

THE DURER POLYHEDRON AND A COMPOSITE THREE-DIMENSIONAL MODEL OF A SYMMETRICAL GEOMETRIC OBJECT

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Abstract

Two variants of the geometry of the Durer polyhedron, conventionally named as "traditional" and "alternative", are investigated. The geometric properties of both variants of polyhedra are revealed. According to the results of the field experiment, it was revealed that the "alternative" version of the polyhedron has the property of complementarity.

1 Introduction

The Durer polyhedron is a polyhedron depicted in Albrecht Durer's engraving "Melancholia I" (Figure 1). A massive polyhedron is located at the base of the staircase and dominates the left half of the engraving. The exact geometry of this body is currently the subject of unfinished scientific debate.



Figure 1 – Albrecht Durer's engraving "Melancholia I" (1514)

2 The main part

We have investigated two variants of the geometry of the Durer polyhedron. One (conventionally called "traditional" by us [5]) has the geometric parameters of six large faces: the length of the larger side of the face corresponds to the smaller side in relation to the square of the golden proportion (1), and the average side to the smaller side in relation to the golden proportion (2). The larger and smaller sides of the figure in question form an angle between each other in 108° , and the larger sides are 72° (Figure 2, left). In addition, the following equalities are also established for the "traditional" Durer polyhedron ((3), (4)). The "alternative" polyhedron we propose has a geometry of six large faces, which is characterized by equalities (5) and (6). It can be constructed from four Fibonacci triangles – when two large triangles are connected by large catheters, and the other two smaller ones are adjacent by their hypotenuses to the smaller catheters of the large Fibonacci triangles (Figure 2, right). (The Fibonacci triangle is a right-angled triangle with a ratio of legs equal to the golden ratio [1, 2, 3, 4]). The larger and smaller sides of the face of the "alternative" polyhedron form an angle of $\approx 63.435^\circ$... Thus, a geometric shape with the following properties (5), (6) is obtained.

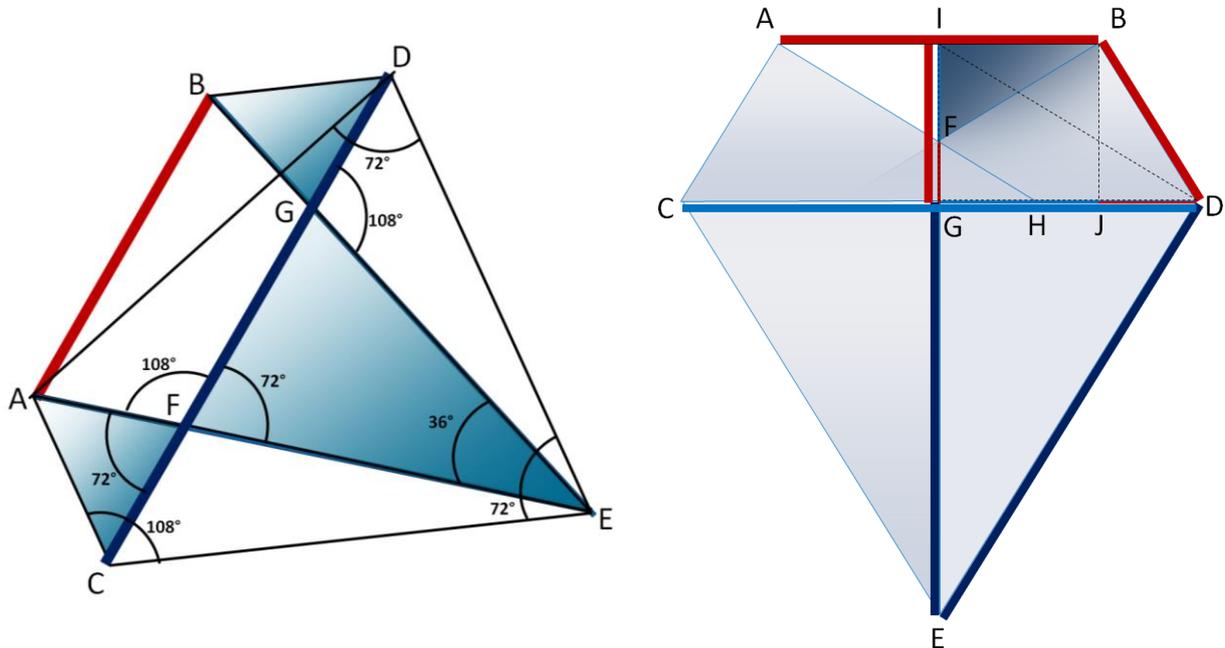


Figure 2 – The face of the Durer polyhedron. There are two ways to represent the face of a polyhedron: on the left – the traditional one, on the right – the alternative one

$$FD/CF=FE/AF=AD/AC=1,61803\dots^2 \quad (1)$$

$$FG/CF=CD/AB=1,61803\dots \quad (2)$$

$$AD=CE=DE \quad (3)$$

$$CD=AE=BE \quad (4)$$

$$CD/AB=AH/CA=IF/FG=FH/FG=EG/GD=GJ/JD=BJ/JD=1,61803\dots \quad (5)$$

$$ED/DB=EG/GI=1,61803\dots^2 \quad (6)$$

To produce three-dimensional models of two variants of the representation of the Durer polyhedron, two scans were calculated and drawn (see Appendices – Figures P1, P2). Based on them, three-dimensional full-scale models of two Durer polyhedra were made (Figure 3).

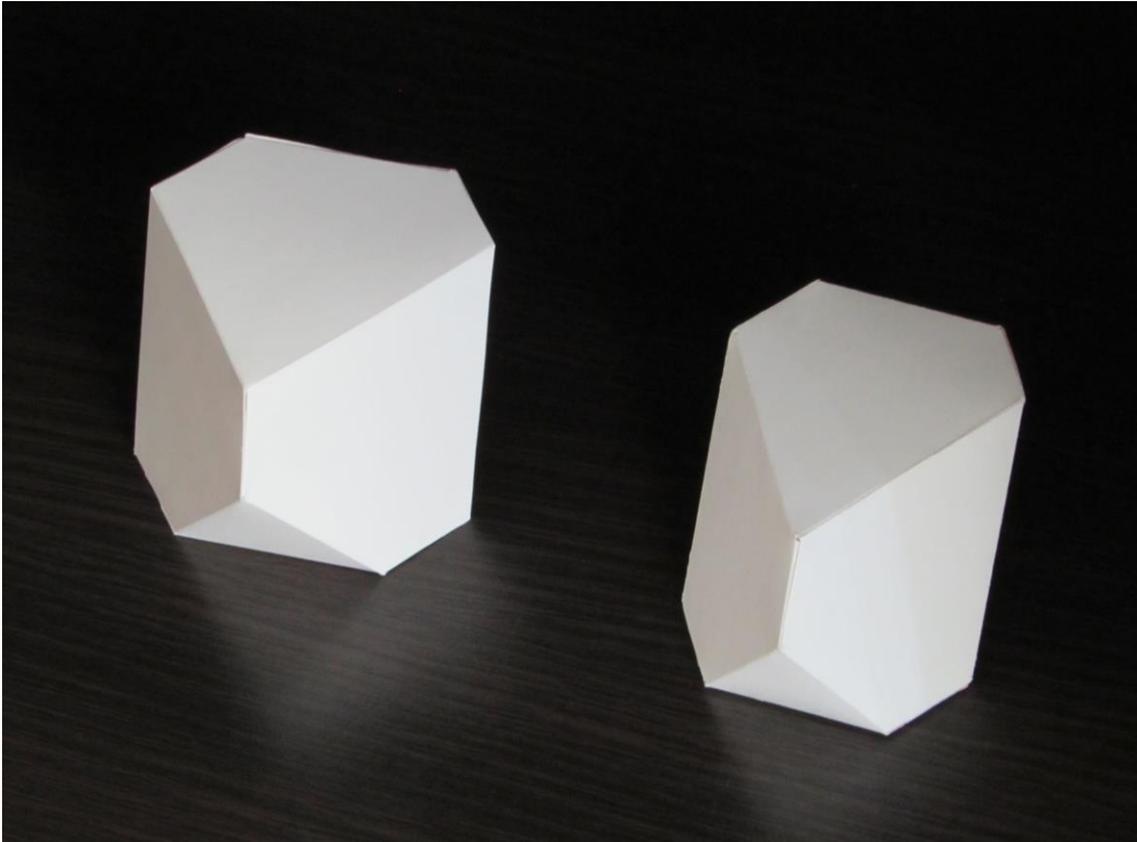


Figure 3 is a three-dimensional model of two representations of the Durer polyhedron: on the left – "traditional", on the right – "alternative" (photo)

Based on the results of the field experiment, it was revealed that the "alternative" representation of the Durer polyhedron has the property of complementarity – polyhedra form a variety of composite geometric objects (both non-symmetrical and symmetrical (figure 4, 5, 6, 7, 8, see Appendices P3, P4, P5, P6, P7, P8)). The inner space, a cavity, of one of the composite models, formed by the triangular faces of twenty "alternative" polyhedra, has the shape of an icosahedron (Figure 7, see Appendices P6). We have added twenty icosahedral models commensurate with the smaller faces of the "alternative" Durer polyhedron to the model of a composite symmetrical geometric object formed from twenty "alternative" Durer polyhedra (Figures 8, see Appendices P7, P8).

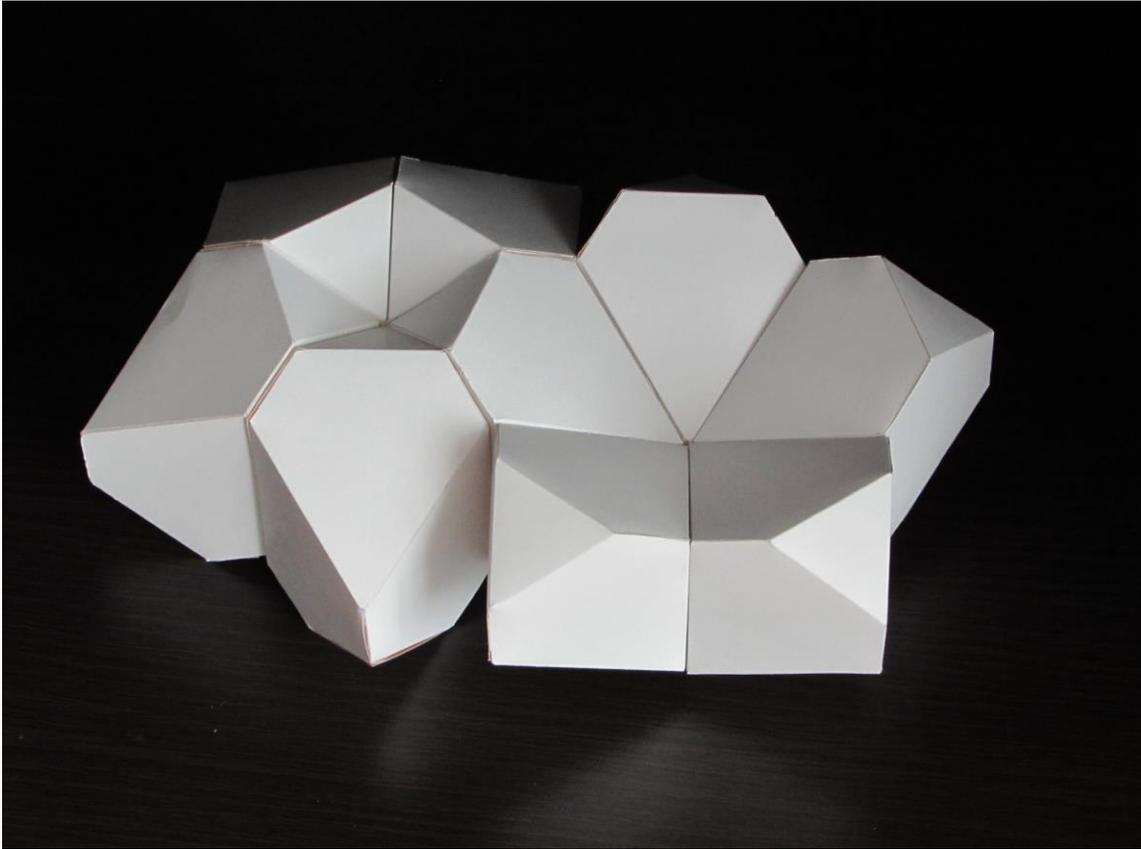


Figure 4 – Three-dimensional model of an object made up of nine Durer polyhedra (photo)



Figure 5 is a three-dimensional model of an object made up of twenty Durer polyhedra.
Side view (photo)



Figure 6 is a three-dimensional model of an object made up of twenty Durer polyhedra.
Side view-top view (photo)



Figure 7 – Three-dimensional model of an object made up of ten Durer polyhedra (photo)



Figure 8 is a three–dimensional model of an object made up of twenty Durer polyhedra and twenty-one icosahedra. Side view-top view (photo)

3 Conclusion

Two variants of the geometry of the Durer polyhedron, conventionally named as "traditional" and "alternative", are investigated. The geometric properties of both variants of polyhedra are revealed. According to the results of the field experiment, it was revealed that the "alternative" version of the polyhedron has the property of complementarity.

References

- [1] Ворон, А.В. Геометрический треугольник Фибоначчи и пирамида Хеопса // «Академия Тринитаризма», М., Эл № 77-6567, публ.23546, 13.07.2017.
- [2] Ворон, А.В. Дуальность и комплементарность треугольников Кеплера, Фибоначчи и их связь с геометрией платоновых тел // «Академия Тринитаризма», М., Эл № 77-6567, публ.24401, 06.04.2018.
- [3] Ворон, А.В. Культурологический и технологический аспект великой пирамиды // «Академия Тринитаризма», М., Эл № 77-6567, публ.24005, 29.11.2017.
- [4] Ворон, А.В. Свойства треугольников Кеплера, Фибоначчи и их связь с геометрией пирамиды Хуфу // «Академия Тринитаризма», М., Эл № 77-6567, публ.24320, 04.03.2018.
- [5] https://www.jacquier.org/Art_contemporain/Yvo_Jacquier-Construction_Polyedre_Durer-3.pdf

APPENDICES

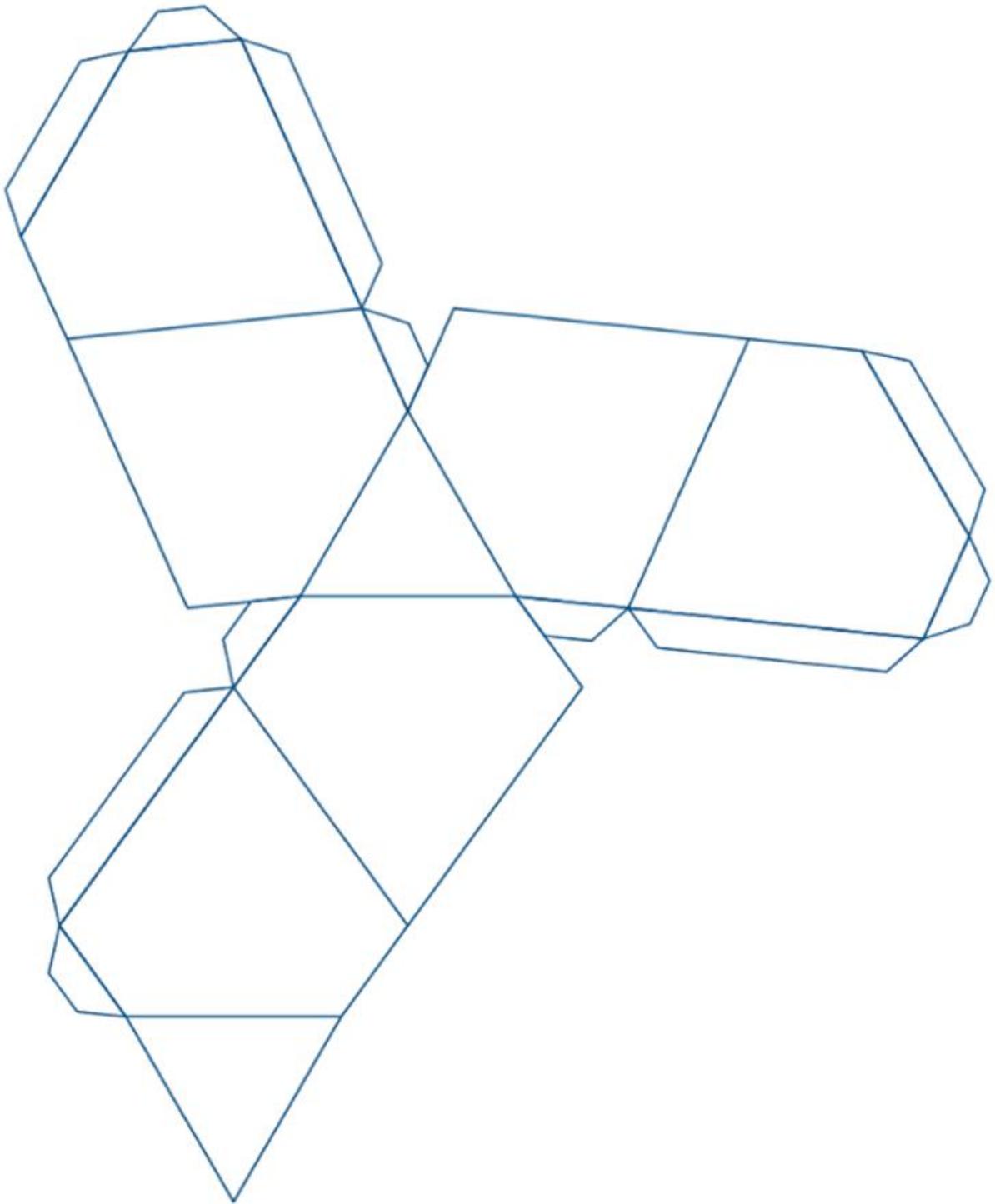


Figure P1 is a scan of the "traditional" representation of the Durer polyhedron with an angular value of 72°

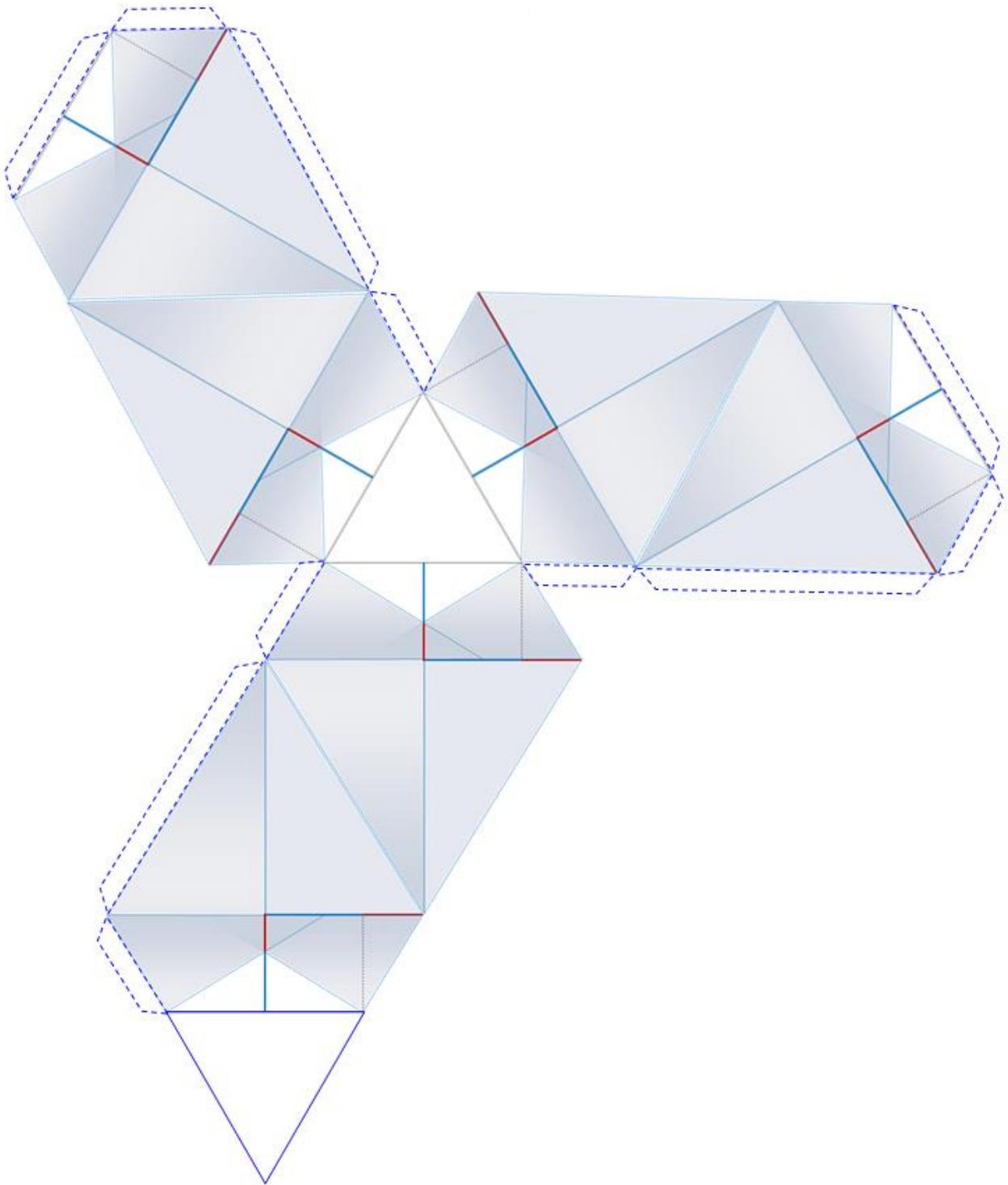


Figure P2 is a scan of an "alternative" representation of the Dürer polyhedron with an angular edge value of $\approx 63.435\dots^\circ$

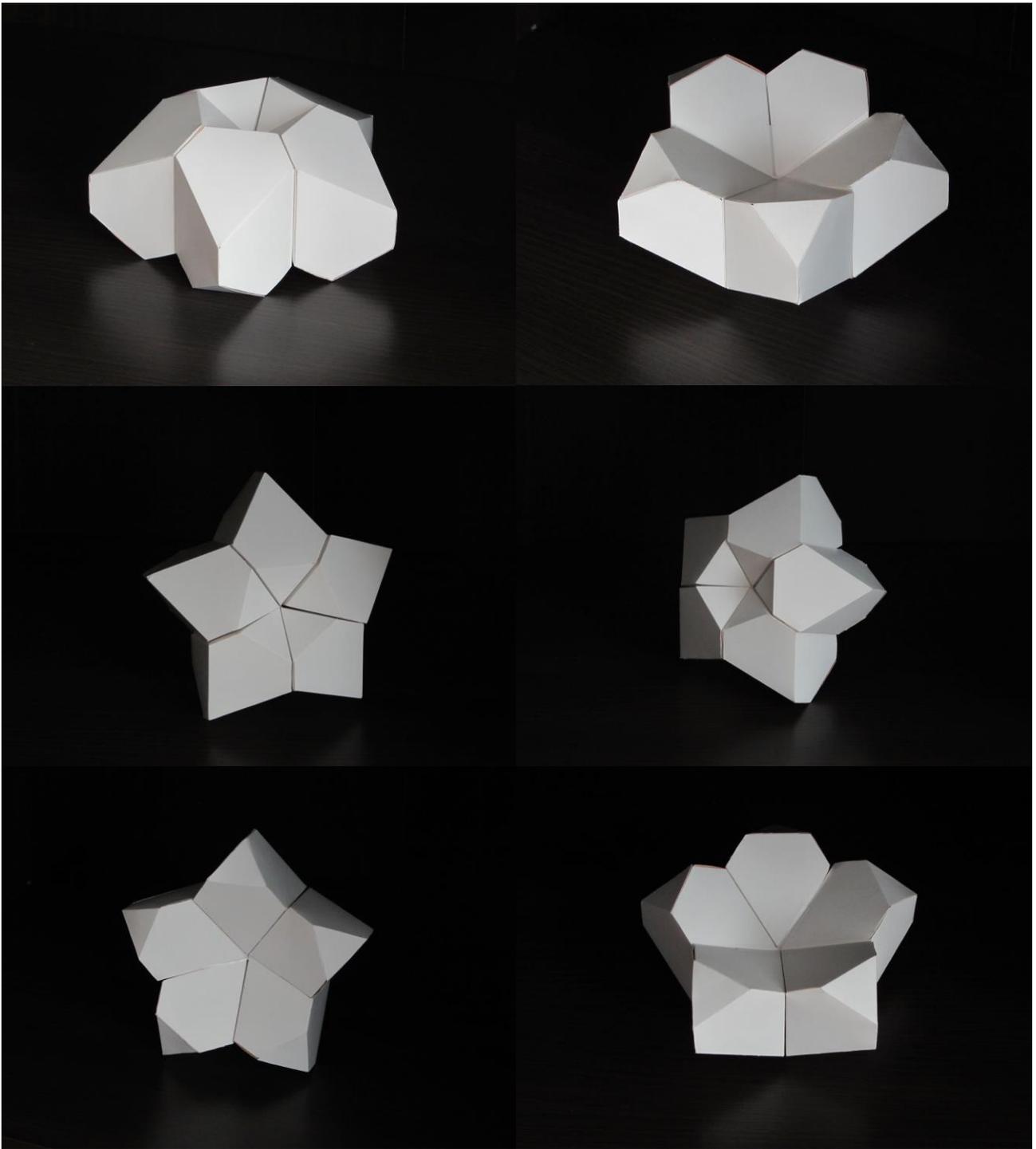


Figure P3 is a three-dimensional model of an object made up of five Dürer polyhedra.
View from various angles (photos)

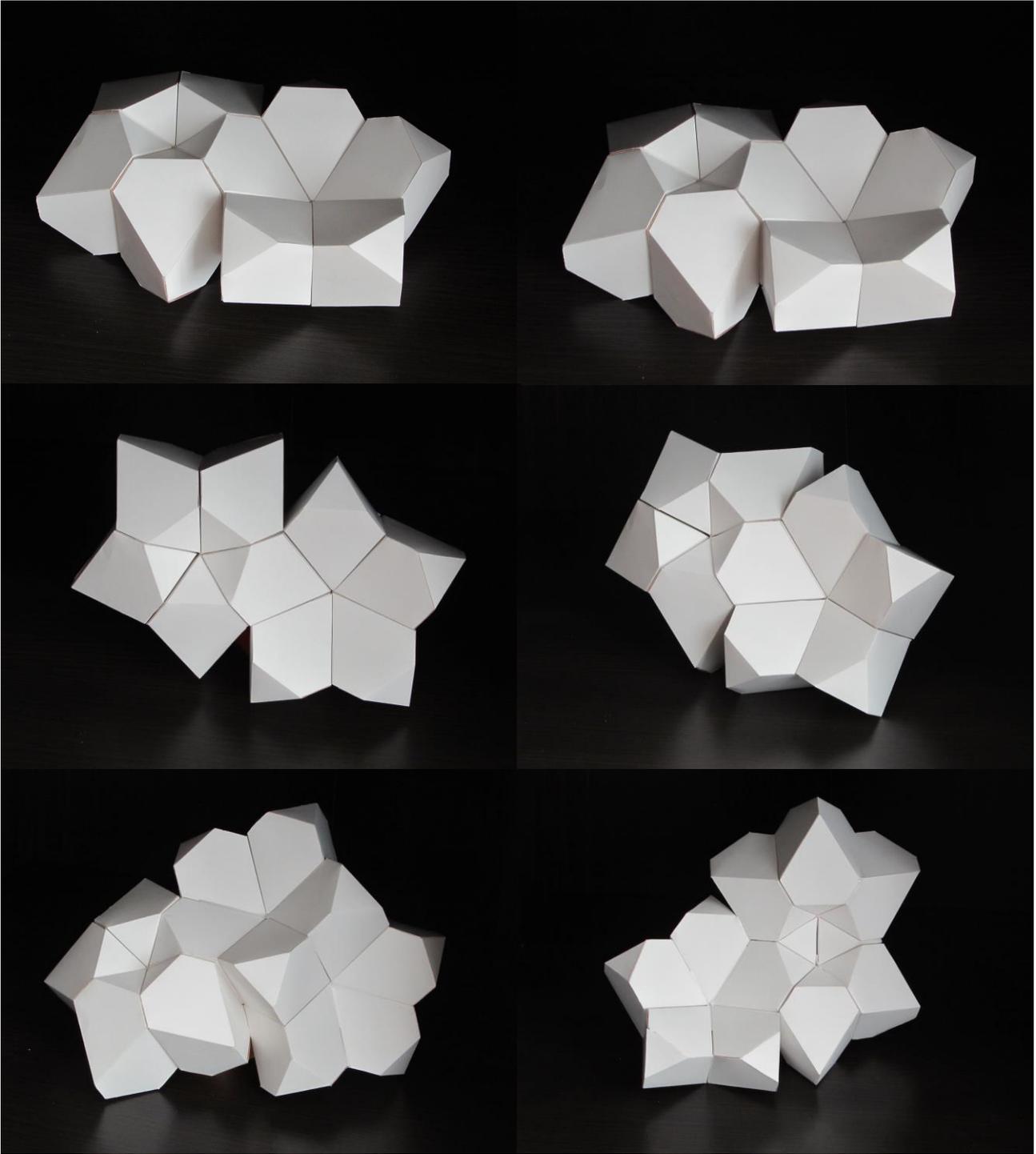


Figure P4 – Three-dimensional models of objects made up of a set of Durer polyhedra.
View from various angles (photos)

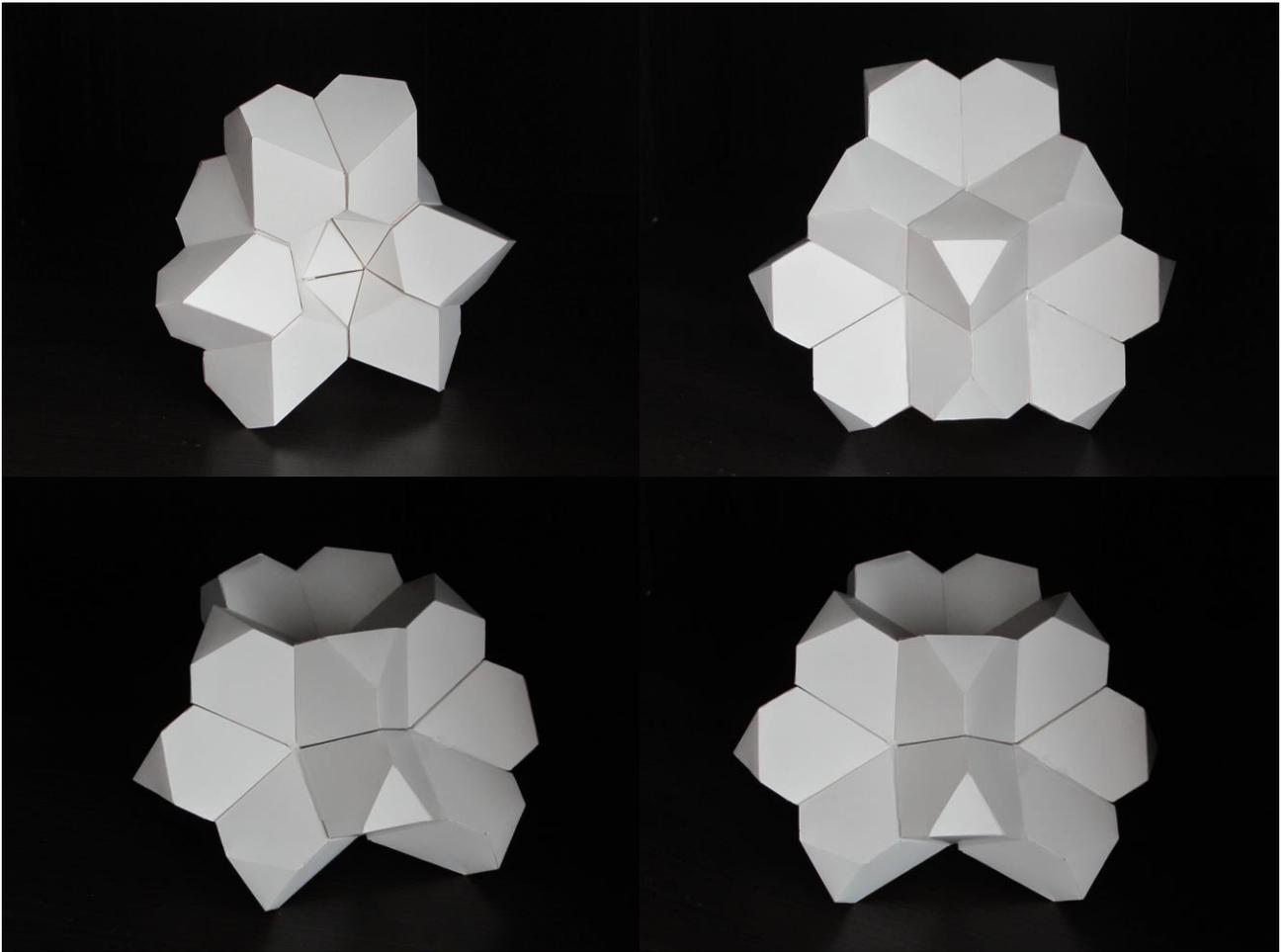


Figure P5 – Three-dimensional models of objects made up of a set of Durer polyhedra.
View from various angles (photos)

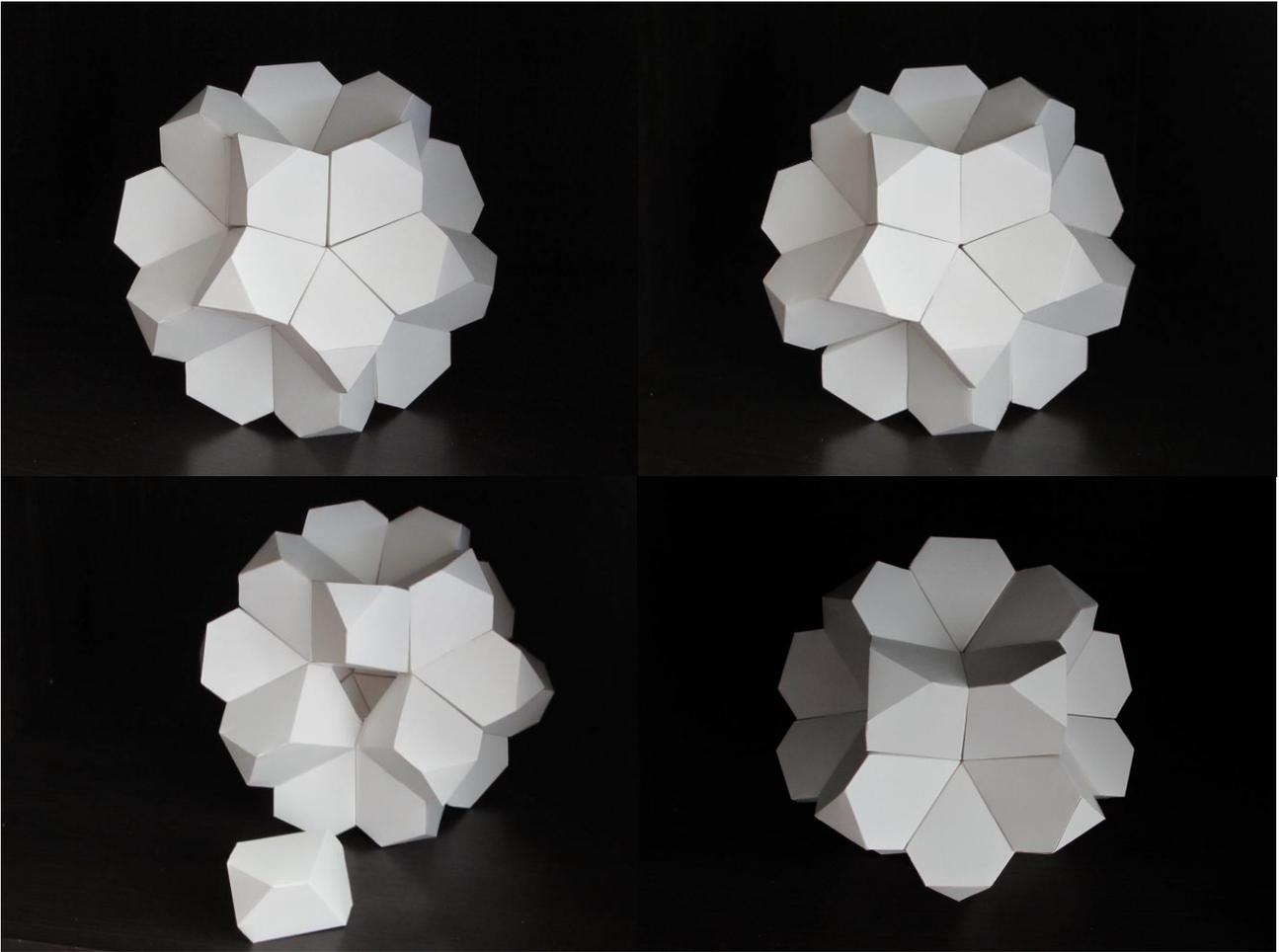


Figure P6 is a three-dimensional model of an object made up of twenty Durer polyhedra.
View from various angles (photos)

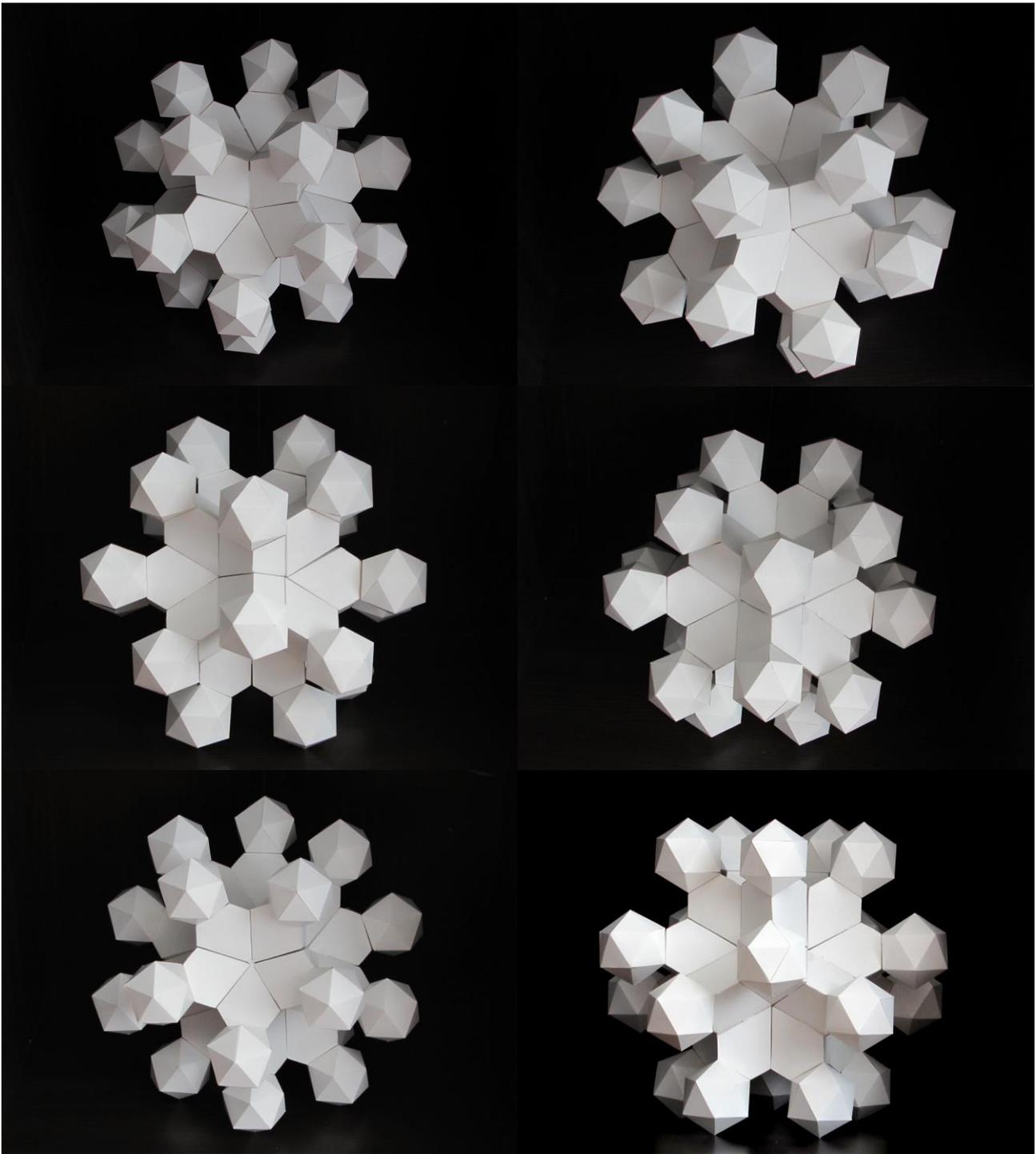


Figure P7 is a three-dimensional model of an object made up of twenty Durer polyhedra and twenty-one icosahedra. View from various angles (photos)

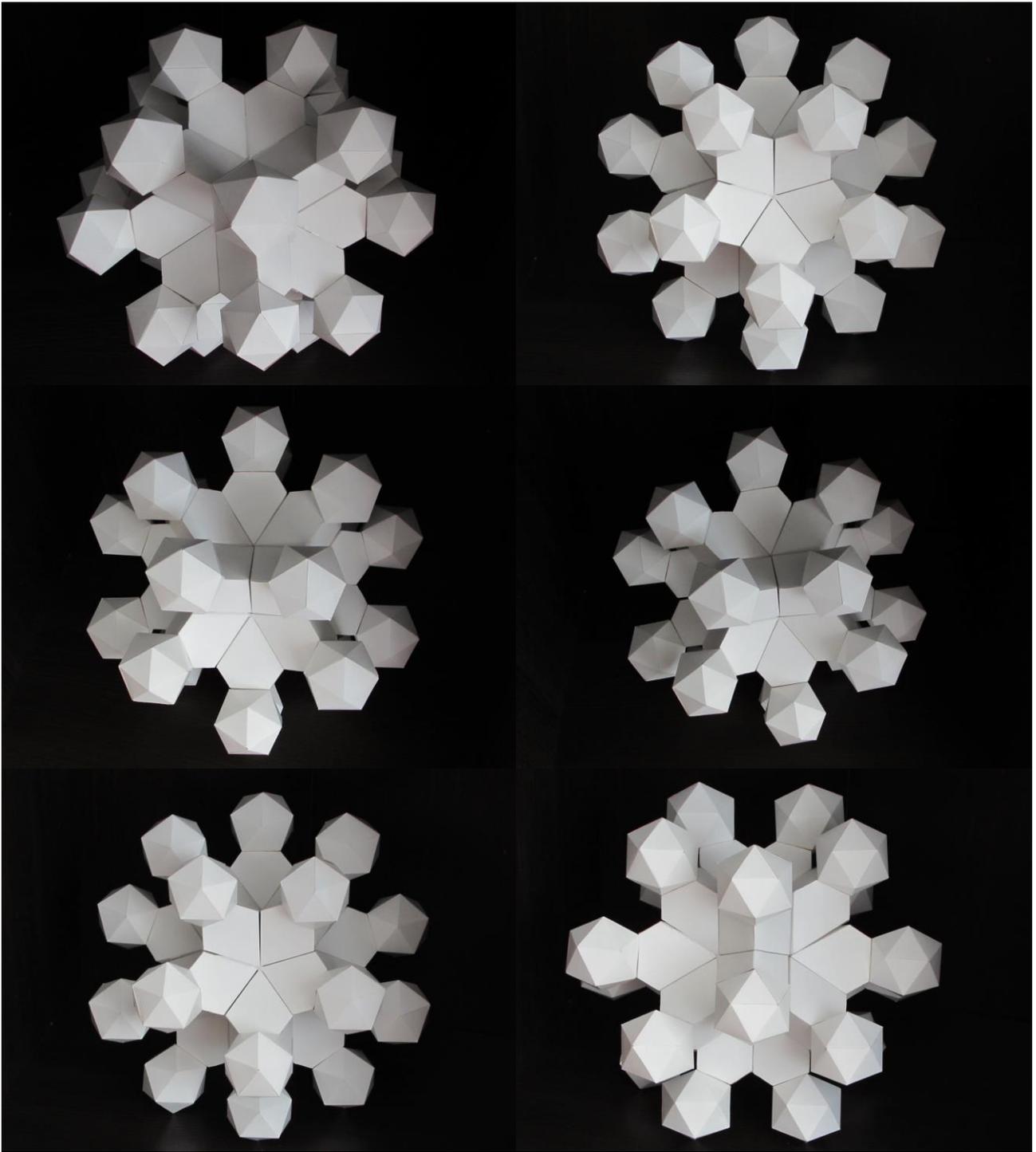


Figure P8 is a three-dimensional model of an object made up of twenty Durer polyhedra and twenty-one icosahedra. View from various angles (photos)