

# Gravitational Redshift of Internal Quantum Clocks: A Zitterbewegung-Based Model

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Zitterbewegung, a high-frequency oscillatory term originally derived from the Dirac equation [1], has traditionally been viewed as a mathematical artifact without physical significance. In contrast, this paper adopts a reinterpretation of Zitterbewegung as a physically real internal motion of the electron. Within this framework, the internal oscillation is confined to a compact photonic shell structure governed by the electron's Compton wavelength [2] and evolves according to the proper time of the system. Building on this model, the author has previously derived a special relativistic estimate for the mean internal velocity, yielding  $v_{e,\text{SR}} = 0.040472c$  [3]. This estimate was subsequently refined by incorporating general relativistic effects, including geodetic precession and curvature-modified critical radius constraints, resulting in a corrected velocity  $v_{e,\text{SR+GR}} = 0.040374c$  at Earth's surface [4]. A key prediction of the model is that this velocity is subject to gravitational modulation: if Zitterbewegung were directly observable, its effective velocity would differ between gravitational potentials, such as those found at Earth's surface and in satellite orbits. This predicted velocity shift mirrors the time dilation corrections required in satellite-based atomic clocks, such as those used in the Global Positioning System (GPS) [5], which account for both special and general relativistic effects.

This study thus provides a novel connection between quantum internal dynamics and spacetime geometry, proposing that general relativistic time dilation affects not only macroscopic clocks but also subatomic internal oscillations. If verified experimentally, this would support the view that Zitterbewegung is a real physical process and offer a new probe for exploring the intersection of quantum mechanics and general relativity.

## I. INTRODUCTION

The phenomenon known as Zitterbewegung, or “trembling motion”, was originally introduced by Schrödinger [6] as a consequence of the Dirac equation for relativistic spin- $\frac{1}{2}$  particles. It arises due to interference between positive and negative energy solutions, leading to a high-frequency oscillatory term in the position operator. Conventionally, this oscillation has been interpreted as a non-physical artifact—a mathematical curiosity without direct physical observability.

However, recent theoretical developments, including those by the present author [7, 8], have challenged this interpretation. By postulating that Zitterbewegung corresponds to a real internal motion of the electron, a new perspective emerges wherein internal dynamics contribute meaningfully to the particle's structure and interactions. This reinterpretation allows the oscillation to be modeled geometrically, constrained within a compact domain bounded by the electron's Compton wavelength, and evolving according to the proper time of the system.

Building on this reinterpretation, the author has proposed a model in which the average Zitterbewegung velocity  $v_e$  is derived from Lorentz kinematics and the electron's anomalous magnetic moment, building on early spin concepts [9] and relativistic corrections [10], with quantum electrodynamics [11] providing the theoretical foundation and recent high-precision measurements [12] offering experimental validation. This leads to a predicted

velocity of  $v_{e,\text{SR}} = 0.040472c$  based on special relativity alone. More recently, this model has been extended to incorporate general relativistic effects [13], specifically those arising from geodetic precession and modifications of the critical radius under curved spacetime. This refinement results in a slightly lower predicted average velocity,  $v_{e,\text{SR+GR}} = 0.040374c$ , at Earth's surface.

These theoretical developments lay the groundwork for a key prediction: if the internal Zitterbewegung motion of electrons were experimentally observable, then differences in gravitational potential—such as those between the Earth's surface and orbital altitudes—should manifest as detectable variations in the effective frequency of this internal motion. This effect would mirror the time correction mechanisms employed in the Global Positioning System (GPS), where atomic clocks onboard satellites experience both special and general relativistic time shifts, necessitating continuous synchronization with ground-based reference clocks.

The aim of this paper is to formalize this prediction within the author's extended model, and to compute the magnitude of the expected frequency shift of Zitterbewegung oscillations between the Earth's surface and satellite altitudes. In doing so, the study not only offers a potential avenue for indirect observation of internal particle dynamics, but also suggests that general relativistic effects extend even to the internal, subatomic scale. This contributes to an emerging perspective in which spacetime geometry influences not just the trajectories of particles, but also the fine structure of their internal degrees of freedom.

This investigation also serves a broader conceptual goal:

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to explore whether the internal structure of the electron, as formulated in the author’s 0-Sphere model, can act as a quantum-mechanical clock sensitive to spacetime geometry. In this model, the electron hosts a simple harmonic oscillator-like mechanism whose frequency defines a natural timescale intrinsic to the particle. If this internal clock is modulated by gravitational time dilation, then Zitterbewegung becomes not just a quantum fluctuation, but a bridge between quantum theory and general relativity—providing a testable realization of a quantum clock affected by curved spacetime, as explored in quantum field theory frameworks [14].

## II. METHODS

The present study builds upon a sequence of prior works in which the author developed the 0-Sphere model of the electron—a theoretical framework positing a compact internal structure composed of two thermal kernels exchanging energy through a photon shell exhibiting Zitterbewegung (ZB). In previous formulations [3, 4, 7, 8, 15], this internal motion was linked to the anomalous magnetic moment via observer-dependent relativistic contraction effects, suggesting that magnetic anomalies could arise without external fields. The current investigation extends this framework by reinterpreting the same internal dynamics not merely as spin-generating motion, but as an intrinsic harmonic oscillator that functions as a quantum clock governed by proper time. This shift in perspective opens the way to analyze how gravitational potentials—via general relativistic time dilation—modulate the Zitterbewegung frequency, thereby allowing the internal motion of the electron to serve as a probe of spacetime curvature.

The methodology employed in this study builds upon the author’s previous theoretical work, which reinterprets the Zitterbewegung (ZB) phenomenon as a physically real internal oscillation of the electron, rather than as a formal byproduct of the Dirac equation, building on geometric interpretations of Zitterbewegung [16, 17]. This foundational assumption allows for a systematic construction of the internal dynamics of the electron using relativistic and geometric principles.

At the core of the author’s framework is a unique equation that connects Zitterbewegung dynamics with Lorentz transformations and the electron’s anomalous magnetic moment. This relationship provides a geometric interpretation of spin-induced motion and yields a predictive estimate for the average Zitterbewegung velocity. Specifically, based on the special relativistic model of internal motion, the predicted mean velocity of the ZB process is given by

$$v_{e,\text{SR}} = 0.040472c, \quad (\text{II.1})$$

where  $c$  is the speed of light in vacuum. This value is derived through analysis of internal oscillatory motion

within a confined photon-sphere-like structure, bounded by the Compton wavelength of the electron.

To extend the model, the effects of general relativity (GR) are incorporated by considering spacetime curvature and geodetic precession. The author’s approach introduces a modified critical radius that determines the allowable curvature of the internal trajectory. When these GR effects are included, the corrected average velocity becomes

$$v_{e,\text{SR+GR}} = 0.040374c. \quad (\text{II.2})$$

The difference between these two values, though small, is physically meaningful and provides a basis for analyzing gravitational corrections to internal particle dynamics.

This relativistic correction scheme is analogous in principle to the well-established framework used in global positioning systems (GPS). In GPS satellites, both special relativistic (SR) and general relativistic (GR) effects are required to correctly synchronize the on-board atomic clocks with those on Earth. Without compensating for time dilation due to orbital velocity (SR) and gravitational potential difference (GR), satellite timing would rapidly diverge from ground-based reference frames. The same logic is applied here: the internal Zitterbewegung clock of the electron is subject to both SR and GR influences, and its effective frequency must be corrected accordingly depending on gravitational context.

By combining these relativistic considerations, the methodology allows for the computation of Zitterbewegung frequency variations across different gravitational potentials, such as those encountered on Earth’s surface and in satellite orbits. The resulting predictions form the theoretical foundation for the observational proposals discussed in later sections. While the author has also examined frame-dragging effects such as those measured by Gravity Probe B [18], the predicted influence on Zitterbewegung dynamics within the 0-Sphere model was found to be several orders of magnitude smaller than the geodetic precession, and is therefore omitted from detailed discussion in the present study.

## III. DISCUSSION

### A. Gravitational Redshift of Quantum Internal Clocks

In conventional treatments of Zitterbewegung (ZB), the effective internal velocity  $v_e$  is derived from special relativistic considerations. Using the electron’s Compton wavelength  $\lambda_{\text{compton}}$ , the characteristic frequency of this internal oscillation is given by

$$\nu_{e,\text{ZB}} = \frac{v_e \cdot c}{\lambda_{e,\text{compton}}}, \quad (\text{III.1})$$

which implies a direct proportionality  $\nu_{e,\text{ZB}} \propto v_e$ . A commonly adopted value under special relativity (SR) is

$v_{e,\text{SR}} = 0.040472c$ , which yields  $\nu_{e,\text{ZB}} \approx 5.00706 \times 10^{18}$  Hz. However, this estimate neglects general relativistic (GR) effects, particularly those stemming from spacetime curvature and geodetic precession near massive bodies like Earth.

This formulation adopts the interpretation, originally advanced in [7], that the Compton wavelength represents the characteristic amplitude of the Zitterbewegung oscillation derived from solutions to the Dirac equation. In this view,  $\lambda_{\text{Compton}}$  acts not merely as a derived length scale, but as a physically significant internal dimension governing the oscillatory behavior of the electron.

In the present model, GR corrections are incorporated by considering (i) the geodetic precession induced by spacetime curvature and (ii) the gravitational modulation of the critical radius bounding the photonic internal structure. The incorporation of both SR and GR effects has been previously examined and published by the author [4]. These lead to a refined estimate of the effective internal velocity, denoted  $v_{e,\text{SR+GR}} = 0.040374c$ . Substituting this into Eq. III.1, we obtain the corrected ZB frequency at Earth's surface:

$$\begin{aligned} \nu_{\text{ZB, Earth}} &= \frac{0.040374 \cdot c}{\lambda_{\text{Compton}}} \\ &= 4.98720 \times 10^{18} \text{ Hz.} \end{aligned} \quad (\text{III.2})$$

This frequency is marginally lower than the SR-only estimate, reflecting the gravitational suppression of internal motion. Since  $\nu_{e,\text{ZB}}$  is linearly proportional to  $v_e$  by construction, any change in  $v_e$  arising from relativistic effects directly modifies the ZB frequency, lending physical significance to even subtle corrections.

Furthermore, because Zitterbewegung is modeled as internal oscillation governed by the proper time  $\tau$  of the particle, gravitational time dilation influences its observed frequency. In a Schwarzschild geometry, the relation between proper time and coordinate time is

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{2GM}{rc^2}}. \quad (\text{III.3})$$

This factor affects the perceived ticking rate of any internal clock, including ZB. To examine this effect in detail, we now consider the specific velocity reduction and compare the Zitterbewegung characteristics between electrons at satellite altitude and those at Earth's surface.

At satellite altitude ( $h = 20,200$  km, corresponding to  $r = R_E + h = 2.6571 \times 10^7$  m), the gravitational time dilation factor becomes:

$$\begin{aligned} \left(\frac{d\tau}{dt}\right)_{\text{sat}} &= \sqrt{1 - \frac{2GM}{r_{\text{sat}}c^2}} \\ &\approx 1 - 1.67 \times 10^{-10}. \end{aligned} \quad (\text{III.4})$$

At Earth's surface ( $r = R_E = 6.371 \times 10^6$  m), we have:

$$\left(\frac{d\tau}{dt}\right)_{\text{Earth}} \approx 1 - 6.96 \times 10^{-10}. \quad (\text{III.5})$$

Since the proper time rate is slower at Earth's surface due to stronger gravitational potential, the intrinsic Zitterbewegung frequency, when measured in coordinate time, would appear reduced compared to satellite altitude. If we consider the Zitterbewegung frequency as measured by a local observer (proper time), it would be:

$$\begin{aligned} \nu_{\text{ZB,proper}} &= \frac{v_{e,\text{SR+GR}}}{\lambda_{\text{Compton}}} \\ &= 4.98720 \times 10^{18} \text{ Hz} \end{aligned} \quad (\text{III.6})$$

However, when this frequency is observed from a distant coordinate frame, the observed frequencies differ due to gravitational redshift [19]:

$$\begin{aligned} \nu_{\text{ZB,Earth}} &= \nu_{\text{ZB,proper}} \cdot \left(\frac{d\tau}{dt}\right)_{\text{Earth}} \\ &\approx 4.98720 \times 10^{18} \cdot (1 - 6.96 \times 10^{-10}) \text{ Hz} \end{aligned} \quad (\text{III.7})$$

$$\begin{aligned} \nu_{\text{ZB,sat}} &= \nu_{\text{ZB,proper}} \cdot \left(\frac{d\tau}{dt}\right)_{\text{sat}} \\ &\approx 4.98720 \times 10^{18} \cdot (1 - 1.67 \times 10^{-10}) \text{ Hz} \end{aligned} \quad (\text{III.8})$$

The frequency difference between satellite and Earth-based electrons is therefore:

$$\begin{aligned} \Delta\nu &= \nu_{\text{ZB,sat}} - \nu_{\text{ZB,Earth}} \\ &\approx 4.98720 \times 10^{18} \cdot 5.29 \times 10^{-10} \\ &\approx 2.64 \times 10^9 \text{ Hz} \end{aligned} \quad (\text{III.9})$$

This represents a fractional frequency change of approximately  $5.29 \times 10^{-10}$ , demonstrating that satellite-based electrons exhibit slightly higher observed Zitterbewegung frequencies when measured from a distant coordinate frame, consistent with gravitational redshift effects where clocks appear to run faster at higher gravitational potentials.

This provides a concrete, **testable prediction** of the extended Zitterbewegung framework: internal oscillatory processes, while intrinsic to the particle, are not entirely decoupled from the curvature of spacetime. The model thus suggests a novel form of gravitational redshift acting on quantum internal degrees of freedom, analogous to how general relativity affects atomic clocks [20].

## B. Implications for Quantum Gravity and Unified Clocks

These results point toward a deeper connection between internal quantum motion and macroscopic gravitational geometry. If Zitterbewegung serves as a clock-like internal process, then its ticking rate responds—however subtly—to the geometry of spacetime. This opens the

door to extending well-established gravitational redshift phenomena into new quantum regimes, and potentially informs future experimental tests linking proper time evolution with quantum substructure.

An important implication of the above findings, though not yet experimentally verified, concerns the fundamental speed of Zitterbewegung oscillations as determined by proper time. The present author posits that, in principle, the Zitterbewegung frequency at Earth’s surface should be inherently slower than that at satellite altitude due to gravitational time dilation. This is because the proper-time rate at lower gravitational potential reduces the effective internal clock rate. From the 0-Sphere model perspective, the intrinsic Zitterbewegung velocity  $v_{e,ZB}$  itself should therefore differ slightly depending on the gravitational context.

Specifically, the Zitterbewegung velocity  $v_{e,ZB}$  can be expressed as a function of proper time  $\tau$  and Compton wavelength  $\lambda_{\text{compton}}$  as:

$$v_{e,ZB} = \lambda_{\text{compton}} \cdot \frac{d\phi}{d\tau}, \quad (\text{III.10})$$

where  $d\phi/d\tau$  denotes the internal angular frequency in proper time. However, when this motion is observed from coordinate time  $t$ , gravitational time dilation modifies the apparent velocity:

$$v_{e,\text{obs}} = \lambda_{\text{compton}} \cdot \frac{d\phi}{dt} = v_{e,ZB} \cdot \frac{d\tau}{dt}. \quad (\text{III.11})$$

Thus, even the internal velocity appears reduced when measured at lower altitudes in a gravitational field.

Nonetheless, the numerical difference in  $v_{e,\text{SR+GR}}$  due to gravitational redshift, on the order of  $10^{-10}$ , is below the current precision of the model’s velocity estimates, which are accurate to five significant figures. This implies that the present model lacks sufficient resolution to capture gravitational modulation of  $v_{e,ZB}$  to this level of fidelity.

However, the author predicts that in future experiments—once the precision of measurement improves to match the scale of general relativistic corrections through advanced optical frequency metrology techniques [21]—the differential slowing of Zitterbewegung at lower gravitational potentials should become detectable, much like the relativistic corrections observed in atomic clocks. From the standpoint of the 0-Sphere model, even a single electron could exhibit measurable frequency shifts in Zitterbewegung depending on its altitude. This opens the possibility that Zitterbewegung may serve as an ultra-fine internal probe of spacetime curvature, extending the conceptual scope of gravitational redshift down to the quantum-electrodynamical scale.

The potential connection to quantum gravity effects [22] suggests that such measurements could provide new insights into the interface between quantum mechanics and general relativity. If the internal structure of particles responds measurably to gravitational fields, this could inform theoretical frameworks attempting to unify these fundamental theories.

### C. Relativistic Constraints on TPE Dynamics

In the 0-Sphere model, the electron’s internal energy dynamics consist of thermal potential energy (TPE) stored in the two kernels and the kinetic energy of a photon shell that oscillates between them. Importantly, the kinetic excitation of the photon shell must propagate at the speed of light  $c$ , as required by special relativity. Therefore, when the observed Zitterbewegung velocity  $v$  is significantly less than  $c$  (e.g.,  $v \approx 0.04c$ ), the time required for TPE to convert into propagating kinetic energy becomes proportionally longer.

Since the total energy—comprising the thermal potential energy and the kinetic energy of the photon shell—is expected to remain conserved within the electron as a closed system, any delay in the energy conversion process would necessarily extend the oscillation period. This implies that the relativistic redshift of proper time causes a delay in the energy transfer process, indicating that the internal clock rate governed by Zitterbewegung slows down due to gravitational time dilation, which in turn reduces the effective rate at which thermal potential energy is released.

Moreover, gravitational redshift not only slows the internal clock rate but also lowers the intrinsic angular frequency  $\omega$  of the Zitterbewegung process. Since this frequency governs the oscillatory dynamics of the photon shell responsible for mediating kinetic energy, a reduction in  $\omega$  implies a corresponding decrease in the frequency of any radiated energy. This mechanism provides a natural explanation for the observed redshift of photons emitted from lower gravitational potentials, linking internal proper-time dynamics to externally measurable spectral shifts.

Such a constraint introduces a natural mechanism by which Zitterbewegung dynamics could slow down in deeper gravitational wells—not only due to spacetime curvature affecting proper time, but also because energy conversion from TPE to radiation-like motion is fundamentally bounded by  $c$ . Whether this transition proceeds in a continuous or quantized manner remains an open question, closely tied to the energy-scale discretization discussed below. Currently, the model does not specify whether the energy transfer follows a continuous or quantized pathway; this question remains unresolved.

In parallel, the hypothesis that the electron is not a true point particle but instead possesses a finite kernel radius introduces further foundational questions regarding its internal structure. Specifically, this study concludes that the decay velocity of the kernel is not instantaneous, implying that external observers would detect redshift effects due to the gravitational potential experienced by the kernel itself. This delay in decay—caused by spacetime curvature—provides a physical basis for treating the electron’s oscillatory behavior as analogous to an atomic clock.

A previous study estimated the effective radius of

the electron’s kernel to be approximately  $3.43 \times 10^{-25}$  metres, based on a general relativistic reinterpretation of the muon decay process [4]. This finding suggests that the kernel is not a mathematical point but a physical entity with finite spatial extent, and that its decay occurs over a non-zero, albeit extremely short, timescale. The presence of a finite radius implies that the electron may exhibit internal structure beyond the standard model description. Within the present framework, the thermal potential energy (TPE) associated with the kernel is not yet classified as either a discrete or continuous quantity. Determining the physical nature of this energy remains a central objective for future theoretical investigation.

#### IV. CONCLUSION

In standard quantum field theory, the Zitterbewegung term derived from the Dirac equation has long been regarded as an artificial oscillation without direct physical observability. It has often been interpreted as a mathematical artifact arising from the interference between positive and negative energy solutions. In contrast, the present study reinterprets Zitterbewegung as a physically real internal oscillation of the electron, with a well-defined frequency and velocity tied to the internal structure described by the 0-Sphere model.

By incorporating general relativistic corrections—specifically, geodetic precession and the adjustment of the critical radius—the author has previously refined the effective Zitterbewegung velocity from the special relativistic estimate of  $v_{e,SR} = 0.040472c$  [3] to a corrected value  $v_{e,SR+GR} = 0.040374c$  at Earth’s surface [4]. The study further predicts that this velocity would be slightly higher at satellite altitudes due to gravitational time dilation, in accordance with general relativity.

The core prediction of this work is as follows: if Zitterbewegung, as a physical phenomenon, were to be experi-

mentally observed—especially through frequency-sensitive measurements in differing gravitational potentials such as ground-based and satellite-based platforms—a measurable discrepancy in the internal oscillation frequency would emerge. This discrepancy would reflect gravitational modulation of the internal dynamics and provide empirical evidence that Zitterbewegung is not merely a mathematical term in the Dirac equation but a real dynamical process subject to the structure of spacetime.

More fundamentally, this study suggests that the electron’s internal motion, as modeled by the 0-sphere framework, embodies a simple harmonic oscillator whose frequency plays the role of a clock. This internal frequency, subject to gravitational redshift, provides a natural link between quantum field dynamics and general relativistic curvature. Thus, Zitterbewegung may not only represent a subatomic phenomenon, but also offer a concrete mechanism for integrating time in quantum systems with proper time in curved spacetimes—a potential cornerstone for future unified models of quantum gravity.

Furthermore, the author proposes that the true, proper-time-based Zitterbewegung velocity is inherently slower in deeper gravitational potentials, such as at Earth’s surface, compared to higher altitudes. This suggests that the quantity  $v_{e,SR+GR}$  computed at ground level is itself redshifted relative to what would be observed in a higher gravitational potential. Although the current numerical precision (on the order of  $10^{-5}$ ) is insufficient to resolve this relativistic correction (on the order of  $10^{-10}$ ), future advances in measurement precision may make this difference detectable.

If such ultra-high precision measurements of electron internal dynamics become feasible, the Zitterbewegung frequency could serve as a quantum-scale probe of spacetime curvature, similar to how atomic clocks are used today. In that case, even a single electron could function as a **relativistic quantum clock**, exhibiting frequency shifts across gravitational gradients. This would reinforce the physical reality of Zitterbewegung and strengthen the case for its integration into the broader framework connecting quantum theory and general relativity.

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