# de Broglie waves in gravity

David L. Berkahn, James M. Chappell and Derek Abbott

[a] *School of Electrical and Electronic Engineering, The University of Adelaide, North Tce, Adelaide, 5000, SA, Australia*

**Abstract**

The de Broglie wave hypothesis introduced the concept of wave-particle duality and led to the development of wave mechanics. We show from first principles, how the de Broglie wave will be modified by the presence of a gravitational potential. We find a relationship between de Broglie waves and the gravitational potential. A straightforward experiment is described to accurately verify this relationships.

*Keywords:* Equivalence principle, de Broglie waves, gravitational potential, mass

## 1. Introduction

de Broglie [1], introduced the idea in 1923, that every particle has an associated wave, with a wavelength

$$\lambda = \frac{h}{p}, \tag{1}$$

where $p = \gamma m v$ is the relativistic particle momentum, and $h$ is Planck's constant. The further development of the de Broglie wave hypothesis then led to the development of wave mechanics. In formulating his wave theory for matter, de Broglie's starting point was noticing that time and energy in special relativity (SR), enter into the Lorentz transformations inverse to each another [1]. Earlier, in 1912, Einstein had used the thought experiment involving a rotating frame of reference and the principle of equivalence, to deduce that time slows in a gravitational field [2, 3]. Clearly then, due to time, as well as length changes in gravity, de Broglie waves will also be modified by gravity. The goal of this paper is to make these various relationships explicit in terms of a gravitational potential.

*1.1. de Broglie waves in gravity*

Considering Einstein's relation $E = h\nu = mc^2$, de Broglie observed that the frequency change due to time dilation, is

$$\nu'_D = \frac{m_0 c^2}{h}\sqrt{1-\beta^2}, \tag{2}$$

whereas based on the relativistic energy relation $h\nu = \gamma mc^2$, he observed

$$\nu'_E = \frac{1}{h}\frac{mc^2}{\sqrt{1-\beta^2}}, \tag{3}$$

where $\beta = \nu/c$. This apparent contradiction brought him to his theorem of phase harmony.

Now, the Gullstrand-Painlevé coordinate system [4, 5] can be used to write the Schwarzschild solution with an alternate set of coordinates [6] from the perspective of a 'rain frame' [7]. With this representation, spacetime can be interpreted as flowing into a black hole with a velocity given by the escape velocity of $v = \sqrt{\frac{2GM}{r}}$. This then gives the gravitational time dilation factor

$$t = \frac{t'}{\sqrt{1-\frac{2MG}{rc^2}}}, \tag{4}$$

based on the relativistic formula from SR of $t = \frac{t'}{\sqrt{1-\frac{v^2}{c^2}}}$, where $t'$ is associated with the clock in the gravitational field. Hence, by applying the same process for $E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$, we derive the energy relation

$$E' = \frac{m_0 c^2}{\sqrt{1-\frac{2MG}{rc^2}}}, \tag{5}$$

. where $E'$ lies above the surface of $M$ at $r$ and $m_0$ lies at $\infty$.

Now, in gravity, we have from the time dilation Eq. (4), the frequency relation

$$\nu' = \nu\sqrt{1-\frac{2MG}{rc^2}}, \tag{6}$$

where $\nu'$ is the frequency of light originating from the surface of the gravitating mass $M$ at $r$ and measured locally by an observer at infinity, sometimes notated by the less succinct but clearer nomenclature $O_\infty \nu s$. The frequency $\nu$ originates locally and is also measured locally by any observer. For the observer at infinity, less wave fronts arrive per second on their clock, originating from $r$, compared to his local frequency measurements.

Now for equal comparison of frequency, we again apply the relations to $E = hv = mc^2$ to Eq. (5) and Eq. (6) to give Eq. (7) and Eq. (8) respectively, finding,

$$\nu' = \frac{1}{h}\frac{m_0c^2}{\sqrt{1-\frac{2MG}{rc^2}}} \tag{7}$$

and

$$\nu' = \frac{m_0c^2}{h}\sqrt{1-\frac{2MG}{rc^2}}. \tag{8}$$

These two frequencies enter into the gravitational field differently, since Eq. (7) shows frequency increase with decreasing $r$, while Eq. (8) shows the inverse. They are therefore not the same type of wave. We can identify this phenomena, once again, with de Broglie's hypothesis of phase harmony, and see that it also exists in gravity.

### *1.2. Wavelength*

From Eq. (7) we have then for the wavelength

$$\lambda = \frac{h}{m_0v}\sqrt{1-\frac{2MG}{rc^2}}, \tag{9}$$

in the low velocity limit. As the De Broglie wavelength decreases in gravity, we would therefore expect electron orbitals to also shrink. We note that as $r \to \infty$, we recover the de Broglie result, of $\lambda = \frac{h}{mv}$ .

For the special case of zero velocity (refer Appendix A), we find

$$\lambda = \frac{h}{m_0c}\sqrt{\frac{rc^2}{2MG}-1}, \tag{10}$$

where we are assuming $r \geq \frac{2MG}{rc^2}$, the Schwarzschild radius.

Since $\lambda$ decreases when $r$ decreases, then the de Broglie wavelength of a massive particle such as an electron also decreases the closer it is to a larger mass $M$, approaching zero wavelength at the Schwarzschild radius. When $r \to \infty$, we approach flat space, and reproduce de Broglie's result of the wavelength going to infinity.

*1.3. Experimental tests*

In order to test the theory, we can undertake various X-ray diffraction experiments, where electrons can be fired perpendicular to the field, at different gravitational potentials. These results should confirm Eq. (9), that the de Broglie wavelength decreases with altitude decrease.

## 2. Discussion and Conclusion

We derived a simple relation for the de Broglie wave in gravity. We found the wavelength decreases with an increase in gravitational potential, as shown in Eq. (9). The phenomena also appears consistent with de Broglie's phase harmony. An experiment has been presented to test for the effect. Although further research needs to be done, this shows how quantum and gravitational effects could be intertwined at a fundamental level.

## Appendix A. Deriving the de Broglie wavelength

We have $E = (\gamma - 1)m_0c^2 + m_oc^2 = K + m_oc^2$, where $K$ is the kinetic energy. Squaring we have $E^2 = (K + m_0c^2)^2 = K^2 + 2Km_0c^2 + m_o^2c^4$ and equating this to $p^2c^2 + m_0^2c^4$ we have $K^2 + 2Km_0c^2 = p^2c^2$ and so $p = \frac{1}{c}\sqrt{K(K + 2m_0c^2)} = m_0c\sqrt{(\gamma - 1)(\gamma + 1)} = m_0c\sqrt{\gamma^2 - 1}$, which can be substituted into the de Broglie equation

$$\lambda = \frac{h}{p} = \frac{h}{m_0c\sqrt{\gamma^2 - 1}}. \tag{A.1}$$

For our treatment, we again use the Gullstrand-Painlevé coordinate system, allowing the mapping $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \to \frac{1}{\sqrt{1-\frac{2MG}{rc^2}}}$, giving

$$\lambda = \frac{h}{m_0c}\frac{1}{\sqrt{\frac{1}{1-\frac{2MG}{rc^2}} - 1}} = \frac{h}{m_0c}\frac{1}{\sqrt{\frac{\frac{2MG}{rc^2}}{1-\frac{2MG}{rc^2}}}} = \frac{h}{m_0c}\sqrt{\frac{rc^2}{2MG} - 1}. \tag{A.2}$$

## References


[1] L. de Broglie, C.r. hebd. séanc. acad. sci., paris, 177, 506, 548, 630, Ph.D. thesis, Thesis de Doctoral (1923).

[2] A. Einstein, The Special and General Theory, Princeton Univ. Press, New Jersey, 2015.

[3] A. Einstein, Die grundlage der allgemeinen Relativitätstheorie [Adp 49, 769 (1916)], Annalen der Physik 14 (S1 1) (2005) 517–571.

[4] A. Gullstrand, Allgemeine Lösung des statischen Einkörperproblems in der Einsteinschen Gravitationstheorie., Ark. Mat. Astron. Fys. 16 (8) (1922) 15.

[5] P. Painlevé, Le mecanique classique et la theorie de la relativite., L'Astronomie 36 (1922) 6–9.

[6] G. Lemaître, L'univers en Expansion, in: Annales de la Société Scientifique de Bruxelles, Vol. 53, 1933, p. 51.

[7] E. F. Taylor, J. A. Wheeler, Exploring Black Holes: Introduction to General Relativity, Addison Wesley Longman, New York, 2000.