

Title: An Algebraic Derivation to Determine Lorentzian Broadening from Voigt Function in Plasma Diagnostics

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Abstract: This paper presents an algebraic derivation to express Lorentzian broadening,  $\Delta\lambda_L$  as a function of the Voigt,  $\Delta\lambda_V$  and Gaussian widths,  $\Delta\lambda_G$  providing a practical method for extracting electron density from spectroscopic data in plasma diagnostics.

In plasma spectroscopy, spectral lines are commonly modeled using the Voigt profile — a convolution of Lorentzian and Gaussian broadening. To estimate electron density, the Lorentzian component is especially important, as it reflects Stark broadening effects. However, many studies and fitting tools express the Voigt width as the subject of the equation, which can make it difficult for beginners to isolate the Lorentzian contribution [1, 2]. This paper provides a simple algebraic derivation to rearrange the standard expression and make the Lorentzian width as the subject, allowing clearer access to electron density calculations from spectroscopic data.

Starting equation (Eqn. 4 in [1]):

$$\Delta\lambda_v = \left[ \left( \frac{\Delta\lambda_L}{2} \right)^2 + \Delta\lambda_G^2 \right]^{\frac{1}{2}} + \frac{\Delta\lambda_L}{2}$$

Rearranged:

$$\Delta\lambda_v - \frac{\Delta\lambda_L}{2} = \left[ \left( \frac{\Delta\lambda_L}{2} \right)^2 + \Delta\lambda_G^2 \right]^{\frac{1}{2}}$$

Square both sides:

$$\left( \Delta\lambda_v - \frac{\Delta\lambda_L}{2} \right)^2 = \left( \frac{\Delta\lambda_L}{2} \right)^2 + \Delta\lambda_G^2$$

Expand the left-hand side:

$$\Delta\lambda_v^2 - \Delta\lambda_v\Delta\lambda_L + \frac{\Delta\lambda_L^2}{4} = \frac{\Delta\lambda_L^2}{4} + \Delta\lambda_G^2$$

Cancel the common terms:

$$\Delta\lambda_v^2 - \Delta\lambda_v\Delta\lambda_L = \Delta\lambda_G^2$$

Rearrange to solve for  $\Delta\lambda_L$ :

$$\Delta\lambda_L = \frac{\Delta\lambda_v^2 - \Delta\lambda_G^2}{\Delta\lambda_v}$$

## References

- [1] Torres J, van de Sande MJ, van der Mullen JJAM, Gamero A, Sola A. Stark broadening for simultaneous diagnostics of the electron density and temperature in atmospheric microwave discharges. *Spectrochimica Acta Part B: Atomic Spectroscopy*. 2006;61:58-68.
- [2] Nikiforov AY, Leys C, Gonzalez MA, Walsh JL. Electron density measurement in atmospheric pressure plasma jets: Stark broadening of hydrogenated and non-hydrogenated lines. *Plasma Sources Sci Technol*. 2015;24:034001.