Euler Perfect Box (Final)

Proof

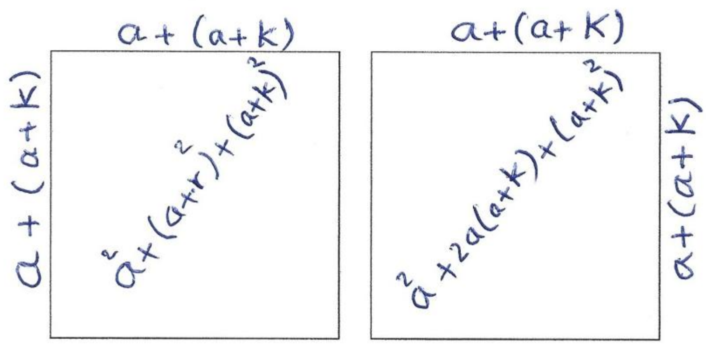
$Taha's\ Complete\ Square\ Fact\ (TCSF)$

is

$The\ Only\ Tool\ to\ Solve\ Euler\ Perfect\ Box$

$$let\ a^2+(a+r)^2+(a+k)^2 \in N_S \Rightarrow [a+(a+k)]^2$$

$$\because a^2+2a(a+k)+(a+k)^2 \in N_S \Rightarrow [a+(a+k)]^2$$



$$\because The\ above\ sequares\ have\ same\ sides$$

$$\therefore\ a^2+(a+r)^2+(a+k)^2 = a^2+2a(a+k)+(a+k)^2 \ldots (Areas)$$

$$\therefore (a+r)^2 = 2a(a+k) \Leftrightarrow a = r = k$$

$$(a+a)^2 = 2a(a+a)$$

$$4a^2 = 2a(2a)$$

$$4a^2 = 4a^2$$

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Abstract

$$[a, b, c, d, e, f, g \in N_+ \Leftrightarrow Euler\ Perfect\ Box] \ldots (EPB)$$

$let\ a, b, c, d, e, f, r, k \in N_+, \&\ let\ N_{Square} = N_S$.

$Is\ g \in N_+$?

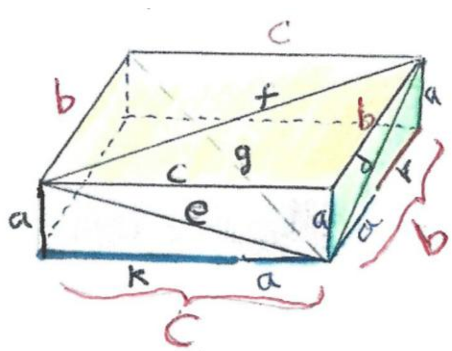
$A)$

$Let\ a < b < c, \&\ [(a < r < k), Or\ (a > r, and\ r < k)] \Rightarrow a \neq r \neq k$

$\&\ let\ b = a + r, c = a + k$

$\because\ g^2 = d^2 + c^2, \ldots Pythagorean\ theorem$

$\&\ d^2 = a^2 + b^2, \ldots Pythagorean\ theorem$



$\therefore g^2 = a^2 + b^2 + c^2 \ldots by\ substitution\ of\ d^2$

$\therefore g^2 = a^2 + (a + r)^2 + (a + k)^2 \ldots by\ substitution\ of\ b, \&\ c$

$let\ g^2 = [a^2 + (a + r)^2 + (a + k)^2] \in N_S$

$\because [a^2 + 2a(a + k) + (a + k)^2] \in N_S$

$\therefore\ (a + r)^2 = 2a(a + k) \Leftrightarrow a = r = k \ldots (TCSF)$.

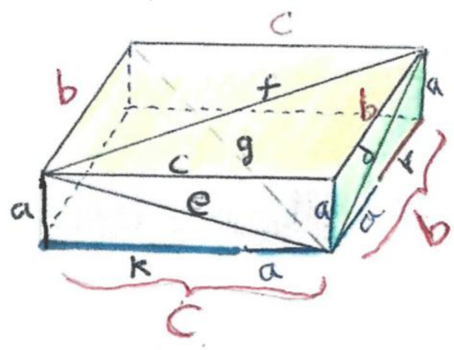
$Contradiction\ to\ a \neq r \neq k$

$\therefore\ (a + r)^2 \neq 2(a + k)\ for\ all\ a, r, k \in [N_+ / \{a = r = k\}]$.

$\therefore g^2 = [a^2 + (a + r)^2 + (a + k)^2] \notin N_S \Rightarrow g \notin N_+$

$\Longrightarrow$ *The Euler Perfect Box does not exist*$\ldots (EPB)$

$B)\ \ a < b < c, let\ b = a + r, \&\ \ a = r$



$\because\ d^2 = a^2 + b^2$

$d^2 = a^2 + (a + r)^2$

$d^2 = a^2 + (a + a)^2$

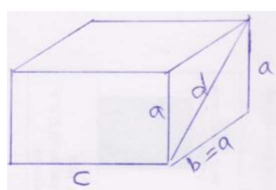
$d^2 = a^2 + (2a)^2$

$d^2 = a^2 + 4a^2$

$d^2 = 5a^2 \Longrightarrow d = a\sqrt{5} \notin \mathrm{N}_+ \Longrightarrow\ d \notin N_+$

$\Longrightarrow$ *The Euler Perfect Box does not exist… (EPB)*

$C)\ if\ a = b < c$

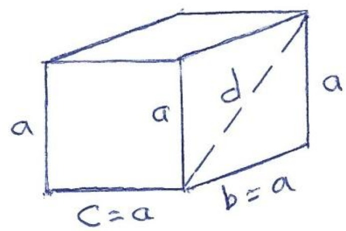


$\because\ d^2 = a^2 + b^2$

$\therefore\ d^2 = a^2 + a^2 \Longrightarrow d^2 = 2a^2 \Longrightarrow d = \sqrt{2}\,a\ \Longrightarrow d \notin \mathrm{N}_+$

$\Longrightarrow$ *The Euler Perfect Box does not exist … (EPB)*

$D)\ if\ a = b = c$



$\because\ d^2 = a^2 + b^2$

$\therefore\ d^2 = a^2 + a^2 = 2a^2$

$\therefore\ d^2 = 2a^2 \implies d = a\sqrt{2} \implies d \notin \mathrm{N}_+$

$\implies$ *The Euler Perfect Box does not exist*… (EPB)