White dwarfs and the Chandrasekhar limit: perspectives from kinetic theory and thermodynamics

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Abstract: In the Standard Model of gaseous stars, temperature plays an indispensable role in generating the gas pressure which prevents 'gravitational collapse'. Yet, as stars age in this model, changes in thermonuclear fuel lead to decreased temperatures and associated internal pressures. Gravitational forces between gas particles begin to dominate and stellar collapse results. The process results in ultra-dense compact objects including white dwarfs, neutron stars, and black holes. The Chandrasekhar limit plays a central role in the theory of white dwarfs by constraining dwarf mass. These transformations have been described using thermodynamic expressions. Yet, within any given thermodynamic relation, not only must units balance on each side, but so too must thermodynamic character. Whether or not equilibrium conditions are established, temperature must always be intensive in macroscopic thermodynamics and mass must always be extensive. The theory of temperatures and pressures within gaseous stars is constructed from the kinetic theory of an ideal gas, by which temperature is introduced, in combination with gravitational and Coulomb forces. The resulting thermodynamic relations impart non-intensive character to temperature, non-extensive character to mass, and thermodynamically unbalanced luminosity relations. Consequently, the theory of gravitational collapse of gaseous stars to form compact stellar objects is not valid. Stars cannot be gaseous in nature. Rather, they must be comprised of condensed matter, most likely metallic hydrogen, and therefore essentially incompressible.

Key words: White Dwarf; Chandrasekhar limit; Schönberg-Chandrasekhar mass; Mass-Luminosity Relation; Degenerate Electron Gas; Gravitational Collapse; Intensive and Extensive Thermodynamic Coordinates.

1. INTRODUCTION

The theory of gaseous stars is founded on the ideal gas law. In such a gas, no forces are experienced between the particles except when they collide elastically with one another and with the walls of their container.[1-9] Gas pressure is due entirely to the particle collisions with the walls.[2-11] The ideal gas law does not apply to charged particles or gravitationally bound particles. Nevertheless, in astrophysics, Coulomb forces and gravitational forces are introduced between particles of an ideal gas to generate thermodynamic relations to account for stellar processes. Yet in any proposed thermodynamic expression, not only must units balance but so must thermodynamic character. Failure to adhere to thermodynamic balance results in violations of the laws of thermodynamics. In fact, Landsberg argued that thermodynamic balance is so important that it should be regarded as the Fourth Law of Thermodynamics.[12] Canagaratna noted that "if one side of an equation is extensive (or intensive), then so

must be the other side."[13] The laws of thermodynamics must also hold wherever local thermodynamic equilibrium is utilised.[12]

In this respect, temperature is always intensive, in accordance with the 0th and 2nd laws of thermodynamics. Mass is always extensive. Consider a homogeneous system in thermal equilibrium. Divide the system into two identical parts geometrically similar to the original system. Those properties that are halved are extensive (e.g. mass, volume, internal energy); those which remain the same are intensive (e.g. temperature, density, pressure);[10, 11] those which change but not by half are neither extensive nor intensive, but they still possess thermodynamic character.[14] Mathematically, extensive thermodynamic coordinates are homogeneous functions of degree 1; intensive thermodynamic coordinates are homogeneous functions of degree 0. Luminosity is neither intensive nor extensive, being a homogeneous function of degree ⅔.[14] The quotient of two different extensive thermodynamic coordinates produces an intensive thermodynamic coordinate; density is an example, as it represents mass divided by volume. Pure numbers and physical constants of nature have no thermodynamic character because they can never be thermodynamic coordinates. The ideal gas law, $PV = NkT$, is thermodynamically balanced; the left side is extensive and the right side is extensive.

By way of an introductory example from astronomy and astrophysics, a crude value for the central temperature T_c of a white dwarf on the assumption of constant density is,[10]

$$T_c = \left(\frac{3\bar{\kappa}\rho}{4ac} \frac{L}{4\pi R} \right)^{1/4} , \tag{1}$$

where $\bar{\kappa}$ is the Rosseland mean opacity, a the radiation constant, ρ the constant density, L the luminosity and R the radius of the white dwarf. The thermodynamic character of the right side of this equation is governed by the luminosity and the stellar radius. Luminosity is neither extensive nor intensive as it is a homogenous function of degree ⅔ and radius is a homogeneous function of degree ⅓ for a spherical source so that their quotient is homogeneous of degree ⅓.[14] Consequently the right side is homogeneous of degree 1/12, so it is not intensive. Temperature however must always be intensive (homogeneous of degree 0) as demanded by the laws of thermodynamics. Equation (1) is therefore invalid. Yet from Eq. (1) the astronomers assert that hydrogen "cannot be present in appreciable amounts below the surface layers of a white dwarf."[10] They argue further that thermonuclear reactions are not responsible for the production of energy radiated by white dwarfs so that their cores must consist of particles incapable of fusion and are comprised, in most cases, of completely ionised oxygen and carbon nuclei.[10] These arguments are specious since Eq. (1) is inadmissible due to its violation of the laws of thermodynamics.

The theory of white dwarfs produces temperatures which are not intensive, masses which are not extensive, and equations for luminosities that are not thermodynamically balanced. Consequently, the theory of white dwarfs is invalid; hence, so too the associated Chandrasekhar limit.

2. DEGENERATE MATTER AND WHITE DWARFS

The astronomers maintain that white dwarfs, neutron stars and black holes are ultra-dense compact objects formed by gravitational collapse of gaseous stars.[10,11] To support a white dwarf against self-compression, called 'gravitational collapse', an electron degeneracy pressure is advanced due to an 'electron gas'. The Fermi energy of this 'gas' is,[10, 11]

$$\varepsilon_F = \frac{\hbar^2}{2m_e}\left[\frac{3\pi^2 Z\rho}{Am_H}\right]^{2/3},$$ (2)

where Z is the number of protons, m_e the electron mass, m_H the hydrogen atom mass, A the number of nucleons in the nuclei of the white dwarf, and ρ is the density. Note that the presence of electrons and positive ions implies Coulomb interactions. It is then argued that an electron will be unable to transition to an unoccupied quantum state if the thermal energy of the electron, given by the ideal gas term $3kT/2$, is lower than the Fermi energy. As a result, the electron gas will be degenerate, thus[10, 11]

$$\frac{3kT}{2} < \frac{\hbar^2}{2m_e}\left[\frac{3\pi^2 Z\rho}{Am_H}\right]^{2/3}.$$ (3)

The right side of this inequality introduces Coulomb interactions into the ideal gas law because the kinetic energy of an ideal gas particle is given by,

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT,$$ (4)

where $\overline{v^2}$ is the mean square of the speeds of the ideal gas particles. Although inequality (3) is intensive on each side, it violates the kinetic theory of the ideal gas because there are no forces between the neutral particles of the ideal gas except when they collide elastically with one another and the walls of their container. It is improper to apply the mean square speed kinetic energy of the neutral ideal gas particles to the case of the charged particles of the supposed electron gas and its Fermi energy.

Setting $Z/A = \frac{1}{2}$ in inequality (3) gives,[10]

$$\frac{T}{\rho^{2/3}} < \frac{\hbar^2}{3km_e}\left[\frac{3\pi^2}{2m_H}\right]^{2/3} = D = 1.3 \times 10^5\,\text{K cm}^2\,\text{g}^{-2/3}.$$ (5)

The degeneracy condition is then rendered as,

$$\frac{T}{\rho^{2/3}} < D,$$ (6)

from which it is concluded that the smaller the quotient on the left, the greater the degeneracy of the hypothesised electron gas acting as an ideal gas. However, since inequality (3) is inadmissible, expressions (5) and (6) are also inadmissible. Nevertheless, inequality (6) serves to argue complete electron degeneracy for white

3

dwarfs. For instance, in the case of Sirius B, it is asserted that the central density is ρ_c = 3.0 × 10⁶ g/cm³ and the central temperature is T_c = 27,000 K, so that by expression (6),[10]

$$\frac{T_c}{\rho_c^{2/3}} = 3.6 \times 10^3 \,\text{K cm}^2 \,\text{g}^{-2/3} \ll D, \tag{7}$$

and therefore Sirius B is claimed to be completely degenerate. This conclusion has no scientific basis because inequality (6) is inadmissible.

To estimate the electron degeneracy pressure the Pauli Exclusion Principle is invoked to limit one electron to each quantum state. The Heisenberg Uncertainty Principle is expressed in the form,

$$\Delta x \, \Delta p \approx \hbar. \tag{8}$$

The pressure integral for the ideal gas is given by,[5,10,11,15]

$$P = \frac{1}{3} \lim_{\alpha \to \infty} \int_0^\alpha n_p \, pv \, dp, \tag{9}$$

where v is the speed of ideal gas particles, p the particle momentum, and n_p is the number density of the ideal gas particles. The crude assumption is then made that the momentum of all the electrons is the same. As a result, Eq. (9) becomes,[10]

$$P \approx \frac{1}{3} n_e \, pv, \tag{10}$$

where n_e is the number density of the electrons. Because expression (10) evolves from Eq. (9) for the ideal gas, it stands in violation of the kinetic theory of the ideal gas by substituting charged particles (hence inclusive of Coulomb forces) for neutral particles between which there are no interactions other than the very brief elastic collisions. In the ideal gas, in the absence of external forces, the particles move uniformly in straight lines, but charged particles, since they are subject to Coulomb forces and consequently accelerated, do not move in straight lines with uniform velocities, again in violation of the kinetic theory of the ideal gas.[2-7]

Owing to the Pauli Exclusion Principle, the uncertainty in the positions of the electrons cannot be greater than their separation, which is $n_e^{-1/3}$. Setting $\Delta x = n_e^{-1/3}$ in expression (8), gives

$$\Delta p \approx \frac{\hbar}{n_e^{-1/3}} = \hbar n_e^{1/3}. \tag{11}$$

From expression (11) the electron momentum p is estimated as,[10]

$$p \approx \Delta p = \hbar n_e^{1/3}. \tag{12}$$

Now n_e is given by,[10]

$$n_e = \frac{Z\rho}{Am_H}, \tag{13}$$

where Z is the number of protons and A the number of nucleons in the white dwarf's nuclei. Since it follows from the foregoing that,

$$v = \frac{p}{m_e} \approx \frac{\hbar}{m_e}\left(\frac{Z\rho}{Am_H}\right)^{1/3}, \tag{14}$$

and using Eqs. (12) and (13), an estimate of electron degeneracy pressure of the supposed nonrelativistic electron gas is obtained from the inadmissible Eq. (10),

$$P \approx \frac{\hbar^2}{3m_e}\left(\frac{Z\rho}{Am_H}\right)^{5/3}. \tag{15}$$

From this it is concluded that "Electron degeneracy pressure is responsible for maintaining hydrostatic equilibrium in a white dwarf."[10] This conclusion is erroneous owing to the invalidity of the analysis advanced in violation of the kinetic theory of an ideal gas.

3. THE CHANDRASEKHAR LIMIT

By means of a variation on the invalid theme on the ideal gas leading to Eq. (15), it is advanced that the pressure of a completely degenerate nonrelativistic electron gas can be expressed as,[10]

$$P = \frac{\left(3\pi^2\right)^{2/3}\hbar^2}{5m_e}\left(\frac{Z\rho}{Am_H}\right)^{5/3}. \tag{16}$$

Equation (16) is invalid for the very same reason Eq. (15) is invalid. Nonetheless it is used to estimate a relation between white dwarf mass and radius. Taking the central pressure as,

$$P \approx \frac{2\pi G\rho^2 R^2}{3}, \tag{17}$$

the mean density at,

$$\rho = \frac{M}{V} = \frac{3M}{4\pi R^3}, \tag{18}$$

then equating Eq. (17) to Eq. (16) and using Eq. (18), gives,

$$R \approx \frac{\left(18\pi\right)^{2/3}\hbar^2}{10Gm_e\sqrt[3]{M}}\left(\frac{Z}{Am_H}\right)^{5/3}. \tag{19}$$

From this it is concluded that for white dwarfs, $MR^3 = $ const. or $MV = $ const. So that the more massive a white dwarf the smaller it is.[10] Owing to the invalidity of the use of the ideal gas law, Eq. (19) is invalid, so the mass-volume relation for white dwarfs is false.

It follows from Eq. (14) that the electrons of the electron gas would acquire speeds greater than c, the speed of light in vacuum. It is therefore claimed that special relativity must intervene to prevent this from occurring. Furthermore, there is a maximum mass for white dwarfs, a mass such that the speed of light is not exceeded by the electrons of the electron gas. Thus, the mass of a white dwarf is smaller than the mass-volume relation Eq. (19) would predict. In other words, by invoking special relativity, there is an upper limit to the mass that can be supported by electron degeneracy pressure; due to Chandrasekhar. To obtain an estimate of the electron degeneracy pressure, instead of using Eq. (14), set $v = c$ in Eq. (10), then,[10]

$$P \approx \frac{1}{3} n_e pc. \tag{20}$$

Then putting Eqs. (12) and (13) into Eq. (20), the estimate of the relativistic electron degeneracy pressure is,[10]

$$P = \frac{\hbar c}{3} \left(\frac{Z\rho}{Am_H} \right)^{4/3}. \tag{21}$$

A slightly different expression for estimate of relativistic electron degeneracy pressure is also advanced as,[10]

$$P = \frac{\left(3\pi^2\right)^{1/3} \hbar c}{4} \left(\frac{Z\rho}{Am_H} \right)^{4/3}. \tag{22}$$

Note that this equation has the form,

$$P = K\rho^\gamma, \tag{23}$$

where γ is the quotient of specific heats and K is a constant. From Eq. (22) it is concluded that the white dwarf is in unstable equilibrium. As a consequence, even "[t]he smallest departure from equilibrium will cause the white dwarf to collapse as electron degeneracy pressure fails."[10] Moreover, it is also concluded that the pressure of the degenerate electron gas is entirely independent of temperature. Thereby, the mechanical structure of the white dwarf becomes decoupled from its thermal properties.[10] General Relativity is also invoked and is said to "raise the critical value of γ for dynamical instability slightly above 4/3."[10] However, Eqs. (21) and (22) have no scientific basis owing to their improper derivation in using the ideal gas law.

An estimate of the Chandrasekhar limit is obtained from the foregoing by equating the central pressure to Eq. (22), thus,

$$\frac{2\pi G\rho^2 R^2}{3} = \frac{\left(3\pi^2\right)^{1/3} \hbar c}{4} \left(\frac{Z\rho}{Am_H} \right)^{4/3}. \tag{24}$$

Hence,

$$\rho = \frac{9}{16R^3 \sqrt{2\pi}} \left(\frac{\hbar c}{G} \right)^{3/2} \left(\frac{Z}{Am_H} \right)^2. \tag{25}$$

6

Since $\rho = 3M/4\pi R^3$, Eq. (25) for the Chandrasekhar mass limit $M \equiv M_{CH}$ is,[10]

$$M_{CH} = \frac{3\sqrt{2\pi}}{8}\left(\frac{\hbar c}{G}\right)^{3/2}\left(\frac{Z}{Am_H}\right)^2. \qquad (26)$$

Equations (24), (25) and (26) are invalid because they result from combining gravitational and Coulomb forces with the ideal gas law. The Chandrasekhar limit is therefore false.

With the foregoing in mind, the original derivation of the Chandrasekhar limit can be examined and characterised. Chandrasekhar advanced this finite limit for mass,[15]

$$M_3 = -4\pi\left(\frac{2A}{\pi G}\right)^{3/2}\frac{1}{B^2}\left(\xi^2\frac{d\theta_3}{d\xi}\right)_{\xi=\xi_1(\theta_3)}, \qquad (27)$$

where,

$$A = \frac{\pi m_e^4 c^5}{3h^2}, \qquad B = \frac{8\pi m_e^3 c^3 \mu_e m_H}{3h^3},$$

and μ_e is the weighted mean molecular weight on element abundances, ξ and $d\theta_3/d\xi$ are dimensionless numbers evaluated at $\xi = \xi_1(\theta_3)$. The terms ξ and θ are related via the Lane-Emden equation of index n,[15]

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n. \qquad (28)$$

The solution to this differential equation for index n is denoted by θ_n. Using Eq. (27) Chandrasekhar gave his limit as,

$$M_{CH} = \frac{M_3}{\beta_\omega^{3/2}}, \qquad (29)$$

where $\beta_\omega = 960/(960 + \pi^4)$ "on the basis of the perfect gas equation" for electron gas pressure.[15] Putting in the values of all the physical constants Chandrasekhar gives,

$$M_{CH} = 1.156M_3 = \frac{6.65M_\odot}{\mu_e^2}, \qquad (30)$$

where $M_\odot$ is the Sun's mass. As a result, Eq. (30) gives the Chandrasekhar limit in solar masses. Of Eq. (29) Chandrasekhar asserted that "[w]e have thus proved that the maximum mass of a stellar configuration which, consistent with the physics of degenerate matter, can be regarded as wholly degenerate, is $M = M_3\beta_\omega^{-3/2}$."[15] His claim is erroneous. Bowers and Deeming render Eq. (30) as,[11]

$$M_{CH} = \frac{5.80 M_{\Theta}}{\mu_e^2} . \tag{31}$$

On its face, Eq. (29) does not reveal its true character, particularly when presented in the form of Eq. (30). The violations of the ideal gas law and the laws of thermodynamics are obscured in the final mathematical expression for the Chandrasekhar limit. They are no less invalid.

Astrophysics asserts that the maximum mass of a white dwarf, the so-called Chandrasekhar limit, is about 1.4 solar masses, confined within a radius about that of Earth, resulting in a density of about 10^9 kg/m^3; "a million times denser than water".[16] According to Eddington[17], "[i]f stellar matter at the density of platinum has still the compressibility of a perfect gas, the limiting density must be much higher. … matter 2000 times denser than platinum is not only possible, but is actually present in the universe."[2] From these invalid arguments the astronomers advance their mass-radius relation for white dwarfs: "The more massive a white dwarf star is, the *smaller* it is."[16] Owing to the invalidity of the Chandrasekhar limit the claimed mass-radius relationship for white dwarfs is untenable.

4. ENVELOPE STRUCTURE
The total kinetic energy K of an ideal gas containing N particles is,

$$K = \frac{3}{2} NkT . \tag{32}$$

If the volume of the gas is V, then the energy density of the ideal gas is,

$$\frac{K}{V} = \frac{3}{2} \frac{N}{V} kT . \tag{33}$$

Now

$$N = M / \mu m_H , \tag{34}$$

where M is the mass of the ideal gas, μ is the molecular weight of the ideal gas particle and m_H is the mass of the hydrogen atom. Putting Eq. (34) into Eq. (33) gives,

$$\frac{K}{V} = \frac{3}{2} \frac{M}{V \mu m_H} kT = \frac{3}{2} \frac{\rho kT}{\mu m_H} , \tag{35}$$

where ρ is the ideal gas density. According to the astrophysicists, "the thermodynamic internal-energy density of a system of low-temperature electrons and ions in the envelope of a white dwarf may be written as",[11]

$$u = \frac{3}{5} n_e E_F + \frac{\pi^2}{4} n_e \frac{(kT)^2}{E_F} + \frac{3}{2} \frac{\rho kT}{\mu m_p} , \tag{36}$$

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[2] Eddington calls an ideal gas a perfect gas.

where n_e is the electron number density and E_F is the electron Fermi energy. The term m_p is the proton mass. The first term on the right side of Eq. (36) is the energy density of a degenerate electron gas. Note that the third term on the right side of Eq. (36) is just Eq. (35) for an ideal gas, which pertains only to charge-neutral particles since there are no Coulomb forces between the particles of an ideal gas. Nevertheless, the ideal gas law is applied to charged particles and the third term above is referred to as "the energy density associated with a classical ideal gas of ions at temperature T".[11] Equation (35) does not apply to ions. To apply it to ions is a direct violation of the kinetic theory of an ideal gas. Consequently Eq. (36) is invalid.

The inadmissible use of Eq. (35) also manifests in the determination of density as a function of temperature at the core-envelope border of a white dwarf. The following relation is advanced,[11]

$$\rho_1 = \left[\frac{5 k m_e m_p^{2/3}}{\hbar^2 \left(3\pi^2\right)^{2/3}} \right]^{3/2} \left(\frac{\mu_e^{5/2}}{\mu^{3/2}} \right) T_1^{3/2}. \tag{37}$$

Although this equation is thermodynamically balanced it has no scientific merit because it is derived by inappropriately equating the electron degeneracy pressure to the pressure for an ideal gas, the latter having no relevance to charged particles. The approximate electron degeneracy pressure is given by,[11]

$$P_e = \frac{\left(3\pi^2\right)^{2/3} \hbar^2}{10\pi^2 m_e} \left(\frac{\rho}{m_p \mu_e} \right)^{5/3}. \tag{38}$$

Using the proton mass in the pressure equation of an ideal gas gives,

$$P = \frac{\rho k T}{\mu m_p}. \tag{39}$$

Equating Eq. (38) to Eq. (39) and solving for density in terms of temperature gives,

$$\rho_1 = \left[\frac{10\pi^2 k m_e m_p^{2/3}}{\hbar^2 \left(3\pi^2\right)^{2/3}} \right]^{3/2} \left(\frac{\mu_e^{5/2}}{\mu^{3/2}} \right) T_1^{3/2}, \tag{40}$$

which is similar to Eq. (37) and has exactly the same thermodynamic character thereof. Furthermore, the quantity μ in Eq. (39) is the mean molecular weight of the ideal gas and although the quantity μ_e is the mean molecular weight of the supposed degenerate gas, μ_e is not defined in the same way as μ. It is defined by,[11]

$$n_e = \frac{\rho}{m_p \mu_e}, \tag{41}$$

where n_e is the electron number density. The definition of μ_e has nothing to do with an ideal gas. The invalidity of Eq. (37) becomes stark by considering the mass of the envelope. According to Bowers and Deeming the approximate scale height l of the envelope is,[11]

9

$$l \sim \frac{P}{\rho g} \approx \frac{kT}{\mu g}, \tag{42}$$

where,

$$g = \frac{MG}{R^2}. \tag{43}$$

Expression (42) employs the ideal gas law. Indeed, the pressure of a column of gas of height l is,

$$P = \rho g l. \tag{44}$$

Equating Eq. (44) to Eq. (39) for the pressure of an ideal gas and solving for l gives,

$$l = \frac{kT}{\mu m_p g}. \tag{45}$$

By means of Eq. (42) gravity is combined with the ideal gas. However, there are no gravitational forces between the particles of an ideal gas so the equating of Eq. (44) to Eq. (39) is a violation of the kinetic theory of an ideal gas. Consequently Eq. (42) is inadmissible. Nevertheless, according to modern theory, the mass M_s of the envelope is,[11]

$$M_s \le 4\pi R^2 l \rho_1. \tag{46}$$

Substituting Eq. (37), Eq. (42) and Eq. (43) into Eq. (46) for the envelope mass gives,[11]

$$M_s \le \frac{4\pi R^4 k}{\mu MG} \left[\frac{5km_e m_p^{2/3}}{\hbar^2 \left(3\pi^2\right)^{2/3}} \right]^{3/2} \left(\frac{\mu_e^{5/2}}{\mu^{3/2}} \right) T_1^{5/2}. \tag{47}$$

The left side of this inequality is extensive because mass is a homogeneous function of degree 1. The right side however is homogeneous degree ⅓ owing to the component R^4/M where M is the mass of the core, which is extensive, and the stellar radius R is homogeneous degree ⅓. Temperature is intensive so $T_1^{5/2}$ is also intensive because temperature is a homogeneous function of degree 0. Equation (47) is therefore invalid. This also demonstrates that gravity cannot be combined with the ideal gas without violating both the kinetic theory and the laws of thermodynamics.

Writing the equation for hydrostatic equilibrium as,[10, 11]

$$\frac{dP}{dr} = -\frac{MG}{r^2}\rho, \tag{48}$$

and the radiative transfer equation as,[10, 11]

$$\frac{dT}{dr} = -\frac{3\bar{\kappa}\rho}{16\pi ac}\frac{L}{r^2 T^3},$$ (49)

then dividing Eq. (48) by Eq. (49) gives,[10, 11, 18]

$$\frac{dP}{dT} = \frac{16\pi acMG}{3\bar{\kappa}L}T^3.$$ (50)

Since the mean opacity $\bar{\kappa}$ has the form $\bar{\kappa} = \kappa_o \rho T^{-3.5}$ for Kramers' opacity, where κ_o, is a constant, Eq. (50) becomes,

$$\frac{dP}{dT} = \frac{16\pi ac\, MG}{3\kappa_o \rho L}T^{6.5}.$$ (51)

The ideal gas law is then invoked in this form,[10, 11]

$$P = \frac{\rho kT}{\mu m_H}.$$ (52)

Solving Eq. (52) for ρ and substituting into Eq. (51) gives,[10, 11]

$$PdP = \left[\frac{16\pi ac}{3}\frac{MG}{\kappa_o L}\frac{k}{\mu m_H}\right]T^{7.5}dT.$$ (53)

Expression (53) is inadmissible because it combines the ideal gas law with gravity. Integrating Eq. (53) produces,[10, 11]

$$P = \left[\frac{2}{8.5}\frac{16\pi ac}{3}\frac{MG}{\kappa_o L}\frac{k}{\mu m_H}\right]^{1/2}T^{4.25}.$$ (54)

Pressure on the left is intensive, but the right side of Eq. (54) is not intensive owing to the term M/L. This is because mass is homogeneous degree 1 and luminosity is homogeneous degree ⅔.[14] Since temperature is intensive, $T^{4.25}$ is also intensive. Thus, Eq. (53) is thermodynamically unbalanced and therefore invalid. Yet, from this result, it is asserted that "[t]he density follows from the ideal gas law".[11] Equating Eq. (54) to Eq. (52) gives the density,[10, 11, 18]

$$\rho = \left(\frac{4}{17}\frac{16\pi ac}{3}\frac{GM}{L}\frac{\mu m_H}{\kappa_o k}\right)^{1/2}T^{13/4}.$$ (55)

Density and temperature are both intensive, so $T^{13/4}$ is also intensive. The right side is not intensive however owing to the quotient M/L, which is homogeneous degree ⅓. Thus, Eq. (55) is invalid.

These invalid equations are supplemented by an invalid equation for the temperature in the envelope. Putting $\bar{\kappa} = \kappa_o \rho T^{-3.5}$ into Eq. (49) gives,[10, 11]

$$\frac{dT}{dr} = -\frac{3\kappa_o L \rho}{16\pi ac}\frac{T^{-6.5}}{r^2}.$$

(56)

Solving Eq. (52) for ρ and substituting into Eq. (56) gives,

$$\frac{dT}{dr} = -\frac{3\kappa_o L P^2 \left(\mu m_H\right)^2}{16\pi ack^2}\frac{T^{-8.5}}{r^2}.$$

(57)

Substituting Eq. (54) into Eq. (57) gives,

$$\frac{dT}{dr} = -\frac{2}{8.5}\frac{\mu m_H}{k}\frac{MG}{r^2}.$$

(58)

Integrating finally gives the temperature in the envelope,[10]

$$T = \frac{2}{8.5}\frac{\mu m_H}{k}MG\left(\frac{1}{r} - \frac{1}{R}\right).$$

(59)

In this case, the left side is intensive, but the right side is not intensive by noting that mass M is homogeneous degree 1 (extensive) and spherical radius is homogeneous of degree ⅓ (neither intensive nor extensive). As a result, the right side is homogeneous degree ⅔. Consequently Eq. (59) is invalid. This outcome also reaffirms that it is inadmissible to combine gravity with the ideal gas law.

Similar processes are claimed where stellar surfaces are convective. The adiabatic temperature gradient is given by,[10, 11]

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}.$$

(60)

Using Eq. (48) for hydrostatic equilibrium and Eq. (52) for the ideal gas, gravity is once again inadmissibly combined with the ideal gas to express Eq. (60) as,[10]

$$\frac{dT}{dr} = -\left(1 - \frac{1}{\gamma}\right)\frac{\mu m_H}{k}\frac{MG}{r^2}.$$

(61)

Taking γ constant and integrating from R to r yields,[10]

$$T = GM\left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{\mu m_H}{k}\right)\left(\frac{1}{r} - \frac{1}{R}\right).$$

(62)

12

Once again, the result is thermodynamically unbalanced. The left side is intensive, but the right side is not intensive because the mass M is extensive and the radius is homogeneous degree ⅓. Consequently Eq. (62) is false.

Let $R_c = \xi R$ be the radius of the degenerate core of the white dwarf where R is the radius of the star; hence $0 < \xi < 1$. The numerical quantity ξ is called the condensation of a star. Then according to Chandrasekhar, the temperature of the nondegenerate gaseous envelope, wherein "the perfect gas law, $p \propto \rho T$" is obeyed, is given by,[15]

$$T = \frac{4}{17} \frac{\mu H}{k} \frac{GM}{R} \left(\frac{1}{\xi} - 1 \right),$$

(63)

where H is the proton's mass. This equation suffers from the same violation of the laws of thermodynamics: the right side of Eq. (63) is homogeneous degree ⅔ whereas the left side is homogeneous degree 0.

Another element in the Standard Model of the white dwarf envelope relates the core temperature T_c to the effective surface temperature T_{eff}. This is accomplished via invalid luminosity expressions. Substituting Eq. (59) into Eq. (55) and evaluating at the core-envelope interface, a constant envelope luminosity relation is obtained using T_1 and ρ_1. Since the core is isothermal, the core temperature is set equal to T_1 and, by Eq. (37), the density ρ_1 is eliminated to give,[11]

$$L = \frac{64\pi\sigma}{3} \frac{2}{8.5} \left(\frac{\mu}{k} \right)^4 K_1{}^5 \frac{MG}{\kappa_o} T_c^{3.5},$$

(64)

where $4\sigma = ac$ and $K_1 = 10^{13}$ in cgs units, for Kramers' opacity. Since temperature is intensive, $T_c^{3.5}$ is also intensive. The left side of Eq. (64) is homogeneous degree ⅔ (neither extensive nor intensive) whereas the right side is extensive since mass M is extensive (homogenous degree 1).[14] Equation (64) is therefore invalid. Nevertheless, taking an approximate Kramers' opacity as,[11]

$$\kappa_o = 4.43 \times 10^{25} \left(\overline{g} / t \right) Z,$$

(65)

where the constant $\overline{g}$ is the Gaunt factor and the constant t is the guillotine factor, Eq. (65) is substituted into Eq. (64) to obtain,[11]

$$L = 5.7 \times 10^{25} \frac{t/\overline{g}}{Z} \left(\frac{\mu}{\mu_e^2} \right) \left(\frac{M}{M_\Theta} \right) T_c^{3.5},$$

(66)

"with temperature in units of 10^6 K".[11] The constant M_Θ is the solar mass. Once again, the left side of this equation is homogeneous degree ⅔ whereas the right side is extensive (homogeneous degree 1). Equation (66) is therefore invalid.

Finally, to obtain a relation between T_c and T_{eff}, Eq. (66) is equated to the Stefan-Boltzmann law[11] to give,

$$5.7 \times 10^{25} \frac{\left(t/\overline{g}\right)}{Z} \left(\frac{\mu}{\mu_e^2}\right) \left(\frac{M}{M_\Theta}\right) T_c^{3.5} = 4\pi R^2 T_{eff}^4 . \tag{67}$$

The right side of this equation is homogeneous degree ⅔ whereas the left side is extensive (homogeneous degree 1). Hence Eq. (67) is certainly false.

Using $\rho^2 = T^3/D^3$ where $D = 1.3 \times 10^5$ K cm^2 g$^{-2/3}$ is a constant, density is eliminated from Eq. (55) to give the luminosity of the white dwarf in terms of its interior temperature T_c as,[10]

$$L = \frac{4D^3}{17} \frac{16\pi ac}{3} \frac{Gm_H}{\kappa_o k} \mu M T_c^{7/2} . \tag{68}$$

The left side of this equation is neither intensive nor extensive as it is homogeneous of degree ⅔ but the right side is extensive due to the mass M, homogeneous of degree 1. It is clearly evident that Eq. (68) is not thermodynamically balanced, so it too is invalid.

From these results, it is claimed that the luminosity of a white dwarf does not follow the Stefan-Boltzmann law when expressed in terms of the interior temperature and that the surface of a white dwarf cools more slowly than the supposed isothermal interior.[10] These conclusions are false given the invalidity of the foregoing thermodynamic equations.

5. COOLING OF WHITE DWARFS
The thermal energy of a white dwarf available for radiation is said to be the total kinetic energy of an ideal gas,[10]

$$U = \frac{3}{2} \frac{M}{Am_H} kT_c , \tag{69}$$

because "the average thermal energy of a nucleus is $\frac{3}{2}kT$".[10] Here Am_H is the mass of a nucleus. The quotient M/Am_H is the number of nuclei, N. A rough characteristic time scale for cooling of a white dwarf is then obtained by dividing the thermal energy U given by Eq. (69) by the luminosity given by Eq. (68), [10]

$$\tau_{cool} = \frac{3Mk}{2Am_H C T_c^{5/2}} , \tag{70}$$

$$C \equiv \frac{4D^3}{17} \frac{16\pi ac}{3} \frac{\mu m_H GM}{\kappa_o k} ,$$

Since Eq. (68) is invalid Eq. (70) is necessarily invalid.

As the white dwarf supposedly cools, the thermal energy described by Eq. (69) is given up to luminosity, Eq. (68), over time, thus,[10]

$$-\frac{d}{dt}\left(\frac{3}{2}\frac{M}{Am_H}kT_c\right)=CT_c^{7/2},\tag{71}$$

where C is defined in Eq. (70). Let $T_c = T_o$ at $t = 0$. Integration of Eq. (71) gives the white dwarf core temperature as a function of time,[10]

$$T_c(t)=T_o\left(1+\frac{5}{3}\frac{Am_H CT_o^{5/2}}{Mk}t\right)^{-2/5}.\tag{72}$$

Using the definition of C in Eq. (70) and setting $\tau_o = \tau_{cool}$, Eq. (71) becomes,[10]

$$T_c(t)=T_o\left(1+\frac{5}{2}\frac{t}{\tau_o}\right)^{-2/5}.\tag{73}$$

Here τ_o "is the time scale for cooling at the initial temperature T_o, that is, $\tau_o = \tau_{cool}$ at time t_o."[10] Equations (71) through to (73) are all invalid because they are all derived using the invalid Eq. (70).

Using C as defined in Eq. (70), Eq. (68) can be written as,[10]

$$L = CT_c^{7/2}.\tag{74}$$

Substituting Eq. (72) into Eq. (74) gives luminosity as a function of time,[10]

$$L(t)=L_o\left(1+\frac{5}{3}\frac{Am_H C^{2/7}L_o^{5/7}}{Mk}t\right)^{-7/5}=L_o\left(1+\frac{5}{2}\frac{t}{\tau_o}\right)^{-7/5}.\tag{75}$$

where $L_o = CT_o^{7/2}$ is the initial luminosity, according to Eq. (74). Equations (74) and (75) are false since they too are derived from invalid Eqs. (68) and (70).

From equations (73) and (75) an elaborate theory of processes in the cooling of a white dwarf has been devised including crystallisation into a lattice of ionised carbon and oxygen atoms pervaded by a degenerate electron sea, and a tortuous evolutionary wandering over the H-R diagram, culminating in a diamond-like[10, 16] body-centred cubic "cold, dark, Earth-sized sphere of crystallised carbon and oxygen floating through the depths of space".[10] These conclusions have no scientific merit owing to the invalidity of the foregoing equations from which they have been contrived.

According to Bowers and Deeming the cooling time, in years, for a white dwarf is,[11]

$$t = 8.9\times10^7\,\frac{Z^{2/7}}{A}\left(\frac{\bar{g}}{t}\right)^{2/7}\left(\frac{\mu_e^2}{\mu}\right)^{2/7}\left(\frac{M}{M_\Theta}\right)^{5/7}\left(\frac{L_\Theta}{L}\right)^{5/7},\tag{76}$$

where $M_\odot$ is the solar mass, $L_\odot$ the solar luminosity and t on the right side is the guillotine factor, not time. This equation is also invalid as will be proven in §6 below.

5. THE SCHÖNBERG-CHANDRASEKHAR MASS

Laboratory experience attests that gases have only positive heat capacities. In combining the ideal gas law with gravity in conjunction with the virial theorem, the erroneous conclusion that the stars have negative heat capacities is reached. It is said that self-compression by 'gravitational collapse' converts gravitational potential energy into two parts, radiated energy and thermal energy; the latter raising the temperature of the star.[10, 11, 15] This alleged process is implicated in the evolution of main sequence stars. The maximum fraction of a star's mass in its isothermal core in order to remain in hydrostatic equilibrium is given by the Schönberg-Chandrasekhar mass,[10, 11]

$$\left(\frac{M_{ic}}{M}\right)_{SC} \approx 0.37\left(\frac{\mu_e}{\mu_{ic}}\right)^2, \tag{77}$$

wherein M_{ic} is the mass of the isothermal core, M the mass of the star, μ_e and μ_{ic} the mean molecular weights of the envelope and the core respectively. The mass enveloping the core is $M - M_{ic}$. If the left side of Eq. (77) is less than the right side then it is said that the core will collapse due to its own gravity, i.e. self-compress. The derivation of the Schönberg-Chandrasekhar mass involves terms that violate the laws of thermodynamics. Consequently the Schönberg-Chandrasekhar mass relation is necessarily invalid. To prove the invalidity of the Schönberg-Chandrasekhar mass relation, consider the "generalised form of the virial theorem for stellar interiors in hydrostatic equilibrium,"[10]

$$2K_{ic} + U_{ic} = 4\pi R_{ic}^3 P_{ic}. \tag{78}$$

The subscript ic denotes 'isothermal core'. The total kinetic energy of the core's gas particles is given by the ideal gas law,[10]

$$K_{ic} = \frac{3}{2}N_{ic}kT_{ic}, \tag{79}$$

where N_{ic} is the number of gas particles, which in turn is given by,[10]

$$N_{ic} = \frac{M_{ic}}{\mu_{ic}m_H}, \tag{80}$$

where μ_{ic} is the mean molecular weight throughout the core. The total gravitational potential energy is,[10]

$$U_{ic} = -\frac{3}{5}\frac{GM_{ic}^2}{R_{ic}}. \tag{81}$$

Using the ideal gas law in combination with gravitational potential energy by substituting Eqs. (79), (80) and (81) into Eq. (78), the pressure at the surface of the isothermal core is,[10]

$$P_{ic} = \frac{3}{4\pi}\left(\frac{M_{ic}kT_{ic}}{R_{ic}^3\mu_{ic}m_H} - \frac{1}{5}\frac{GM_{ic}^2}{R_{ic}^4}\right). \tag{82}$$

Eq. (82) demonstrates once again that it is inadmissible to combine the ideal gas law with gravity. The left side of Eq. (82) is intensive (homogeneous degree 0). The first group of terms within the parentheses on the right side is intensive but the second group of terms is homogeneous degree ⅔. First, the right side is not intensive so the equation is not thermodynamically balanced. Secondly, an element of homogeneous degree ⅔ is being subtracted from an element that is homogeneous degree 0, which is also forbidden by the laws of thermodynamics: thermodynamic coordinates that are not of the same thermodynamic character cannot be added or subtracted.[14, 19-21] Thirdly, in reality, potential energy cannot contribute any pressure to a gas. According to the kinetic theory of an ideal gas, pressure is a dynamic process due to the elastic collisions of the gas particles with the walls of their container. Potential energy plays no part. Moreover, there is no potential energy of any kind between the particles of an ideal gas.

Setting the derivative dP_{ic}/dM_{ic} of Eq. (82) equal to zero yields the radius at which P_{ic} is maximum,

$$R_{ic} = \frac{2}{5}\frac{GM_{ic}\mu_{ic}m_H}{kT_{ic}}. \tag{83}$$

This equation is also invalid. The left side is homogeneous degree ⅓ whereas the right side is homogeneous degree 1. Nevertheless, to obtain the maximum surface pressure, Eq. (83) is inserted into Eq. (82) to obtain,[10]

$$P_{ic,\max} = \frac{375}{64\pi}\frac{1}{G^3M_{ic}^2}\left(\frac{kT_{ic}}{\mu_{ic}m_H}\right)^4. \tag{84}$$

Given its ætiology, Eq. (84) is also false; easily verified by noting that the left side is intensive, but the right side is not.

Similarly, the pressure $P_{ic,env}$ at the core-envelope interface is,[10]

$$P_{ic,env} \approx \frac{81}{4\pi}\frac{1}{G^3M^2}\left(\frac{kT_{ic}}{\mu_e m_H}\right)^4. \tag{85}$$

The left side of Eq. (85) is intensive but the right side is not. Hence Eq. (85) is also invalid. Equating these erroneous equations the following is obtained,

$$\left(\frac{M_{ic}}{M}\right)_{SC} \approx 0.54\left(\frac{\mu_e}{\mu_{ic}}\right)^2, \tag{86}$$

which is close to Eq. (77). Owing to its derivation Eq. (86) is invalid and similarly also Eq. (77). The Schönberg-Chandrasekhar mass is a fallacious concept, just like the Chandrasekhar limit and the Eddington limit.[20]

This result is due to an erroneous application of the virial theorem: "Let us apply the virial theorem to a perfect gas configuration in gravitational equilibrium."[15] The primary reason is that a gas is not a system that is in gravitational equilibrium. In the absence of a container of some sort, gases always expand. Uncontained, they are never gravitationally bound.

6. EVOLUTION OF WHITE DWARFS

In order to obtain dwarf luminosity as a function of time, this equation is advanced for the luminosity due to thermal energy release,[11]

$$L = -\int_0^M dm \, c_v \frac{dT}{dt},$$ (87)

where c_v is the specific heat at constant volume. Then "assuming that the ions remain an ideal gas we may express the specific heat c_v as"[11],

$$c_v = c_v^i = \frac{3}{2} \frac{k}{A m_p} = \frac{3}{2} \frac{n_i k}{\rho},$$ (88)

where n_i is the ion number density. Equation (88) is inadmissible because the argument is based on the false assumption that the ideal gas law can be applied to charged particles. The assumption is false because there are no Coulomb forces between the particles of an ideal gas. The far right side is obtained by multiplying numerator and denominator of the penultimate group of terms by n_i. Note that $n_i = N_i/V$ where N_i is the number of ions and V the volume. Then the product $A m_p N_i$ is the total ionic mass: hence $A m_p N_i / V = \rho$. It is then assumed that $T = T_c$, because the core is isothermal. Substitution of Eq. (88) into Eq. (87) is then used to obtain,[11]

$$L = -\frac{3}{2} \frac{k}{A m_p} \frac{dT_c}{dt} \int_0^M dm = -\frac{3}{2} \frac{kM}{A m_p} \frac{dT_c}{dt}.$$ (89)

Next, equations (66) and (89) are then equated. Integrating the resultant differential equation and solving for time gives,[11]

$$t = \frac{kM_\Theta}{2.5 \times 10^{-8}} \left(\frac{\bar{g}}{t} \right) \left(\frac{\mu_e^2}{\mu} \right) \frac{Z}{A m_p L_\Theta T_c^{5/2}}.$$ (90)

In the quotient $(\bar{g}/t)$ the quantity t is the guillotine factor, not time. Equation (90) is false because Eq. (66) is false.

Finally, eliminating T_c from Eq. (90) by using Eq. (66) yields,[11]

$$t = 8.9 \times 10^7 \frac{Z^{2/7}}{A} \left(\frac{\bar{g}}{t} \right)^{2/7} \left(\frac{\mu_e^2}{\mu} \right)^{2/7} \left(\frac{M}{M_\Theta} \right)^{5/7} \left(\frac{L_\Theta}{L} \right)^{5/7},$$ (91)

which is Eq. (76) above. Equation (91) is false because Eq. (66) is false. Using $(t/\overline{g}) = 10$ and $\mu_e \approx \mu \approx 2$, it is declared that "the time required for the luminosity to be reduced by a factor of 10 is nearly 10^{10} years. The final cooling times are therefore comparable to the age of the universe."[11] This conclusion is without scientific merit due to the invalidity of the equations employed.

"The evolutionary track of a white dwarf is given by the mass-radius relation and the definition of the effective temperature $L \sim R^2 T_{eff}^4$."[11] Using the alleged relation $M \sim R^{-3}$ [10, 11] and $L \sim R^2 T_{eff}^4$ of the Stefan-Boltzmann law, eliminating R yields,

$$L = M^{-2/3} T_{eff}^4. \tag{92}$$

Hence,

$$\log L = 4 \log T_{eff} - \frac{2}{3} \log M. \tag{93}$$

Define a constant C as,

$$C = -\log L_\Theta + 4 \log T_\Theta - \frac{2}{3} \log M_\Theta, \tag{94}$$

wherein L_Θ, T_Θ and M_Θ are the solar luminosity, temperature and mass respectively. Then Eq. (93) becomes,[11]

$$\log \frac{L}{L_\Theta} = 4 \log \frac{T_{eff}}{T_\Theta} - \frac{2}{3} \log \frac{M}{M_\Theta} + C. \tag{95}$$

Equation (92) is homogeneous degree ⅔ on the left side but homogeneous degree -⅔ on the right side so it is invalid. Consequently Eq. (95) is false as well. Note that here it is by Eq. (92) that violation of thermodynamic balance is introduced into the Stefan-Boltzmann law. Therefore the mass-radius relation for white dwarfs is again proven false.

From the mass-radius relation the astronomers assert that the density of a white dwarf has the peculiar property that its density is proportional to the square of its mass,[10]

$$\rho \propto M^2. \tag{96}$$

Relation (96) is necessarily false owing to the invalidity of the white dwarf mass-radius relation.

7. CONCLUSION

The theory of white dwarfs produces violations of the laws of thermodynamics by making temperature non-intensive and mass non-extensive. Furthermore, in deriving thermodynamic relations for white dwarfs, temperatures are assigned to potential energies even though potential energy cannot make any contribution to temperature; potential energy does not have a temperature. In the ideal gas law, temperature is due entirely to the kinetic energy of the gas particles. The combination of gravitational forces and Coulomb forces with the kinetic theory of an ideal gas violates the premises of the kinetic theory.

The use of Eddington's mass-luminosity relation both in the theory of white dwarfs and generally relative to gaseous stars is improper because this relation implies nonintensive temperature and is therefore certainly false.[20] Similarly, the luminosity expressions advanced for white dwarfs are also inadmissible, because, once again, they are not thermodynamically balanced.

Gases are not gravitationally bound internally since they always expand when released from their containers. There are no gravitational forces between the particles of an ideal gas. Application of the virial theorem to the ideal gas is therefore inadmissible. Yet, in the words of Chandrasekhar, "[f]or a cloud of density so low that the ideal gas laws may be assumed, all forces except the gravitational forces may be neglected."[15] This false premise produces nonintensive temperature. White dwarfs are not in fact 'dwarf' – they are not ultra-dense compact bodies. Chandrasekhar's own derivation of the Chandrasekhar limit for the upper bound on the mass of a white dwarf is invalid owing to its violation of the kinetic theory of an ideal gas.

Unfortunately, when astrophysicists are confronted with the reality that their theories stand in violation of the laws of physics, they merely assert that their theories need not follow the laws of physics, as if the laws need not be satisfied. This is the case with respect to the density expression $\rho \sim M^2$ in relation to white dwarfs, along with the negative heat capacities of gaseous stars. Such violations should have made evident the falsity of these theories. Instead, false theories are being maintained despite the laws of physics and ratiocination. It is also evident that the violations of the laws of thermodynamics and of the kinetic theory of an ideal gas have simply been ignored or unrecognised. Inexplicably, even Landsberg, who recognised the requirement for thermodynamic balance in thermodynamic expressions, accepted violations of the laws of thermodynamics to allow the existence of black holes. For instance, he disregarded the thermodynamic imbalance of the Hawking black hole temperature,[21]

$$T = \frac{\hbar c^3}{8\pi kGM}.$$

The left side is intensive but the right side is homogeneous degree -1. As a result, when two identical masses (black holes), hence at the same temperature, coalesce, the temperature of the resultant mass is half that of the two coalescing masses, in violation of the 0th law of thermodynamics. Hawking's black hole temperature is invalid.[22] In all these cases however, the laws of physics must be obeyed and not simply disregarded to preserve dogmatic theories.

The theory of the main sequence stars and white dwarfs has been examined by Robitaille from an observational perspective.[23-30] He has questioned Eddington's application of the Stefan-Boltzmann law to these stars[17] by which the lower luminosity of white dwarfs was explained by decreasing the stellar radius, thereby resulting in the currently accepted ultra-dense nature of these objects and predicted associated gravitational red shifts. Robitaille has cogently argued that Eddington inappropriately assumed stellar emissivities of 1 without proper justification.[24] An emissivity of 1 characterizes a black body. More importantly, Robitaille has dissected the modern red-shift arguments, revealing that individual hydrogen lines (H_α, H_β, H_γ, etc) in white

dwarfs do not share the same red shifts[25, 26] and that none of the metallic lines in these stars have confirmed the red shifts obtained with the hydrogen lines. He has shown that the red shifts observed in the hydrogen lines are Stark effects predicted to occur as a result of the known presence of quasi-hydrogen molecules and cations in white dwarfs[27]. Robitaille has highlighted that the stars are composed of metallic hydrogen and he assigns a hexagonal planar lattice to the main sequence stars.[23] He argues that the lower luminosity of white dwarfs is due to an altered photospheric lattice structure which produces a lower emissivity. He disputes the current theory that supernovae are formed when white dwarfs accrete material, move beyond the Chandrasekhar limit, and explode.[29, 30] Robitaille advances that supernovae are the result of exfoliation in normal stars when intercalate zones rapidly expand resulting in the ejection of their outer envelope.[29] He gives a similar mechanism for the formation of planetary nebulae.[30] His arguments are supported by the proof herein that the Chandrasekhar limit is invalid.

Finally, with the proof that the theory of thermonuclear reactions in gaseous stars is false,[31] astronomy and astrophysics have no alternative but to concede that the stars are condensed matter. Gaseous stars do not exist.

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