Inertial force in the d'Alambertian formalism of mechanics

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Abstract. We argue that the inertial force is a real force of gravitational origin. Specifically, it is a passive induction force that arises when a test particle accelerates relative to the gravitational field of the Universe. In the classical approximation this force depends linearly on the acceleration and has the opposite direction, allowing us to identify it with the classical inertial force. Using the d'Alembertian formalism of mechanics, we demonstrate that the inertial force arises from the interaction of the particle with the cosmic gravitational field, thus providing a consistent implementation of Mach's principle.

1.   Introduction

Inertia is a property of material bodies that enables them to resist changes in their state of motion, specifically changes in their velocity vector. In classical mechanics, this property is quantified by inertial mass m_i, which is operationally defined as the ratio of the net applied force to the resulting acceleration *.

On the other hand, gravitational mass m_g, more appropriately called gravitational charge, characterizes both the intensity of the gravitational field that a body generates (active gravitational mass) and the force it experiences in the presence of an external gravitational field (passive gravitational mass). Based on the equivalence principle (Segura, 2023a), we postulate that inertial mass and gravitational mass are proportional, and by appropriately choosing the value of the universal gravitational constant G, both can be made numerically equal. We complete this agreement by adopting units for G that guarantee the dimensional coincidence between both masses.

The equivalence principle does not rule out the possibility that the proportionality constant between inertial mass and gravitational mass is a function of time, which would have observational implications in non-classical contexts **, therefore in general

$$m_i = \chi(t) m_g$$

valid for all bodies in the Universe. $\chi(t)$ is the coefficient of inertia, which depends on time but not position. Experience shows us that $\chi(t)$ varies very slowly, so its variation will only be detectable in cosmic time intervals. We can consider it constant during the motion of a test body.

From a conceptual point of view, we define an inertial reference frame as one in which the law of inertia holds: an isolated body remains at rest or in uniform and rectilinear motion. In contrast, a non-inertial reference frame is accelerated relative to an inertial frame.

According to the principle of Special Relativity, all inertial reference frames are equivalent to formulating physical laws, whether they relate to mechanical, electromagnetic, or any other

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* The equivalence between mass and energy extends the property of inertia to non-material systems, although its physical meaning is controversial.
** Not all possible formulations of the equivalence principle are compatible with a time-dependent inertia coefficient. However, the universality of free fall remains valid, from which we deduce that the trajectory of a free particle in a gravitational field is a geodesic (Segura, 2023a).

phenomena; this implies that it is impossible to determine an inertial frame's absolute velocity through physical experiments.

However, no general principle of relativity postulates complete equivalence between inertial and non-inertial frames. In particular, the law of inertia does not hold in non-inertial frames. Nevertheless, the general principle of covariance (Tolman, 1987, pp. 166–174) remains valid, according to which physical laws retain their mathematical form under any space-time transformation. In other words, the fundamental laws of physics can be expressed by tensor equations defined in four-dimensional space-time, which are invariant under any change of coordinates.

We can, however, impose the formal validity of the laws of mechanics in non-inertial systems by introducing fictitious forces (Segura, 2019), which do not correspond to true interactions since they do not arise from the action of external physical bodies or fields. However, we call them forces because, like real forces, we obtain them as the product of the body's mass and an acceleration. Their introduction is necessary to preserve the usual form of mechanical laws in non-inertial frames. However, this does not imply that such systems are equivalent to inertial ones. In particular, the acceleration of a non-inertial system is an absolute magnitude, in the sense that we can determine it through mechanical or other experiments.

We will see later that this distinction between inertial and non-inertial frames has a profound justification within the framework of Mach's principle. This principle provides an explanation for the inequivalent nature of reference frames. In particular, it clarifies why inertial frames have a privileged status: in them, the laws of mechanics take their simplest form.

It is common to identify fictitious forces with inertial forces. As discussed later, the latter are real forces because they result from interactions with physical systems.

2.   D'Alambertian formalism of mechanics

The Newtonian formulation of the second law of dynamics states that

$$\mathbf{F} = m_i \mathbf{a} \tag{1}$$

where $\mathbf{F}$ is the sum of the forces applied to the body of inertial mass m_i, and $\mathbf{a}$ is the acceleration acquired by the body and measured with respect to an inertial reference system. However, we can formulate it differently using d'Alembert's principle. Both formulations are mathematically identical but conceptually very different.

D'Alembert's principle in classical mechanics has two postulates:

1) The force of inertia is real * and acts on everybody accelerated with respect to an inertial reference system; it is proportional to the acceleration and in the opposite direction; the coefficient of proportionality is the inertial mass

$$\mathbf{F}_i = -m_i \mathbf{a}. \tag{2}$$

2) Principle of dynamic equilibrium, according to which the sum of the forces applied to a body and the force of inertia is identically zero

$$\mathbf{F} + \mathbf{F}_i = 0.$$

(2) is a mathematical formula identical to (1), and therefore, from both formulations, we deduce the same equations of motion. The difference is that the d'Alambertian formulation requires the existence and reality of the inertial force, and we use this circumstance to develop Mach's principle.

Since we consider the force of inertia to be real, it must be associated with the action of another body or a physical field. However, d'Alembert's principle does not indicate the physical origin of this force.

From equation (2), we observe that the force of inertia manifests itself only when an external force acts that induces acceleration on the body. Consequently, the force of inertia only exists when a body accelerates relative to an inertial reference frame; we understand it as a reaction associated with the change in the body's state of motion.

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* Inertial forces can be measured experimentally (Graneau & Graneau, 2006, pp. 157-164)..

2. Mach's principle

In the following arguments, we consider the Universe divided into two components: on the one hand, nearby objects characterized by an irregular spatial distribution —such as the Sun, the Moon, or the nucleus of our galaxy— and on the other, the external Universe, consisting of a homogeneous and isotropic distribution of matter by the cosmological principle, which we will henceforth call the Cosmos.

Due to their irregular distribution, nearby bodies exert a net gravitational force on local bodies, which, to a first approximation, is described by Newton's theory of gravity. A fundamental question then arises: Does the rest of the Universe —the Cosmos— exert any influence on local phenomena?

Within the Newtonian framework, the Cosmos is an infinite and isotropic medium. Although each of the cosmic bodies exerts a gravitational force on a local test body, the vector superposition of these forces, due to the spherical symmetry of the system, results in a zero net force. Consequently, within the Newtonian framework, the Cosmos has no dynamic effect on local phenomena *. We can state that the bodies in our immediate surroundings would be isolated from the gravitational action of the rest of the Universe.

Conceptually simple experiments led Ernst Mach to postulate that the Cosmos exerts some influence on local bodies. Although Mach did not explicitly formulate a concrete theory, we will refer here to the assertion that the inertial force acting on an accelerated body (relative to an inertial frame) is the result of the interaction with the rest of the Universe as Mach's principle **. From this perspective, inertial mass is not an intrinsic property of the body but instead emerges as a relational property, a product of interaction with the Universe as a whole.

From this interpretation, the Cosmos, considered on average and according to the cosmological principle, defines a global inertial reference system. In contrast, a non-inertial system is accelerated with respect to this cosmic background.

It is important to note that, in his critique of Newtonian mechanics (Mach, 1960, pp. 271–297), Mach intertwined two conceptually distinct ideas. On the one hand, Mach's principle already mentioned, and on the other, the notion of the relationalism of motion, according to which all mechanical quantities (position, velocity, acceleration) have no absolute meaning and only acquire meaning when defined in relation to other bodies.

In this context, the notion of an absolute space, as postulated by Newton, is unacceptable. The second Machian conception gives rise to what we call relational mechanics, which interprets all dynamical quantities as relative (Schrödinger, 1995, pp. 145–158). Although both ideas —Mach's principle and relational mechanics— originate in a common criticism of Newtonian foundations, they are conceptually distinct and, as will be analyzed later, present elements that may be mutually incompatible. Mach's principle, if true, would be of great importance because we can deduce cosmic properties from local phenomena.

3. D'Alembert-Mach principle

We can combine the principles of dÀlembert and Mach in a single formulation. From the statement of this last principle, we understand that the action of gravitational origin of the whole Cosmos $\mathbf{F}_g$ on a test body is the force of inertia, then we formulate the principle of dynamic equilibrium as

$$\mathbf{F} + \mathbf{F}_g = 0 \tag{3}$$

$\mathbf{F}$ are the applied forces, excluding the gravitational forces exerted by the Cosmos. We will refer to expression (3) as the d'Alembert-Mach principle and use this formulation as the basis for further development.

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* In other words, in the Newtonian conception, the gravitational potential of the Cosmos is independent of the position of the body. It, therefore, does not produce a net gravitational force since the divergence of the cosmic potential is zero because it is independent of the position.

** There are several ways to define Mach's principle, but not all of them are equivalent (Bondi & Samuel, 1996), (Assis, 1999, p. 121).

4.   Gravitational induction

The induction force is produced by the motion of either the source of the field or the test particle on which the force acts. If the motion of the source causes the induction, we call it active induction, and when the motion of the test particle produces it, we call it passive induction. Both types of induction can co-occur if both the source and the particle are in motion.

Active induction arises because the interaction propagates at the speed of light; if the force were action at a distance, there would be no active induction. Active induction occurs whenever the source is in motion, whether uniform or accelerated. In the latter case, sources can radiate energy. Passive induction does not exist in electromagnetism, but it does exist in a gravitational field, which suggests that the force of inertia has a gravitational origin. Our fundamental hypothesis is the identification of the force of inertia with the force of passive induction caused by the Cosmos, which, as we will see later, is a force of interaction of the test particle with the cosmic gravitational field, that is, with a pre-existing field.

Active induction is a well-known phenomenon in electromagnetism, and its theory is fully developed (Panofsky & Phillips, 1972, pp. 341–376). With some differences, we extend it to gravitation. The idea that the force of gravitational induction is responsible for inertia arose in Sciama's famous article "The origin of inertia" (Sciama, 1953). Since Mach's principle is one of the most recurrent topics in physics, it is surprising that very few authors have pursued Sciama's idea. (Martín, Rañada and Tiemblo, 2007), (Veto, 2013), (Hessén, 2012), and (Segura, 2028), as well as Sciama, have attempted to derive the force of inertia from active induction. For this, they assume that the acceleration of a body with respect to the Universe is relative; that is, the accelerated movement of the Universe with respect to the test particle is equivalent to the movement with the same acceleration and in the opposite direction of the particle with respect to the Universe. However, this idea is incorrect because acceleration is not a relative magnitude but rather refers to an inertial reference system; that is, it is an absolute magnitude *.

5.   Equation of motion in a gravitational field

We derive the electromagnetic potentials from Maxwell's equations and complete the theory with the Lorentz equation of motion. In contrast, General Relativity is a nonlinear theory, and we can derive the equation of motion from its field equations. Therefore, it is not an additional element, as it is in electromagnetism.

If we follow the Newtonian formalism of the second law of dynamics and assume equality between inertial and gravitational mass, the equation of motion of a free particle in a gravitational field is the geodesic equation

$$\frac{d^2 x^k}{d\tau^2} + \Gamma_{pq}^k \frac{dx^p}{d\tau} \frac{dx^q}{d\tau} = 0 \qquad (4)$$

τ is the particle's proper time, and Γ_{pq}^k is the affine connection of gravitational origin (Latin indices range from 0 to 3 and Greek from 1 to 3) **. Note that $d^2 x^\alpha / d\tau^2$ is the Cartesian component of the acceleration. Using the coordinate time instead of the proper time, we transform equation (4) (Segura, 2013, pp. 48-51)

$$\frac{d^2 x^\alpha}{dt^2} = -\Gamma_{00}^\alpha c^2 - 2c\Gamma_{0\beta}^\alpha \frac{dx^\beta}{dt} - \Gamma_{\gamma\beta}^\alpha \frac{dx^\beta}{dt} \frac{dx^\gamma}{dt} + \left(\Gamma_{00}^0 c + 2\Gamma_{0\beta}^0 \frac{dx^\beta}{dt} + \frac{1}{c}\Gamma_{\gamma\beta}^0 \frac{dx^\gamma}{dt} \frac{dx^\beta}{dt} \right) \frac{dx^\alpha}{dt}. \qquad (5)$$

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* By assuming that active induction is responsible for inertia, we employ the technique of retarded potentials, a well-known concept in electromagnetism (Landau & Lifshitz, 1971, pp. 158-160). We then need to explain why the inertial force changes at the very instant that there is a change in the relative motion of the source when, strictly speaking, a time should elapse before the disturbance reaches the source particle. Another absurd situation that arises with this approach is that when a test body accelerates, bodies in the Universe could radiate energy in the form of gravitational waves. These difficulties have been overcome through action-at-a-distance theories of gravity or using advanced potentials.

** The affine connection appears in the equation of motion for three reasons: the use of curvilinear coordinates, the non-inertial nature of the reference frame, and the gravitational field. We choose an inertial reference frame with Cartesian coordinates so that the affine connection has only one gravitational origin, and we can

Developing (5) up to the second order with respect to the inverse of c, we find (Weinberg, 1972, pp. 212-222) (Segura, 2013, pp. 48-51)

$$\mathbf{a} = -\nabla\phi - 4\frac{\partial \mathbf{A}}{\partial t} + 4\mathbf{u}\wedge\left(\nabla\wedge\mathbf{A}\right) - \nabla\left(\frac{2\phi^2}{c^2} + \frac{\psi}{c^2}\right) + 3\frac{\mathbf{u}}{c^2}\frac{\partial\phi}{\partial t} + 4\frac{\mathbf{u}}{c^2}\left(\mathbf{u}\nabla\right)\phi - \frac{u^2}{c^2}\nabla\phi \approx$$

$$\approx -\nabla\phi - 4\frac{\partial\mathbf{A}}{\partial t} + 4\mathbf{u}\wedge\left(\nabla\wedge\mathbf{A}\right) - \frac{1}{c^2}\nabla\psi + 4\frac{\phi}{c^2}\mathbf{a} + 3\frac{\mathbf{u}}{c^2}\frac{\partial\phi}{\partial t} + 4\frac{\mathbf{u}}{c^2}\left(\mathbf{u}\nabla\right)\phi + \frac{u^2}{c^2}\mathbf{a}. \qquad (6)$$

In (6), there are three potentials: ϕ is the Keplerian potential, $\mathbf{A}$ is the vector or gravitomagnetic induction potential, and ψ is the inductive scalar potential; $\mathbf{u}$ is the velocity of the test particle and, in first approximation, $\mathbf{a} = -\nabla\phi$ *. These potentials are related to the components of the metric tensor of the gravitational field

$$g_{00} = \frac{2\phi}{c^2} + \frac{2\phi^2}{c^4} + \frac{2\psi}{c^4}; \quad g_{0\alpha} = -\frac{4}{c}A^\alpha; \quad g_{\alpha\beta} = -\delta_{\alpha\beta} + 2\delta_{\alpha\beta}\frac{\phi}{c^2}.$$

From Einstein's field equations, we find the expressions for the gravitational potentials. In the case of a weak gravitational field, such as the one we are considering, the vector potential $\mathbf{A}$ is formally identical to the electromagnetic one; that is, it depends on the velocity of the source. The second scalar potential ψ, which has no equivalent in electromagnetism, is equally inductive, that is, also dependent on the velocity of the source. Note that ϕ is the Keplerian gravitational potential that causes the particle's motion, which is why we have set $\mathbf{a} = -\nabla\phi$ in (6). We are not considering the gravitational potential created by the entire Universe or cosmic gravitational potential, therefore, the potential ϕ that appears in (6) is that produced by nearby bodies.

In (6), there are three types of terms: the first summand is the Keplerian term; the second, third and fourth terms are those of active induction, that is, generated by the motion of the source; and the remaining terms are the passive induction terms, that is, those arising from the motion of the test particle.

In (6), the first term is of zero order, and all the remaining terms are of second order, which are perturbative terms and originate the gravitoelectromagnetic effects, especially the planetary perturbations (Segura, 2014, pp. 105-120).

6.   Calculation of the inertial force

We consider a body of sufficiently large mass, M, inside a spherical shell of negligible thickness and having a uniformly distributed mass. At a point inside the spherical shell, the gravitational potential is

$$\phi = \phi_M + \phi_S$$

ϕ_M is the potential created by M, and ϕ_S is the potential of the shell, which is the same at every point inside. Let us assume that both M and the shell are at rest relative to an inertial frame. The gravitational force acting on a test particle of gravitational mass m_g inside the spherical shell is (6)

$$\mathbf{F}_g = m_g\left[-\nabla\phi - 4\frac{\phi}{c^2}\nabla\phi + 3\frac{\mathbf{u}}{c^2}\frac{\partial\phi}{\partial t} + 4\frac{\mathbf{u}}{c^2}\left(\mathbf{u}\nabla\right)\phi - \frac{u^2}{c^2}\nabla\phi\right] =$$

$$= m_g\left[-\nabla\phi_M - 4\frac{\phi_M}{c^2}\nabla\phi_M + 3\frac{\mathbf{u}}{c^2}\frac{\partial\phi_M}{\partial t} + 4\frac{\mathbf{u}}{c^2}\left(\mathbf{u}\nabla\right)\phi_M - \frac{u^2}{c^2}\nabla\phi_M\right] + m_g\left(-4\frac{\phi_S}{c^2}\nabla\phi_M + 3\frac{\mathbf{u}}{c^2}\frac{\partial\phi_S}{\partial t}\right), \qquad (7)$$

where the inductive potentials $\mathbf{A}$ and ψ are zero. We have divided the terms into two groups: the forces arising from the mass M and the forces produced by the spherical shell. Note that the spatial variations of ϕ_S are zero. The first group of (7) contains the Keplerian term and the perturbative forces that originate the gravitoelectromagnetic effects.

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then express the gravitational force as (4).

* In (6), the acceleration we use is the coordinate. The acceleration defined by the distance and proper time of the point in the field where the observer is would make more physical sense. However, as we will see later, the use of either acceleration does not alter our final results. When developing the gradient of the scalar potential, second-order active induction terms arise (Panofsky & Phillips, 1972, pp. 354-357).

We now consider a simplified model of the Universe, consisting of consecutive spherical shells of negligible thickness, which obey the cosmological laws of the standard model. The total gravitational potential originating in the Universe ϕ_c is then the sum of the potentials of each of the spherical shells. Summing the gravitational forces produced by each of the shells, we determine by (7) the total force exerted by the Cosmos

$$\mathbf{F}_c = m_g \left(4\frac{\phi_c}{c^2}\mathbf{a} + 3\frac{\mathbf{u}}{c^2}\frac{\partial \phi_c}{\partial t} \right)$$

where we have taken into account that the cosmic potential is the same at every point and that the acceleration acquired by the particle is only produced by the force exerted by the body *M*, therefore $\mathbf{a} \approx -\nabla \phi_M$. If we also consider that the particle is not relativistic and accept that the cosmic potential varies very slowly with time, we find that the cosmic gravitational force acting on a body of mass mg is

$$\mathbf{F}_c = 4m_g \frac{\phi_c}{c^2}\mathbf{a} \qquad (8)$$

and which, according to our initial hypothesis, we identified with the inertial force. We note that (8) is also derived from the Einstein-Infeld-Hoffmann equations of motion, which in turn are obtained from the gravitational field equations.

We have deduced (8) for a particle subjected exclusively to a gravitational field. However, it is logical to assume that the same cosmic or inertial force will arise when a force of another origin accelerates the body. We conclude that, in general, the inertial force acting on an accelerated body is of cosmic origin and equal to (8).

By identifying the inertial force with (8), we relate the inertial and gravitational masses

$$m_i = -4m_g \frac{\phi_c}{c^2}$$

Since, at present, the inertial and gravitational masses are equal, it must be true that the cosmic gravitational potential is $-c^2/4$ approximately, as calculated by other authors using simplified procedures.

We can understand that force (8) derives from a cosmic vector potential *

$$\mathbf{A}_c = -\frac{\phi_c}{c^2}\mathbf{u} \qquad (9)$$

and in similarity with (6) we find the inertial force by

$$\mathbf{F}_g = m_g \left[-4\frac{\partial \mathbf{A}_c}{\partial t} + 4\mathbf{u} \wedge \left(\nabla \wedge \mathbf{A}_c \right) \right].$$

From (8) we deduce the following properties of the inertial force:

* The inertial force acting on a body accelerated relative to an inertial frame is of gravitational origin and is the passive induction force exerted by the Universe.

* By identifying the inertial force with the induction force, we confirm that inertia is the result of the action of the entire Universe. That is, inertia is not an innate property of bodies but is acquired through interaction with the cosmic gravitational field, as required by Mach's principle.

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* Using the Keplerian expression of the potential, (9) is

$$\mathbf{A}_c = \frac{G}{c^2}\int \frac{\rho \mathbf{u}}{d}dV$$

ρ is the density of matter. We verify that the vector potential is the same as that found by assuming the inertia generated by active induction, that is, by the overall motion of the entire Universe. However, in this case, other inductive terms arise from the gradients of the scalar potential and the inductive scalar potential. This identity of both types of inductive potentials occurs if the Cosmos moves with uniform motion with respect to the test particle by applying the principle of Relativity, which tells us that speed is relative; however, if the motion is accelerated, we can no longer make the previous identification because acceleration is an absolute magnitude.

* The inertial force, in the classical approximation in which it is valid (8), depends only on acceleration.

* In the classical approximation, the induction force, like the inertial force, is proportional to acceleration and has the opposite direction (note that in (8), the cosmic potential is negative).

* There is no induction force when the body is in uniform, rectilinear motion relative to an inertial reference frame.

* The Universe is an inertial reference frame since the law of inertia holds concerning it; that is, the inertial force is zero. In a certain sense, we can identify Newton's absolute space with the Universe.

* The coefficient of inertia $\chi(t)$ is universal, the same everywhere in the Universe since it does not depend on position and varies with time.

* The force of active induction is not the cause of inertia, as several authors have proposed, although active induction can produce forces on the test particle.

* Mach's principle links a local property of matter, such as inertial mass, with a cosmic property, such as its gravitational potential. Local measurements, therefore, reveal cosmic characteristics.

* In a dynamic Universe, such as the Standard Model, the cosmic potential varies over time, which means a variation in inertial mass, which should produce observable effects. Note that in our reasoning, we assume that gravitational mass is an invariable magnitude of a body, as is the case with electric charge.

* Local phenomena are not isolated from the action of the Cosmos.

* In our hypothesis, the Universe fulfils the same function as Newton's absolute space, namely, defining the "good" reference or inertial frames. We can say that the Universe is the materialization of Newtonian absolute space.

* If we identify the inertial force as a passive induction force, it is modified instantaneously when the acceleration of the test body changes, which is not the case if we assume that inertia is the result of active induction.

* Inertial mass varies over time as a result of the change in cosmic potential.

* All gravitational sources in the observable Universe contribute to the formation of inertial mass.

7.   Cosmic potential

To obtain practical results from (8), it is necessary to find the cosmic potential ϕ_c, which poses certain problems. From the simplified cosmic model chosen, we assume that the cosmic gravitational potential is

$$\phi_c = -G \int \frac{\rho}{d} dV.$$

It is logical to think that the density of the source of gravity (matter, radiation and vacuum) must be calculated at the retarded time, and dV is the retarded proper volume element. We encounter more problems in understanding the meaning of the distance d that appears in the denominator, as it can be identified with any of the various proper distances used in cosmology (Hogg, 2000).

To avoid an excessive extension of this investigation, we reserve for a later work the calculation of the cosmic potential and some of its consequences.

8.   Conclusions

We use d'Alembert's formalism of the second law of mechanics combined with Mach's principle. We identify the force of inertia as a real force acting on anybody accelerated relative to an inertial frame.

We show that in gravitation, there are two types of induction, i.e., forces originating from motion: active induction, which results from the motion of the field source, which also exists in electromagnetism, and passive induction, which is forces arising from the motion of the particle. This type of induction is absent in electromagnetism.

We demonstrate that passive gravitational induction arises from the field equations, while

passive induction arises from the equation of motion.

One of the various passive induction forces depends linearly on the acceleration of the test particle and has the opposite direction. We calculate this force in a simplified model of the Universe and verify that we can identify it with the inertial force. Therefore, the inertial force originates in the gravitational action of the entire Universe, as follows from Mach's principle.

More specifically, the inertial force results from the test particle's interaction with the pre-existing cosmic gravitational field. Thus, a change in the particle's acceleration instantaneously modifies the inertial force, which would not occur if, as some authors have done, we assume that the origin of inertia is the active induction force.

We note that other authors find expressions similar to (8); however, what distinguishes our result is its distinct deduction, based on what we have termed passive gravitational induction.

To effectively calculate the inertial force, we must calculate the gravitational potential of the Cosmos, which raises several problems, the solution of which we leave for further investigation.

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