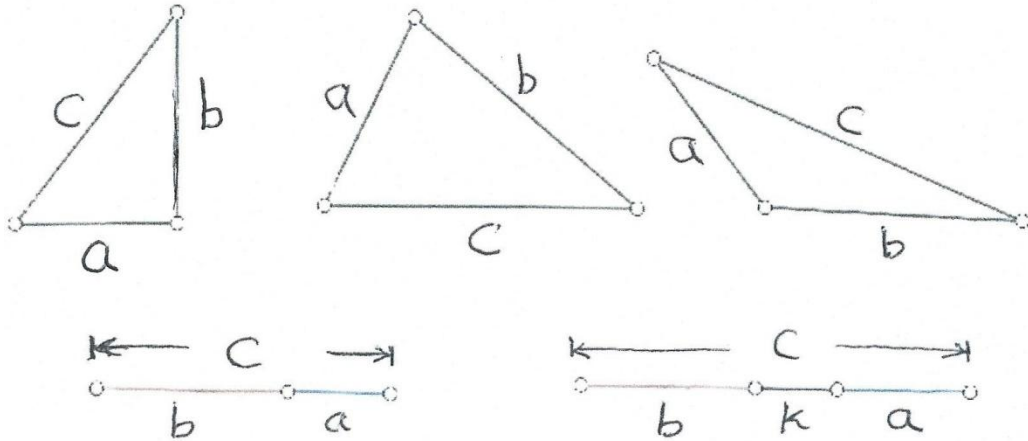
1 – Taha's Logic Equations Fact

$a = b$ (True or False) & $c = d$ (True) $\Rightarrow (a - c = b - d)$ False $\Leftrightarrow a = b$ (False)

2 – Taha's Coefficient Fact: $[a + b = c \Rightarrow ax + by = cz] \Leftrightarrow x = y = z$

3- Taha's (N_+) & Three Sided Geometric Shapes Fact below:

$N_+ = (\text{RTs}) \cup (\text{ATs}) \cup (\text{OTs}) \cup (\text{Segments } c = a + b) \cup (\text{Segments } c > a + b)$



In all shapes

1st) if $a = b$ in all above five geometric shapes

is $a^n + b^n = c^n$?

let $a^n + b^n = c^n$

$a^n + a^n = c^n$

$2a^n = c^n$

$\therefore c = \sqrt[n]{2} a \notin N_+$

$\therefore a^n + b^n \neq c^n$

2nd) if $a = b = c$ in Acute Triangle

is $a^n + b^n = c^n$?

let $a^n + b^n = c^n$

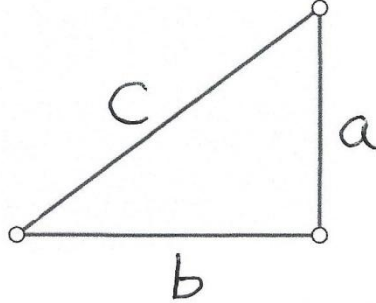
$\therefore a^n + a^n = a^n$

$a^n = 0 \Rightarrow a = 0 \dots$ Contradiction to $a \in N_+$

$\therefore a^n + b^n \neq c^n$

3rd) if $a < b < c$ in all above five geometric shapes

Shape1: Right Triangle



A) is $a^n + b^n = c^n$? $a < b < c$

let $a^n + b^n = c^n \dots eq1$

$\therefore a^2 + b^2 = c^2 \dots eq2$

$\therefore a^n - a^2 + b^n - b^2 = c^n - c^2$ (by subtraction)

$a^n (1 - a^{2-n}) + b^n (1 - b^{2-n}) = c^n (1 - c^{2-n}) \dots eq3$

Taha's Coefficient Fact in eq3: $1 - a^{2-n} = 1 - b^{2-n} = 1 - c^{2-n} \Leftrightarrow a^n + b^n = c^n$

$\therefore 1 - a^{2-n} = 1 - b^{2-n} \Rightarrow a = b \dots$ contradiction to $a < b$

$\therefore a^n + b^n \neq c^n$

B) is $a^n + c^n = b^n$? $a < b < c$

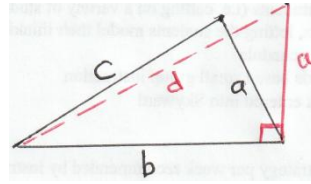
let $a^n + c^n = b^n$

$\therefore c^n < b^n \dots$ (subtracted a^n from 1st side)

$\therefore c < b \dots$ Contradiction to $c > b$

$\therefore a^n + c^n \neq b^n$

Shape2: Acute Triangle



A) is $a^n + b^n = c^n$? when $a < b < c$

let $a^n + b^n = c^n \dots eq1$

$\therefore a^2 + b^2 = d^2 \dots eq2$

$\therefore a^n - a^2 + b^n - b^2 = c^n - d^2 \dots$ by subtraction (eq1 - eq2)

$a^n (1 - a^{2-n}) + b^n (1 - b^{2-n}) = c^n \left(1 - \frac{d^2}{c^n}\right) \dots eq3$

Taha's Coefficient Fact in eq3: $1 - a^{2-n} = 1 - b^{2-n} = 1 - \frac{d^2}{c^n} \Leftrightarrow a^n + b^n = c^n$

$\therefore 1 - a^{2-n} = 1 - b^{2-n} \Rightarrow a = b \dots$ contradictio to $a < b$

$\therefore a^n + b^n \neq c^n$

B) is $a^n = b^n + c^n$? when $a < b < c$

let $a^n = b^n + c^n$

$\therefore a^n > c^n \dots$ subtracted b^n from 2nd side

$\therefore a > c \dots$ contradiction to $a < b < c$

$\therefore a^n \neq b^n + c^n$

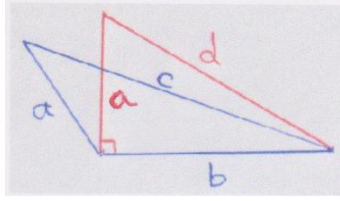
C) is $a^n = b^n + c^n$? when $a = b = c$

Proof:

let $a^n = b^n + c^n \Rightarrow a^n = a^n + a^n \Rightarrow 1 = 2$ false

$\therefore a^n \neq b^n + c^n$

Shape3: Obtuse Triangle



A) is $a^n + b^n = c^n$? when $a < b < c$.

let $a^n + b^n = c^n \dots eq1$

$\therefore a^2 + b^2 = d^2 \dots eq2$

$\therefore a^n - a^2 + b^n - b^2 = c^n - d^2$

$\therefore a^n(1 - a^{2-n}) + b^n(1 - b^{2-n}) = c^n \left(1 - \frac{d^2}{c^n}\right) \dots eq3$

Taha's Coefficient Fact in eq3: $1 - a^{2-n} = 1 - b^{2-n} = 1 - \frac{d^2}{c^n} \Leftrightarrow a^n + b^n = c^n$

$\therefore 1 - a^{2-n} = 1 - b^{2-n} \Rightarrow a = b \dots$ contradiction to $a < b$

$\therefore a^n + b^n \neq c^n$

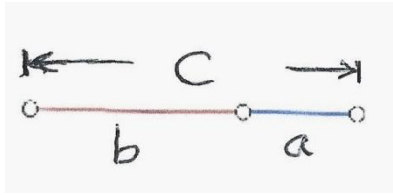
B) is $c^n + b^n = a^n$? When $a < b < c$.

let $c^n + b^n = a^n$

$\therefore c^n < a^n \Rightarrow c < a \dots$ Contradiction to $a < b < c$

$\therefore c^n + b^n \neq a^n$ the same way for $c^n + a^n \neq b^n$

Shape 4: $c = a + b$



is $a^n + b^n = c^n$? When $c = a + b$

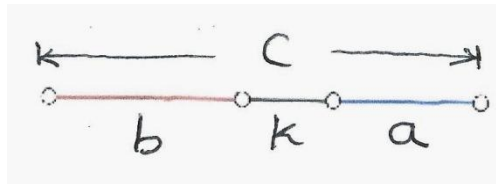
$$\because c = a + b$$

$$\therefore c^n = (a + b)^n$$

$$\therefore c^n > a^n + b^n$$

$$\therefore c^n \neq a^n + b^n$$

Shape 5: $c > a + b$



is $a^n + b^n = c^n$? When $c > a + b$

$$\because c > a + b$$

$$\therefore c^n > (a + b)^n$$

$$\therefore c^n > a^n + b^n$$

$$\therefore c^n \neq a^n + b^n$$