Solargons, solar stratovases, solarhedrons, and other light concentrators

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Abstract. We invent the "solarhedron," "solargon," and "solar stratovase" solar concentrator designs which will *uniformly* illuminate a flat solar cell with $F(k)^2$, F(k), and k suns worth of radiation respectively, for any desired integer k≥1, where F(k)≥1 is a certain roughly-linearly-increasing function apparently obeying $2k-2.6 \le F(k) \le 2k-1$. We prove the solargon and stratovase both enjoy *optimal* efficiency for any fixed mirror-reflectivity value in (0, 1].

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1. "Solargon," "solarhedron," and "stratovase" light concentrators

Many photovoltaic solar cells work best when their surfaces are **uniformly** illuminated. If the illumination as a function of distance x along the solar cell were a Dirac delta function, then while the total incoming solar power on a unit length of solar cell would the same as from uniform illumination, the latter will produce steady electric power, while the former would vaporize a small portion of the solar cell, destroying it while outputting zero electric power! Unfortunately, some published and commercially sold solar concentrator designs (e.g. R.E.Winston's "compound parabolic concentrator," CPC) indeed *do* produce Dirac delta function illumination profiles whenever the sun reaches particular bad positions. Mirror-based solar concentrators are useful provided the cost per unit area is substantially smaller for mirrors than solar cells. As of year 2021, solar panels cost about $80 per square meter while flat mirror glass costs about $2 per square meter.

Solargon. Here is a **F(k):1 solar concentrator design** [k≥1 an arbitary integer, F(k)≥1 is the halfwidth of the solargon, a roughly-linearly-increasing function [whose precise value will equal x_{k-1} in the below table] producing exactly-uniform illumination on a flat solar cell. We model the solar cell as the interval (-1,1) on the x-axis and regard the sun as located vertically overhead at y=+∞. (Extended into 3D, this would be an infinitely long convex-trough prism with infinitely-long rectangular strips as faces, which should be rotated over the course of each day to track the sun.) The mirror is **convex polygonal** with 2k-1 edges, and x-negation symmetric. The edges are numbered 1-k, 2-k, ..., -1, 0, 1, ..., k-2, k-1. Edge 0 is the solar cell itself (not a mirror).

The **angle** θ_j between the line containing polygon-edge j and the y-axis, for j=0,1,2,3,... equals 90, 30, 15, arccot(3)/2≈9.2174744, ... degrees, and the θ_j monotonically decrease and ultimately approach 0, with (for j≥2) each θ_j equalling *half* the angle-to-vertical of the line that joins the righthand vertex (1,0) of the solar cell, to the lower endpoint of edge j (which also is the upper endpoint of edge j-1).

The **length** $L_j=2|\cos(2\theta_j)|\csc(\theta_j)$ of edge j for j=0,1,2,3,... equals 2, 2, $2(1+\sqrt{3})\sqrt{6}\approx 6.692130$, $6[(10+3\sqrt{10})/5]^{1/2}\approx 11.84505$, ...

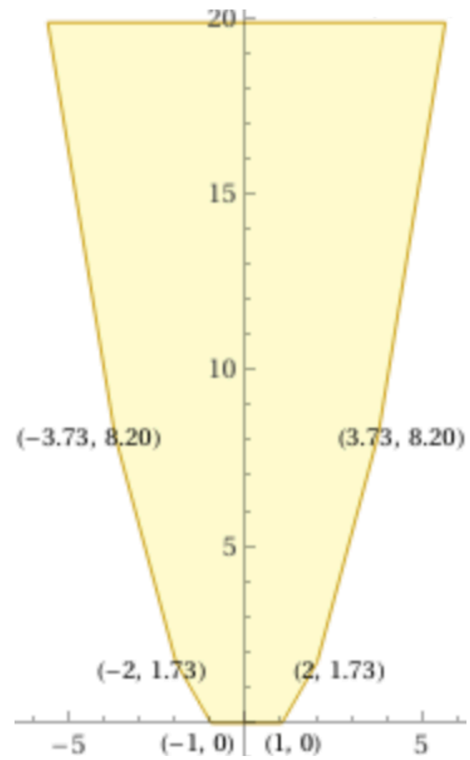
Polygon edge j≠0 reflects sunlight onto the upper surface of the solar cell via exactly one bounce, uniformly illuminating it by rays which hit the solar cell at angle $2\theta_{|j|}$ to the vertical.

The (x,y) **polygon vertex coordinates** with x>0 are (1,0), (2,√3), (2+√3,3+3√3), (5.62942, 19.8883), ... that is (1, 0), (2, 1.73205), (3.73205, 8.19615), (5.62942, 19.8883), ... For |j|=1,2,...,9 these vertex coordinates happen to be fit fairly well by the parabola $y=0.65887x^2-0.90485$, with maximum vertical error 0.2460. The focal point of this parabola is (0, -0.52541), which lies beneath the solar cell. An even better curvefit arises by using the minimum of *two* parabolas, namely $y=0.66209x^2-0.057891|x|-0.72902$, now with maximum vertical error≈0.1248. The focal points of these parabolas are (±0.04372, -0.3527) which again lie beneath the solar cell.

Pseudocode to compute all these things:

```
x0=1;  y0=0;  θ0=π/2;  θ1=π/6;  L0=2;
for j=1,2,3,... do {
  Lj = 2 |cos(2θj)| / sinθj;
  xj = xj-1 + Lj sinθj;
  yj = yj-1 + Lj cosθj;
  θj+1 = arctan( (xj-1)/yj ) / 2;
}
```

j	x	y	y/x^2	θ (deg)	L
0	1.0	0.0	0.0	90.0	2.0
1	2.0	1.73205	0.433013	30.0	2.0
2	3.73205	8.19615	0.588457	15.0	6.692130
3	5.62942	19.88825	0.627581	9.217474	11.845049
4	7.57734	36.84877	0.641785	6.551744	17.072011
5	9.54622	59.08395	0.648345	5.060229	22.322181
6	11.52562	86.59537	0.651877	4.115244	27.582532
7	13.51101	119.38355	0.653987	3.465138	32.848240
8	15.50012	157.44873	0.655344	2.991288	38.117110
9	17.49169	200.79099	0.656267	2.630884	43.387998
10	19.48498	249.41041	0.656923	2.347685	48.660255
20	39.45515	1025.85078	0.658987	1.129165	101.411227
30	59.44536	2330.01370	0.659360	0.743019	154.176789
40	79.44049	4161.89962	0.659489	0.553633	206.945790
50	99.43758	6521.50860	0.659549	0.441171	259.716135
60	119.43565	9408.84065	0.659581	0.366681	312.487142
70	139.43427	12823.89577	0.659600	0.313710	365.258525
80	159.43324	16766.67397	0.659613	0.274111	418.030142
90	179.43243	21237.17524	0.659622	0.243388	470.801914
100	199.43179	26235.39959	0.659628	0.218858	523.573793
200	399.42891	105242.41240	0.659647	0.109000	1051.294954
300	599.42795	237021.73302	0.659651	0.072571	1579.017397
400	799.42747	421573.36146	0.659652	0.054393	2106.740157



Poetically speaking, the "**solargon**" is a bilaterally symmetric convex polygonal approximation to a parabola whose j^{th} sidelength is asymptotically (for large j) about 5.25j, while the angles θ_j of those sides to the vertical asymptotically roughly obey $j\theta_j \approx 21.7$ degrees. These asymptotics presumably are solely of theoretical interest because in practice I doubt anybody would want k>6. Let the interior of this convex polygon be y>H(x) where H(x) is the piecewise-linear continuous "height function."

Two solargon variants involving either two edge-halvings or a solar-cell doubling. We also may double the length of the solar cell from 2 to 4, leaving all else unchanged (i.e. the two halves of the solargon are simply translated bodily -1 and +1 in the x direction). Then the solar rays from the left half of the solargon illuminate only the left half of the solar cell, not the whole cell, and similarly for the right half. For that variant the old concentration-ratio F(k) needs to be replaced by $F_{doub}(k)=[F(k)+1]/2$. Another variant: instead of using edges j={1-k, 2-k, ..., -1, 0, 1, ..., k-2, k-1} of the original solargon, we can *halve* the leftmost and rightmost edge line-segments. For it the old concentration-ratio F(k) needs to be replaced by $F_{halv}(k)=[F(k-1)+F(k)]/2$.

The 3D solarhedron mirror shape is y=max(H(x),H(z)) where H(x) is the solargon height-function. The solarhedron will uniformly illuminate the upper surface of a square solar cell, namely the square y=0, |x|≤1, |z|≤1, but now with $F(k)^2$:1 concentration ratio for any desired integer k≥1. [We instead may use either of the solargon's edge-halving/doubling variants to define H(x), in which case we get two corresponding solarhedron variants.] This is a convex polyhedron with 4k-3 faces, each a trapezoid, and the same dihedral order-8 symmetry group as the square solar cell itself. Unlike the (≤1)-bounce-only light paths for the solargon, with the solarhedron there also are 2-bounce paths. Also, the solarhedron requires *two*-dimensional sun tracking (e.g. from a Dobsonian mount) to keep the "positive y axis" always pointing at the sun. In contrast, the solargon had only needed 1-dimensional sun tracking, accomplishable by rotation about a fixed north-south axial line ("z axis"), to make the "xy plane" always contain the sun in its y>0 halfplane at y=+∞. With perfect sun tracking and perfectly reflective mirrors, both the solargon and solarhedron are 100% efficient in the sense that every solar ray that hits them, hits the solar cell; and also both illuminate the solar cell perfectly uniformly.

And even with **imperfectly reflective** mirrors (which we model as: fraction R of the light-energy is reflected, the remaining 1-R is lost, for some constant R with 0<R≤1), the solargon's efficiency still seems best possible since (1) all light rays reflect at most once and (2) the maximum possible amount of them directly hit the solar cell with no reflection at all. (Also valid for the two solargon variants.) I do not know whether the solarhedron is optimal with imperfect mirrors, albeit it definitely is not optimal for all sufficiently-small mirror-reflectivities (e.g. reflectivity R≤0.5 should suffice, for all large-enough k; this non-optimality may be shown by contrast versus the strato-vase we shall describe momentarily). But since all light rays bounce at most twice off the solarhedron, its efficiency cannot be very suboptimal when mirror-reflectivity is large. Specifically, achieved efficiency always exceeds R times optimal efficiency.

Solar stratovase. Now for each integer k≥1, we design an **axially-symmetric k:1 solar concentrator for a flat circular-disk solar cell**, which illuminates the solar cell exactly uniformly, and whose efficiency is best possible (for the same reason as the solargon) even with imperfectly reflective mirrors. We need to specify, piecewise, the continuous mirror-curve height function y(r). The solar cell will be the disk 0≤r≤1, and hence y(r)=0 for 0≤r≤1. For each integer j with 1≤j≤k-1, when $j^{1/2}<r<(j+1)^{1/2}$ we make y(r) obey this differential equation:

$$dy/dr = \cot\Big(\arctan\big((r-[j+1-r^2]^{1/2})\, y^{-1} \big) / 2 \Big) = \big([(y^2 + j+1 - 2[j+1-r^2]^{1/2} r)\, y^{-2}]^{1/2} + 1 \big)\, y \,/\, (r-[j+1-r^2]^{1/2}).$$

How I derived that differential equation: Vertical sunlight rays reflect at (r,y) to hit the solar cell at (x,0) with 0≤x≤r, where to assure areally-uniform illumination we demand x and r be related by r dr = -x dx so

$dx/dr = -r/x$ so $xdx=-rdr$ and $d(x^2)=-d(r^2)$ with solution $x(r) = [j+1-r^2]^{1/2}$ for $j^{1/2}<r<(j+1)^{1/2}$ mapping to $0<x<1$ with $x(r)$ monotonically decreasing from 1 to 0 while $r-x(r)$ monotonically increases with $r-x \geq j^{1/2}-1 \geq 0$. If the slope $y'(r)$ obeys $\arctan([r-x(r)] / y) = 2\text{arccot}(y')$ then the desired reflection happens (equal-angle reflection law). Hence $y' = \cot(\arctan([r-x(r)] / y) / 2)$, hence $y' = \cot(\arctan([r-(j+1-r^2)^{1/2}] y^{-1}) / 2)$. The right hand side simplifies to the algebraic function of y and r rightmost above.

Note, $y(r)$ is continuous but $y'(r)$ jumps *dis*continuously when r crosses $\sqrt{j}$ (but is continuous away from those points). Although the two right hand sides mathematically are exactly equal, numerical difficulties in evaluating both tend to cause slight differences between them when both are evaluated in limited-precision arithmetic when j=1 for r slightly above 1 and y slightly above 0. When $r=\varepsilon+\sqrt{j}$ we have $\lim_{\varepsilon\to 0+} y'(\varepsilon+\sqrt{j}) = \cot(\arctan([j^{1/2}-1]y^{-1})/2))$ and (by continuity) $\lim_{\varepsilon\to 0+} y(\varepsilon+\sqrt{j}) = \lim_{\varepsilon\to 0+} y(\sqrt{j}-\varepsilon)$ which are perfectly fine initial conditions for the differential equation for each j=2,3,4,... *However* when j=1 our formulas for y' formally yield 0/0 when $r=\sqrt{j}$. This turns out to be the harbinger of a very interesting and rare mathematical phenomenon: ***non*****uniqueness and *****non*****existence, combined with arbitrarily-close "almost existence."** I've never seen this occur "in the wild" before, and will explain what happens. Consider starting the stratovase $y(r)$ curve *not* from the point $(r,y)=(1,0)$, but rather $(1+\varepsilon,0)$ for some arbitrarily small, but positive, ε, with initial condition necessarily $y'(1+\varepsilon)=1$. If you do that, then a unique stratovase curve results (its uniqueness follows from the Picard-Lindelöf theorem as usual), call it C_ε. Because $\varepsilon>0$, this curve is not truly a stratovase, i.e. does not truly yield uniform illumination of the solar cell. But if ε is small, then for all practical purposes nobody will be able to tell the difference. If we then vary ε to make it approach 0 from above, the C_ε do *not* approach a limit curve. Rather, apparently $y(\sqrt{2})\to\infty$ when $\varepsilon\to 0+$, with the approach to infinity behaving like $|\log\varepsilon|$. I.e, apparently, no limit curve exists!

If a "true stratovase" curve C exists it must be tangent to a **vertical line** at r=y=0. [**Proof:** For $r=1+\varepsilon$ slightly above 1, we want the vertical solar rays to reflect to hit the solar cell at $r=1-\varepsilon$ (here neglecting terms of order ε^2) because the areas $(2\varepsilon\pm\varepsilon^2)\pi$ of the two annuli defined by the r=1 and $r=1\pm\varepsilon$ circles are equal up to errors of order ε^2. To make that happen we want the stratovase's initial slope $T=\lim_{\varepsilon\to 0+}y'(1+\varepsilon)$ to obey $\arctan(T)-\arctan(T/2)=\pi/2-\arctan(T)$, or equivalently $2\arctan(T)=\pi/2+\arctan(T/2)$, which is unsatisfied by any finite $T\geq 0$ but is solved by $T=\infty$, $\arctan(T)=\pi/2$, i.e. initially vertical slope. Indeed by taking the tan of both sides and applying the tangent addition formula $\tan(\alpha+\beta)=(\tan\alpha+\tan\beta)/(1+\tan\alpha\tan\beta)$ we find $2T/(1+T^2)=2/T$ hence $T/(1+T^2)=1/T$ hence $T^2=1+T^2$ which for any finite T is a contradiction but is satisfied by $T=\infty$. **Q.E.D.**] That suggests that it would be better to describe C not as $y(r)$ but rather to make r be a function $r(y)$ of y. Then for each integer j with $1\leq j\leq k-1$, when $j^{1/2}<r<(j+1)^{1/2}$ we make $r(y)$ obey this differential equation:

$$dr/dy = \tan\Big(\arctan((r-[j+1-r^2]^{1/2}) y^{-1}) / 2 \Big) = (r-[j+1-r^2]^{1/2}) y^{-1} / ([(y^2 + j+1 - 2[j+1-r^2]^{1/2}r) y^{-2}]^{1/2} + 1)$$

and $dr/dy=0$ when y=0 and r=1. But then when y gets a little positive, and r stays 0, we still find $dr/dy=0$, causing the solution of this differential equation and initial condition to be simply $r(y)=0$ for all $y\geq 0$, i.e. just a vertical line, which is useless. And by the Picard-Lindelöf theorem this exact solution is unique! So we have proven the

Stratovase theorem. No "true stratovase" C exists, but curves C_ε "arbitrarily close to being a stratovase" (when $\varepsilon\to 0+$) do exist.

Unfortunately, I do not know any closed form solution of the C_ε differential equation. If nobody can find one, it must be solved numerically. So practical people should just pick some small $\varepsilon>0$ then find C_ε using numerical ODE-solving methods, then be satisfied with that. The following table arises from ε=1/100 using order-4 Runge Kutta method with stepsize $\Delta r=10^{-5}$. Notice that the $\mathbf{C_{0.01}}$ **curve** often is *not* concave-∪, e.g. for each integer j≥2 apparently y''(r)<0 when $j+0.1<r^2<j+0.2$, and also y''(1.26)<0. But y(r) is concave-∪ for a large majority of r. And y(r) always is continuous and increasing:

r^2	r	y	dy/dr	r^2	r	y	dy/dr
1.0201	1.01	0.0	1.0	5.699999839	2.387467244	5.449947	6.088911
1.099999362	1.048808544	0.056894	1.718401	5.800008518	2.408320684	5.574557	5.855854
1.200003483	1.095446705	0.143042	1.938907	5.900005617	2.428992716	5.692783	5.568500
1.300003541	1.140176978	0.231911	2.022571	6.000000000	2.449489743	5.802488	4.940139
1.400003979	1.183217638	0.319733	2.052208	6.100003502	2.469818516	5.966134	7.969782
1.500004085	1.224746539	0.405040	2.052217	6.200002726	2.489980467	6.125179	7.805912
1.600005507	1.264913241	0.487133	2.032450	6.300007047	2.509981483	6.279623	7.636510
1.700006697	1.303843049	0.565615	1.996870	6.400007082	2.529823528	6.429412	7.460047
1.800000732	1.341641059	0.640181	1.945495	6.500003921	2.549510526	6.574470	7.274299
1.900009510	1.378408325	0.710456	1.871923	6.600009501	2.569048365	6.714682	7.075830
2.000000000	1.414213562	0.775303	1.688637	6.700006337	2.588437045	6.849809	6.858924
2.099999549	1.449137519	0.913274	3.905811	6.800007784	2.607682455	6.979504	6.612308
2.200005215	1.483241455	1.044891	3.811484	6.900007609	2.626786556	7.103051	6.306866
2.300003448	1.516576226	1.170306	3.711896	7.000000000	2.645751311	7.218277	5.633999
2.400007819	1.549195862	1.289700	3.607182	7.100004486	2.664583361	7.384063	8.721286
2.500009023	1.581141684	1.403193	3.496793	7.200002597	2.683282057	7.545578	8.553120
2.600000906	1.612451831	1.510867	3.379328	7.300008986	2.701852880	7.702811	8.378857
2.699999834	1.643167622	1.612746	3.251965	7.400009405	2.720295830	7.855679	8.196888
2.800002638	1.673320841	1.708704	3.108856	7.499999708	2.738612734	8.004081	8.004874
2.900004965	1.702940094	1.798326	2.934421	7.600006331	2.756810899	8.147910	7.799149
3.000000000	1.732050808	1.879989	2.561263	7.700006172	2.774888497	8.286900	7.573673
3.100007649	1.760683858	2.030581	5.193935	7.800006560	2.792849183	8.420684	7.316528
3.200005919	1.788856036	2.175047	5.061110	7.900005166	2.810694784	8.548541	6.996900
3.300004523	1.816591457	2.313543	4.924884	8.000000000	2.828427125	8.668306	6.288452
3.399999948	1.843908877	2.446174	4.784248	8.100006810	2.846051090	8.835794	9.419880
3.500003889	1.870829733	2.573017	4.637659	8.200005939	2.863565250	8.999284	9.248439
3.600003130	1.897367421	2.694061	4.482751	8.300005620	2.880973034	9.158742	9.070448
3.700008817	1.923540698	2.809238	4.315437	8.400004688	2.898276158	9.314093	8.884203
3.800005510	1.949360282	2.918301	4.127754	8.500002282	2.915476339	9.465228	8.687246
3.900001914	1.974842250	3.020706	3.898919	8.600007903	2.932577007	9.612004	8.475783
4.000000000	2.000000000	3.114520	3.407951	8.700001221	2.949576448	9.754151	8.243489
4.100001950	2.024846155	3.271378	6.239960	8.800002447	2.966479807	9.891312	7.977872
4.199999955	2.049390142	3.422716	6.091144	8.900001839	2.983287086	10.022754	7.646726
4.300008260	2.073646127	3.568622	5.938242	9.000000000	3.000000000	10.146333	6.908981
4.400004756	2.097618830	3.709096	5.780003	9.100008899	3.016622101	10.315218	10.075556
4.500008791	2.121322416	3.844163	5.614544	9.200009255	3.033151703	10.480334	9.901517

4.600002343	2.144761605	3.973733	5.439046	9.300001792	3.049590430	10.641627	9.720549
4.700009642	2.167950563	4.097691	5.248663	9.400007399	3.065943150	10.799048	9.530844
4.800006216	2.190891649	4.215704	5.034044	9.500007634	3.082208240	10.952453	9.329886
4.899999752	2.213594306	4.327145	4.770820	9.600004001	3.098387323	11.101679	9.113757
5.000000000	2.236067978	4.429939	4.200334	9.700008367	3.114483644	11.246499	8.875823
5.100006200	2.258319331	4.590773	7.150592	9.800002579	3.130495580	11.386498	8.603220
5.200000043	2.280350860	4.746583	6.992709	9.900009166	3.146428001	11.520990	8.262454
5.300001604	2.302173235	4.897417	6.829962	10.000000000	3.162277660	11.647817	7.500080
5.400003337	2.323790726	5.043253	6.660928	100.00000000	10.00000000	166.47346	33.32470
5.500008697	2.345209734	5.184043	6.483539	999.99999999	31.62277660	1833.81411	115.98955
5.600002277	2.366432394	5.319664	6.294655	9999.900007706	99.999500037	18884.24967	378.88762

No-corner stratovase. It also is possible to devise a different kind of stratovase now *without* corners [i.e. enjoying a *continuous* derivative y'(r) at all r>1]. (I tend to doubt this is an advantage; the corners probably usefully increase stiffness.) For the original stratovase, vertical solar rays at r>1 "raster scanned" the solar cell as r increased, always in an inward direction. The idea of the corner-free alternative stratovase is instead to use "**bidirectional** raster scanning." The two kinds of stratovase are identical for $r\le\sqrt{2}$ but differ for $r>\sqrt{2}$. For each *odd* integer j with $1\le j\le k-1$, when $j^{1/2}<r<(j+1)^{1/2}$ we make y(r) obey the original differential equation. But for *even* integer j, when $j^{1/2}<r<(j+1)^{1/2}$ we instead make y(r) obey this differential equation:

$$dy/dr \;=\; \cot\Big(\arctan\big((r-[r^2-j]^{1/2})\, y^{-1} \big) / 2 \Big) \;=\; (r + [r^2-j]^{1/2}) \Big([2(r-[r^2-j]^{1/2})r+y^2]^{1/2}-j\Big)\, j^{-1}$$

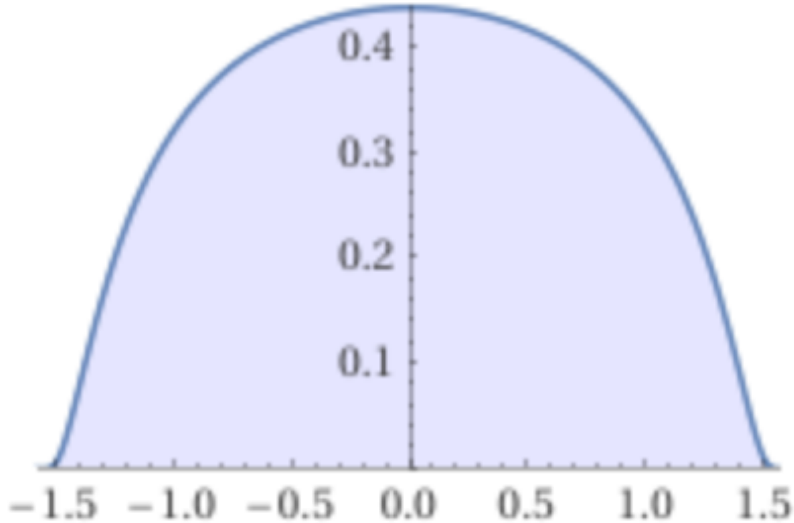
Properties of stratovase solutions C_ε. Their y(r) are monotonically increasing and continuous. It is differentiable except when $r=\sqrt{j}$ for integer $j\ge1$, where the original stratovase has corners; but the no-corners version does not have corners and has a continuosu derivative everywhere. Asymptotically for large j, one can argue from the differential equation that (for both types) $r\sim\sqrt{j}$ and $y\sim cj$ for some constant c>0, i.e. C_ε asymptotically agrees with a paraboloid. The derivatives y'(r) also asymptotically are linear, although apparently oscillatory perturbations added to linear functions, with oscillation-amplitudes tending(?) to constants.

Three kinds of "uniform illumination." The stratovases (ignoring the slight issue about $\varepsilon>0$) and the solarhedron illuminate their solar cells "weakly uniformly," meaning each point on the cell's surface experiences the same incoming optical power flux. The original solargon illuminates its solar cell "strongly uniformly," meaning that, in addition, each point on the cell's surface sees identical *angular distributions* of incoming photons. For the two variant solargons, these angular distributions are not always identical, but can be made identical by performing an x-axis sign-negation if necessary, which for many practical purposes *is* "identical."

2. *Stationary* solar concentrators

2D model. We regard the solar cell as the interval (-1,1) on the East-West x-axis, and light as coming in from the upper infinite semicircle in the xy plane. (One could also consider 3D models.)

During the course of a day, and depending on the weather, your geographic location and altitude, and the amount of atmospheric aerosols, the light and its distribution *change*. However, if we time-average over multiyear periods, we get some fixed distribution. This light-ray distribution is *not* uniform. Rather, the light power flux shining from angle θ east or west of vertical behaves proportionally to something roughly like



0.63466747exp(-0.367sec(0.99416 θ)) for $|\theta|<\pi/2$ (otherwise zero power). Here the overall scaling constant in front has been chosen to make this expression a normalized probability density (plotted at right), while the other two constants were chosen to agree with allegedly-typical values quoted in Flanders & Creed's article in the June 2008 *Sky and Telescope* magazine. Better such curves can be created by measuring the brightnesses of a selection of "standard stars" at different times throughout the night using different colors; these curves can be used to correct the brightness of pixels in telescope images. (In fact, the "Vera Rubin observatory" claims their "auxiliary telescope" does exactly that almost every day at their location in the Elqui Province of Chile, but annoyingly they currently refuse to publish the resulting curves on the internet in any readily assimilable form. The curves at your location undoubtably will differ from, and probably usually exhibit greater extinction at all angles than, the curve at the Rubin's 2673m altitude location.) This probability density p(θ) has *entropy* $S=-\int_{|\theta|<\pi/2} p(\theta)\ln(p(\theta))d\theta\approx1.01245$ nats, which is **ΔS**≈0.13228 nats less than the maximum possible entropy $S_{unif}=\int_{|\theta|<\pi/2}\pi^{-1}\ln(\pi)d\theta=\ln\pi\approx1.14473$ nats, achieved by the uniform (rectangular) density.

This nonuniformity allows us, by clever shaping of stationary mirrors, to concentrate more light (in a time-averaged sense) onto a flat solar cell than it would have received without mirrors. The **question** is: what shape for those mirrors maximizes total output energy?

Frankly, I regard this question as more an *experimental* than mathematical question; and to the extent it is mathematical, then it is probably best answered via computational "experiments" with ray-tracing computer programs and optimization. Nevertheless we shall discuss some exact-formula shape-recipes. The first thing to realize is that the nonuniformity of the light distribution is *crucial* to allow any light concentration to be achievable at all. If (to illustrate this point) the Earth were not a sphere with axial tilt≈23.44°, but rather an infinite circular cylinder with axial tilt=0 with *no* atmosphere (sun also a hot blackbody infinite cylinder), then the inhabitants of cylindro-Earth would be unable to obtain any light concentration factor>1 at all, via any stationary mirror arrangement at all – they'd be best off simply using solar cells laid flat on the surface. The source of this claim is the "entropy cannot decrease" law of thermodynamics:

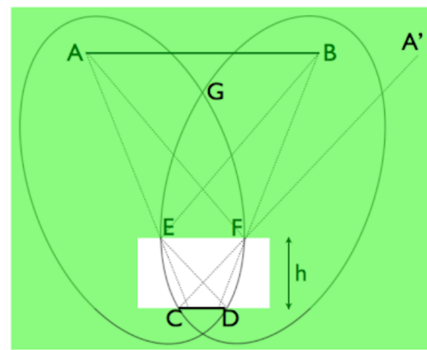
Entropy theorem. No stationary mirror arrangement can produce any light concentration factor greater than exp(ΔS); and that could only be achieved if the mirrors somehow altered the photon *direction-angle* distribution to become uniform[-π/2,π/2], while shrinking their *position* distribution by factor exp(ΔS).

(Entropy is the logarithm of "phase space volume" for the classical "frictionless billiards" model; phase space volume cannot change by "Liouville's theorem" for Hamiltonian systems.)

So for the 2D light distribution model we quoted above based on Sky & Telescope magazine, an upper bound on the maximum possible light-concentration factor is exp(0.13228)≈1.14143. It hardly seems worth devoting great money and effort to obtain, at best, 14% improvement! I contend that the entire story foisted on us for the last 50+ years that stationary (i.e. *not* sun-tracking) mirror East-West solar-concentrator arrangements are a good idea, worth paying money for, has been basically a **fraud**.

Winston's design and why I do not like it. Winston suggested a particular mathematical class of mirror designs called "CPCs," based on two parabolic arcs, which he proved it "optimal" in this (not tremendously relevant) sense: his solar cell width is $\sin\theta$ times the width of the CPC's light-input portal, where 2θ is the angular width of the input-photon direction-wedge; and $\sin\theta$ is *minimum possible* if all input-portal photons in that direction angle-interval must hit the flat solar cell.

Unfortunately, a perfectly-built perfectly-reflective CPC would cause *focusing*, twice per day for an East-West CPC, of sunlight onto a line at the edge of the solar cell, causing it to melt – a fact CPC manufacturers do *not* provide prominent health and safety warnings about. The distribution of illumination on the solar cell near those times of day would be extremely nonuniform – as opposed to if you simply used a flat solar cell with no mirrors at all, in which case it would be uniform always. This kind of nonuniformity probably would cause your solar cell to work badly even if it did not melt, quite plausibly defeating whatever efficiency gains could be got by concentrating the light.



Bilaterally-symmetric compound ellipse concentrator. All light rays emitted from source line segment AB passing between E and F, will hit solar cell line-segment CD. Ellipse-arc mirror EC has foci B,D; ellipse-arc mirror FD has foci A,C. Three-parameter family: h & 2 coords of B. "Naive optimality": solar cell power maximized for ideal blackbody source if choose h to maximize mean EQF angle for Q on segment AB.

A 3-parameter class of designs based on two ellipse-arcs, that I prefer. See picture explaining the design. It should be obvious that (with a hot blackbody line segment AB as the source of illumination) the optimum value of the height parameter h lies *strictly between* zero and the height of point G. In other words, this concentrator *works* in this setting, to get more photons to land on the solar cell, than in the no-mirrors case h=0. This class of designs *never* focuses light to infinite intensity anywhere on the solar cell if illuminated by parallel light rays from any direction. Also note that some of the photons hypothetically emitted from the line segment BA' that extends AB – this extension is part of the sky but not part of our blackbody – could also reach the solar cell; and that the true sky light distribution is *not* that produced by the pictured line-segment blackbody source model. Therefore, the parameter set optimal for the pictured blackbody source, will not be the optimal parameter set for the true sky light distribution. What I suggest is that you, via computer ray tracing, solar cell modeling under nonuniform illumination conditions, and imperfectly-reflective mirror modeling, *model* (or directly measure) the solar cell output arising from the true sky light-distribution for any given choice of the 3 parameters, then find the optimal choice of parameters.

My computations suggest the best parameter-triple choice will indeed yield performance better than you would have gotten with a bare solar cell – but only about 1% better! (A CPC with acceptance angle 159.1 degrees should outperform a bare solar cell.) And the performance gain obtainable in this way probably is not worth the extra manufacturing, complexity, maintenance, and/or materials costs.

Probably a **more productive approach** than the above "East-West model" would be to note that the sun always lies within angle 23.44° North or South from Earth's equatorial plane. Therefore, orient your stationary solar cells perpendular to that plane, and surround them on both their *North* and *South* sides by a stationary mirror concentrator arrangement like that pictured, designed to have "acceptance angle" range of, say, ±24°, which is a subset (fractionally 28.9%) of the full ±90° range. Then the entropic concentration upper-bound is $\exp(-\ln(0.289))=0.289^{-1}\approx 3.46$, while Winston's CPC achieves $\csc(24°)\approx 2.4586$. A factor≈2.2 performance gain should be attainable in practice, at least in high-altitude areas with clear skies.

Lessons from Biology. Time lapse videos show that some plants attempt to reorient their leaves during the course of each day to do sun-tracking – and I am speaking of common houseplants, not unusual plants specialized for high altitudes. Also many attempt to grow, over time spans longer than 1 day, to maximize whole-year solar collection efficiency.

But there is no plant I am aware of that uses mirrors. That presumably is because, for plants, mirrors are *not* substantially cheaper per square-centimeter, than solar collectors, i.e. leaves. However, *microscopically*, plant "chromatophores" employ structures made of cheap molecules to gather light, then convert that light to usable chemical energy using rarer more-expensive molecules. In that sense plants could be regarded as performing solar "concentration" at macromolecular length scales. But since those length scales are much smaller than the wavelength of the light, this is not very relevant to solar concentrator constructs much larger than the wavelength of light.

Given present day technical and economic realities for artificial solar cells and mirrors and sun-tracking mechanisms, as compared with plant biological pseudo-economics, it therefore should be clear that

1. Sun-tracking is *worthwhile* for large scale solar power, even in common low-altitude locations.
2. Given that you already are tracking the sun, you probably should use something like the solargon, solarhedron, etc. to lower costs per watt; and the use of non-tracking solar concentrators designed for *stationary* solar cells, no longer has interest.

3. References

Julio Chaves: Introduction to Nonimaging Optics, Second Edition, CRC Press 2015. TJ812.C427.

Tony Flanders & Phillip J. Creed: Transparency and atmospheric extinction, Sky & Telescope 10 June 2008.

Roland E. Winston, Juan C. Minano, Pablo G. Benitez: Nonimaging Optics, Elsevier 2005. TJ812.W56.