Generalized Probability Theory

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Abstract: Disordered events can be divided into two types: disordered deterministic events - generalized events, disordered random events - random events - narrow events; Establishing probability axioms, boundary axioms, etc., proposing four important original works: probability method, boundary, subdivision probability, Xiong's sieve; Establish General Probability Theory. Using analytical methods to solve mathematical problems such as the prime number theorem, Riemann hypothesis, twin prime numbers, densest K-tuple prime numbers, K-tuple prime numbers, Goldbach's conjecture, Gaussian lattice points, etc.

Part 1　　　Generalized probability basis

Chapter 1　　　　Probability method

1. 1 Basic concept

1.1.1 C function

General probability theory is divided into two parts: general probability number theory —— probability method, applied in problems such as prime numbers, integers, and digit; Generalized Probability Sieving Theory —— Sieving Method, Applied in Prime Number Problems

Definition 1.1.1:

C1，The limit of the function C(x) is the constant C;

C2，The function C(x) and the constant C fluctuate within a small range of relative error;

C3，The limit of the function C(x) and the constant C exist greater than, less than and other describable relations;

In the above three cases, the function C(x) is called a C constant function, or a C relational function, collectively referred to as a C function; When C=1, it is called the function of 1; You can mark C，or 1， or relation symbol, etc. in the upper left corner, i.e. ， ${}^{1}1$ ${}^{1}\mathrm{C}$ **,** ${}^{\mathrm{C}}1$ **,** ${}^{\mathrm{C}}\Pi(x)$ **,** ${}^{1}\mathrm{C}\Pi(x)$ **,** ${}^{\leq}C$ **,** ${}^{\geq}C$ **, O(C), etc., for example:**

$$\lim_{x\to\infty}\frac{\pi(x)}{\Pi(x)}=\mathrm{C}，\left|\lim_{x\to\infty}\frac{\pi(x)}{\Pi(x)}-\mathrm{C}\right|\ll\mathrm{C}，\frac{\pi(x)}{\Pi(x)}=\begin{cases}{}^{\mathrm{C}}1\\ {}^{1}\mathrm{C}\end{cases}，\quad |o(\mathrm{C})|={}^{\leq}\mathrm{C} \qquad (1.1.1)$$

Note 1: C function symbol, do not participate in calculation;

Note 2: When the limit C is a definite value, the C value does not have a decimal point, or the C value has more digits after the decimal point; When the limit C varies in a small range, the number of digits after the decimal point of the C value is less

Inference 1.1.2:1 function approximate expression formula —— 1 function formula

$$\begin{cases} \Pi\left({}^{1}x\right) \\ \Pi\left(x+\varepsilon x\right) \end{cases} = \Pi\left(x\right) +^{1} \varepsilon x \frac{d\Pi\left(x\right)}{dx} = \Pi\left(x\right) +^{1} \varepsilon\ \Pi\left(x\right) =^{1} \Pi\left(x\right) \qquad (1.1.2)$$

Definition 1.1.3 Using approaching, basically, almost equal, most likely value etc. language description: the limit of the ratio of the π(x) and the Π(x) is equal to 1, Then, the Π(x) is called the asymptotic solution of the π(x) ; it can be expressed $\pi\left(x\right)$ **by the** ${}^{1}\Pi\left(x\right)$ **with 1 function symbol can be used, then, it can be expressed as formula (1.1.1)**

1.1.2 Actual value, theoretical value

Definition 1.1.4 Usually, lowercase or more lowercase letters start with a string of variables and parentheses p(x), pp(x), representing the actual value of an unordered event;

Similarly,uppercase or more uppercase letters start with a string of variables and parentheses, representing the theoretical value of an unordered event

Common value classification: actual value , subset value , set value ,probability value, sieve value , generalized value, narrow value .etc $\pi(x)$, $\pi(x_i)$, $\overline{\pi(x_i)}$, $\Pi(x_i)_{\mathrm{P}}$, $\Pi(x_i)_{\mathrm{S}}$, $\Pi(x)_{\mathrm{G}}$, $\Pi(x)_N$; Can text description, character annotation

1.1.3 Generalized quantity, Narrow quantity

Definition 1.1.5: There are two types of disordered events: disordered deterministic events —— generalized events, disordered random events —— random events —— narrow events ;

There are two types of quantities for generalized events: generalized quantities and narrow quantities;Among them, generalized quantity is the most likely value of the actual quantity of a generalized event

Narrow events only have narrow quantities;Among them, narrow

quantity refers to the most likely value of the actual quantity of narrow events;

Probability can be divided into two categories: generalized probability —— the probability to closest reality value in an unordered deterministic event; Narrow probability refers to traditional probability;

Statistical values are divided into two categories: generalized probability statistical values —— generalized values —— the most likely value of the actual value, and narrow probability statistical values —— narrow values;

There are two types of boundaries: generalized boundaries and narrow boundaries

The application of traditional probability in the problem of generalized events —— unordered deterministic events, and the addition of functions to establish this probability theory system, is called generalized probability theory —— generalized probability;

At present, the generalized probability structure, as shown in the diagram below, can be developed in the direction of other unordered deterministic events

$$\left\{\begin{array}{c}\text{Generalized probability}\\ \text{generalized events}\end{array}\right. \in \left\{\begin{array}{l}\text{Probability：tnarrow probability，narrow events，}\\ \qquad\text{unordered random event}\\ \text{Probability+unordered deterministic event}\\ \text{Probability+boundary}\\ \text{Probability+sieve}\end{array}\right.$$

There are two types of disordered events

1.Disordered random events, disordered random test, can be called narrow events, narrow tests;

2. Disorder determination events, disorder determination inspect, can be called generalized events, generalized tests;

Note：A generalized test, which can be called a generalized inspect, inspect whether an unordered deterministic event is a target event, for example, to inspect whether an integer is prime

Definition 1.1.6 When the distribution of a disordered event is uncertain, it is a random event in the traditional sense，it is not repeatable, cannot specify validation , can be called narrow event；The value of a narrow event is called a narrow value (e.g., probability, statistic, mean square error, boundary, etc.).; In the t-th narrow event, the occurrence frequency of the target event is called the narrow frequency pp(t); The most likely value of the narrow frequency, called the narrow probability $P(t)_N$ **; In the x narrow events, the statistical value of the narrow frequency and the narrow probability is called the actual value , and the narrow value** $\pi(x)$，$\Pi(x)_N$**, then**

$$\pi(x)=\sum_{t=1}^{x} pp(t) =^1 \Pi(x)_N =^1 \int_1^x P(t)_N \, dt \begin{pmatrix} \text{narrow event} \\ \text{狭义事件} \end{pmatrix} \tag{1.1.3}$$

The theory of narrow events, called narrow probability theory, is just traditional probability theory

Definition 1.1.7：When the distribution of an unordered event is determined, it can be called an unordered determined event, or a generalized event: It has repeatability and is convenient to specify verification; In the t- th generalized event, the occurrence frequency of the target event is called the generalized frequency $pp(t)_G$; The most probable value of the generalized frequency, called the generalized probability $P(t)_G$; In x generalized events, the statistical value of generalized frequency， generalized probability， called the actual value，generalized value $\pi(x)$， $\Pi(x)_G$, then

$$\pi(x) = \sum_{t=1}^{x} pp(t)_G =^1 \Pi(x)_G =^1 \int^x P(t)_G \, dt \quad (\text{generalized event}) \qquad (1.1.4)$$

The theory of studying generalized events is called generalized probability theory.For example, the distribution of prime numbers, the distribution of the hour point in the Gaussian circle, etc., are generalized events

Note：In probability theory dealing with random events, the values calculated are narrow values

There are two kinds of generalized events, which are described as follows:

In the generalized event, if it can be proved, or the data can be shown, the ratio of the actual value to the narrow value —— the generalized narrow transformation coefficient C, with the increase of

the statistical range, gradually approaching 1, is called the generalized narrow event, otherwise, it is called the non-generalized narrow event

1.1.4 Set value, coefficient

Define 1.1.8: The data from in the set,calculated as specified,the value obtained,it called set value; The ratio of two values, called a coefficient, is introduced as follows

Set frequency values, set statistics values (set values) , set boundaries, set coefficients $\overline{pp(t_i)}$, $\overline{\pi(x_i)}$, $\uparrow\downarrow\overline{\pi(x_i)}$, $\overline{C(x_i)}$;

Collecting actual narrow statistical coefficients and broad narrow conversion coefficients; Collecting actual narrow boundary coefficients, broad narrow boundary transformation coefficients, and the degree of correlation between multiple probability events —— correlation coefficients $\overline{C_N^A(x_i)}$, $C_N^G(k)$, $\uparrow\downarrow\overline{C_N^A(x_i)}$, $\uparrow\downarrow C_N^G(k)$, C_k

Definition 1.1.9: The unordered event of in $x_1 \le x_i \le x_{\uparrow i} = x$**, the actual value, the actual boundary** $\pi(x_i)$, $\uparrow\downarrow\pi(x_i)$ **of the target unordered event , The data set is composed , and the average value of** $\pi(x_i)$, $\uparrow\downarrow\pi(x_i)$ **is calculated, which is called the set value set boundary** $\overline{\pi(x_i)}$, $\uparrow\downarrow\overline{\pi(x_i)}$;

They divided by the narrow value ， by the narrow boundary $\Pi(x)_N$， $\uparrow\downarrow\Pi(x)_N$,etc，is called the set value coefficient , and is called the set boundary coefficient $\overline{C_N^A(x_i)}$， $\uparrow\downarrow\overline{C_N^A(x_i)}$， etc， then

$$\left\{\begin{matrix}\overline{\pi(x_i)}\\ {}^1\Pi(x)_G\end{matrix}\right. = \sum_{i=\downarrow i}^{\uparrow i}\frac{\pi(x_i)}{\uparrow i} \quad \left[\left\{\begin{matrix}\uparrow\downarrow\overline{\pi(x_i)}\\ {}^1\uparrow\downarrow\Pi(x)_G\end{matrix}\right. = \sum_{i=\downarrow i}^{\uparrow i}\frac{\uparrow\downarrow\pi(x_i)}{\uparrow i}, \quad \begin{matrix}|x_{\uparrow i}-x_i|\ll x_{\uparrow i}\\ x_1\le x_i\le x_{\uparrow i}=x\end{matrix}\right] \quad (1.1.5)$$

$$\left\{\begin{matrix}{}^1\overline{C_N^A(x_i)}\\ C_N^G(k)\end{matrix}\right. = \frac{\Pi(x)_G}{\Pi(x)_N} = \frac{{}^1\pi(x)}{\Pi(x)_N} = \frac{{}^1 1}{\uparrow i}\sum_{i=1}^{\uparrow i}\frac{\pi(x_i)}{\Pi(x_i)_N} = ^1 \sum_{i=1}^{\uparrow i}\frac{C_N^A(x_i)}{\uparrow i}$$

$$=^1 \sum_{i=1}^{\uparrow i}\frac{C_N^A(x_i)}{\uparrow i} \quad \begin{pmatrix}|x-x_i|\ll x\\ x_1\le x_i\le x_{\uparrow i}=x\end{pmatrix}$$

$$\left[\left\{\begin{matrix}{}^1\uparrow\downarrow\overline{C_N^A(x_i)}\\ \uparrow\downarrow C_N^G(k)\end{matrix}\right. = \frac{\uparrow\downarrow\Pi(x)_G}{\uparrow\downarrow\Pi(x)_N} = \frac{{}^1\uparrow\downarrow\pi(x)}{\uparrow\downarrow\Pi(x)_N} = \frac{{}^1 1}{\uparrow i}\sum_{i=1}^{\uparrow i}\frac{\uparrow\downarrow\pi(x_i)}{\uparrow\downarrow\Pi(x_i)_N}\right]$$

$$\left[=^1 \sum_{i=1}^{\uparrow i}\frac{\uparrow\downarrow C_N^A(x_i)}{\uparrow i} \quad \begin{pmatrix}|x-x_i|\ll x\\ x_1\le x_i\le x_{\uparrow i}=x\end{pmatrix}\right] \quad (1.1.6)$$

Inference 1.1.10: Set value inference

$$\overline{\pi(x_i)} = \sum_{i=1}^{\uparrow i}\frac{\pi(x_i)}{\uparrow i} =^1 \pi(x) =^1 \Pi(x)_G \qquad (|x-x_i|\ll x = x_{\uparrow i}) \quad (1.1.7)$$

Proof：Since, $|x-x_i|\ll x$, so， $x_i =^1 x$, referring to formula (1.1.2)of 1 function formula, referring to formula （1.1.4） of definition of generalized value, get

$$\overline{\pi(x_i)} = \sum_{i=1}^{\uparrow i}\frac{\pi(x_i)}{\uparrow i} = \sum_{i=1}^{\uparrow i}\frac{\pi({}^1x)}{\uparrow i} = \pi({}^1x) =^1 \pi(x) =^1 \Pi(x)_G$$

This inference proof is completed

Definition 1.1.11 : t-th disordered event, the frequency of the occurrence of the target event is $pp(t)$; from the frequency set $pp(t_i)$ that some points t_i $(|t-t_i| \ll t)$ near t, calculate the average value of $pp(t_i)$,is called the set frequency $\overline{pp(t_i)}$, then

$$\overline{pp(t_i)} = \sum_{i=1}^{\uparrow i} \frac{pp(t_i)}{\uparrow i} \qquad (|t-t_i| \ll t) \tag{1.1.8}$$

Inference 1.1.12: Set frequency inference

$$\frac{\overline{pp(t_i)}}{P(t)_N} = \sum_{i=1}^{\uparrow i} \frac{pp(t_i)}{P(t)_N \uparrow i} =^1 \frac{P(t)_G}{P(t)_N} \begin{cases} =^1 1: & \text{Narrow Generalized event} \\ \neq^1 1: & \text{Other} \qquad (|t-t_i| \ll t) \end{cases} \tag{1.1.9}$$

Proof: Similar to the proof of set value inference of formula (1.1.7), omitted

1.1.5 Disorder rate, Randomness rate, Uncertainty rate

Definition 1.1.13: In any set composed of events, when there are two types of events in following, it is called mixed event set: A, unordered event; B, known events.The proportion of type A events in mixed events for different situations. is called disorder rate, random rate and uncertainty rate

1.2 Probability axiom

1.2.1 Generalized value 4-step method

Five elements of generalized event

Certainty1：When the test is repeated, the same determined result is obtained [convenient verification result], which is called generalized value;

Randomness2 ： Describing the unordered characteristics of generalized events using traditional probability —— narrow probability can be referred to as randomness;

Transformability3： Generalized narrow transformation coefficient;.

Relevance4：Describe the degree of association between generalized events;

Bias5：The actual values of several adjacent intervals have slightly more deviation in the same direction compared with the deviation -- error of their respective generalized values. Slightly smaller reverse bias occurs, which is called bias;

【Narrow events, no bias】

Summary: Narrow events have only narrow values; The generalized event has the generalized value, the narrow value, and the generalized narrow transformation coefficient.

Generalized value 4-step method

1. Calculate the narrow value corresponding to the narrow event;

2. The generalized narrow transformation coefficients are calculated;

3. Multiply the first two values to obtain the generalized value - the most probable value of the actual value

4. Verify with actual data, calculate the comprehensive error, calculate the potential coefficient error (which can be used to check if important influencing factors are missed), and correct it. Calculate the potential fluctuation error, calculate the set boundary, etc.

Definition 1.2.1: Generalized Event Main Coefficients: Metens Generalized Narrow Coefficient, Metens Coefficient, Generalized Narrow Coefficient, Actual Narrow Coefficient, Correlation Coefficient, Correction Screening Coefficient, Set Actual Narrow Boundary Coefficient, Generalized Narrow Boundary Coefficient $M_N^G(k)$ **, M(k),** $C_N^G(k)$ **,** $C_N^A(k)$ **,** C_k **, Correction(x) ,** $\uparrow\downarrow\overline{C_N^A(x_i)}$ **,** $\uparrow\downarrow C_N^G(k)$ **.etc , such as**

$$\mathrm{M}_N^G(k) = C_N^G(k) * \mathrm{M}(k) \qquad (1.2.1)$$

Generalized events have both broad and narrow properties, known as the duality of broad and narrow, similar to the wave particle duality of matter. Provided ideas for solving generalized events

Note: The correlation between narrow events can be referred to as

narrow correlation; The correlation between generalized events can be referred to as generalized correlation

Definition 1.2.2: The elements of disordered events are divided into two types: target events and non-target events;

Event1: Unordered random events, The t th unordered random event: the set containing all permutations and combinations before it occurs; After it occurs, it becomes a set of permutations and combinations that can appear; In the disorder random event, the occurrence probability of the target event is called the narrow probability $P(t)_N$

Narrow probability theory,is the current probability theory , the study of unordered random events that have not yet occurred,narrow probability of occurrence of target event , generally, it is a full arrangement;

Event2.: Unordered deterministic event, On the basis of unordered random event, the event that meets the specified conditions such as already happened is constituted;For example, the natural numbers in x are all combinations of prime numbers and cannot appear;

Disorder determines events, usually in partial order;

In the disordered deterministic event, the probability of the occurrence of the target event ,it is called generalized probability $P(t)_G$；

Generalized probability theory,study the disordered deterministic events that have occurred;

In general,due to specified constraints, let disorder determine the event, and only contain some elements in the disordered random event set; On the contrary, disordered random events contain all the elements in the set of disordered definite events；

Statistical analysis was conducted on numerous disordered events t to discover common patterns: whether disordered events are determined or not [in a broad or narrow sense], whether they are repeated or not [determined or random], and whether they do not affect the actual set value [approaching the broad value], which is proportional to the narrow value

Potential reasons: Two types of events, target events and non target events, are randomly identified and occur almost proportionally. Therefore, the ratio of multiple similar values of the target event approaches a constant:

A1, set frequency [approaching generalized probability], ratio to

narrow probability —— set frequency coefficient, reaching constant

A2, the ratio of the actual value to the narrow value -- aggregate statistical coefficient, the approach constant;

A3, the ratio of the set boundary to the narrow boundary -- the set boundary coefficient, the approach constant;

On the basis of this law, the relations of generalized events and narrow events are put forward as follows

1.2.2 Generalized event， narrow event

A1, Narrow quantity: can not be marked, or in the expression string at the bottom right, marked N;

A2, generalized quantity: when it is obviously suitable for or belongs to a generalized quantity, it can not be marked, otherwise, there is at least one expression string in the lower-right corner, marked G;

Note: As long as there is a quantity, the lower-right corner of the expression string, marked G, all additions and subtractions are generalized quantities.

There are two kinds of events: generalized event and narrow event.

Generalized events: not only have generalized quantities such as generalized probabilities, generalized values, and generalized

boundaries, but also have narrow quantities such as narrow probabilities, narrow values, and narrow boundaries;

Narrow event: only narrow probability, narrow value, narrow boundary and other narrow quantities.

Generalized events can be divided into two categories generalized narrow events ,generalized no narrow events, which are defined as follows

Definition 1.2.3: When the ratio of the actual value π(x) of the generalized event to the narrow value $\Pi(x)_N$ approaches 1, it is called the generalized narrow event; otherwise, it is called the generalized non-narrow event, then

$$\frac{\pi(x)}{\Pi(x)_N} = \mathrm{C}\begin{cases} =^1 1: \text{generalized narrow event} & 广义狭义事件 \\ \neq^1 1: \text{generalized no narrow event} & 广义非狭义事件 \end{cases} \qquad (1.2.2)$$

Definition 1.2.4: In a generalized event, the most likely value of frequency 11 $pp(t)$ is called the generalized probability $P(t)_{\mathrm{G}}$; When the set frequency $\overline{pp(t_i)}$ approaches the narrow probability $P(t)_N$: $\overline{pp(t_i)} =^1 \mathrm{C}P(t)_N$, it's called a generalized narrow event; Otherwise, it's called a generalized non- narrow event, then

$$\frac{\overline{pp(t_i)}}{P(t)_N} = \sum_{i=1}^{\uparrow i} \frac{pp(t_i)}{P(t)_N \uparrow i} =^1 \text{C} \begin{cases} =^1 1: \text{generalized narrow event} & \text{广义狭义事件} \\ \neq^1 1: \text{generalized no narrow event} & \text{广义非狭义事件} \end{cases}$$

$$\left(\left| t - t_i \right| \ll t \quad , \quad t_1 \le t_i \le t_{\uparrow i} \right) \tag{1.2.3}$$

Set quantity can be divided into: set frequency, set statistic value —— set value, set statistic coefficient —— set coefficient, set boundary, set boundary coefficient, etc

1.2.3 C Function Judgment Theorem

Definition 1.2.5: If the change of the function, similar to the wave propagation direction, is relatively large within a limited distance, from near to far, at the same time to measure multiple points, the height of the water surface wave; The height of the adjacent points, most of the decrease, a few increase, the starting height is higher than the end height phenomenon, called wave decline function; The monotone decreasing function and wave decreasing function are collectively referred to as decreasing function

Similarly, a monotone increasing function and a wave increasing function can be defined, collectively referred to as an increasing function

Theorem 1.2.6: C function judgment theorem, series value in x, actual value of an unordered event, actual subset value, generalized value,

narrow value, narrow subset value, ratio, ratio subset value $\pi(x)$ **,** $\pi(x_i)$ **,** $\Pi(x)_G$**,** $\Pi(x)_N$ **,** $\Pi(x_i)_N$ **,** $C(x)$ **,** $C_N^A(jx_i)$**, if the following formula**

$$\left|\frac{\pi(jx_i)}{\Pi(jx_i)_N}-\frac{\pi(jx_{i-1})}{\Pi(jx_{i-1})_N}\right|=\begin{cases}\left|C(jx_i)-C(jx_{i-1})\right| \\ \delta(ji)<\delta((j-1)i)\end{cases}\begin{pmatrix} j=1,2,3,,,\infty \\ x_1\le x_i\le x_{\uparrow i}=x\end{pmatrix} \qquad (1.2.4)$$

It holds in a sufficiently large finite range, and the δ(i) value in the increasing interval has multiple wave decays, then

$$\pi(x)=C_N^A(x)\Pi(x)_N=^1 C\Pi(x)_N=^1\Pi(x)_G \quad (x_1\le x_i\le x_{\uparrow i}=x) \qquad (1.2.5)$$

Conclusion 1: Ratio C(x) is a C constant function——C function;

Conclusion 2: If the ratio C(x) is a constant function of 1, then the narrow value is the generalized narrow value;

Conclusion 3: If the ratio C(x) is a non-1 constant function, then the narrow value is a non-generalized narrow value;

Proof: According to the statement of this theorem, it can be obtained that in the infinite large range, with the infinite increase of the value, the delta (ji) value of formula (1.2.4) will decrease the infinite wave and become a non-negative infinitesimal —— zero, that is

$$\left|\frac{\pi(jx_i)}{\Pi(jx_i)_N}-\frac{\pi(jx_{i-1})}{\Pi(jx_{i-1})_N}\right|=\begin{cases}\left|C_N^A(jx_i)-C_N^A(jx_{i-1})\right| \\ \delta(ji)<\delta((j-1)i)<,,,,,<\delta(i)\end{cases}$$

$$\lim_{j\to\infty}\left|\frac{\pi(jx_i)}{\Pi(jx_i)_N}-\frac{\pi(jx_{i-1})}{\Pi(jx_{i-1})_N}\right|=\lim_{j\to\infty}\left|C_N^A(jx_i)-C_N^A(jx_{i-1})\right|=\lim_{j\to\infty}\delta(ji)=0$$

The above formula satisfies the definition of the formula of C

function (1.1.1), the formula and conclusion of this theorem are obtained, and the proof is completed

Note 1: The way to obtain the narrow value : traditional probability, conjecture, empirical formula;

Note 2: With the help of this theorem, data can not only verify conjectures, but also become evidence to prove conjectures;

Note 3: With the help of data, it is possible to judge whether Ramanujan's 4,000 formulas are valid or not

1.2.4 3-dimensional generalized events, 4-dimensional generalized events

Definition 1.2.7: In a 3-dimensional space (x, y, z), an unordered deterministic event that can repeatedly occur and be easily specified for verification is called a 3-dimensional unordered deterministic event or a 3-dimensional generalized event; Unordered deterministic events that are independent of time, such as prime numbers and pi.

Definition 1.2.8: Disorderly deterministic events in 4-dimensional spacetime (x, y, z, t) are called 4-dimensional spacetime generalized events or 3-dimensional narrow events ——3-dimensional random

events. That is, random events. In 3D space (x, y, z), it cannot be repeated and it is inconvenient to specify verification, such as explosions, poker games, etc. The result or state of an event that occurs is related to time t.

In the universe, all moving matter, at a certain moment t, is a uniquely determined disordered event —— a 4-dimensional spatiotemporal disordered deterministic event, or a 4-dimensional generalized event; Therefore, all disordered events in the universe are four-dimensional generalized events

1.2.5 Probability axiom

Axiom 1.2.9: In a generalized event, within a reasonable range, the generalized target event tends towards an average distribution, and, the ratio of the actual value $\pi_1(x)$, the generalized value $\Pi_1(x)_G$, to its narrow value $\Pi_1(x)_N$ approach constant C_1; The constant can be called the actual narrow transformation coefficient, or the generalized narrow transformation coefficient;

The frequency $PP_1(x)$ of the generalized event , the ratio of the average frequency $\overline{PP_1(x_i)}$ of the composition , the generalized probability $P_1(x)_G$ to its narrow probability $P_1(x)_N$ approach constant C_1;

Among them, C_1 can be referred to as the generalized narrow sense transformation coefficient;

The ratio of the actual value $\pi_2(x)$, the generalized value $\Pi_2(x)_G$ of a generalized event of the same type, to its narrow value $\Pi_2(x)_N$ of the same type approach constant C_2 ;

The frequency $PP_2(x)$ of the generalized event of the same type , the ratio of the average frequency $\overline{PP_2(x_i)}$ of the composition , the generalized probability $P_2(x)_G$ to its narrow probability $P_2(x)_N$ approach constant C_2.etc, then

$$\frac{\pi_1(x)}{\Pi_1(x)_N C_1} = \frac{{}^1\Pi_1(x)_G}{\Pi_1(x)_N C_1} = \frac{{}^1\overline{PP_1(x_i)}}{P_1(x)_N C_1} = \frac{{}^1P_1(x)_G}{P_1(x)_N C_1}$$
$$= \frac{{}^1\pi_2(x)}{\Pi_2(x)_N C_2} = \frac{{}^1\Pi_2(x)_G}{\Pi_2(x)_N C_2} = \frac{{}^1\overline{PP_2(x_i)}}{P_2(x)_N C_2} = \frac{{}^1P_2(x)_G}{P_2(x)_N C_2} =^1 1 \qquad (1.2.6)$$

Among them

$$\overline{PP_1(x)} = \frac{\pi_1(x)}{x}, \quad P_1(x)_G = \frac{{}^1\Pi_1(x)_G}{x}, \quad P_1(x)_N = \frac{{}^1\Pi_1(x)_N}{x}$$

$$\overline{PP_2(x)} = \frac{\pi_2(x)}{x}, \quad P_2(x)_G = \frac{{}^1\Pi_2(x)_G}{x}, \quad P_2(x)_N = \frac{{}^1\Pi_2(x)_N}{x}$$

This convention is called the probability axiom

1.2.6 Generalized probability coefficient corollary

Corollary 1.2.10: In the generalized events , frequency of target quantity , subset frequency , generalized probability $pp(t)$, $\overline{pp(t_i)}$,

$P(t)_G$ **of target quantity , and ratio of narrow probability** $P(t)_N$ **, it's called the subset frequency coefficient , probability coefficient** $\overline{C(t_i)}_F$ **,** $C_p(t_i)_G$ **, etc., are coefficients that approaches one another, namely**

$$\overline{C(t_i)}_F = \frac{\overline{pp(t_i)}}{P(t_i)_N} =^1 C(t)_p =^1 \frac{P(t)_G}{P(t)_N} =^1 C \quad (|t-t_i| \ll t) \qquad (1.2.7)$$

Proof: Since the generalized probability $P(t)_G$ **is the most probable value of frequency** $pp(t)$**, it is also the most probable value of the mean of frequency** $pp(t)$ **—— the subset frequency** $\overline{pp(t_i)}$**, namely**

$$P(t)_G =^1 \overline{pp(t_i)} \quad (|t-t_i| \ll t)$$

The above formula, divided by $P(t)_N$**, refers to the formula (1.2.6) of the probability axiom, obtains the corollary, and the proof is complete**

1.2.7 Generalized statistical coefficient corollary

Inference 1.2.11: Actual value , set value, generalized value , narrow value $\pi(x)$ **,** $\overline{\pi(x_i)}$ **,** $\Pi(x)_G$ **,** $\Pi(x)_N$ **of the same unordered target event, are statistics value that frequency** $pp(t)$ **, set frequency , generalized probability , narrow probability** $pp(t)$ **,** $\overline{pp(t_i)}$ **,** $P(t)_G$ **,** $P(t)_N$ **of the occurrence of this target event, the ratio of these statistics value divided by** $\Pi(x)_N$ **, are called the statistical**

coefficient $C_N^A(k)$ **,** $\overline{C_N^A(x_i)}$ **,** $C_N^G(k)$ **,** $\overline{C_{PN}^{pp}(x_i)}$ **,** $C_{PN}^{PG}(k)$ **.etc., they approach each other，then**

$$\frac{\pi(x)}{\Pi(x)_N} =^1 \frac{\overline{\pi(x_i)}}{\Pi(x_i)_N} = \frac{^1\Pi(x)_G}{\Pi(x)_N} = \frac{^1\overline{pp(x_i)}}{P(x_i)_N} = \frac{^1P(x)_G}{P(x)_N} \qquad (x_1 \le x_i \le x_{\uparrow i} = x)$$

$$=^1 C_N^A(k) =^1 \overline{C_N^A(x_i)} =^1 C_N^G(k) =^1 \overline{C_{PN}^{pp}(x_i)} =^1 C_{PN}^{PG}(k) \qquad (1.2.8)$$

Proof : Similar to the proof of formula (1.2.7) of generalized probability coefficient corollary above, omitted

1.3 Probability axiom corollary

With the help of probability axioms, the following three inferences are obtained,and

1.3.1 Probability axioms , function prime number corollary

Inference 1.3.1：Within a reasonably large range of $|x-H(t)| \ll x$ **, natural number x, function H (t), frequency , generalized probability , narrow probability , of appears prime number ,prime number theorem** $pp(x)$ **,** $pp(H(t))$**,** $P(x)_G$ **,** $P(H(t))_G$**,** $P(x)_N$ **,** $P(H(t))_N$ **,** $\pi(H(x))$ **,** $\Pi(H(x))_G$ **, then**

$$\begin{cases} {}^{1}P(x)_{N} \\ P(H(t))_{N} \end{cases} = \left(1-\frac{{}^{1}0.5}{\sqrt{x}}\right)\frac{1}{\ln x} = \left(1-\frac{{}^{1}0.5}{\sqrt{H(t)}}\right)\frac{1}{\ln H(t)} \quad \left[|x-H(t)| \ll x\right] \quad (1.3.1)$$

$$\begin{cases} {}^{1}\pi(H(x)) \\ \Pi(H(x))_{G} \end{cases} =^{1} \int^{x} \mathrm{C}P(H(t))_{N}\, dt = \left(1-\int^{x}\frac{{}^{1}0.5}{\sqrt{H(t)}}\frac{dt}{x}\right)\int^{x}\frac{\mathrm{C}dt}{\ln H(t)} \quad (1.3.2)$$

Proof: According to "Probability Prime Number Theory", or from the formula (1.2.4) of probability axioms, the formula (5.1.14) of prime generalized probability, two results of this inference are obtained

1.3.2 Probability axioms , integer point corollary

The integer point is also called lattice point, and the integer point axiom , is also called lattice point axiom

Inference 1.3.2: Within the reasonable size of the random region,the random integer point, the narrow probability and the narrow probability of occurrence,approaching equal,and is generalized narrow event

Proof: The integer point is a lattice arranged by equidistant rules 1 apart , in the reasonable size range of the random region, the generalized number and the narrow number of the integer point of the random region are approach to the area of the random region, and the corollary is obtained, the proof is completed

1.3.3 Probability axioms , digital corollary

Inference 1.3.3: At any position x of an irrational number, any digit i in base k occurs, and its narrow probability $P(x,i)_N$, equal to 1/k, approaches the generalized probability $P(x,i)_G$, namely

$$P(x,i)_N = \frac{1}{k} =^1 P(x,i)_G \qquad (0 \le i \le k-1) \tag{1.3.3}$$

Conclusion: Irrational number is a broad narrow event

Proof: Any number i in K-base : 0≤i≤ K-1, as a narrow event, or as a generalized event, appears in any position x, there is no particularity - equal probability, its narrow probability, or generalized probability, are all equal to 1/k ,this inference is eobtained, the proof is complete

This theorem analyzes the solution and obtains statistical data support for 200 billion digits of pi digits 【see Probability Prime Number Theory】

1.4 Function approximate formula

$$\int^x \mathrm{u}(t,\ln t)dt =^1 \mathrm{u}(x,\ln x-1) \tag{1.4.1}$$

$$\int^x \frac{f(t)dt}{\mathrm{u}(\ln t)} =^1 \int^x f(t)\overline{\frac{1}{\mathrm{u}(\ln t)}}dt =^1 \int^x \frac{f(t)}{x}dt \int^x \frac{dt}{\mathrm{u}(\ln t)} \tag{1.4.2}$$

$$\ln(1+t) = t -^1 0.5t^2 \tag{1.4.3}$$

$$\int^{x} {}^{1}C(t)\Pi(t)dt = \int^{x} {}^{1}C(0.5x)\Pi(t)dt = {}^{1} C(0.5x)\int^{x}\Pi(t)dt \qquad (1.4.4)$$

Chapter 2 Boundary

2.1 Boundary basic

2.1.1 Boundary axioms

In early 1983, the author decided to study the Goldbach Conjecture, calculate and analyze solutions using the average method, and enter probability theory to establish boundaries, which was a prerequisite for this method to solve the problem; More than 10 years of original boundary creation failed, but in probability theory, discovered the law of double logarithms and potential boundary functions; Customize boundary axioms, establish value domain boundaries, and obtain 200 billion bits of numerical statistical data support for pi.

Inspired by the boundary of the range, the boundary of the domain is established, so that Cramer's conjecture is proved by probability method: the maximum distance between adjacent primes in x (lnx)^2 【see the Theory of Probabilistic primes】.

Recently, the maximum length of K-generated primes in x,

including the maximum length of 2-generated primes and the maximum distance between adjacent primes, has been obtained directly by sieve method, and the precision has been improved

https://www.zhihu.com/question/313650837/answer/623965043

zhihu.com/question/313650837/answer/623965043

两相邻素数的最大间距能够多大?

假设 p_n 是第 n 个素数，我们记 $G(x) = \max_{p_n \le x}\{p_{n+1} - p_n\}$ ，于是我们想知道 $G(x)$ 的大小。Erdős在1938年悬赏10000美元，希望可以证明下述猜想:

$$G(x) \ge c\frac{\log x \log\log x \log\log\log\log x}{(\log\log\log x)^2}$$

其中不等式对任意常数 c 成立。这个猜想最终被Ford, Green, Konyagin, Tao以及Maynard各自独立的证明。后来这两组人开始一起合作，最终在2018年证明了如下结果，也是目前最好结果:

$$G(x) \gg \frac{\log x \log\log x \log\log\log\log x}{\log\log\log x}$$

Further, the set boundary is established, and the actual data can not only verify the conjecture， but can determine the distribution range of theoretical values, also be used as evidence to prove the conjecture. When the statistical coefficient, the boundary coefficient, cannot be determined theoretically, with the help of the set statistical coefficient, the set boundary coefficient, the actual data, it is possible to determine the mean value of the statistical coefficient —— the most likely value of the mean value —— the generalized value, and the maximum distribution range around the mean value —— the boundary

With the aid of the logarithm theorem, it is possible to define,

analyze, and statistically obtain narrow boundaries, set boundaries, etc., in a limited range

There are two kinds of boundary, range boundary and definition domain boundary 【see the Theory of Probability Prime Numbers】.

The so-called boundary,generally refers to statistical values —— actual values,the most likely value around it——the most likely value of the maximum deviation of the generalized value

Axiom 2.1.1: If a finite and enough number of disordered events occur, the sum of the probability of the occurrence of the target event is small enough. Convention: The target event actually occurs zero times, the boundary obtained from this is called the axiom of sufficiently small boundary

Axiom 2.1.2: If an unordered event occurs infinite times and the target event only occurs finite times, it is agreed that the unordered event occurs finite times and the target event occurs zero times. The boundary obtained from this is called the finite order boundary axiom

Random event —— narrow event, narrow error——random error, narrow error maximum —— narrow boundary;

Disorderly deterministic events —— generalized events,

generalized errors, narrow errors, generalized boundary, narrow boundary

Based on a large amount of data from disordered events,comparing fluctuation errors —— generalized events,random error —— narrow event

A1, similarity, their average, all approach zero;

A2, The difference, fluctuation error potential is weak, not strict, local law; Random errors are completely irregular;

A3: The fluctuation error is almost proportional to the narrow error

2.1.2 Introduction to boundaries

Definition 2.1.3: Target statistical value —— range —— actual value $\pi(x)_G$, $\pi(x)_N$, of variable x, the error $\pi(x)_G-\Pi(x)_G$, $\pi(x)_N-\Pi(x)_N$ of this value relative to its most probable value —— mathematical expectation —— generalized value , narrow value $\Pi(x)_G$, $\Pi(x)_N$, called the generalized error, narrow error $\Delta\pi(x)_G$, $\Delta\pi(x)_N$, then:

$$\Delta\pi(x)_N=\pi(x)_N-\Pi(x)_N \tag{2.1.1}$$

$$\Delta\pi(x)_G=\pi(x)-\Pi(x)_G={}^{1}\Delta\Pi(x)_G={}^{1}\mathrm{C}\Delta\pi(x)_N \tag{2.1.2}$$

The most probable value of the maximum value of the generalized error, the narrow error, are called the generalized boundary, the narrow boundary $\uparrow\downarrow \Pi(x)_G$ **,** $\uparrow\downarrow \Pi(x)_N$ **, Boundary coefficient** $\uparrow\downarrow C$ **, then:**

$$\uparrow\downarrow \pi(x)_N = \max\left|\Delta\pi(x)_N\right| \overset{1}{=} \max\left|\pi(x)_N - \Pi(x)_N\right| \overset{1}{=} \uparrow\downarrow \Pi(x)_N \qquad (2.1.3)$$

$$\uparrow\downarrow \pi(x)_G = \max\left|\Delta\pi(x)_G\right| \overset{1}{=} \uparrow\downarrow \Pi(x)_G \overset{1}{=} \uparrow\downarrow C \uparrow\downarrow \Pi(x)_N \qquad (2.1.4)$$

Definition 2.1.4: In the domain of definition, the set composed of some point errors, the calculated mean square error and the calculated boundary are called the set mean square error and the set boundary $\overline{\sigma(x_i)}_G$ **,** $\overline{\sigma(x_i)}_N$ **,** $\uparrow\downarrow \overline{\pi(x_i)}_G$ **,** $\uparrow\downarrow \overline{\pi(x_i)}_N$ **, otherwise, the mean square error and the boundary** $\sigma(x)_G$ **,** $\sigma(x)_N$ **,** $\uparrow\downarrow \pi(x)_G$ **,** $\uparrow\downarrow \pi(x)_N$ **, are expressed as follows:**

$$\begin{cases} \uparrow\downarrow \overline{\pi(x_i)}_G \\ {}^{1}\uparrow\downarrow \Pi(x)_G \end{cases} = \max\left|\pi(x)_G - \overline{\pi(x_i)}_G\right| = \max\left|\pi(x)_G - \sum_{i=1}^{\uparrow i}\frac{\pi(x_i)_G}{\uparrow i}\right|$$

$$\overset{1}{=} \overline{\sigma(x_i)}_G \sqrt{2\ln\ln x} \overset{1}{=} \sigma(x)_G \sqrt{2\ln\ln x} \quad \left(\left|x_{\uparrow i} - x_i\right| \ll x = {}^{1}x_{\uparrow i}\right) \qquad (2.1.5)$$

In a generalized event with symmetric distribution, half of the difference between its maximum values $\uparrow \pi(x)_G$ **and minimum**

values $\downarrow\pi(x)_G$ is called the boundary; The ratio of the actual boundary, generalized boundary $\uparrow\downarrow\pi(x)_G$, $\uparrow\downarrow\Pi(x)_G$, and its narrow boundary $\uparrow\downarrow\Pi(x)_N$ is called the boundary coefficient $^1\uparrow\downarrow\mathrm{C}$, which is:

$$\begin{cases}\uparrow\downarrow\overline{\pi(x_i)}_G\\ {}^1\uparrow\downarrow\Pi(x)_G\end{cases}=\max\left|\pi(x)-\overline{\pi(x_i)}_G\right|=\max\left|\pi(x)-\sum_{i=1}^{\uparrow i}\frac{\pi(x_i)_G}{\uparrow i}\right|$$

$$={}^1\uparrow\downarrow\mathrm{C}\overline{\sigma(x_i)}_N\sqrt{2\ln\ln x}={}^1\uparrow\downarrow\mathrm{C}\sigma(x)_N\sqrt{2\ln\ln x}\quad\left(\left|x_{\uparrow i}-x_i\right|\ll x={}^1x_{\uparrow i}\right)\quad(2.1.6)$$

2.1.3 Set Absolute Value Boundary

Theorem 2.1.5: Several points are uniformly distributed near a straight line, and the height of these points - the generalized value - is equal to the difference between the height of these points and the corresponding height of the line [the average height close to these points] - the generalized error $\Delta\pi(x_i)_G$, which is twice the average absolute value of the generalized error, and approaches the maximum deviation of these points from the height of the line - the actual boundary $\uparrow\downarrow\pi(x)_G$, called the absolute value set boundary $\uparrow\downarrow\overline{\pi(x_i)}_{\mathrm{A}}$, that is:

$$\begin{cases}{}^1\uparrow\downarrow\pi(x)_G\\ \uparrow\downarrow\Pi(x)_G\end{cases}=\begin{cases}{}^1\uparrow\downarrow\overline{\pi(x_i)}_{\mathrm{A}}\\ {}^1 2\overline{\left|\Delta\pi(x_i)_G\right|}\end{cases}={}^1 2\sum_{i=1}^{\uparrow i}\left|\frac{\Delta\pi(x_i)_G}{\uparrow i}\right|\quad\left[x_1\le x_i\le x_{\uparrow i}=x\right]\qquad(2.1.7)$$

Generalized value narrow logarithmic boundary, narrow value

narrow logarithmic boundary, generalized value absolute value error set boundary coefficient, narrow value absolute value error set boundary coefficient $\uparrow\downarrow \Pi(x)_{I\ N}^{G}$**，** $\uparrow\downarrow \Pi(x)_{I\ N}^{N}$**，** $^{1}\uparrow\downarrow \overline{C_{I\ G}^{AG}(x_i)}$**，** $\uparrow\downarrow \overline{C_{IN}^{AN}(x_i)}$**；**

广义值狭义重对数边界，狭义值狭义重对数边界，广义值绝对值误差集合边界系数，狭义值绝对值误差集合边界系数 $\uparrow\downarrow \Pi(x)_{I\ N}^{G}$**，** $\uparrow\downarrow \Pi(x)_{I\ N}^{N}$**，** $^{1}\uparrow\downarrow \overline{C_{I\ G}^{AG}(x_i)}$**，** $\uparrow\downarrow \overline{C_{IN}^{AN}(x_i)}$**；**

Similarly, the definition of the boundary of the set of absolute values on this line can be extended to any curve - generalized events; If the absolute value of the generalized value actual error $\left|\Delta\pi(x_i)_G\right|$ **approaches a uniform distribution within 【0，**$\uparrow\downarrow \pi(x)$**】——【0，**$\uparrow\downarrow C \uparrow\downarrow \Pi(x)_{I\ N}^{G}$**】, then**

$$\begin{cases} {}^{1}\uparrow\downarrow \pi(x) \\ \uparrow\downarrow \overline{\pi(x_i)_A} \end{cases} =^{1} 2\sum_{i=1}^{\uparrow i}\left|\frac{\Delta\pi(x_i)_G}{\uparrow i}\right| =^{1} \begin{cases} {}^{1}2\uparrow\downarrow \overline{C_{IG}^{AG}(x_i)}\uparrow\downarrow \Pi(x)_{I\ N}^{G} \\ {}^{1}2\uparrow\downarrow \overline{C_{IN}^{AN}(x_i)}\uparrow\downarrow \Pi(x)_{I\ N}^{N} \end{cases}$$

$$\left\{\overline{C_{IG}^{AG}(x_i)} = \sum_{i=1}^{\uparrow i}\left[\left|\Delta\pi(x_i)_G\right| \div \uparrow\downarrow \Pi(x_i)_{I\ N}^{G}\right] \div \uparrow i\right\} \quad \left[x_1 \le x_i \le x_{\uparrow i} = x\right]$$

$$\left\{\overline{C_{IN}^{AN}(x_i)} = \sum_{i=1}^{\uparrow i}\left[\left|\Delta\pi(x_i)_N\right| \div \uparrow\downarrow \Pi(x_i)_{I\ N}^{N}\right] \div \uparrow i\right\} \quad \left[x_1 \le x_i \le x_{\uparrow i} = x\right] \quad (2.1.8)$$

Proof: Assuming that for each value point, the distance from the line —— the absolute value of the height difference —— the absolute value of the error is within【0, boundary】. If the calculated boundary is close to an equal probability distribution, but is larger than the actual boundary in a normal distribution but smaller than the logarithmic boundary, then

$$\begin{cases} {}^{1}\uparrow\downarrow \pi(x)_G \\ \uparrow\downarrow \overline{\pi(x_i)}_{\text{A}} \end{cases} =^{1} 2\overline{\left|\Delta\pi(x_i)_G\right|} =^{1} 2\sum_{i=1}^{\uparrow i}\left|\frac{\Delta\pi(x_i)_G}{\uparrow i}\right| =^{1} \sum_{i=1}^{\uparrow i}\frac{2\left|\Delta\pi(x_i)_G\right|}{\uparrow\downarrow \Pi(x_i)_{IN}^{G}}\frac{\uparrow\downarrow \Pi(x_i)_{IN}^{G}}{\uparrow i}$$

$$=^{1} 2*0.5\uparrow\downarrow C\uparrow\downarrow \Pi(x)_{IN}^{\text{G}} =^{1}\uparrow\downarrow C\uparrow\downarrow \Pi(x)_{IN}^{\text{G}}$$

$$=^{1} 2\sum_{i=1}^{\uparrow i}\frac{\left|\Delta\pi(x_i)_G\right|\div\uparrow i}{\uparrow\downarrow \Pi(x_i)_{IN}^{G}}\uparrow\downarrow \Pi(x)_{IN}^{G} =^{1} 2\overline{\left|\frac{\Delta\pi(x_i)_G}{\uparrow\downarrow \Pi(x_i)_{IN}^{G}}\right|}\uparrow\downarrow \Pi(x)_{IN}^{G}$$

$$=^{1} 2\overline{\text{C}_{\text{IG}}^{\text{AG}}(x_i)}\uparrow\downarrow \Pi(x)_{IN}^{G} \quad \left[x_1 \le x_i \le x_{\uparrow i} = x\right]$$

$$\left\{\overline{\text{C}_{\text{IG}}^{\text{AG}}(x_i)} = \sum_{i=1}^{\uparrow i}\left[\ \left|\Delta\pi(x_i)_G\right|\div\uparrow\downarrow \Pi(x_i)_{IN}^{G}\right]\div\uparrow i\right\}$$

Obtain the first expression of this theorem;

Similarly, the second expression of this theorem can be obtained, and the proof is complete.

Summary: Any boundary obtained through the use of actual data and specified theoretical statistical methods (such as logarithmic set boundary, absolute value set boundary) is called a narrow set boundary and belongs to a type of narrow boundary;

There are two types of generalized boundaries

A1, Generalized narrow boundary: Generalized boundary approaches narrow boundary

$$\uparrow\downarrow \Pi(x)_G =^{1}\uparrow\downarrow C\uparrow\downarrow \Pi(x)_N =^{1}\uparrow\downarrow \pi(x) \quad (C=1) \qquad (2.1.9)$$

A2， Generalized non narrow boundary: The generalized boundary does not approa the narrow boundary

$$\uparrow\downarrow \Pi(x)_G =^{1}\uparrow\downarrow C\uparrow\downarrow \Pi(x)_N =^{1}\uparrow\downarrow \pi(x) \quad (C\neq 1) \qquad (2.1.10)$$

Summary: There are two types of generalized quantities:

A1. Generalized narrow quantity: generalized quantity approaches narrow quantity;

A2. Generalized non-narrow quantities: generalized quantities do not approach narrow quantities, but generalized quantities approach proportional to narrow quantities; Therefore, the generalized quantity can be expressed by the product of the narrow quantity and the coefficient.

Definition 2.1.6: A monotone increasing or decreasing function is called a monotone function; If the center line of the distribution area of the wave function π(x) [approaching average] ——The monotonic change of generalized value $\Pi(x)_G$ is called monotonic increasing fluctuation function or monotonic decreasing fluctuation function.

Below, the default actual value π (x) and so on are monotonically increasing fluctuation functions.

Note 1: In the boundary calculation, the high-order small error generated by approximate calculation is far less than the fluctuation of the actual boundary. If it is ignored, it is still connected with an equal sign;

Note 2: Boundary values are approximate values,

approximate function symbols, with or without;

Note 3: The actual boundary fluctuates greatly. Therefore, to calculate the theoretical boundary, only the main value expressed by the limit function can be calculated, and the estimation (which can be estimated) can be omitted, and the equal sign is still used, indicating that it is close to equality.

2.2 Iterated logarithm range boundary

2.2.1 Set mean square error approach theorem

Definition 2.2.1 : In x, the error of some points, the set formed and the mean square deviation calculated are called the mean square deviation of the set, otherwise, it is called the mean square deviation..

Analysis: Different numbers and values of error set points (and data) result in different values of the calculated set mean square error; But as the number of statistical points in the error set approaches the total number of statistical points, the set mean square error approaches the mean square error; Therefore, the set mean square deviation and mean square deviation tend to approach each other; And, the mean square error is a definite value, strictly prove as follows

Theorem 2.2.2: Set mean square error approach theorem, For disordered event x_i, the error subset $\Delta\pi\left(x_i\right)_{\mathrm{G}}$, $\Delta\pi\left(x_i\right)_N$ is obtained, the calculated any error set mean-square error $\sigma\left(\Delta\pi\left(x_i\right)_{\mathrm{G}}\right)$, $\sigma\left(\Delta\pi\left(x_i\right)_N\right)$, they all approach the same, and approaching them total set mean square error —— actual mean-square error $\sigma\left(\Delta\pi\left(x\right)_{\mathrm{G}}\right)$, $\sigma\left(\Delta\pi\left(x\right)_N\right)$.etc,namely

$$\begin{cases}\sigma\left(\Delta\pi\left(x_i\right)_{\mathrm{G}}\right) \\ {}^{1}\sigma\left(\Delta\pi\left(x\right)_{\mathrm{G}}\right)\end{cases} = \sqrt{\sum_{i=1}^{\uparrow i}\frac{\Delta\pi\left(x_i\right)_G^2}{\uparrow i}} =^{1} \sqrt{\sum_{i=1}^{x}\frac{\Delta\pi\left(j\right)_G^2}{x}} \quad \begin{pmatrix}\Delta\pi\left(x_i\right)_{\mathrm{G}} = \pi\left(x_i\right)_{\mathrm{G}} - \Pi\left(x_i\right)_{\mathrm{G}} \\ 1\le j\le x,\ x_1\le x_i\le x_{\uparrow i}=x\end{pmatrix} \tag{2.2.1}$$

$$\begin{cases}\sigma\left(\Delta\pi\left(x_i\right)_N\right) \\ {}^{1}\sigma\left(\Delta\pi\left(x\right)_N\right)\end{cases} = \sqrt{\sum_{i=1}^{\uparrow i}\frac{\Delta\pi\left(x_i\right)_N^2}{\uparrow i}} =^{1} \sqrt{\sum_{i=1}^{x}\frac{\Delta\pi\left(j\right)_N^2}{x}} \quad \begin{pmatrix}\Delta\pi\left(x_i\right)_N = \pi\left(x_i\right)_N - \Pi\left(x_i\right)_N \\ 1\le j\le x,\ x_1\le x_i\le x_{\uparrow i}=x\end{pmatrix} \tag{2.2.2}$$

Proof: Within x, the error set composed of error data of several points or all points, the calculated mean square error is called the set mean square error, as follows

$$\lim_{x_i \to x} \sigma\left(\Delta\pi\left(x_i\right)_{\mathrm{G}}\right) = \lim_{x_i \to x} \sqrt{\frac{1}{\uparrow i} \sum_{i=1}^{\uparrow i} \Delta\pi\left(x_i\right)_G^2} =^1 \lim_{x_i \to x} \sqrt{\frac{1}{\uparrow i} \sum_{i=1}^{\uparrow i} \left[\pi\left(x_i\right) - \Pi\left(x_i\right)_{\mathrm{G}}\right]^2}$$

$$=^1 \lim_{x_i \to x} \sqrt{\frac{1}{x} \sum_{j=1}^{x} \left[\pi\left(j\right) - \Pi\left(j\right)_{\mathrm{G}}\right]^2} =^1 \sqrt{\frac{1}{x} \sum_{j=1}^{x} \Delta\pi\left(j\right)_G^2} =^1 \sigma\left(\Delta\pi\left(x\right)_G\right)$$

The formula (2.2.1)　is obtained

For the same reason，the formula (2.2.2)　is obtained ，this theorem is obtained

2.2.2　　　Iterated logarithm theorem　重对数定理

Lemma 2.2.3：　i≤x times Bernoulli random trials b(i), equal　the narrow probability P, the number π(x)　of target events【can be expressed by η(x)　】，mean square error σ(x), then ，when x approaches infinity, the following formula 【 proof omitted[4王]P177】

$$\begin{cases} \pi\left(x\right) \\ \eta\left(x\right) \end{cases} = \sum_{i=1}^{N} b\left(i\right) =^1 xP \quad , \quad \begin{cases} \dfrac{\sigma\left(x\right)}{\sqrt{xP\left(1-P\right)}} \end{cases} =^1 \sqrt{\pi\left(x\right)\left[1 - \frac{\pi\left(x\right)}{x}\right]} \qquad (2.2.3)$$

$$\left|\pi\left(x\right) - xP\right| \le \sqrt{2xP\left(1-P\right)\ln\ln x} = \sqrt{2\ln\ln x}\,\sigma\left(x\right) \qquad (2.2.4)$$

Only the limited times is not valid; But the following formula is not true for an infinite number of times

$$\left|\pi\left(x\right) - xP\right| \le \left(1+\varepsilon\right)\sqrt{2xP\left(1-P\right)\ln\ln x} = \sqrt{2\ln\ln x}\,\sigma\left(x\right) \qquad (2.2.5)$$

2.2.3 Boundary positive correlation mean square error

Definition 2.2.4 : A variety of disordered events and target quantities generated by the same disordered factor are called homogeneous disordered events and homogeneous quantities;

For example, in the class function kx+l, the number π(kx+l) of prime numbers , is the class quantity; In the same generalized event, the generalized value and the narrow value are the same quantity

Inference 2.2.5: Boundary positive correlation mean square error inference, within a limited range of test cycles, Bernoulli test —— narrow event, actual boundary, narrow value narrow logarithmic boundary $\uparrow\downarrow\pi(x)$ **,** $\uparrow\downarrow\Pi(x)_{NI}$ **, is proportional to** $\sqrt{2\ln\ln x}$ **, almost positively correlated with the narrow mean squared** $\sigma(x)_N$**, then**

$$\uparrow\downarrow\pi(x)_N \overset{1}{=} \uparrow\downarrow\Pi(x)_{I\ N}^{N} \overset{1}{=} \sqrt{2\ln\ln x}\,\sigma(x)_N \qquad (2.2.6)$$

Proof: According to the formulas (2.2.4) and (2.2.5) of the iterated logarithm theorem, and the axiom 2.1.2——1 boundary axiom, this inference holds within a finite range; Due to being the

boundary of narrow events, with the help of the law of iterated logarithm and narrow values, it can be referred to as the narrow value narrow iterated logarithm boundary. $\uparrow\downarrow \Pi(x)_{NI}$

Which generalizes this principle to the set of similar disordered events

Inference 2.2.6: Proportional inference of the mean square deviation of the function boundary, conducting finite experiments on the actual boundary of the same type of unordered event H (x), including its generalized boundary, generalized log set boundary, generalized log set boundary, narrow log set boundary, and narrow log boundary $\uparrow\downarrow \pi\left(H(x)\right)$ **,** $\uparrow\downarrow \Pi\left(H(x)\right)_G$ **,** $\uparrow\downarrow \overline{\pi\left(H\left(x_i\right)\right)}_{GI}$ **,** $\uparrow\downarrow \Pi\left(H(x)\right)_{GI}$ **,** $\uparrow\downarrow \overline{\pi\left(H\left(x_i\right)\right)}_{NI}$ **,** $\uparrow\downarrow \Pi\left(H(x)\right)_{NI}$ **, is almost proportional to** $\sqrt{2\ln\ln x}$ **, proportional to their mean square deviation** $\sigma\left(\Delta H\left(x_i\right)_G\right)$ **,** $\sigma\left(\Delta H(x)_G\right)$ **,** $\sigma\left(H\left(x_i\right)_N\right)$ **,** $\sigma\left(\Delta H(x)_N\right)$ **, proportional coefficient - boundary coefficient** $\uparrow\downarrow \mathrm{C}_I^G$ **,** $\uparrow\downarrow \mathrm{C}_I^N$ **,** $\uparrow\downarrow \overline{\mathrm{C}_I^G\left(x_i\right)}$ **,** $\uparrow\downarrow \overline{\mathrm{C}_I^N\left(x_i\right)}$ **, then:**

$$\begin{cases} {}^1\uparrow\downarrow \pi\left(H(x)\right)_G \\ \uparrow\downarrow \Pi\left(H(x)\right)_G \end{cases} = \begin{cases} {}^1\uparrow\downarrow \mathrm{C}_I^G \uparrow\downarrow \Pi\left(H(x)\right)_{I\,N}^G \\ {}^1\uparrow\downarrow \mathrm{C}_I^N \uparrow\downarrow \Pi\left(H(x)\right)_{I\,N}^N \end{cases} = \begin{cases} {}^1\uparrow\downarrow \overline{\pi\left(H\left(x_i\right)\right)}_I^G \\ {}^1\uparrow\downarrow \overline{\pi\left(H\left(x_i\right)\right)}_I^N \end{cases}$$

$$= \begin{cases} {}^1\uparrow\downarrow \overline{\mathrm{C}_I^G\left(x_i\right)}\sqrt{2\ln\ln x}\,\sigma\left(\Delta H\left(x_i\right)_G\right) \\ {}^1\uparrow\downarrow \overline{\mathrm{C}_I^N\left(x_i\right)}\sqrt{2\ln\ln x}\,\sigma\left(\Delta H\left(x_i\right)_N\right) \end{cases} \quad \left(x_1 \le x_i \le x_{\uparrow i} = x\right) \qquad (2.2.7)$$

Proof: Because the shape of the probability distribution function of the same kind of disordered events is similar, the mean square error determined by the shape of the probability distribution function is almost proportional to the boundary. Similar to the principle of similar triangle, corresponding edge growth is proportional to the formula (2.2.6) of Bernoulli's narrow test boundary, the inference is obtained, and the proof is completed

2.2.4 Narrow iterated logarithm range boundary

Bernoulli tests are used for narrow events, such as test scores; Bernoulli inspects are used for generalized events, such as whether a given integer is prime.

Definition 2.2.7: In Bernoulli's test——narrow sense event, run a finite number of tests, the range boundary established by means of the logarithm theorem, mean square error and boundary axiom is called the Bernoulli log-mean square range boundary, referred to as the log-logarithm range boundary

Theorem 2.2.8: Perform i ≤x times Bernoulli test——narrow sense event b(i) , B(i) , the narrow probability of 0 【or 1】 occurring in the target event is equal to P 【or 1-P】;

The actual value , the narrow value, the narrow set value, the narrow subset error, the narrow error of the occurrence times of the target event $\pi(x)_N$ **,** $\Pi(x)_N$ **,** $\overline{\pi(x_i)}_N$ **,** $\Delta\pi(x_i)_N$ **,** $\Delta\Pi(x)_N$ **;**

The above error corresponds to the narrow mean square error, the narrow subset mean square error $\sigma(x)_N$ **,** $\sigma(\Delta\Pi(x)_N)$ **,** $\overline{\sigma(x_i)}_N$ **;**

The mean square error, corresponding to the narrow value of the narrow log-log range set boundary, the narrow value of the narrow iterated logarithm range boundary $\uparrow\downarrow\overline{\pi(x_i)}_{I\ N}^{N}$ **,** $\uparrow\downarrow\Pi(x)_{I\ N}^{N}$ **, etc;**

Appear t times target event, the narrow probability density function $P(x,P,t)_N$ **, get**

$$\Pi(x)_N = xP \quad \left[\Delta\Pi(x)_N = \pi(x)_N - \Pi(x)_N\right] \qquad (2.2.8)$$

$$P(x,P,t)_N = \frac{x!(1-P)^{x-t} P^{\ t}}{(x-t)!\ t!} \qquad (2.2.9)$$

$$\begin{cases} {}^1\sigma(x)_N \\ \overline{\sigma(x_i)}_N \end{cases} = \begin{cases} {}^1\sigma(\Delta\Pi(x)_N) \\ {}^1\sqrt{xP_N(1-P_N)} \end{cases} ={}^1 \sqrt{\Pi(x)_N\left[1-\frac{\Pi(x)_N}{x}\right]} \qquad (2.2.10)$$

$$\begin{cases} {}^1\uparrow\downarrow\pi(x)_N \\ \uparrow\downarrow\Pi(x)_{I\ N}^{N} \end{cases} = \begin{cases} {}^1\sqrt{2\ln\ln x}\ \sigma(x)_N \\ {}^1\sqrt{2\ln\ln x}\ \overline{\sigma(x_i)}_N \end{cases} ={}^1 \sqrt{2\Pi(x)_N\left[1-\frac{\Pi(x)_N}{x}\right]\ln\ln x} \qquad (2.2.11)$$

Proof: Referring to the definition 2.2.6 of numerical domain boundaries and the inference 2.2.4 of boundary mean square deviation positive correlation in "Probability Prime Number Theory" and related definitions, the proof process is briefly described

2.2.5 Generalized event iterated logarithm range boundary

Definition 2.2.9: The range boundary established by finite order generalized event test with the help of logarithm theorem, set mean square error, mean square error, set mean square error approach theorem and boundary axiom is called the generalized iterated logarithm range boundary

Theorem 2.2.10: i ≤ x Bernoulli generalized test b(i), B(i), the probability of 0 [or 1] of the target event is equal to P [or 1-p], the actual value of the target event statistic , the actual boundary of the actual value ↑↓π(x), the boundary of the generalized iterated logarithm set, the boundary of the generalized iterated logarithm range, Narrow iterated logarithm set range boundary, narrow iterated logarithm range boundary, narrow range boundary

$\pi(x)$, $\uparrow\downarrow\pi(x)$, $\uparrow\downarrow\Pi(x)_G$, $\uparrow\downarrow\overline{\pi(x_i)}_I^G$, $\uparrow\downarrow\overline{\pi(x_i)}_I^N$, $\uparrow\downarrow\Pi(x)_{I\ N}^G$, $\uparrow\downarrow\Pi(x)_{I\ N}^N$ **.etc.; Corresponding boundary system: generalized iterated logarithm** set range boundary coefficient, narrow **iterated logarithm** set range boundary coefficient, narrow range **boundary coefficient** $\uparrow\downarrow C_I^G$, $\uparrow\downarrow C_I^N$, $\uparrow\downarrow\overline{C_I^G(x_i)}$, $\uparrow\downarrow\overline{C_I^N(x_i)}$**, etc., then:**

$$\pi(x)=\sum_{i=1}^{x}b(i)=\sum_{i=1}^{x}B(i)\pm\uparrow\downarrow\sum_{i=1}^{x}B(i)=\Pi(x)_G\pm{}^1\uparrow\downarrow\pi(x) \quad (2.2.12)$$

$$\begin{cases}\uparrow\downarrow\overline{\pi(x_i)}_I^G \\ {}^1\uparrow\downarrow\pi(x)\end{cases}={}^1\uparrow\downarrow\sqrt{\frac{2}{\uparrow i}\sum_{i=1}^{\uparrow i}\frac{\Delta\pi(x_i)_G^2\ln\ln x_i}{\uparrow\downarrow\Pi(x_i)_G^2}}\uparrow\downarrow\sqrt{2\Pi(x)_G\left[1-\frac{\Pi(x)_G}{x}\right]\ln\ln x}$$

$$\left[\Delta\pi(x_i)_G=\pi(x_i)-\Pi(x_i)_G\ ,\ \uparrow\downarrow\Pi(x)_G=\uparrow\downarrow\sqrt{2\Pi(x)_G\ln\ln x}\right]$$

$$={}^1\uparrow\downarrow\overline{C_I^G(x_i)}\uparrow\downarrow\Pi(x)_G={}^1\uparrow\downarrow C\sqrt{2\Pi(x)_G\ln\ln x}\quad\left(x_1\le x_i\le x_{\uparrow i}=x\right)\quad (2.2.13)$$

$$\begin{cases}\uparrow\downarrow\overline{\pi(x_i)}_I^G \\ {}^1\uparrow\downarrow\pi(x)\end{cases}={}^1\uparrow\downarrow\sqrt{\frac{2}{\uparrow i}\sum_{i=1}^{\uparrow i}\frac{\Delta\pi(x_i)_G^2\ln\ln x_i}{\uparrow\downarrow\Pi(x_i)_N^2}}\sqrt{2\Pi(x)_N\left[1-\frac{\Pi(x)_N}{x}\right]\ln\ln x}$$

$$\left[\Delta\pi(x_i)_G=\pi(x_i)-\Pi(x_i)_G={}^1\pi(x_i)-C(x_i)\Pi(x_i)_N={}^1\pi(x_i)-\overline{\pi(x_i)}\right]$$

$$={}^1\uparrow\downarrow\overline{C_I^N(x_i)}\uparrow\downarrow\Pi(x)_N={}^1\uparrow\downarrow C\sqrt{2\Pi(x)_N\ln\ln x}\quad\left[x_1\le x_i\le x_{\uparrow i}=x\right]\quad (2.2.14)$$

Proof: By definition, formula (2.2.12) are obtained

According to the Probability Axiom , the ratio of statistical values in a generalized narrow value approach to ratio of the boundary in a generalized narrow value;

From the formula (2.2.6) of boundary mean square error proportional theorem, the formula (2.2.1) of set mean square

error approach theorem， get

$$\uparrow\downarrow \mathrm{C}_I^G =^1 \uparrow\downarrow \overline{\mathrm{C}_I^G\left(x_i\right)} = \frac{\overline{\uparrow\downarrow \pi\left(x_i\right)_I^G}}{\uparrow\downarrow \Pi\left(x_i\right)_G} =^1 \frac{\overline{\sqrt{2\ln\ln x_i}\ \sigma\left(\Delta\pi\left(x_i\right)_G\right)}}{\uparrow\downarrow \Pi\left(x_i\right)_G}$$

$$=^1\uparrow\downarrow\ \sigma\left(\frac{\Delta\pi\left(x_i\right)_G}{\uparrow\downarrow \Pi\left(x_i\right)_G}\sqrt{2\ln\ln x_i}\right) =^1\uparrow\downarrow\ \sigma\left(\Delta\uparrow\downarrow \mathrm{C}\left(x_i\right)_G\sqrt{2\ln\ln x_i}\right)$$

$$=^1\uparrow\downarrow\sqrt{\frac{2}{\uparrow i}\sum_{i=1}^{\uparrow i}\Delta\uparrow\downarrow \mathrm{C}\left(x_i\right)_G^2\ln\ln x_i} =^1\uparrow\downarrow\sqrt{\frac{2}{\uparrow i}\sum_{i=1}^{\uparrow i}\frac{\Delta\pi\left(x_i\right)_G^2\ln\ln x_i}{\uparrow\downarrow \Pi\left(x_i\right)_G^2}}$$

By referring to the above formula , referring to the formula (2.2.4) of the iterated logarithm theorem，the formula (2.2.11) of the generalized range boundary $\uparrow\downarrow \Pi\left(x\right)_{I\ N}^{\mathrm{G}}$,get

$$\uparrow\downarrow \pi\left(x\right) =^1 \uparrow\downarrow \overline{\pi\left(x_i\right)}_I^{\mathrm{G}} =^1 \max\left|\pi\left(x\right) - \Pi\left(x\right)_{\mathrm{G}}\right| =^1 \uparrow\downarrow C_I^{\mathrm{G}}\uparrow\downarrow \Pi\left(x\right)_{I\ N}^{\mathrm{G}}$$

$$=^1\uparrow\downarrow \overline{C_I^{\mathrm{G}}\left(x_i\right)}\uparrow\downarrow \Pi\left(x\right)_{I\ N}^{\mathrm{G}} =^1\uparrow\downarrow\sqrt{\frac{2}{\uparrow i}\sum_{i=1}^{\uparrow i}\frac{\Delta\pi\left(x_i\right)_G^2\ln\ln x_i}{\uparrow\downarrow \Pi\left(x_i\right)_G^2}}\sqrt{2\ln\ln x}\ \sigma\left(\Delta\Pi\left(x\right)_{\mathrm{G}}\right)$$

$$=^1\uparrow\downarrow\sqrt{\frac{2}{\uparrow i}\sum_{i=1}^{\uparrow i}\frac{\Delta\pi\left(x_i\right)_{\mathrm{G}}^2\ln\ln x_i}{\uparrow\downarrow \Pi\left(x_i\right)_G^2}}\uparrow\downarrow\sqrt{2\Pi\left(x\right)_{\mathrm{G}}\left[1-\frac{\Pi\left(x\right)_{\mathrm{G}}}{x}\right]\ln\ln x}$$

The formula (2.2.13) for the generalized values the iterated logarithm set range boundary is obtained.

Similarly，the formula (2.2.14) for the generalized iterated logarithm set range boundary is obtained, and the proof is complete

2.2.6 Set statistical values

In the set boundary value formula, for example, when the set boundary coefficient is transformed by removing boundary symbols such as ↑↓, it becomes the corresponding set statistical value coefficient, the corresponding set statistical value formula, the corresponding formula expression, etc

Analyze the data of prime number theorem within 10 ^ 29 , the following conjecture is proposed

Conjecture 2.2.11: The value domain boundary of the iterated logarithm -set of the generalized event, the value domain boundary of the iterated logarithm -set of the narrow value, the value domain boundary of the absolute value of the generalized value, and the value domain boundary of the absolute value of the narrow value $\uparrow\downarrow\overline{\pi(x_i)}_I^{\mathrm{G}}$ **,** $\uparrow\downarrow\overline{\pi(x_i)}_I^{N}$ **,** $\uparrow\downarrow\overline{\pi(x_i)}_{\mathrm{A}}^{\mathrm{G}}$ **,** $\uparrow\downarrow\overline{\pi(x_i)}_{\mathrm{A}}^{N}$ **, are the upper limit and lower limit** $\uparrow\downarrow\pi(x)_G$ **,** $\uparrow\downarrow\Pi(x)_G$ **of the event boundary, namely:**

$$\begin{cases} {}^1\uparrow\downarrow\overline{\pi(x_i)}_{\mathrm{A}}^{\mathrm{G}} \\ {}^1\uparrow\downarrow\overline{\pi(x_i)}_{\mathrm{A}}^{N} \end{cases} < \begin{cases} \uparrow\downarrow\pi(x) \\ {}^1\uparrow\downarrow\Pi(x)_G \end{cases} < \begin{cases} {}^1\uparrow\downarrow\overline{\pi(x_i)}_{I}^{\mathrm{G}} \\ {}^1\uparrow\downarrow\overline{\pi(x_i)}_{I}^{N} \end{cases} \quad \left(x_1 \le x_i \le x_{\uparrow i} = x\right) \qquad (2.2.15)$$

2.3 Boundary Application

2.3.1 Binomial distribution double logarithmic domain boundary

Lemma 2.3.1：i ≤x, zero symmetric binomial distribution test, denoted as λ(i), Λ(i); Their values, equal to -0.5 or equal to the probability P of 0.5, are denoted as P=0.5;

$$P = P_N = \frac{\Pi(x)_N}{x} = \frac{1}{2} \tag{2.3.1}$$

$$P_G = \frac{\Pi(x)_G}{x} = \mathrm{C}P_N = \frac{\mathrm{C}}{2} \tag{2.3.2}$$

The actual value, generalized value, narrow value, and generalized narrow value transformation coefficient of the occurrence times of the target event are counted $\sum_{i=1}^{x}\lambda(i)$, $\sum_{i=1}^{x}\Lambda(i)_G$, $\sum_{i=1}^{x}\Lambda(i)_N$, **C;**

The actual value , generalized value , the narrow value of their mean square error $\sigma\left(\sum_{i=1}^{x}\lambda(i)\right)$, $\sigma\left(\sum_{i=1}^{x}\Lambda(i)\right)_G$, $\sigma\left(\sum_{i=1}^{x}\Lambda(i)\right)_N$;

Their actual boundary, generalized boundary, narrow boundary $\uparrow\downarrow\sum_{i=1}^{x}\lambda(i)$, $\uparrow\downarrow\sum_{i=1}^{x}\Lambda(i)_G$, $\uparrow\downarrow\sum_{i=1}^{x}\Lambda(i)_N$ **, then**

$$\sigma\left(\sum_{i=1}^{x}\Lambda(i)_G\right)=\sqrt{\Pi(x)_G\left[1-\frac{\Pi(x)_G}{x}\right]}=\frac{c}{2}\sqrt{x} \quad (2.3.3)$$

$$\sigma\left(\sum_{i=1}^{x}\Lambda(i)_N\right)=\sqrt{\Pi(x)_N\left[1-\frac{\Pi(x)_N}{x}\right]}=\frac{\sqrt{x}}{2} \quad (2.3.4)$$

$$\uparrow\downarrow\sum_{i=1}^{x}\Lambda(i)_G={}^{1}\uparrow\downarrow\sigma\left(\sum_{i=1}^{x}\Lambda(i)_G\right)\sqrt{2\ln\ln x}={}^{1}\uparrow\downarrow c\sqrt{\frac{x\ln\ln x}{2}} \quad (2.3.5)$$

$$\uparrow\downarrow\sum_{i=1}^{x}\Lambda(i)_N={}^{1}\uparrow\downarrow\sigma\left(\sum_{i=1}^{x}\Lambda(i)_N\right)\sqrt{2\ln\ln x}={}^{1}\uparrow\downarrow\sqrt{\frac{x\ln\ln x}{2}} \quad (2.3.6)$$

$$\sum_{i=1}^{x}\lambda(i)_G\pm\uparrow\downarrow\sum_{i=1}^{x}\lambda(i)_G={}^{1}\uparrow\downarrow\sum_{i=1}^{x}\Lambda(i)_G={}^{1}\uparrow\downarrow C\sqrt{\frac{x\ln\ln x}{2}} \quad (2.3.7)$$

$$\sum_{i=1}^{x}\Lambda(i)_N\pm\uparrow\downarrow\sum_{i=1}^{x}\Lambda(i)_N={}^{1}\uparrow\downarrow\sum_{i=1}^{x}\Lambda(i)_N={}^{1}\uparrow\downarrow\sqrt{\frac{x\ln\ln x}{2}} \quad (2.3.8)$$

Proof： When the probability P=0.5, the Bernoulli test of this inference condition is equivalent to the zero-point symmetric binomial distribution, from formula (2.3.1) ,formula (2.3.2), get

$$\sigma(x)_G=\sqrt{\Pi(x)_G\left[1-\frac{\Pi(x)_G}{x}\right]}=\sqrt{xP_G\left(1-\frac{xP_G}{x}\right)}=\sqrt{\frac{C}{2}x\left(1-\frac{C}{2}\right)}=\frac{c}{2}\sqrt{x}$$

$$\sigma(x)_N=\sqrt{\Pi(x)_N\left[1-\frac{\Pi(x)_N}{x}\right]}=\sqrt{xP_N\left(1-\frac{xP_N}{x}\right)}=\sqrt{\frac{x}{2}\left(1-\frac{1}{2}\right)}=\frac{\sqrt{x}}{2}$$

By referring to formula (2.2.6) of boundary mean square error positive correlation reasoning, formula (2.3.5) of generalized

boundary, formula (2.3.6) of narrow boundary and formula (2.3.7) , formula (2.3.8) of binomial statistical value expression are obtained, and the proof is completed

2.3.2 Weighted set boundary

Definition2.3.2: The result of processing the data in a set using a specified calculation method (such as mean, weighted average, mean square error) is called the set value

1. There are two types of set values

1.1. 1 function set value;

1.2. Function set value = 1 function set value * function;

2. Calculation weighted set boundary

Theorem 2.3.3: Count the set values of the target quantity, and use the generalized value, narrow value, generalized boundary, narrow boundary, or the product of both as weights to weight the average of the set values of the target quantity; Moreover, the set boundary is composed of the set error boundary coefficient multiplied by the boundary. Therefore, the set error boundary coefficient is weighted and averaged using reasonable methods such as boundary,

boundary square, and boundary square root. Referring to formula (2.1.8)， formula (2.2.13), that is:

$$\uparrow\downarrow \overline{\mathrm{C_{IG}}\left(x_i\right)} =^1 \uparrow\downarrow \sqrt{2\sum_{i=1}^{\uparrow i} \frac{\Delta\pi\left(x_i\right)_G^2 \ln\ln x_i}{\uparrow\downarrow \Pi\left(x_i\right)_G^2} \uparrow\downarrow \Pi\left(x_i\right)_G \div \sum_{i=1}^{\uparrow i} \uparrow\downarrow \Pi\left(x_i\right)_G}$$

$$=^1 \uparrow\downarrow \sqrt{2\sum_{i=1}^{\uparrow i} \frac{\Delta\pi\left(x_i\right)_N^2 \ln\ln x_i}{\uparrow\downarrow \Pi\left(x_i\right)_N^2} \uparrow\downarrow \Pi\left(x_i\right)_G \div \sum_{i=1}^{\uparrow i} \uparrow\downarrow \Pi\left(x_i\right)_G}$$

$$\left[x_1 \le x_i \le x_{\uparrow i} = x\right] \qquad (2.3.9)$$

$$\uparrow\downarrow \overline{\mathrm{C_{IG}^{AG}}\left(x_i\right)} = \sum_{i=1}^{\uparrow i}\left[\left|\frac{\Delta\pi\left(x_i\right)_G}{\uparrow\downarrow \Pi\left(x_i\right)_G}\right| \sqrt{\uparrow\downarrow \Pi\left(x_i\right)_G} \div \sum_{i=1}^{\uparrow i}\sqrt{\uparrow\downarrow \Pi\left(x_i\right)_G}\right]$$

$$= \sum_{i=1}^{\uparrow i}\left[\left|\frac{\Delta\pi\left(x_i\right)_G}{\uparrow\downarrow \Pi\left(x_i\right)_G}\right| \sqrt{\uparrow\downarrow \Pi\left(x_i\right)_N} \div \sum_{i=1}^{\uparrow i}\sqrt{\uparrow\downarrow \Pi\left(x_i\right)_N}\right]$$

$$\left[x_1 \le x_i \le x_{\uparrow i} = x\right] \qquad (2.3.10)$$

$$\begin{cases} ^1\uparrow\downarrow \pi\left(x\right) \\ \uparrow\downarrow \pi\left(x_i\right) \end{cases} = \begin{cases} ^1\uparrow\downarrow \overline{\mathrm{C_{IG}}\left(x_i\right)} \\ ^1\uparrow\downarrow \overline{\mathrm{C_{IG}^{AG}}\left(x_i\right)} \end{cases} \uparrow\downarrow \sqrt{2\Pi\left(x\right)_{\mathrm{G}}\left[1 - \frac{\Pi\left(x\right)_{\mathrm{G}}}{x}\right]\ln\ln x} \qquad (2.3.11)$$

Instructions ： Due to the proportional relationship between statistical errors, generalized boundaries, and narrow boundaries; Therefore, using generalized boundaries and narrow boundaries as weights, weighted average —— boundary weighted set value is more reasonable

2.3.3 Boundary of Boundary

Definition 2.3.4: The so-called boundary generally refers to the statistical value - the actual value, and the most likely value around it - the maximum deviation of the generalized value - the generalized boundary; Since the boundary is the most likely value - the generalized boundary, then the boundary of the boundary, which is the most likely value of the maximum deviation of the actual boundary around the generalized boundary - can be called a 2-level boundary, and so on, n-level boundaries can be defined; Not only are their definitions similar, but their relationship is also similar

$$\pi(x)=\sum_{i=1}^{x}b(i)=\sum_{i=1}^{x}B(i)\pm\uparrow\downarrow\sum_{i=1}^{x}B(i)=\Pi(x)_G\pm^1\uparrow\downarrow\Pi(x)_G$$

$$\left[\uparrow\downarrow\Pi(x)_G=\uparrow\downarrow\sqrt{2\Pi(x)_G\ln\ln x}\right] \tag{2.3.11}$$

$$\uparrow\downarrow\pi(x)=\uparrow\downarrow\Pi(x)_G\pm^1\uparrow\downarrow^2\Pi(x)_G$$

$$\left[\uparrow\downarrow^2\Pi(x)_G=\uparrow\downarrow\sqrt{2\uparrow\downarrow\Pi(x)_G\ln\ln x}=\uparrow\downarrow\sqrt{2\ln\ln x\sqrt{2\ln\ln x\Pi(x)_G}}\right] \tag{2.3.12}$$

$$\uparrow\downarrow^2\pi(x)=\uparrow\downarrow^2\Pi(x)_G\pm^1\uparrow\downarrow^3\Pi(x)_G$$

$$\left[\uparrow\downarrow^3\Pi(x)_G=\uparrow\downarrow\sqrt{2\ln\ln x\sqrt{2\ln\ln x\sqrt{2\ln\ln x\Pi(x)_G}}}\right] \tag{2.3.13}$$

$$\uparrow\downarrow^n\pi(x)=\uparrow\downarrow^n\Pi(x)_G\pm^1\uparrow\downarrow^{n+1}\Pi(x)_G$$

$$\left[\uparrow\downarrow^{n+1}\Pi(x)_G=\uparrow\downarrow\left[(2\ln\ln x)\wedge\left(\frac{1}{2}+\frac{1}{2^2}+....+\frac{1}{2^n}\right)\right]\Pi(x)_G^{1/2\wedge n}\right]$$

$$\left[\uparrow\downarrow^{n+1}\Pi\left(x\right)_G=\uparrow\downarrow\left[\left(2\ln\ln x\right)\wedge\left(1-\frac{1}{2^{n}}\right)\right]\Pi\left(x\right)_{G}^{1/2\wedge n}\right] \qquad (2.3.13)$$

$$\pi\left(x\right)=\Pi\left(x\right)_G\pm\uparrow\downarrow\Pi\left(x\right)_G\pm\uparrow\downarrow^{2}\Pi\left(x\right)_G\pm\cdots\cdots\pm\uparrow\downarrow^{n}\Pi\left(x\right)_G \qquad (2.3.14)$$

Obviously, both level 2 and multi-level boundaries are much smaller than level 1 boundaries and can be ignored. However, this definition has given us a quantitative understanding of the maximum fluctuation of level 1 boundaries

Part 2 Generalized probability sieve theory

Chapter 3 K-tuple Prime Numbers, sieve method

3.1 1-tuple Prime Numbers —— prime number theorem, sieve method

3.1.1 3 types of sieves

1 Eratosthenes sieve

About 250 BC, by the Greek mathematician Eratothenes (born 275 BC) proposed that in the natural numbers within n, using the natural numbers not greater than the root of n to try to divide n, you can determine the number of composite numbers in n, you can determine the number of prime numbers in n

2 Euler sieve

On the basis of the sieve of Eratosthenes, in the natural numbers within n, the number of composite numbers in n can be determined by dividing n by prime numbers not greater than the root of n

The above two kinds of sieve, can be manually or programmed statistics, can calculate the actual value of the prime number within the specified range, can not calculate the theoretical value

3 Xiong's sieve

Definition 3.1.1: With the help of probability, prime number screening, Mertens formula, prime number theorem, sets and other tools, the composite number in the target event can be filtered out to determine the number of prime numbers and other analytical solutions within a specified range 【Such as x】. The resulting sieve is called Xiong's sieve, abbreviated as sieve method;

Xiong's sieve was the first to provide analytical solutions and could not count the actual value，abbreviated as sieving method

3.1.2 Element prime number sieve

Definition 3.1.2：In the set E∈e(x,t) composed of natural numbers t ≤x, the subset element e(x,t) is composed of an integer variable t, which is called a single element; A set E composed of single elements

is called a unit set; The prime number theorem obtained by sieve the composite number in the unit set E by the prime number is called the single elements prime number sieve prime number theorem

The set E∈e(x,t) composed of natural numbers t≤x, which are all composite numbers greater than the prime p(i) factor, is called the set without p(i), and the subset e(x,t) number and the set formed by it are denoted as Without(x,p(i))

In the set E∈e(x,t) composed of natural numbers t≤x, the set consisting of composite numbers with prime p(i) factors is called the set containing p(i), and the subset e(x,t) numbers and the set composed of them are denoted as Contain(x,p(i))

Refers to the sieve prime number theorem, Mertens formula, and coefficients of generalized and narrow transformations, known as Mertens generalized narrow coefficients $M_N^G(1)$

3.1.3 Sieve Prime Number Theorem, Mertens Generalized Narrow Coefficient $M_N^G(1)$

Obviously, the generalized value of the screening method=the generalized value of probability ≈ the actual value;

The narrow value of sieving method may not necessarily be equal to the narrow value of probability method;

Generalized and narrow coefficients can be divided into two types: sieve method generalized and narrow coefficients, and probability method generalized and narrow coefficients;

To simplify expression, the difference between the sieve method value and the probability method value is generally not made in the expression symbol when clearly labeled or not easily misunderstood

Theorem 3.1.3: the 1-tuple prime numbers —— Probability method of prime number theorem The generalized narrow coefficient $C_N^G(1)_{\mathrm{P}}$ of probability method approaches 1; It can be proven that the generalized narrow coefficient $C_N^G(K)_{\mathrm{S}}$ of the K-tple prime number sieving method approaches 1, then:

$$C_N^G(1)_{\mathrm{P}} =^1 \mathrm{C}_N^{\mathrm{G}}(K)_{\mathrm{S}} =^1 1 \qquad (3.1.0)$$

Proof: K-tuple prime numbers are probability events of the same kind. According to the probability axiom, the narrow value $\Pi(x)_N$ of the probability method can be obtained from formula (5.1.2) and formula (5.1.21) [or $\Pi(x)$ in Probability Prime Theory]. The general value $\Pi(x)_{\mathrm{G}}$ of the probability law prime theorem in formula (3.1.3) can be obtained, and the general narrow coefficient $C_N^G(1)_{\mathrm{P}}$ of the probability method prime theorem in formula (3.1.0) can be obtained.

The expression in front of this theorem is obtained

Similarly, using the screening method, the generalized narrow coefficient properties of K-prime numbers are the same. Based on the actual data of several K-prime numbers that have been obtained, it has been proven that the generalized narrow coefficient $C_N^G(K)_S$ **of these K-tuple prime numbers approaches 1. Therefore, the generalized narrow coefficient** $C_N^G(K)_S$ **of all of the K-tuple prime number screening method all approaches 1. The expression behind this theorem is obtained, and the proof is completed**

Theorem 3.1.4 sieving method, within a limited range, a sufficiently large natural number x, the actual value , the generalized value, the narrow value, the integer narrow double logarithmic boundary, the Mertens generalized narrow coefficient, the Mertens coefficient, the generalized narrow coefficient, the densest boundary, the boundary coefficient of the prime number theorem ： $\pi(x)$**,** $\Pi(x)_G$**,** $\Pi(x_i)_N$**,** $\uparrow\downarrow\Pi(x)_{IN}^{I}$**,** $M_N^G(1)$**,** $M(1)$**,** $C_N^G(1)_S$**,** $\uparrow\downarrow\Pi(x)_G$**,** $\uparrow\downarrow C_1$ **, then**

$$\Pi(x)_N =^1 x\prod_{p\le\sqrt{x}}\frac{p-1}{p} =^1 \frac{2li(x)}{e^{\gamma}} \tag{3.1.1}$$

$$M_N^G(1)=M(1)C_N^G(1)_S=\frac{\Pi(x)_G}{\Pi(x)_N} =^1 M(1) =^1 \frac{e^{\gamma}}{2} \quad \left[\begin{cases} C_N^G(1)_P \\ C_N^G(1)_S \end{cases} =^1 1\right] \tag{3.1.2}$$

$$\pi(x)=\Pi(x)_G \pm \uparrow\downarrow\Pi(x)_G = li(x)-\frac{{}^1\sqrt{x}}{2\ln x} \pm^1 \uparrow\downarrow C_1\sqrt{2\Pi(x)_G \ln\ln x} \tag{3.1.3}$$

$$\uparrow\downarrow \Pi\left(x_i\right)_{IN}^{I} = \frac{{}^{1}1}{2}\uparrow\downarrow\sqrt{\frac{\sqrt{x}\ln\ln\sqrt{x}}{2\ln\sqrt{x}}} \qquad (3.1.4)$$

Euler's constant: γ≈0.5772156649015328606065120

Proof: From the generalized boundary $\uparrow\downarrow\pi(x)_{G}$ of formula (2.2.13), obtain the generalized boundary $\uparrow\downarrow\pi(x)_{G}$ of formula (3.1.3);

The i-th prime number p(i) is used as the narrow prime number sieve, and the composite number in x is sieved. The maximum prime of the sieve $p(\uparrow i)=\sqrt{x}$, since there is no p (0),then

$$2 \le p(i) \le p(\uparrow i) = \sqrt{x} \qquad (3.1.5)$$

$$\text{Without}\left(x, p(0)\right) = x \qquad (3.1.6)$$

The p (i) factor is within $(2,\sqrt{x})$, and the total number of particles should be screened out (x, p (i)); Due to the disorder and equal probability of the narrow prime number sieve, the distribution of rounding random functions (generated by the remainder) resulting from the division and rounding errors is almost equivalent to the Bernoulli distribution random function ± b (i)=0, b, 2b,,,, [p (i) -1] b, with a total of p (i) values [obviously: $\mathrm{b}=\frac{1}{p(i)}$]; And, the average value $\overline{\pm b(i)}$ of ± b (i), then

$$\overline{\pm b(i)} =^{1} \frac{1-P(i)_N}{2} =^{1} \frac{p(i)-1}{2p(i)}$$

On the basis of the mean value $\overline{\pm b(i)}$, the probability of occurrence of -0.5 and 0.5, which are both 0.5, and the combination

of zero symmetric binomial distribution ±λ(i): $\overline{\pm b(i)} \pm \Lambda(i)\frac{p(i)-1}{2p(i)}$ **,**

instead of taking the mean value $\overline{\pm b(i)}$ **, the fluctuation value and the boundary ↑↓b(i) of integer random function ±b(i), get**

$$\pm b(i) = \overline{\pm b(i)} \pm \uparrow\downarrow b(i) = \overline{\pm b(i)} \pm \Lambda(i)\frac{p(i)-1}{2p(i)} = \frac{p(i)-1}{2p(i)} \pm^{1} \frac{\Lambda(i)}{2}\frac{p(i)-1}{2p(i)}$$

$$=^{1} \overline{\pm b(i)} =^{1} \frac{1-P(i)_N}{2} =^{1} \frac{p(i)-1}{2p(i)} \quad [\text{由}(3.1.8)] \qquad (3.1.7)$$

Within x, there are no integer numbers less than p(i) factor Without (x, p(i)), and there are integer numbers containing p(i) factor Containing (x, p (i)); For every p (i) integer, there is one that is divided by the p(i) factor and filtered out. Therefore, the exclusion narrow probability $P(i)_N$ **of the composite number is obtained**

$$P(1)_N = \frac{1}{p(1)} = \frac{1}{2} \qquad (3.1.8)$$

$$\text{Without}(x, p(1)) = \text{Without}(x, p(0)) - \text{Contain}(x, p(1))$$

$$= \text{Without}(x, p(0)) - \left[\text{Without}(x, p(0))P(1)_N\right] \quad [\text{By}(3.1.8)]$$

$$= \text{Without}(x, p(0)) - \left(\text{Without}(x, p(0))\frac{1}{p(1)} - \pm b(1)\right) \quad [\text{By}(3.1.7)]$$

$$=^{1} \text{Without}(x, p(0))\frac{p(1)-1}{p(1)} +^{1} \frac{p(1)-1}{2p(1)} =^{1} \frac{x}{2} \quad [\text{By}(3.1.6)] \qquad (3.1.9)$$

The narrow probability $P(t, p(i))$ **,** $P(i)_N$ **of integer t being divisible and sieved by prime p (i), because for every consecutive p (i) integers, only one is divisible by p (i), then**

$$P\left(t,p(i)\right)=\frac{1}{p(i)} \quad \begin{bmatrix} 3\le t\le x \\ 3\le p(i)\le\sqrt{x} \end{bmatrix} \tag{3.1.10}$$

$$P(i)_N=P\left(t,p(i)\right)=\frac{1}{p(i)} \quad \begin{bmatrix} 3\le t\le x \\ 3\le p(i)\le\sqrt{x} \end{bmatrix} \tag{3.1.11}$$

$$\text{Without}\left(x,p(i)\right)=\text{Without}\left(x,p(i-1)\right)-\text{Contain}\left(x,p(i-1)\right)$$

$$=\text{Without}\left(x,p(i-1)\right)-\left[\text{Without}\left(x,p(i-1)\right)P(i)_N\right] \quad \left[\text{By}(3.1.11)\right]$$

$$=\text{Without}\left(x,p(i-1)\right)-\left(\text{Without}\left(x,p(i-1)\right)\frac{1}{p(i)}-\pm b(i)\right) \quad \left[\text{By}(3.1.7)\right]$$

$$=\text{Without}\left(x,p(i-1)\right)\frac{p(i)-1}{p(i)}+\frac{p(i)-1}{2p(i)}\pm^{1}\frac{\Lambda(i)}{2} \quad \begin{bmatrix}\text{Iteration}\\ \text{迭代}\end{bmatrix}$$

$$=\left[\text{Without}\left(x,p(i-2)\right)\frac{p(i-1)-1}{p(i-1)}+\frac{p(i-1)-1}{2p(i-1)}\pm^{1}\frac{\Lambda(i-1)}{2}\right]\frac{p(i)-1}{p(i)}$$

$$+\frac{p(i)-1}{2p(i)}\pm^{1}\frac{\Lambda(i)}{2} \quad \begin{bmatrix}\text{Iteration}\\ \text{迭代}\end{bmatrix}$$

$$=\text{Without}\left(x,p(i-2)\right)\prod_{p(i-1)\le p}^{p\le p(i)}\frac{p-1}{p}+\sum_{j=i-1}^{i}\prod_{p(j+1)\le p}^{p\le p(i)}\frac{p(j)-1}{2p(j)}\frac{p-1}{p}\pm^{1}\sum_{j=i-1}^{i}\frac{\Lambda(j)}{2} \tag{3.1.12}$$

$$=\text{Without}\left(x,p(i-3)\right)\prod_{p(i-2)\le p}^{p\le p(i)}\frac{p-1}{p}+\sum_{j=i-2}^{i}\prod_{p(j+1)\le p}^{p\le p(i)}\frac{p(j)-1}{2p(j)}\frac{p-1}{p}\pm^{1}\sum_{j=i-2}^{i}\frac{\Lambda(j)}{2}$$

……………………………………………………………

$$=\text{Without}\left(x,p(0)\right)\prod_{p(1)\le p}^{p\le p(i)}\frac{p-1}{p}+\sum_{j=1}^{i}\prod_{p(j+1)\le p}^{p\le p(i)}\frac{p(j)-1}{2p(j)}\frac{p-1}{p}\pm^{1}\sum_{j=1}^{\pi(p(i))}\frac{\Lambda(j)}{2}$$

【Binomial boundary (2.3.8)，By sum (3.1.21)】

$$=^{1}\text{Without}\left(x,p(0)\right)\prod_{p\le p(i)}\frac{p-1}{p}+\frac{{}^{1}0.5p(i)}{\ln p(i)}\pm\frac{{}^{1}\uparrow\downarrow}{2}\sqrt{\frac{\pi\left(p(i)\right)\ln\ln\pi\left(p(i)\right)}{2}} \tag{3.1.13}$$

When the prime number p(i) traverses a prime number that is not greater than the square root of x: $p(i)\le\sqrt{x}$ **, the narrow boundary** $\uparrow\downarrow\Pi(x)_{IN}^{I}$ **of the integer is obtained by calculating the sieve method**

from the above equation

$$\uparrow\downarrow\Pi(x)_{IN}^{I}=\frac{^{1}1}{2}\uparrow\downarrow\sqrt{\frac{\pi(p(i))\ln\ln\pi(p(i))}{2}}^{p(i)=\sqrt{x}}=\frac{^{1}1}{2}\uparrow\downarrow\sqrt{\frac{\sqrt{x}\ln\ln\sqrt{x}}{2\ln\sqrt{x}}}$$

Obtain formula (3.1.4);

Obviously, taking the integer boundary $\uparrow\downarrow\Pi(x)_{IN}^{I}$ is a high-order small quantity of the generalized boundary $\uparrow\downarrow\Pi(x)_{G}$ in formula (3.1.3); Similarly, the rounding boundary for all prime number problems is a high-order small quantity of the generalized boundary $\uparrow\downarrow\Pi(x)_{G}$, and rounding fluctuations and boundary calculations will be ignored in the future.

•

When the prime number p(i) traverses a prime number that is not greater than the square root of x: $p(i)\leq\sqrt{x}$, using formula (3.1.6) for Without (x, p(0)) and formula (3.1.13) for Without (x, p(i)) (ignoring boundaries, etc.), among the natural numbers within (1, x), the prime number theorem is obtained by sieving out composite numbers with first-order properties. The narrow value $\Pi(x)_{N}1$ of the first-order property sieving method is as follows

$$\Pi(x)_{N}1=\text{Without}\left(x,p(i)\approx\sqrt{x}\right)\quad[\text{由}(3.1.13)]$$

$$=^{1}\text{Without}\left(x,p(0)\right)\prod_{p\leq\sqrt{x}}\frac{p-1}{p}=^{1}\Pi(x)_{N}\qquad(3.1.14)$$

Obtain the first partial result of formula (3.1.1);

In order to improve the accuracy of the above formula, modify the range of sieving out twin composite numbers ,change from (1, x)

——Without(x,p(0)) of formula (3.1.6) to micro interval (t,t+Δt)——ΔWithout(t,p(0));

$$\Delta\text{Without}\left(t,p(0)\right)=\text{Without}\left(t+\Delta t,p(0)\right)-\text{Without}\left(t,p(0)\right)$$

$$=^{1} t+\Delta t-t=^{1}\Delta t \qquad \left[由(3.1.6)\right] \qquad (3.1.15)$$

Statistics the one-time sieve method twin prime number micro narrow value $\Delta\Pi(t)_{SN}1$ **in the micro interval Δt , and, in 2≤t≤x , sum this sieve method twin prime number micro narrow value** $\Delta\Pi(t)_{SN}1$ **of each micro interval, a more accurate the sieve method twin prime number narrow value** $\Pi(x)_{SN}$ **is obtained;**

Therefore, modify the narrow definition of the number of prime numbers in the first-order sieving method of formula (3.1.14) from $\Pi(x)_{N}1$ **to** $\Delta\Pi(t)_{N}1$**, and refer to formula (3.1.18) using the Mertens formula, get**

$$\Delta\Pi(t)_{N}1=^{1}\Delta\text{Without}\left(t,p(0)\right)\prod_{p\le\sqrt{t}}\frac{p-1}{p}=^{1}\frac{2e^{-\gamma}\Delta t}{\ln t} \qquad \left[由(3.1.15)\right] \qquad (3.1.16)$$

$$\Pi(x)_{N}=\sum_{t=2}^{x}\Delta\Pi(t)_{N}1=^{1}\sum_{t=2}^{x}\frac{2e^{-\gamma}\Delta t}{\ln t}=^{1}\int_{2}^{x}\frac{2e^{-\gamma}dt}{\ln t}=^{1}\frac{2li(x)}{e^{\gamma}}$$

The latter formula (3.1.1) of $\Pi(x)_{N}$ **is obtained;**

From the above formula $\Pi(x)_{N}$ **, from formula (5.1.21)** $\Pi(x)_{G}$ **, from the sieve method Meitens generalized narrow coefficient, from the Meitens coefficient of formula (3.1.17), from the sieve method generalized narrow coefficient** $M_{N}^{G}(1)$ **,** $M(1)$ **,** $C_{N}^{G}(1)_{S}$**, get**

$$M_N^G(1)=M(1)C_N^G(1)_S=\frac{\Pi(x)_G}{\Pi(x)_N}=^1\left[li(x)-\frac{\sqrt{x}}{2\ln x}\right]\div\frac{2li(x)}{e^\gamma}=^1\frac{e^\gamma}{2}=^1 M(1)$$

$$C_N^G(1)_S=^1 1$$

Obtain the $M_N^G(1)$, $M(1)$, $C_N^G(1)_S$ of formula (3.1.2);

From the generalized boundaries of formula (2.2.13) and formula(2.2.14), obtain the generalized boundary $\uparrow\downarrow\Pi(x)_G$ of the prime number theorem for formula (3.1.3), as well as the sieving value π (x) for the prime number theorem;

Referring to Section 1 of Chapter 5, with the help of formula (2.2.13), formula (2.2.14), and the generalized boundary formula, as well as the boundary coefficient formula, and using the data in Table 5.1.2, the boundary can be calculated, and the boundary coefficient $\uparrow\downarrow C_1$ can be calculated. The calculation process of $\uparrow\downarrow\overline{C(x_i)}_G$ in formula (5.1.25) is omitted, and the generalized boundary $\uparrow\downarrow\Pi(x)_G$ of prime number theorem and the sieving value $\pi(x)$ of prime number theorem in formula (3.1.3) are obtained. The process is omitted, and the proof is completed

3.1.4 Mertens formula

In 1874, Mertens proved the following theorem concerning prime numbers

Lemma 3.1.5 Mertens formula: The prime p in x satisfies

$$\prod_{p\le x}\frac{p-1}{p}=\left[1+\mathrm{O}\left(\frac{1}{\ln x}\right)\right]\frac{e^{-\gamma}}{\ln x}=\frac{{}^{1}0.5}{M(1)\ln x}\quad\left[M(1)={}^{1}\frac{e^{\gamma}}{2}\right]\tag{3.1.17}$$

$$\prod_{p\le\sqrt{x}}\frac{p-1}{p}=\left[1+\mathrm{O}\left(\frac{1}{\ln x}\right)\right]\frac{2e^{-\gamma}}{\ln x}=\frac{{}^{1}1}{M(1)\ln x}\quad\left[M(1)={}^{1}\frac{e^{\gamma}}{2}\right]\tag{3.1.18}$$

Euler's constant γ≈0.577215664901532860606512O;

3.1.5 Prime number theorem lemma

Lemma 3.1.6

$$\int_{{}^{1}2}^{p(i)}\frac{\sqrt[-2]{t}\,dt}{\ln t}={}^{1}\frac{2\sqrt{p(i)}}{\ln p(i)}\tag{3.1.19}$$

$$\sum_{j=1}^{i}\prod_{p(j+1)\le p}^{p(i)}\frac{p(j)-1}{2p(j)}\frac{p-1}{p}=\frac{{}^{1}0.5p(i)}{\ln p(i)}\tag{3.1.20}$$

prove:

$$\int_{{}^{1}2}^{p(i)}\frac{\sqrt[-2]{t}\,dt}{\ln t}=\int_{{}^{1}2}^{p(i)}\frac{2d\sqrt{t}}{\ln t}=\frac{2\sqrt{t}}{\ln t}\Bigg|_{t={}^{1}2}^{t=p(i)}+\int_{{}^{1}2}^{p(i)}\frac{2\sqrt[-2]{t}\,dt}{\ln^{2}t}={}^{1}\frac{2\sqrt{p(i)}}{\ln p(i)}$$

Obtain formula (3.1.19);

$$\sum_{j=1}^{i}\prod_{p(j+1)\le p}^{p(i)}\frac{p(j)-1}{2p(j)}\frac{p-1}{p}=\sum_{j=1}^{i}\frac{p(j)-1}{2p(j)}\left(\prod_{p\le p(i)}\frac{p-1}{p}\div\prod_{p\le p(j)}\frac{p-1}{p}\right)$$

$$=\prod_{p\le p(i)}\frac{p-1}{p}\sum_{j=1}^{i}\frac{p(j)-1}{2p(j)}\div\prod_{p\le p(j)}\frac{p-1}{p}\quad\left[\begin{array}{c}\text{By Mertens' formula}(3.1.17)\\ \text{引用梅腾斯公式}(3.1.17)\end{array}\right]$$

$$=\frac{{}^{1}e^{-\gamma}}{\ln p(i)}\sum_{j=1}^{i}\frac{p(j)-1}{2p(j)}\div\left[\frac{{}^{1}e^{-\gamma}}{\ln p(j)}\right]={}^{1}\sum_{j=1}^{i}\frac{\ln p(j)}{2\ln p(i)}={}^{1}\int_{2}^{p(i)}\frac{\ln tP(t)}{2\ln p(i)}dt$$

$$=\int_{2}^{p(i)}\frac{{}^{1}\ln t}{2\ln p(i)}\frac{dt}{\ln t}=\frac{{}^{1}0.5p(i)}{\ln p(i)}$$

Obtain formula (3.1.20) to prove completion

Lemma 3.1.7:

$$\sum_{j=1}^{i}\prod_{p(j+1)\le p}^{p(i)}\frac{p(j)-1}{2p(j)}\frac{p-1}{p}=\frac{{}^{1}0.5p(i)}{\ln p(i)} \qquad (3.1.21)$$

prove:

$$\sum_{j=1}^{i}\prod_{p(j+1)\le p}^{p(i)}\frac{p(j)-1}{2p(j)}\frac{p-1}{p}=\sum_{j=1}^{i}\frac{p(j)-1}{2p(j)}\left(\prod_{p\le p(i)}\frac{p-1}{p}\div\prod_{p\le p(j)}\frac{p-1}{p}\right)$$

$$=\prod_{p\le p(i)}\frac{p-1}{p}\sum_{j=1}^{i}\frac{p(j)-1}{2p(j)}\div\prod_{p\le p(j)}\frac{p-1}{p}\quad\left[\begin{array}{c}\text{By Mertens' formula}(3.1.17)\\ \text{引用梅腾斯公式}(3.1.17)\end{array}\right]$$

$$=\frac{{}^{1}e^{-\gamma}}{\ln p(i)}\sum_{j=1}^{i}\frac{p(j)-1}{2p(j)}\div\left[\frac{{}^{1}e^{-\gamma}}{\ln p(j)}\right]={}^{1}\sum_{j=1}^{i}\frac{\ln p(j)}{2\ln p(i)}={}^{1}\int_{2}^{p(i)}\frac{\ln tP(t)}{2\ln p(i)}dt$$

$$=\int_{2}^{p(i)}\frac{{}^{1}\ln t}{2\ln p(i)}\frac{dt}{\ln t}=\frac{{}^{1}0.5p(i)}{\ln p(i)}$$

Deden Lemma, Prove Completion

Lemma 3.1.8:

$$\sum_{j=1}^{i}\prod_{p(j+1)\le p}^{p(i)}\frac{p(j)-1}{2p(j)}\frac{p-1}{p}=\frac{{}^{1}0.5p(i)}{\ln p(i)} \qquad (3.1.22)$$

prove:

$$\sum_{j=1}^{i}\prod_{p(j+1)\le p}^{p(i)}\frac{p(j)-1}{2p(j)}\frac{p-1}{p}=\sum_{j=1}^{i}\frac{p(j)-1}{2p(j)}\left(\prod_{p\le p(i)}\frac{p-1}{p}\div\prod_{p\le p(j)}\frac{p-1}{p}\right)$$

$$=\prod_{p\le p(i)}\frac{p-1}{p}\sum_{j=1}^{i}\frac{p(j)-1}{2p(j)}\div\prod_{p\le p(j)}\frac{p-1}{p}\quad\left[\begin{array}{c}\text{by Mertens' formula}(3.1.17)\\ \text{引用梅腾斯公式}(3.1.17)\end{array}\right]$$

$$=\frac{{}^{1}e^{-\gamma}}{\ln p(i)}\sum_{j=1}^{i}\frac{p(j)-1}{2p(j)}\div\left[\frac{{}^{1}e^{-\gamma}}{\ln p(j)}\right]={}^{1}\sum_{j=1}^{i}\frac{\ln p(j)}{2\ln p(i)}={}^{1}\int_{2}^{p(i)}\frac{\ln tP(t)}{2\ln p(i)}dt$$

$$=\int_{2}^{p(i)}\frac{{}^{1}\ln t}{2\ln p(i)}\frac{dt}{\ln t}=\frac{{}^{1}0.5p(i)}{\ln p(i)}$$

Prove Completion

3.2 Densest 2-tuple prime number —— Twin prime numbers, sieve method

3.2.1 Twin prime numbers concept, Prime sieve

Definition 3.2.1: A subset of integers t and t+2 is called a twin subset t(t,t+2); The set formed by t(t,t+2) is called a twin set T(t,t+2) ∈ t(t,t+2);

When the two integers t and t+2 in the twin subset t(t,t+2) are both prime numbers, they are called twin primes, otherwise, they are called twin composite numbers;

Under the condition of narrow probability, the twinned composite numbers in the twinned set T(t,t+2) are sieved by prime numbers, which is called the double element narrow prime number sieve, simply called the prime number sieve .

Minimum densest 2-tuple Prime Number —— Minimum Twin Prime Number (3.5)

3.2.2 Twin prime numbers sieve method analytical solution ,

Mertens generalized narrow coefficient $M_N^G(2)$

Definition 3.2.2: t ≤ x, using prime numbers within the square of t, for the set T (t, t+2) composed of a subset of twin integers (t, t+2), under narrow probability conditions, the subset of twin composite numbers is filtered out to obtain the narrow sense number of twin prime numbers, which is called the narrow sense value of the screened twin prime numbers; By using this narrow value and multiplying it with the sieve method Mertens' generalized narrow coefficient, obtain the twin prime number sieve method generalized value analysis solution;

In 1921, Hardy Litwood proposed the formula (3.2.23) for the analysis solution conjecture of twin prime numbers,proved by the following theorem

Theorem 3.2.3： Sieving method, sufficiently large natural number x, 5 ≤ t ≤ x, the actual value, generalized value, narrow value, Mertens generalized narrow coefficient, Mertens coefficient,

generalized narrow coefficient, correlation coefficient, densest boundary, densest boundary coefficient of number of twin prime numbers ——the twin prime theorem $\pi_2(x)$ **,** $\Pi_2(x)_G$ **,** $\Pi_2(x)_N$ **,** $M_N^G(2)$ **,** $M(2)$ **,** $C_N^G(2)_S$ **,** C_2 **,** $\uparrow\downarrow\Pi_2(x)_G$ **,** $\uparrow\downarrow C_2$ **, then**

$$\Pi_2(x)_N =^1 2C_2 x\prod_{p\le\sqrt{x}}\left(1-\frac{1}{p}\right)^2 =^1 \int_5^x \frac{8C_2 dt}{e^{2\gamma}\ln^2 t} \qquad (3.2.1)$$

$$\begin{cases} M_N^G(2) \\ {}^1 M_N^G(1)^2 \end{cases} = \begin{cases} C_N^G(2)_S M(2) \\ {}^1 C_N^G(1)_S^2 M(1)^2 \end{cases} =^1 \frac{e^{2\gamma}}{2^2} \quad \left[C_N^G(2)_S =^1 C_N^G(1)_S^2 =^1 1\right] \qquad (3.2.2)$$

$$\pi_2(x)=\Pi_2(x)_G \pm\uparrow\downarrow\Pi_2(x)_G =^1 \int_5^x \frac{2C_2 dt}{\ln^2 t} \pm\uparrow\downarrow C_2\sqrt{2\Pi_2(x)_G \ln\ln x} \qquad (3.2.3)$$

$$C_2=\prod_{3\le p}\left[1-\frac{1}{(p-1)^2}\right]\approx 0.66016118158468695739278 12 \qquad (3.2.4)$$

2 C_2 **≈2*0.66016118158468695739278 12**

≈1.32032236316937391478556 24

Conclusion: Both data table 3.2.1 and the formula for sieving method results formula (3.2.2) can prove that $C_N^G(2)_S =^1 C_N^G(1)_S^2 =^1 1$ **and sieving method twin prime numbers are generalized narrow events**

Proof: Within x, the prime number p (i) is selected from the set of twin integers by filtering out the subset of twin composite numbers,

The number Without(x, p (i)) of twin integer subsets without a factor less than p (i),

The number Contain(x, p (i)) of twin composite subsets containing the p (i) factor;The maximum prime number $p(\uparrow i)=\sqrt{x}$ **in this sieve, then**

$$2 \le p(i) \le p(\uparrow i) = \sqrt{x} \quad (x \gg 1) \qquad (3.2.5)$$

The minimum twin prime number is (3, 5), that is, the starting point of the twin integer (x, x+2) is 5. Within (5, x), there is no p (0) prime number sieve, then

$$\text{Without}(x, p(0)) = \text{S2(x)} = x - 5 + 1 =^{1} x \qquad (3.2.6)$$

In the prime number theorem screening method, formula (3.1.7) for obtaining ± b (i) is described in the same way, ignoring its fluctuations and the high-order small rounding boundaries generated by these fluctuations. The first prime number p (1) is sieved out, and the narrow probability $P(1)_N$ of even numbers in natural numbers is obtained,get

$$\pm b(i) =^{1} \overline{\pm b(i)} =^{1} \frac{1 - p_2(i)_N}{2} =^{1} \frac{p(i) - 1}{2p(i)} \quad [\text{By}(3.1.7)] \qquad (3.2.7)$$

$$P(1)_N = \frac{1}{p(1)} = \frac{1}{2} \qquad (3.2.8)$$

$$\text{Without}(x, p(1)) = \text{Without}(x, p(0)) - \text{Contain}(x, p(1))$$

$$= \text{Without}(x, p(0)) - \left[\text{Without}(x, p(0)) P(1)_N\right] \quad [\text{By}(3.2.8)]$$

$$= \text{Without}(x, p(0)) - \left(\text{Without}(x, p(0)) \frac{1}{p(1)} - \pm b(1)\right) \quad [\text{By}(3.2.7)]$$

$$=^{1} \text{Without}(x, p(0)) \frac{p(1) - 1}{p(1)} +^{1} \frac{p(1) - 1}{2p(1)} =^{1} \frac{x}{2} \quad [\text{By}(3.2.6)] \qquad (3.2.9)$$

In the twin subset t(t,t+2), the narrow sieving probability of two integers t,t+2, separately and simultaneously divisible by the prime

p(i) is denoted as $P(t,p(i))_N$ **,** $P(t+2,p(i))_N$ **,** $P_2(i)_N$ **, since for every consecutive p(i) integers, only one is divisible by p(i), then**

$$P(t,p(i))=P(t+2,p(i))=\frac{1}{p(i)}\quad\begin{bmatrix}3\le t\le x-4\\3\le p(i)\le\sqrt{x}\end{bmatrix}\qquad(3.2.10)$$

$$P(i)_N=\begin{cases}P(t,p(i))+\\P(t+2,p(i))\end{cases}=\frac{2}{p(i)}\quad\begin{bmatrix}3\le t\le x-4\\3\le p(i)\le\sqrt{x}\end{bmatrix}\qquad(3.2.11)$$

$$\text{Without}(x,p(i))=\text{Without}(x,p(i-1))-\text{Contain}(x,p(i))$$

$$=\text{Without}(x,p(i-1))-\left[\text{Without}(x,p(i-1))P(i)_N\right]\quad[\text{By}(3.2.11)]$$

$$=\text{Without}(x,p(i-1))-\left(\text{Without}(x,p(i-1))\frac{2}{p(i)}-\pm b(i)\right)\quad[\text{By}(3.2.7)]$$

$$=\text{Without}(x,p(i-1))\frac{p(i)-2}{p(i)}+^1\frac{p(i)-1}{2p(i)}\quad(\text{By above formula})$$

$$=\left[\text{Without}(x,p(i-2))\frac{p(i-1)-2}{p(i-1)}+^1\frac{p(i-1)-1}{2p(i-1)}\right]\frac{p(i)-2}{p(i)}+^1\frac{p(i)-1}{2p(i)}$$

【By above formula，由上式】

$$=\text{Without}(x,p(i-2))\prod_{p(i-1)\le p}^{p(i)}\frac{p-2}{p}+^1\sum_{j=i-1}^{i}\prod_{p(j+1)\le p}^{p(i)}\frac{p(j)-1}{2p(j)}\frac{p-2}{p}\qquad\begin{matrix}(3.2.12)\\ [\text{By}(3.1.12)]\end{matrix}$$

$$=\text{Without}(x,p(i-3))\prod_{p(i-2)\le p}^{p(i)}\frac{p-2}{p}+^1\sum_{j=i-2}^{i}\prod_{p(j+1)\le p}^{p(i)}\frac{p(j)-1}{2p(j)}\frac{p-2}{p}$$

……………………………………………………………

$$=\text{Without}(x,p(1))\prod_{p(2)\le p}^{p(i)}\frac{p-2}{p}+^1\sum_{j=2}^{i}\prod_{p(j+1)\le p}^{p(i)}\frac{p(j)-1}{2p(j)}\frac{p-2}{p}\quad\begin{bmatrix}\text{factorial}(3.2.18)\\\text{sum}(3.2.20)\end{bmatrix}$$

$$=\text{Without}(x,p(1))4C_2\prod_{p\le p(i)}\left(1-\frac{1}{p}\right)^2+^1\frac{0.5p(i)}{\ln^2 p(i)}$$

$$=^1\text{Without}(x,p(1))4C_2\prod_{p\le p(i)}\left(1-\frac{1}{p}\right)^2\qquad(3.2.13)$$

When prime numbers p(i) traverses a prime number that is not

greater than the square root of x: $p(i) \le \sqrt{x}$ **, by the above formula , in natural numbers within (1,x), by one-time sieving out tiwn composite numbers ,the one-time twin primes number narrow value** $\Pi_2(x)_N 1$ **by the sieve method in one go is obtained as follows**

$$\Pi_2(x)_N 1 = \text{Without}\left(x, p(i) \approx \sqrt{x}\right) \quad \left[\text{By}\left(3.2.13\right)\right]$$

$$=^1 \text{Without}\left(x, p(1)\right) 4C_2 \prod_{p \le^1 \sqrt{x}} \left(1 - \frac{1}{p}\right)^2 =^1 2C_2 x \prod_{p \le^1 \sqrt{x}} \left(1 - \frac{1}{p}\right)^2 =^1 \Pi_2(x)_N \quad (3.2.14)$$

Obtain the results of the first part of formula (3.2.1) ;

In order to improve the accuracy of the above formula, modify the range of sieving out twin composite numbers ,change from (5, x) ——Without(x,p(1)) of formula (3.2.6) to micro interval (t,t+Δt)——Δ Without(t,p(1)), by formula (3.2.9), obtain

$$\Delta \text{Without}\left(t, p(1)\right) = \text{Without}\left(t + \Delta t, p(1)\right) - \text{Without}\left(t, p(1)\right)$$

$$=^1 \frac{t + \Delta t}{2} -^1 \frac{t}{2} =^1 \frac{\Delta t}{2} \quad \left(5 \le t \le x\right) \qquad (3.2.15)$$

statistics the one-time sieve method twin prime number micro narrow value $\Delta\Pi_2(t)_N 1$ **in the micro interval Δt , and, in 5≤t≤x , sum this sieve method twin prime number micro narrow value** $\Delta\Pi_2(t)_N 1$ **of each micro interval, a more accurate the sieve method twin prime number narrow value** $\Pi_2(x)_N$ **is obtained;**

The narrow value of the number of twin prime numbers in formula (3.2.14) is changed from $\Pi_2(x)_N 1$ **to** $\Delta\Pi_2(t)_N 1$ **using the first-order sieve method. By the formula (3.1.18) of the Mertens**

formula and the above formula, get

$$\Delta\Pi_2(t)_N 1 =^1 \Delta\text{Without}(t, p(1))4C_2 \prod_{p(2)\le p}^{\sqrt{t}} \left(1-\frac{1}{p}\right)^2$$

$$=^1 \frac{\Delta t}{2} 4C_2 \left(\frac{2e^{-\gamma}}{\ln t}\right)^2 =^1 \frac{8C_2\Delta t}{e^{2\gamma}\ln^2 t} \quad \begin{bmatrix} 5 \le t \le x \\ \text{By}(3.2.15) \end{bmatrix} \qquad (3.2.16)$$

$$\Pi_2(x)_N =^1 \sum_{t=5}^{x} \Delta\Pi_2(t)_N 1 =^1 \sum_{y=5}^{x} \frac{8C_2\Delta t}{e^{2\gamma}\ln^2 t} =^1 \int_5^x \frac{8C_2 dt}{e^{2\gamma}\ln^2 t}$$

Because the smallest twin prime number is (3，5), the lower limit of summation or integration in the above equation is 5;

Obtain the latter part of $\Pi_2(x)_N$ in formula (3.2.1);

Because the narrow value $\Pi(x)_N$ of the prime number theorem sieving method of formula (3.1.1) includes $\prod_{p\le\sqrt{x}} \frac{p-1}{p}$, prime number theorem Mertens' generalized narrow coefficients, Mertens' coefficients, generalized narrow coefficients $M_N^G(1)$, $M(1)$, $C_N^G(1)_S$ of formula (3.1.2) is obtained;

Similarly, because the $\Pi_2(x)_N$ of formula (3.2.1) contains $\prod_{p\le\sqrt{x}} \left(1-\frac{1}{p}\right)^2$, therefore，twin prime number sieving method Meitens generalized narrow coefficient, Meitens coefficient, generalized narrow coefficient $M_N^G(2)$, $M(2)$, $C_N^G(2)_S$, approaches $M_N^G(1)^2$, $M(1)^2$, $C_N^G(1)_S^2$; By defined of $M_N^G(2)$, $M(2)$, $C_N^G(2)_S$, then

$$\begin{cases} M_N^G(2) \\ {}^1M_N^G(1)^2 \end{cases} = \frac{\Pi_2(x)_G}{\Pi_2(x)_N} = \begin{cases} C_N^G(2)_S M(2) \\ {}^1C_N^G(1)_S^2 M(1)^2 \end{cases} = \frac{{}^1\pi_2(x)}{\Pi_2(x)_N} =^1 \begin{cases} M(2) \\ M(1)^2 \end{cases} =^1 \frac{e^{2\gamma}}{2^2}$$

$C_N^G(2)_S =^1 C_N^G(1)_S^2 =^1 1$

Obtain $M_N^G(2)$, $M(2)$, $C_N^G(2)_S$ **of formula (3.2.2);**

The above results: $C_N^G(2)_S =^1 C_N^G(1)_S^2 =^1 1$ **are supported by data table 3.2.1.**

Quoting the $\Pi_2(x)_N$ **of formula (3.2.1), quoting the formula (3.2.2) of** $M_N^G(2)$, $M(2)$, $C_N^G(2)_S$, **get**

$$\Pi_2(x)_G =^1 M_N^G(2)\Pi_2(x)_N =^1 \frac{e^{2\gamma}}{4}\int_5^x \frac{{}^1 8C_2 dt}{e^{2\gamma}\ln^2 t} =^1 \int_5^x \frac{2C_2 dt}{\ln^2 t}$$

Obtain the twin prime number sieving method generalized values $\Pi_2(x)_G$ **etc. of formula (3.2.3);**

By using the generalized boundaries of formula (2.2.13), formula (2.2.14), and the data in Table 3.2.1, and the calculation process of boundary coefficients $\uparrow\downarrow\overline{C(x_i)}_G$ **similar to the prime number theorem formula (5.1.25), twin boundary coefficients** $\uparrow\downarrow C_2$ **can be calculated; Obtain the generalized boundary** $\uparrow\downarrow\Pi_2(x)_G$ **of twin prime numbers in formula (3.2.3), as well as the actual values** $\pi_2(x)$ **etc. of twin prime numbers. The calculation process is omitted, and prove completion**

3.2.3 Twin primes number lemma

Lemma 3.2.4: When: $1 \ll p(j)$**, then**

$$\prod_{p(j+1)\le p}\frac{p^2-2p}{(p-1)^2}=\prod_{p(j+1)\le p}\left[1-\frac{1}{(p-1)^2}\right]=1-\frac{{}^{1}p(j)^{-1}}{\ln p(j)} \quad (3.2.17)$$

$$\prod_{3\le p\le p(j)}\frac{p-2}{p}=\prod_{3\le p\le p(j)}\left[1-\frac{1}{(p-1)^2}\right]\prod_{3\le p\le p(j)}\left(1-\frac{1}{p}\right)^2 =^{1} 4C_2\prod_{p\le p(j)}\left(1-\frac{1}{p}\right)^2 \quad (3.2.18)$$

$$=^{1}\frac{4C_2e^{-2\gamma}}{\ln^2 p(j)} \quad \begin{bmatrix}\text{By}(3.1.17)\\ \text{由}(3.2.17)\end{bmatrix} \quad (3.2.19)$$

$$\sum_{i=1}^{j}\prod_{p(i+1)\le p}^{p(j+1)\approx\sqrt{x}}\frac{p(i)-1}{2p(i)}\frac{p-2}{p}=^{1}\frac{\sqrt{p(j)}}{2\ln p(j)}\Bigg|^{p(j)\approx\sqrt{x}}=^{1}\frac{\sqrt{x}}{\ln x} \quad (3.2.20)$$

$$\left[C_2=\prod_{3\le p}\left[1-\frac{1}{(p-1)^2}\right]\approx 0.66016118158468695739278\,12 \quad (3.2.4)\right]$$

2C_2 ≈2*0.66016118158468695739278 12

≈1.32032236316937391478556 24

Proof: According to the definition of C_2, formula (3.2.4) is obtained;

$$\prod_{p(j+1)\le p}\left[1-\frac{1}{(p-1)^2}\right]=\exp\left[\sum_{p(j+1)\le p}\ln\left(1-\frac{1}{p^2}\right)\right]=\exp\left(-\sum_{p(j)\le p}\frac{{}^{1}1}{p^2}\right)$$

$$=\exp\left(-\sum_{p(j)\le t}\frac{{}^{1}P(t)}{t^2}\right)=\exp\left(-\int_{p(j)}^{\infty}\frac{{}^{1}dt}{t^2\ln t}\right)=\exp\left(\int_{p(j)}^{\infty}\frac{{}^{1}1}{\ln t}d\frac{1}{t}\right)$$

$$=\exp\left(-\frac{{}^{1}p(j)^{-1}}{\ln p(j)}\right)=1-\frac{{}^{1}p(j)^{-1}}{\ln p(j)}$$

The formula (3.2.17) is obtained ;

$$\prod_{3\le p\le p(j)}\frac{p-2}{p}=\prod_{3\le p\le p(j)}\frac{(p-2)p}{(p-1)^2}\left(\frac{p-1}{p}\right)^2=\prod_{3\le p\le p(j)}\frac{(p-1)^2-1}{(p-1)^2}\left(\frac{p-1}{p}\right)^2$$

$$=\prod_{3\le p\le p(j)}\left[1-\frac{1}{(p-1)^2}\right]\prod_{3\le p\le p(j)}\left(\frac{p-1}{p}\right)^2$$

【The formula (3.2.18) is obtained】

$$=\left\{\prod_{3\le p}\left[1-\frac{1}{(p-1)^2}\right]\div\prod_{p(j+1)\le p}\left[1-\frac{1}{(p-1)^2}\right]\right\}\prod_{p\le p(j)}\left(1-\frac{1}{p}\right)^2\div\left(1-\frac{1}{2}\right)^2$$

$$=\left\{4C_2\div\left[1-\frac{{}^1p(j)^{-1}}{\ln p(j)}\right]\right\}\prod_{p\le p(j)}\left(1-\frac{1}{p}\right)^2\quad\begin{bmatrix}\text{By}(3.2.4)\\ \text{由}(3.2.17)\end{bmatrix}$$

$$=^1 4C_2\prod_{p\le p(j)}\left(1-\frac{1}{p}\right)^2=^1\frac{4C_2e^{-2\gamma}}{\ln^2 p(j)}\quad\left[\text{By}(3.1.17)\right]$$

The formula (3.2.19) is obtained;

$$\prod_{p(i+1)\le p\le p(j)}\frac{p-2}{p}=\prod_{3\le p\le p(j)}\frac{p-2}{p}\div\prod_{3\le p\le p(i)}\frac{p-2}{p}\quad\begin{pmatrix}\text{By } above\ formula\\ \text{引用上式}\end{pmatrix}$$

$$=^1\frac{4C_2e^{-2\gamma}}{\ln^2 p(j)}\div^1\frac{4C_2e^{-2\gamma}}{\ln^2 p(i)}=^1\frac{\ln^2 p(i)}{\ln^2 p(j)}$$

$$\sum_{i=1}^{j}\prod_{p(i+1)\le p\le p(j+1)}\frac{p(i)-1}{2p(i)}\frac{p-2}{p}=\sum_{i=1}^{j}\frac{p(i)-1}{2p(i)}\prod_{p(i+1)\le p\le p(j+1)}\frac{p-2}{p}$$

$$\sum_{i=1}^{j}\frac{p(i)-1}{2p(i)}\frac{\ln^2 p(i)}{\ln^2 p(j)}=^1\sum_{i=2}^{j}\frac{\ln^2 p(i)}{\ln^2 p(j)}\frac{p(i)-1}{2p(i)}\quad\begin{pmatrix}\text{By above formula}\\ \text{引用上式}\end{pmatrix}$$

$$=^1\sum_{t=p(2)}^{p(j)}\frac{\ln^2 t}{\ln^2 p(j)}\frac{t-1}{2t}P(t)=^1\frac{1}{2}\int_{p(1)}^{p(j)}\frac{\ln^2 t}{\ln^2 p(j)}\frac{dt}{\ln t}=^1\frac{1}{2}\int_{p(1)}^{p(j)}\frac{\ln t\,dt}{\ln^2 p(j)}=\frac{{}^1p(j)}{2\ln p(j)}$$

$$\frac{1}{2}\sum_{i=1}^{j}\prod_{p(i+1)\le p}^{p(j+1)\approx\sqrt{x}}\frac{p-2}{p}\frac{p(i)-1}{p(i)}=\frac{{}^1p(j)^{p(j)=\sqrt{x}}}{2\ln p(j)}=^1\frac{\sqrt{x}}{\ln x}\quad\begin{pmatrix}\text{By above formula}\\ \text{引用上式}\end{pmatrix}$$

Get formula (3.2.20), prove completion

3.2.4 Hardy-Littlewood , twin prime number analysis solution conjecture,

Definition 3.2.5: In x, two adjacent prime numbers (p1,p2) , then : p,p+d are separated by d, called binary prime numbers; When: d=2, it can be called the most dense binary prime number, or the twin prime number p,p+2, then

$$5 \le p1+d=p+d=p2 \le x \tag{3.2.21}$$

Two integers separated by d: t,t+d , Form a subset of binary integers t (t, t+d) separated by d, and form a set of binary integers T (x, d) separated by d;

Two integers separated by 2: t,t+2, It is called the densest binary integer, also known as the twin integer, or the subset of twin integers t (t, t+2), and forms the set of twin integers T (x, 2);

Among them, the number of values of the natural number t - the subset t (t, t+2) of the twin integer set T (x, 2), and the number S2 (x), then

$$S2(x)=x-5+1=x-4 \tag{3.2.22}$$

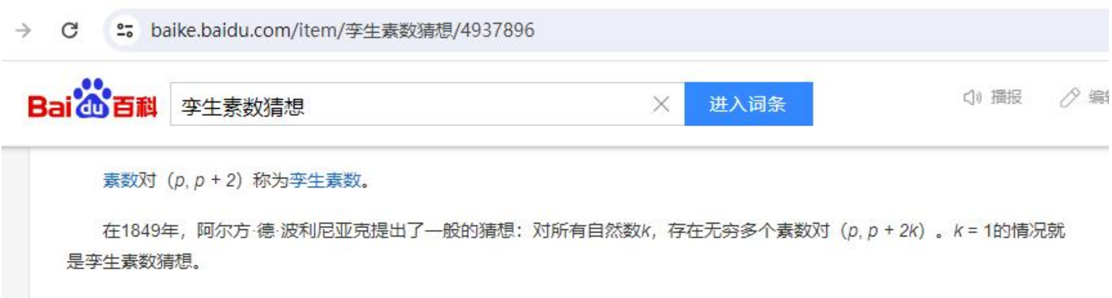
Qualitative description of twin prime conjecture

Euclid (c. 330 BC - 275 BC) proposed 2300 years ago that there are infinitely many prime numbers p and p+2..

https://www.ppkao.com/wangke/daan/cfd2e3673822482fafd96984 57983109

In 1849, French mathematician Alphonse Polignac proposed a more general conjecture (known as the Polignac conjecture) : for all positive integers k, there are infinitely many prime pairs (p, p+2k), which can be called the binary prime conjecture.

【https://baike.baidu.com/item/%E5%AD%AA%E7%94%9F%E7%B4%A0%E6%95%B0%E7%8C%9C%E6%83%B3/4937896】



【 http://www.360doc.com/content/15/0310/02/18559339_453933827.shtml】

孪生素数猜想

孪生素数猜想

- 卢昌海 -

本文撰写于 2003 年 4 月， 是本站的第一篇数学科普， 填补了作为本人兴趣主要组成部分之一的数学在本站的空白。 自那以后， 本文以 “补注” 形式对若干后续进展作了简单提及， 并于 2014 年 9 月进行了不改变基本结构的轻微修订。

孪生素数猜想还有一个更强的形式， 是英国数学家哈代 (Godfrey Hardy) 和李特伍德 (John Littlewood) 于 1923 年提出的， 有时被称为哈代-李特伍德猜想 (Hardy–Littlewood conjecture)， 或强孪生素数猜想 (strong twin prime conjecture)[注一]。 这一猜想不仅提出孪生素数有无穷多组， 而且还给出其渐近分布为：

$$\pi_2(x) \sim 2C_2 \int_2^x \frac{dt}{(\ln t)^2}$$

其中 $\pi_2(x)$ 表示小于 x 的孪生素数的数目， C_2 被称为孪生素数常数 (twin prime constant)， 其数值为：

$$C_2 = \prod_{p\geq 3} \frac{p(p-2)}{(p-1)^2} \approx 0.66016118158468695739278121100145...$$

强孪生素数猜想对孪生素数分布的拟合程度可以由下表看出：

x	孪生素数数目	强孪生素数猜想给出的数目
100,000	1224	1249
1,000,000	8,169	8,248
10,000,000	58,980	58,754
100,000,000	440,312	440,368
10,000,000,000	27,412,679	27,411,417

Thank you for Lu Changhai's personal home page above,

providing the following information

British mathematicians Hardy and Te Li Wood proposed in 1923: Hardy-Te Li Wood's twin prime number analysis solution conjecture, or strong twin prime number conjecture, is defined as follows.

Conjecture 3.2.6: Integers that differ by 2： t,t+2, are called twin integers; The actual number of twin primes —— the twin prime Hardy-Littlewood analysis solution conjecture is $\pi_2(x)$**, then**

$$\pi_2(x) \approx \int_2^x \frac{2C_2 dt}{\ln^2 t} \qquad \left\{ \mathrm{C}_2 = \prod_{3 \le p} \left[1 - \frac{1}{(p-1)^2} \right] \right\} \qquad (3.2.23)$$

2 C_2 **≈2*0.66016118158468695739278121**

≈1.32032236316937391478556242

Note: Hardy Littlewood twin prime analysis solution $\pi_2(x)$**,** $\Pi_2(x)_G$ **conjecture,support formula (3.2.10) of the number of twin prime numbers,and the data for the twin prime numbers in Table 3.2.1**

Lu Changhai Personal homepage:

https://www.changhai.org/articles/science/mathematics/twin_prime_conjecture.php

3.2.5 Twin Prime Number Data

Table 3.2.1: Grass twin prime number data $\pi_2(x)$, $x=10^\wedge i$**, etc**

$$C_N^G(2) =^1 \frac{M_N^G(2)}{M(2)} =^1 \frac{\Pi_2(x)_G}{\Pi_2(x)_N}\frac{1}{M(2)} =^1 \pi_2(x) \div \int_5^x \frac{2C_2 dt}{\ln^2 t} =^1 1$$

i	$\pi_2(x)$	$\Pi_2(x)_G$	$C_N^G(2)$
2	8	10.409	0.7686
3	35	42.668	0.8203
4	205	211.08	0.9712
5	1224	1245.6	0.9827
6	8169	8244.9	0.9908
7	58980	58750	1.0039
8	440312	440364	0.99988
9	3424506	3425302	0.99977
10	27412679	27411387	1.000047
11	224376048	224368646	1.000032
12	1870585220	1870558067	1.000015
13	15834664872	15834583086	1.0000052
14	135780321665	1.3578013446e+11	1.0000014
15	1177209242304	1.17720736071e+12	1.0000016
16	10304195697298	1.03041826541e+13	1.0000012
17	90948839353159	9.09487458786e+13	1.0000010
18	808675888577435	8.08675124537e+14	1.00000094

The above data table shows that the distribution law of $C_N^G(2)$ **values satisfies the judgment theorem of C function. Therefore ,** $C_N^G(2) =^1 1$**, the actual data** $\pi_2(x)$ **supports** $\Pi_2(x)_G$**;**

Among them: $\Pi_2(x)_G =^1 \int_5^x \frac{2C_2 dt}{\ln^2 t}$ **value, obtained from the following website, points calculator**

https://zh.numberempire.com/definiteintegralcalculator.php

$$\left[\Pi_2(x)_G =^1 \int_5^x \frac{2C_2 dt}{\ln^2 t} =^1 \pi_2(x) \qquad (3.2.3)\right]$$

=1.3203223631693739/(log(x)^2)

3.2.6 Twin prime numbers, data source

Thank you, Math China Forum, Xiaocao netizen! On June 10, 2021, at the following website, provide the author with the actual values of twin prime numbers $\pi_2(x)$ within 10 ^ 18:

http://www.mathchina.com/bbs/forum.php?mod=viewthread&tid=2046416&page=1#pid2413883

3.2.7 The expression of the densest K-tuple prime number

Definition 3.2.7: It can be proven that, Within p ≤ x, the number of π 3 (x, p, p+2, p+6) and π 3 (x, p, p+4, p+6), which are the two densest 3-tuple prime numbers (p, p+2, p+6) and (p, p+4, p+6), tend to be equal. They can be abbreviated as; $\pi 3(x,2,6) =^1 \pi 3(x,4,6) =^1 \pi 3(x,6)$**;**

Corresponding to two subsets sub3 (t, 6) of the densest 3-tuple integers, and two subsets sub3 (p, 6) of the densest 3-tuple prime numbers ;

Similarly, the densest K-tuple prime number $\left(p, p+d_1, p+d_2, \cdots\cdots, p+d_{K-1}\right)$**,** $\mathrm{sub}K\left(p, d_{K-1}\right)$**, corresponds to the subset** $subK\left(t, d_{K-1}\right)$ **of densest K-tuple integers;**

类似，最密 K 生素数$\left(p, p+d_1, p+d_2, \cdots\cdots, p+d_{K-1}\right)$**，**$\mathrm{sub}K\left(p, d_{K-1}\right)$**，对应的最密 K 生整数子集**$subK\left(t, d_{K-1}\right)$**；**

Within p ≤ x, the number $\pi K\left(x, d_{K-1}\right)$**,** $\pi_K\left(x, d_{K-1}\right)$ **of densest K- prime numbers, then:**

$$sub3(t,6) = \begin{cases} t, t+2, t+6 \\ t, t+4, t+6 \end{cases} \tag{3.2.24}$$

$$sub3(p,6) = \begin{cases} p, p+2, p+6 \\ p, p+4, p+6 \end{cases} \tag{3.2.25}$$

$$\begin{cases} subK\left(t, d_{K-1}\right) = \left(t, t+d_1, t+d_2, \cdots\cdots, t+d_{K-1}\right) \\ subK\left(p, d_{K-1}\right) = \left(p, p+d_1, p+d_2, \cdots\cdots, p+d_{K-1}\right) \end{cases} \tag{3.2.26}$$

$$\pi K\left(x,d_{K-1}\right)=\pi_K\left(x,d_{K-1}\right)=\sum_{p\le x}\left(p,p+d_1,p+d_2,\cdots\cdots,p+d_{K-1}\right) \qquad (3.2.27)$$

3.2.8 The principle of filtering out K-tuple prime numbers

The principle of filtering out K-tuple prime numbers is also applicable to the densest K-tuple prime numbers

Theorem 3.2.8: If p (i) ≤ K, the spacing between K-tuple integers can take K values, forming K-tuple integers. Among these K integers, there may be 2 or more integers that contain the prime p (i) factor.

Therefore, when K-tuple integers is sieved out by the prime number p(i), the narrow probability of the composite number, must be analyzed and calculated one by one, to get it;

In the K-tuple integer $sub\left(p,d_{K-1}\right)$, Adjacent integer spacing d_J, if p(i)≥K+1, and $p(i)\ge 0.5d_{K-1}$, then, $sub\left(t,d_{K-1}\right)$ is divisible by p(i) at most once; In $subK\left(t,d_{K-1}\right)$, the narrow probability $P\left(p(i),d_{K-1}\right)_N$, $P(i)_N$ that every integer, and all integers, are divisible by prime number p(i), get

$$P\left(p(i),d_j\right)_N=\frac{1}{p(i)} \quad \begin{pmatrix} p(i)\ge 7\ ,\ i\ge 4 \\ 0\le j\le K-1 \end{pmatrix} \qquad (3.2.28)$$

$$P(i)_N = \sum_{j=0}^{K-1} P\left(p(i), d_j\right) = \sum_{j=0}^{K-1} \frac{1}{p(i)} = \frac{K}{p(i)} \quad \begin{bmatrix} p(i) \geq K+1 \\ p(i) \geq 0.5 d_{K-1} \end{bmatrix} \tag{3.2.29}$$

prove:The narrow probability $P\left(p(i), d_{K-1}\right)$ of each K independent integers being filtered out by p (i) is equal,and, equal to 1/ p (i), formula (3.2.28) is obtained;

Similarly, the narrow probability $P(i)_N$ of K-tuple prime numbers being filtered out by p (i) is equal to K times the narrow probability $P\left(p(i), d_{K-1}\right)$ of each integer containing a p (i) factor——, equal to K/ p (i), formula (3.2.29) is obtained,proving completion

Note: The following content will be released in the future