# Corpuscles as packets of strings

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**ABSTRACT**

From Restraint Relativity it is possible to consider a corpuscle as a packet of strings. The variation of the length this packet is equal to the phase speed of the corpuscle as a packet of waves times an universal constant. Considering the interaction of the corpuscle with vacuum we deduce the link between the two packets.

## 1-Revisited Special Relativity:

The introduction of Special Relativity by Einstein in 1905 is due only theoretical thinking that Maxwell equations of the electro-magnetic field should be the same in all inertial frames. Those equations in vacuum are:

$$\nabla^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} \qquad (1)$$

$$\nabla^2\vec{B} = \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2} \qquad (2)$$

Which are two equations of waves with a speed of propagation equal to $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ with:

$\varepsilon_0 = 8.85\ 10^{-12}\ Fm^{-1}$ : the permittivity of vacuum .

$\mu_0 = 4\pi 10^{-7} Hm^{-1}$ : the permeability of vacuum.

But what is vacuum? If vacuum signify that there is nothing but here there is the electromagnetic field and so there is temperature. The only response is that vacuum is when there is nothing and the temperature is zero absolute or approaches asymptotically to zero. If there is any electromagnetic field in vacuum it signify the fundamental state of the field : there is no emission of energy.

In Special Relativity based on the constancy of the speed of light in all inertial frames it is defined:

-Let's have two inertial frames $S(ct, x, y, z)$& $S'(ct', x', y', z')$ with $S'$ is moving in constant speed $v$ and the axes coincide at $t' = t = 0$ than the Lorentz transformations of space and time are as follows [1]:

$$\begin{aligned} ct' &= \gamma(ct - \beta x) \\ x' &= \gamma(x - \beta ct) \\ y' &= y \\ z' &= z \end{aligned} \qquad (3)$$

With $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ & $\beta = \frac{v}{c}$

Considering two events $A(t_A, x_A, y_A, z_A)$ & $B(t_B, x_B, y_B, z_B)$ in the frame $S$ the interval squared :

$$\Delta s^2 = c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 \qquad (4)$$

is invariant under Lorentz transformations.

If $\Delta s^2 > 0$ : than the interval is time-like.

$\Delta s^2 = 0$ : than the interval is light-like

$\Delta s^2 < 0$ : than the interval is space-like.

Lorentz transformations can be also considered as hyperbolic rotation in the 4th space dimensions as the following:

$$ct' = ct\cosh\psi - x\ sinh\psi$$

$$x' = -ct\sinh\psi + x\ cosh\psi \qquad (5)$$

$$y' = y$$

$$z' = z$$

With $th\psi = \beta$ , $cosh\psi = \gamma$, $sinh\psi = \gamma\beta$.

Space & time are considered as a four dimensions continuum that describes physics world with the Minkowski geometry related by relation (4).

The Minkowski geometry is not Euclidian due to the sign minus in (4).

The interval given by equation (4) is the “distance “ between the events $A$ & $B$ measured in a straight line and can be considered as the worldline of a free particle moving in constant speed between the points $A$ & $B$.

The interval measured between the two points $A$ & $B$ in any arbitrary path is:

$$\Delta s = \int_A^B ds \qquad (6)$$

With: $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$ : the line element of the space-time.

For a massive particle we define the proper time $\tau$ as a parameter $t(\tau), x(\tau), y(\tau)$ & $z(\tau)$ in order to determine the position of the particle in the 4th space-time as:

$$d\tau^2 = \frac{ds^2}{c^2} \qquad (7)$$

Which mean that:

$$d\tau = \sqrt{1 - \frac{v^2}{c^2}}.dt = \frac{dt}{\gamma} \qquad (8)$$

The total elapsed proper time interval is:

$$\Delta\tau = \int_A^B \sqrt{1 - \frac{v(t)^2}{c^2}}.\,dt \qquad (9)$$

Where $v(t)$ is the instantaneous speed of the corpuscle along its path.

If we introduce a rest frame in which the corpuscle is in rest along its path the proper time is the time recorded by a clock moving with the corpuscle.

It is convenient to introduce the indexed coordinates $x^i$ $(i = 0,1,2,3)$ so we have:

$x^0 = ct, x^1 = x, x^2 = y, x^3 = z$ & $ds^2 = g_{ij}dx^i dx^j$

where $g_{ij}$ are the covariant metric tensor [1] as the following:

$$[g_{ij}] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad (10)$$

Or in shorthand notation as $[g_{ij}] = diag[1, -1, -1, -1]$.

The contravariant of the metric tensor are the same as: $[g^{ij}] = diag[1, -1, -1, -1]$.

In transforming coordinates $x^i$ in a Minkowski spacetime to a new coordinates $x'^i$ the line & s'element $ds^2$ must take the same form as:

$ds^2 = g_{ij}dx^i dx^j = g_{ij}dx'^i dx'^j$

Which means that the transformation $x^i \to x'^i$ must satisfy:

$$g_{ij} = \frac{\partial x'^k}{\partial x^i}\frac{\partial x'^l}{\partial x^j} g_{kl} \qquad (11)$$

Which implies that the transformations must be linear to be a Lorentz transformations:

$$x'^i = \Lambda^i{}_j x^j + a^i \qquad (12)$$

Where $\Lambda^i{}_j$ & $a^i$ are constants .

Transformations (12) are the inhomogeneous Lorentz transformations (or Poincare transformations). With $a^i$taken equal to zero they are the ordinary Lorentz transformations (or homogeneous transformations).

In a standard configuration of two inertial frames $S$ & $S'$ the matrix transformation can be written as follows:

$$[\Lambda^i{}_j] = \left[\frac{\partial x'^i}{\partial x^j}\right] = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} cosh\psi & -sinh\psi & 0 & 0 \\ -sinh\psi & cosh\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (13)$$

Where : $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ , $\beta = \frac{v}{c}$ & $th\psi = \beta$ , $cosh\psi = \gamma,\ sinh\psi = \gamma\beta$

The inverse transformation of (13) are obtained by replacing $v\ by - v\ \ or\ \ \psi\ \ by - \psi$

The matrix inverse is:

$$[\Lambda_i{}^j] = \left[\frac{\partial x^j}{\partial x'^i}\right]$$

And the relation ship between the two matrices :

$$[\Lambda^i{}_j] = g^{ik} g_{lj} \Lambda_k{}^l$$

And we have:

$$\Lambda^i{}_k \Lambda_j{}^k = \Lambda^i{}_k g^{km} g_{jl} \Lambda^l{}_m = g^{mi} g_{jm} = \delta^i_j$$

In any coordinate system the coordinate basis vectors are tangents to the coordinate curves

In frames $S$ & $S'$ basis vectors are related by:

$$e'^j = \Lambda^j{}_k\, e^k \qquad\qquad e^j = \Lambda_k{}^j\, e'^k$$

And we have;

$$e'^i . e'^j = g^{ij} \qquad\qquad e^i . e^j = g^{ij}$$

Dual basis vectors are defined as:

$$e_i = g_{ik} e^k \qquad\qquad e'_i = g_{ik} e'^k$$

And we have:

$$e_i . e_j = g_{ij} \qquad\qquad e'_i . e'_j = g_{ij}$$

Where the components of $g_{ij}$ are the same as $g^{ij}$ .

We can define a 4-vector $\boldsymbol{v}$ of point $P$ in Minkowski spacetime as :

$$\boldsymbol{v} = v^i \boldsymbol{e_i}$$

At each point $P$ we have a set of orthonormal basis $\boldsymbol{e_i}$ . The square of the length of a vector $\boldsymbol{v}$ is :

$$\boldsymbol{v}.\boldsymbol{v} = v_i v^i = g^{ik} v_i v_k$$

In two inertial frames $S$ & $S'$ with Cartesian coordinates $x^i$ & $x'^i$ the coordinates of a 4-vector $\boldsymbol{v}$ at a point $P$ is :

$\boldsymbol{v} = v^i \boldsymbol{e_i} = v'^i \boldsymbol{e'_i}$

And we have :

$v'^i = \boldsymbol{v}.\boldsymbol{e'^i} = \Lambda^i{}_j v^j$

$v^i = \boldsymbol{v}.\boldsymbol{e^i} = \Lambda_j{}^i v'^j$

The 4-velocity $\boldsymbol{u}$ of a (massive ) corpuscle is as :

$\boldsymbol{u}.\boldsymbol{u} = (\frac{ds}{d\tau})^2 = c^2$

The contravariant components of the 4-velocity are:

$u^i = \boldsymbol{u}.\boldsymbol{e^i} = \frac{dx^i}{d\tau}$

It comes that:

$$[u^i] = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}.(c, \frac{dx^1}{dt}, \frac{dx^2}{dt}, \frac{dx^3}{dt})$$

With : $c = \frac{dx^0}{dt}$, $x^0 = ct$ & $v = \sqrt{\left(\frac{dx^1}{dt}\right)^2 + \left(\frac{dx^2}{dt}\right)^2 + \left(\frac{dx^3}{dt}\right)^2}$

The 4-moment of a massive corpuscle is defined as :

$\boldsymbol{p} = m\boldsymbol{u}$ (14)

With : $m$ : the mass of the corpuscle;

$\boldsymbol{u}$ : the 4-velocity of the corpuscle.

The component of the 4-moment are $p^i = \boldsymbol{p}.\boldsymbol{e^i}$ and we have:

$[p^i] = \left[\frac{E}{c}, p^1, p^2, p^3\right] = (\frac{E}{c}, \vec{p})$ (15)

With $E$ the energy of the corpuscle and $\vec{p}$ its 3 ordinary moment in an inertial frame $S$ .

It comes that:

$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$ & $\vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$ (16)

The squared length of the 4-moment is $p^i p_i$ and so get:

$(\frac{E}{c})^2 - p^2 = m^2c^2$ (17)

Where $p^2 = \vec{p}.\vec{p}$ .

For a photon (massless corpuscle) we can't use the proper time to parametrize its worldline which is a null curve because $d\tau = 0$. We can use an affine parameter $\sigma$ as for a photon moving in the direction $x = ct$ we have:

$$x^i = \sigma u^i \qquad (18)$$

With: $[u^i] = (1,1,0,0)$

The tangent vector of the worldline is:

$$\boldsymbol{u} = \frac{dx^i}{d\sigma}\boldsymbol{e_i} \qquad (19)$$

And we have:

$$\boldsymbol{u}.\boldsymbol{u} = 0 \quad \& \quad \frac{d\boldsymbol{u}}{d\sigma} = \boldsymbol{0} \qquad (20)$$

Equation (20) is the equation of motion of the photon.

Of course in equation (20) we consider that the vector $\boldsymbol{e_i}$ are constant and does not change with the position.

The tangent vector $\boldsymbol{u}$ can be multiplied by any scalar and the equation (20) will be satisfied. The 4-moment of a photon can be defined as:

$$\boldsymbol{p} = \mu \boldsymbol{u} \qquad (21)$$

Where $\mu$ is constant and the components of $\boldsymbol{p}$ are:

$$[p^i] = (\frac{E}{c}, \vec{p})$$

Where $E$ & $\vec{p}$ are respectively the energy and the 3-moment of the photon in a given inertial frame $S$.

It comes that from equation (20):

$$E = pc \qquad (22)$$

The same equation (22) can be obtained for massless corpuscles if we take $m = 0$ in equation (17).

Equation (17) is a more generalized equation (massive corpuscles and massless corpuscles).

For photon we can introduce the 4-wavevector $\boldsymbol{k}$ as :

$$\boldsymbol{p} = \hbar \boldsymbol{k} \qquad (23)$$

And the components of the 4-wavevector are:

$$[k^i] = (\frac{2\pi}{\lambda}, \vec{k}) \qquad (24)$$

Where : $\lambda$ : is the wave-length of the photon

$\vec{k} = \frac{2\pi}{\lambda}\vec{n}$ : the 3-waveevector of the photon

$\vec{n}$ : direction of the propagation of the photon ( 3-vector of one module).

Equation (24) like equation (17) can be generalized for any corpuscle (massless or not). Equating equation (24) to equation (15) we get:

$$\frac{E}{c} = \hbar.\frac{2\pi}{\lambda} \quad (25)$$

$$\vec{p} = \hbar\vec{k} \quad (26)$$

Which mean that massive corpuscle can have waving behavior.

The group speed $v_g$ of such wave behavior of massive corpuscle is defined as :

$$\frac{1}{v_g} = \frac{dk}{d\omega} \quad (27)$$

With : $\omega = 2\pi\nu = 2\pi\frac{c}{\lambda}$ : the frequency of the photon (the frequency associated to the massive corpuscle).

From (25) & (16) we get:

$$E = \hbar\omega = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad (28)$$

And from (26) & (16) we get:

$$\hbar\vec{k} = \vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} \quad (29)$$

From (27) we get:

$$v_g = v \quad (30)$$

The phase speed $v_f$ of such a wave is defined as:

$$v_f = \frac{\omega}{k} \quad (31)$$

We have always:

$$v_g.v_f = v\frac{\omega}{k} = c^2 \quad (32)$$

In equation (23) we had never speak about the constant $\hbar$ : it can be replaced by any other constant and the 4-vector linked in will follow also.

For any corpuscle we introduce the 4-string $\boldsymbol{l}$ as :

$$\boldsymbol{p} = a\boldsymbol{l} \quad (33)$$

And the components of the 4-string are:

$$[l^i] = (l, \vec{l}) \quad (34)$$

With : $s$: the string-length associated to the corpuscle

$\vec{l} = l\vec{n}$

$\vec{n}$ : direction of the propagation of the corpuscle ( 3-vector of one module).

Equating (34) and (15) we get:

$$\frac{E}{c} = al \quad (35)$$

$$\vec{p} = a\vec{l} \quad (36)$$

Let's search the vector $\frac{d\vec{l}}{d\omega}$ (considering the corpuscle as a packet of strings)?

$$\frac{dl}{d\omega} = \frac{dl}{dv}.\frac{dv}{d\omega} = \frac{1}{a}\frac{dp}{dv}.\frac{dv}{d\omega} = \frac{1}{a}\frac{dp}{d\omega} = \frac{\hbar}{a}\frac{dk}{d\omega} = \frac{\hbar}{a}.\frac{1}{v} = \frac{\hbar}{a}.\frac{v_f}{c^2}$$

It comes that:

$$\vec{v}_f = \frac{ac^2}{\hbar}\frac{d\vec{l}}{d\omega} \quad (37)$$

From (35) & (16) we have:

$$E = acl = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad (38)$$

From (36) & (16) we get:

$$a\vec{l} = \vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} = \hbar\vec{k} \quad (39)$$

We have always:

$$\vec{p} = \frac{E}{c^2}.\vec{v} = \frac{as}{c}.\vec{v} = \frac{\hbar\omega}{c^2}.\vec{v} \quad (40)$$

We can also define the 4-vector inertia $\boldsymbol{\xi}$ as:

$$\boldsymbol{p} = \boldsymbol{\xi}c = m\boldsymbol{u} \quad (41)$$

The components of the 4-vector inertia are:

$$[\xi^i] = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}.\left(1, \frac{dx^1}{dx^0}, \frac{dx^2}{dx^0}, \frac{dx^3}{dx^0}\right) = \xi.\left(1, \frac{dx^1}{dx^0}, \frac{dx^2}{dx^0}, \frac{dx^3}{dx^0}\right) \quad (42)$$

With: $x^0 = ct$ & $\xi = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$

In a system of units where $\hbar = c = a = 1$ equation (40) can written in simply form as:

$$\vec{p} = \xi\vec{v} = l\vec{v} = \omega\vec{v} \quad (43)$$

Inertia can be a mass or a length or a frequency or anything else as time for example.

We define the inertial time $\zeta$ as :

$$\zeta = \frac{\xi}{a} \qquad (44)$$

It is evident if we associate to the corpuscle an inertial time at rest $\zeta_0$ than according to equation (8) we have:

$$\zeta_0 = \sqrt{1 - \frac{v^2}{c^2}}\,.\zeta \qquad (45)$$

$d\zeta = dt$ : when the energy of the corpuscle change (46)

From (44) & (45) we get:

$$\zeta_0 = \frac{m}{a} \qquad (47)$$

Of course we can also define the 4-vector inertial time $\boldsymbol{\zeta}$ as:

$$\boldsymbol{\xi} = a\,\boldsymbol{\zeta} \qquad (48)$$

From equations (17) & (25) we get:

$$h^2\nu^2 - p^2c^2 = m^2c^4 \qquad (49)$$

With: $E = h\nu$ : the energy of the corpuscle

$\nu = \frac{\omega}{2\pi}$ : reduced inertial frequency of the corpuscle;

$h = 2\pi\hbar$

Differentiate (49) according to time:

$$h^2\nu d\nu = c^2pdp = \xi c^2\vec{v}.d\vec{p} = h\nu\vec{v}.d\vec{p}$$

Which mean:

$$\frac{d\vec{p}}{dt}.\vec{v} = h\frac{d\nu}{dt} = ac^2\frac{d\zeta}{dt} = \vec{f}.\vec{v} \qquad (50)$$

With: $\vec{f} = \frac{d\vec{p}}{dt}$ : the 3-dimensional force.

If the energy of the corpuscle change then we have always:

$$\vec{f}.\vec{v} = ac^2 \qquad (51)$$

An important case is when the speed of the corpuscle is low compared to $c$ . From equation (49) we get:

$$h\nu \approx mc^2 + \frac{1}{2}m\vec{v}^2 \qquad (52)$$

$$\vec{p} \approx m\vec{v} \qquad (53)$$

$$\vec{f} \approx m\frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt} \quad with\,\vec{p} = m\vec{v} \qquad (54)$$

which mean:

$h d\nu = m\vec{v}.d\vec{v}$

Than:

$h\frac{d\nu}{dt} = \vec{f}.\vec{v}$ (55)

Equation (55) is the same equation as (50) even thought it is obtained by approximation.

The square of (52) is :

$h^2\nu^2 = m^2c^4 + m^2c^2\vec{v}^2 + \frac{1}{4}m^2\vec{v}^4$ (56)

$2h^2\nu\frac{d\nu}{dt} = m^2c^2 2\vec{v}.\frac{d\vec{v}}{dt} + \frac{1}{4}m^2\frac{d\vec{v}^4}{dt}$

$h\frac{d\nu}{dt} = \frac{mc^2}{h\nu}\vec{v}.\vec{f} + \frac{1}{4}m^2\frac{d\vec{v}^4}{dt}.\frac{1}{2mc^2+m\vec{v}^2} \approx \frac{mc^2}{h\nu}\vec{v}.\vec{f} = \vec{f}.\vec{v}$ (57)

which mean :$h\nu \approx mc^2$ for low speed.

From equation(50):

$h\nu^2 = \frac{m^2c^4}{h} + \frac{p^2c^2}{h}$

$\frac{d(h\nu^2)}{dt} = 2\frac{c^2}{h}\vec{p}.\frac{d\vec{p}}{dt} = 2\frac{c^2}{h}.\frac{h\nu}{c^2}\vec{v}.\vec{f} = 2\nu ac^2 = 2h\nu\frac{d\nu}{dt}$ (58)

Which mean:

$ac^2 = h\frac{d\nu}{dt}$ (59)

Than:

$\nu = \frac{ac^2}{h}t + constant$ when the energy of the corpuscle is changing.

But in this case we have: $dt = d\zeta$ so:

$\nu = \frac{ac^2}{h}\zeta$ which is a trivial result (with $constant = 0$) (60)

Equation (51) signify that a corpuscle never can't be in rest otherwise the universal constant $a$ will be equal to zero. Than absolute vacuum never exist in a frame $S$ : there is energy independent from the corpuscle and which don't let the corpuscle become in rest.

**2.Wave-corpuscle duality:**

Constant $h$ was suggested and determined in 1900 by Max Planck in his model of black body radiation [2]. Planck model of black body was a cavity with many oscillators on his wall exchanging energy with radiation inner. Planck determine the mean energy of an oscillator and the radiation law at equilibrium in a temperature $T$ of the cavity and get the values of constants $h$ & $k_B$ (Planck constant and Boltzmann constant) by a statistical manner and referring earlier to F. Kurlbaum & Wien measurements of black body radiation.

In his recent theory of heat radiation by 1911 Planck suggest that his oscillator should have enough time to absorb energy from radiation[3] . The Planck condition for this is that the frequency of the oscillator times time should be a great integer or in other manner:

$$\nu . t \gg 1 \qquad (61)$$

By 1923 De Broglie suggest the wave-corpuscle duality i.e. a corpuscle can have a waving behavior and vice versa: he propose the relation (27) and getting (28) & (29) by referring to Restraint Relativity proposed by A. Einstein in 1905.

In his suggestion De Broglie consider a corpuscle as a packet of waves moving in the same direction of the corpuscle and represented with a wave function having a constant amplitude. De Broglie consider a corpuscle as many monochromatic waves which reinforce each other in a limited region of spacetime and destroy each other away. Mathematically speaking this model can't be adopted because the waves will reinforce each other in many regions of spacetime in the same direction of the corpuscle motion and we can't see where is really the corpuscle. A replacement model of De Broglie one is that a corpuscle should be considered as a pulse and not as a packet of waves because in this case the corpuscle is more identified with the condition that the pulse should not spread so much in time. This condition implies that:

$$\nu . t < 1 \qquad (62)$$

The condition satisfying the two models (Planck & De Broglie) is:

$$\nu . t \approx 1 \qquad (63)$$

Which means that:

$$dt = -\frac{d\nu}{\nu^2} \qquad (64)$$

We take always $dt$ positive and so we can omit the sign minus in (64) or take its absolute value:

$$dt = \frac{d\nu}{\nu^2} \qquad (65)$$

Replace (63) in (55) for low speed corpuscle we get:

$$h\nu^2 = \vec{f} . \vec{v} \qquad (66)$$

Replace (63) in (58) for high speed corpuscle we get:

$$h\nu^2 = \vec{f} . \vec{v} \qquad (67)$$

Equation (67) is the same equation as equation (66) so it is applicable for any kind of oscillator. It is the power absorbed or emitted by an oscillator in a black body.

Planck oscillator is a classic oscillator so equation (66) is the work of a force applied on a corpuscle by unit time i.e. for a corpuscle in motion in a straight line along the axis $x$ for example:

$$\vec{f}.\vec{v} = fv = h\nu^2$$

So the force acting on the corpuscle is:

$$f = \frac{1}{v}.h\nu^2$$

Duality of wave-corpuscle implies that:

$$\frac{1}{v} = \frac{dk}{d\omega}$$

with $v = v_g$ : the group speed of the packet of waves assimilated as a corpuscle;

$k$: wave-vector of the packet of waves

$\omega = 2\pi\nu$ : the frequency of the packet of waves.

So:

$$f = h\nu^2.\frac{dk}{d\omega} = h\nu^2.\frac{dk}{2\pi d\nu} = \hbar\nu^2.\frac{dk}{d\nu} = \frac{d(\hbar k)}{dt}$$

With : $\hbar = \frac{h}{2\pi}$ : reduced Planck constant.

$\hbar k$ : have the dimension of a moment. So:

$f = \frac{dp}{dt}$ with $p = mv$ : is the moment of the corpuscle.

This relation is generalized as:

$$\vec{f} = m\vec{\gamma} \qquad (68)$$

With: $\vec{\gamma} = \frac{d\vec{v}}{dt}$ the acceleration of the corpuscle

$m$ : the mass of the corpuscle.

Equation (68) is the fundamental law of dynamics or the Newton first law.

Planck oscillator can have also very high speed and equation (67) will be applicable. As Planck suggest that his oscillators can absorb or emit energy with quanta as a multiple integer of $h\nu$ at the same time we extend this idea that Planck oscillators can absorb power or emit power with quanta as a multiple integer of $h\nu^2$ .

Between the two models of oscillators (classic and relativist) there is a special frequency $\nu_0$ in the border of our consideration to balance in one side or another and to deduce from Planck model of black body radiation even thought to apply this model for vacuum which can be considered as a black body at zero absolute temperature (no radiation) or approximatively near zero absolute. Replace (65) in (59) we get:

$ac^2 = h\nu^2$ (69)

Which mean that the frequency $\nu$ is near $\nu_0$ changing all time otherwise there is no constant $a$ .

In fact we have exactly:

$a = \frac{h\nu_0^2}{c^2}$ (70)

Even thought the constant $\nu_0$ can be derived from classic approximations.

In general we have:

$$\frac{d\vec{p}}{dt} = \frac{d(a\zeta\vec{v})}{dt} = a\vec{v} + \frac{m}{\sqrt{1-\frac{\vec{v}^2}{c^2}}}\frac{d\vec{v}}{dt} \approx a\vec{v} + m\vec{\gamma} \quad \text{for } \|\vec{v}\| \ll c$$

Than:

$m\vec{\gamma} = \frac{d\vec{p}}{dt} - a\vec{v} = \vec{f} - a\vec{v}$ (71)

All forces applied on the corpuscle are equal to $\vec{f} = \frac{d\vec{p}}{dt}$ plus a permanent force $a\vec{v}$ in the opposite side of its motion which mean an external force which drive the motion of the corpuscle even thought it is in rest. The corpuscle never can't be in rest and will oscillate around its position.

**3.Determination of constant $\nu_0$ :**

Planck oscillators are classic oscillators. They are oscillating charged corpuscles. In a black body they are the electrons of the atoms of the wall of the black body cavity (oscillations of nucleus of atoms are neglected).

According to Bohr model of the atom, the electron in an hydrogen atom is moving in planetary motion (circular) as the speed of the electron is equal to $\alpha c$ where $\alpha = \frac{1}{137}$ the fine structure constant. The vacuum in atoms is the same vacuum in the cosmos. Vacuum which is filled with energy has a certain mechanical impedance and a negative pressure so there is no friction in the motion of any corpuscle as it is given by General Relativity.

Suppose that a corpuscle is in motion from a point $A$ to a point a point $B$, its energy exchanged with vacuum is:

$$\varepsilon = \int_A^B a\vec{v}.\vec{v}dt = \int_A^B a\vec{v}^2\,dt = \int_A^B ac^2\left(1 - \frac{\zeta_0^2}{\zeta^2}\right)d\zeta = ac^2(\zeta_B - \zeta_A) + ac^2\zeta_0^2\left(\frac{1}{\zeta_B} - \frac{1}{\zeta_A}\right) \qquad (72)$$

We have not do any approximation in equation (72). It is done in a 4-space dimensions frame .

Let's take an origin for the energy exchanged which is the point $A$ where the corpuscle is in rest, than:

$$\varepsilon = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 + ac^2\zeta_0^2\left(\frac{\zeta_0-\zeta}{\zeta_0\zeta}\right) = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 + mc^2\left(\sqrt{1-\frac{v^2}{c^2}} - 1\right) = mc^2\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 + \sqrt{1-\frac{v^2}{c^2}} - 1\right] = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\left[1 - 2\sqrt{1-\frac{v^2}{c^2}} + \sqrt{1-\frac{v^2}{c^2}}^2\right) = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)^2 \quad (73)$$

For the electron of Bohr atom we get:

$$\varepsilon = \frac{mc^2}{\sqrt{1-\alpha^2}}\left(1-\sqrt{1-\alpha^2}\right)^2 \approx \frac{1}{4}\alpha^4 mc^2 \quad (74)$$

With: $m = 9{,}1\ 10^{-31}Kg$ the mass of the electron.

If the electron is oscillating in a 4th space dimension than its oscillations are driven by an electromagnetic field in its fundamental state. The energy of an oscillator in its fundamental state is [4]:

$$E = \frac{1}{2}h\nu \quad (75)$$

The mean energy $U$ of the electromagnetic field which drive the electron is :

$$U = \frac{\frac{1}{2}h\nu}{\exp\left(\frac{\nu}{\nu_0}\right)-1} \quad (76)$$

Equation (76) is obtained by statistical manner as for low frequencies the energy of the oscillator is:

$$E \approx \frac{1}{2}h\nu_0 \quad (77)$$

Considering the electron as 4-space dimension corpuscle than we get:

$$\frac{1}{2}h\nu_0 = 4x\frac{1}{4}\alpha^4 mc^2 = \alpha^4 mc^2 \quad (78)$$

Which mean:

$$\nu_0 = 2\frac{\alpha^4 mc^2}{h} \approx 0.7\ 10^{12}\ Hz \quad (79)$$

## 4 -Vacuum energy from cosmology:

The energy density of vacuum as given by General Relativity is as follows[5] :

$$U_0 = \frac{\Lambda.c^4}{8\pi G} \approx 10^{-9} Joule.m^{-3} \quad (80)$$

With :

$\Lambda = 1{,}088\ 10^{-52}m^{-2}$ : cosmological constant ;

$c = 3\ \ 10^8 m.s^{-1}$ : light celerity in vacuum ;

$G = 6{,}67\ \ 10^{-11}\ SI\ units$ : gravitationnel constant ;

The total energy density of the vacuum is then according to the model of the black body theory:

$$U_0 = \int_0^\infty \frac{8\pi\nu^2}{c^3}.U d\nu = \int_0^\infty \frac{4\pi h}{c^3}.\frac{\nu^3}{\exp\left(\frac{\nu}{\nu_0}\right)-1} d\nu = \frac{4\pi^5 h}{15.c^3}.\nu_0^4 \qquad (81)$$

Equating (80) & (81) we get:

$$\nu_0 = \left[\frac{15\,\Lambda.c^7}{32.\pi^6.G.h}\right]^{\frac{1}{4}} \approx 0.7\;10^{12}\,Hz \qquad (82)$$

This is an experimental proof that the electron is a 4-space dimension corpuscle. It means also that extension in space is energy.

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