Remarks on Rivas's Equations of Motion of Spinning Electrons

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Abstract

Rivas's model of spinning particles describes an electron as a point-like center of charge spinning around a point-like center of mass with both points satisfying specific equations of motion. The present remarks comment on various aspects of these equations, including notation, parameters of the spin motion, methods for numerical integration, and a modification of the equations of motion to model the Larmor precession of electrons.

1 Introduction

In the introduction to his book "Kinematical Theory of Spinning Particles," Rivas wrote that it "is an attempt to produce a classical, and also a quantum mechanical, description of spin and the related properties inherent to it [...]" [Riv01, page xix]. Such a description could provide a better understanding of the internal structure of electrons, which to this day many researchers consider an unsolved question. Additionally, a classical description of spinning electrons might also lead to a deeper understanding of the quantum mechanics of electrons.

The present remarks are based on the present author's efforts to implement a numerical integration of Rivas's equations of motion of spinning electrons [Riv01, Riv24] in order to simulate the motion of classical electrons in hydrogen atoms. One of the challenges of this scenario is that the center of charge of the electron moves at the speed of light and orbits the center of mass many thousands of times during one orbit of the center of mass around the nucleus of the hydrogen atom. Thus, the numerical integration has to be quite stable to cover even a single orbit of the electron around the nucleus. At the same time, the exact number of orbits of the spin motion during one orbit around the nucleus is constrained by Einstein's quantization condition [Kra25], thus, the numerical integration has to be rather accurate to be consistent with Einstein's quantization condition. Furthermore, the center of mass of electrons in this scenario reaches relativistic speeds even in the ground state of hydrogen atoms if the ground state is described by linear pendulum-paths as hypothesized by several researchers [Eps16, PW35, Buc06] including Rivas [Riv24, page 18]. Thus, the scenario of a hydrogen atom sets a relatively high bar for a successful numerical integration of Rivas's equations, which might require the use of multiple constants of motion as discussed in Section 4.

Unfortunately, an accurate numerical integration of Rivas's classical equations of motion might not be sufficient to simulate real electrons since the classical motion of electrons as determined by Rivas's equations does not always correctly describe the motion of real electrons as noted by Rivas for the case of Larmor precession of an electron in a uniform magnetic field [Riv01, Riv24]. Therefore, an ad-hoc modification of Rivas's equations is proposed in Section 5 to model this Larmor precession in the hope that this modification is generally useful. Another benefit of the modified equations is that they allow for a plausible interpretation of each term as discussed in Section 5.

2 Notation

The notation employed in the present remarks follows the notation of Rivas [Riv01, Riv24] except for a few name changes. Specifically, Rivas's use of the subscripts "CC" and "CM" referring to the "center of charge" and the "center of mass" is applied to more symbols as summarized in Table 1.

Table 1: Symbols differing from Rivas's notation [Riv01, Riv24].

Symbol here	Rivas's notation	Description
$\mathbf{r}_{\mathrm{CM}}$	$\mathbf{q}$	position of center of mass
$\mathbf{r}_{\mathrm{CC}}$	$\mathbf{r}$	position of center of charge
$\mathbf{v}_{\mathrm{CM}}$	$\mathbf{v}$	velocity of center of mass
$\mathbf{v}_{\mathrm{CC}}$	$\mathbf{u}$	velocity of center of charge
m_{CM}	m	mass resisting acceleration of center of mass
m_{CC}	—	mass resisting acceleration of center of charge
γ_{CM}	$\gamma(v)$	Lorentz factor $1/\sqrt{1-\lvert\mathbf{v}_{\mathrm{CM}}\rvert^2/c^2}$
q_{CC}	e	charge of electron

3 Frequency and Radius of Spin Motion

Rivas concluded that the center of charge of an electron spins around a resting center of mass at a distance of $R_0 = \hbar/(2\,m_{\mathrm{CM}}c)$ with angular frequency $\omega = 2\,m_{\mathrm{CM}}c^2/\hbar$. While this is consistent with the Zitterbewegung of Dirac electrons, it appears to be in conflict with de Broglie's internal clock hypothesis, which assumes an angular frequency of $m_{\mathrm{CM}}c^2/\hbar$. Since a resolution of this disagreement might require different values of R_0 and ω [Kra24b], the equations in the present remarks do not assume specific values of R_0 and ω as long as they fulfill the condition

$$R_0\omega = c, \tag{1}$$

which follows from the assumption that the center of charge moves at the speed of light c.

4 Numerical Integration of Spin Motion

This section describes some methods to improve the stability (but not necessarily the accuracy) of the numerical integration of the motion of spinning electrons.

4.1 Equations of Motion

In the notation introduced in Section 2, Rivas's equations of motion for a spinning electron in an external electromagnetic field in non-relativistic approximation ($\mathbf{v}_{\mathrm{CM}} \ll c$) are [Riv01, page 68][Riv24, page 89]:

$$\frac{\mathrm{d}^2\mathbf{r}_{\mathrm{CM}}}{\mathrm{d}t^2} = \frac{q_{\mathrm{CC}}}{m_{\mathrm{CM}}}\left(\mathbf{E} + \mathbf{v}_{\mathrm{CC}} \times \mathbf{B}\right) \tag{2}$$

$$\frac{\mathrm{d}^2\mathbf{r}_{\mathrm{CC}}}{\mathrm{d}t^2} = \omega^2\left(\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}\right) \tag{3}$$

where the electromagnetic fields $\mathbf{E} = \mathbf{E}(t, \mathbf{r}_{\mathrm{CC}})$ and $\mathbf{B} = \mathbf{B}(t, \mathbf{r}_{\mathrm{CC}})$ are evaluated at $\mathbf{r}_{\mathrm{CC}}$ and

$$\mathbf{v}_{\mathrm{CC}} \stackrel{\mathrm{def}}{=} \frac{\mathrm{d}\mathbf{r}_{\mathrm{CC}}}{\mathrm{d}t}. \tag{4}$$

Furthermore, $\mathbf{v}_{\mathrm{CC}}$ is constrained by

$$\lvert\mathbf{v}_{\mathrm{CC}}\rvert = c. \tag{5}$$

The relativistic equations [Riv24, page 130] are:

$$\frac{\mathrm{d}^2\mathbf{r}_{\mathrm{CM}}}{\mathrm{d}t^2} = \frac{q_{\mathrm{CC}}}{\gamma_{\mathrm{CM}}m_{\mathrm{CM}}}\left(\mathbf{E} + \mathbf{v}_{\mathrm{CC}} \times \mathbf{B} - \frac{1}{c^2}\mathbf{v}_{\mathrm{CM}}\left(\left(\mathbf{E} + \mathbf{v}_{\mathrm{CC}} \times \mathbf{B}\right) \cdot \mathbf{v}_{\mathrm{CM}}\right)\right) \tag{6}$$

$$\frac{\mathrm{d}^2\mathbf{r}_{\mathrm{CC}}}{\mathrm{d}t^2} = \frac{c^2 - \mathbf{v}_{\mathrm{CM}} \cdot \mathbf{v}_{\mathrm{CC}}}{\lvert\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}\rvert^2}\left(\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}\right) \tag{7}$$

where

$$\mathbf{v}_{\rm CM} \stackrel{\rm def}{=} \frac{\mathrm{d}\mathbf{r}_{\rm CM}}{\mathrm{d}t} \tag{8}$$

$$\mathbf{v}_{\rm CC} \stackrel{\rm def}{=} \frac{\mathrm{d}\mathbf{r}_{\rm CC}}{\mathrm{d}t} \tag{9}$$

with the same constraint as in the non-relativistic case:

$$|\mathbf{v}_{\rm CC}| = c. \tag{10}$$

This constraint, the velocity-dependent acceleration, and any non-linear external electromagnetic field (e.g., a Coulomb field) limit the benefits of many advanced numerical integration methods. Therefore, the author chose to implement a straightforward explicit midpoint Euler integration with a relatively small step size (10^{-22} s) as it allows for a straightforward implementation of the constraint $|\mathbf{v}_{\rm CC}| = c$ by rescaling the vector $\mathbf{v}_{\rm CC}$ in each step.

To avoid a diverging value of $\mathbf{v}_{\rm CM}$, the total energy of the electron was constrained as described in the next section.

4.2 Constraining the Total Energy of the Center-of-Mass Motion

In the case of a known static external electromagnetic field, computing the total energy of the center-of-mass motion based on $\mathbf{v}_{\rm CM}$, $m_{\rm CM}$, $\mathbf{r}_{\rm CC}$, and $q_{\rm CC}$ is straightforward if the motion is considered equivalent to the motion of a charged particle in an external electromagnetic field. Given the computed total energy and a constant target value for it, $\mathbf{v}_{\rm CM}$ may be rescaled after each integration step to bring the computed total energy back to the constant target value.

However, care should be taken to avoid this correction if $|\mathbf{v}_{\rm CM}|$ is very small (since the direction of $\mathbf{v}_{\rm CM}$ could be numerically ill-defined in this case), and it is useful to correct only part of the deviation in each step of the integration in order to avoid any overcompensation, which might result in diverging oscillatory deviations. Furthermore, the high-frequency spin motion might add an "uncertainty" to the total energy (depending on the electric potential), which should not be corrected. Thus, as long as the deviation of the computed total energy from its target value is relatively small, the correction should be paused.

In addition to constraining the total energy of the center-of-mass motion and constraining the speed of the center of charge, the radius of the spin motion should also be constrained as discussed next.

4.3 Constraining the Radius of the Spin Motion

Constraining the radius of the spin motion (in the rest frame of a free center-of-mass motion) to a constant target value R_0 is more complicated in the relativistic case due to relativistic length contraction; therefore, the non-relativistic case is discussed first.

4.3.1 Non-Relativistic Case

The non-relativistic spin motion is described by Eq. (3):

$$\frac{\mathrm{d}^2\mathbf{r}_{\rm CC}}{\mathrm{d}t^2} = \omega^2 \left(\mathbf{r}_{\rm CM} - \mathbf{r}_{\rm CC}\right).$$

Even if $|\mathbf{v}_{\rm CC}|$ is constrained to c, Eq. (3) does not limit the distance $|\mathbf{r}_{\rm CM} - \mathbf{r}_{\rm CC}|$ to $R_0 = c/\omega$. In fact, even the radius of curvature is not fixed but varies with the distance $|\mathbf{r}_{\rm CM} - \mathbf{r}_{\rm CC}|$; thus, a small numerical deviation of $|\mathbf{r}_{\rm CM} - \mathbf{r}_{\rm CC}|$ from R_0 may lead to increasingly larger deviations over time. One way of eliminating the dependency of the radius of curvature on the distance $|\mathbf{r}_{\rm CM} - \mathbf{r}_{\rm CC}|$ is to use

Eq. (1), i.e., $R_0\omega = c$, to rewrite Eq. (3) assuming $|\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}| \approx R_0$:

$$\begin{aligned}
\frac{\mathrm{d}^2\mathbf{r}_{\mathrm{CC}}}{\mathrm{d}t^2} &= \omega^2 (\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}) && (11)\\
&= \frac{c^2}{R_0^2}(\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}) && (12)\\
&= \frac{c^2}{R_0}\frac{\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}}{R_0} && (13)\\
&\approx \frac{c^2}{R_0}\frac{\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}}{|\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}|} && (14)
\end{aligned}$$

Together with the constraint $|\mathbf{v}_{\mathrm{CC}}| = c$, the last equation fixes the radius of curvature to R_0. Thus, even if $|\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}|$ has deviated temporarily from R_0, the motion of $\mathbf{r}_{\mathrm{CC}}$ is very likely to return to a circular orbit of radius R_0 around $\mathbf{r}_{\mathrm{CM}}$ if $\mathbf{r}_{\mathrm{CM}}$ is at rest.

4.3.2 Relativistic Case

For relativistic speed $|\mathbf{v}_{\mathrm{CM}}|$, relativistic length contraction affects the spin motion. In the rest frame of a force-free electron that is observed to move with $\mathbf{v}_{\mathrm{CM}}$, the spin motion is always a circular orbit with radius R_0. However, in the observer's frame, the distance between $\mathbf{r}_{\mathrm{CM}}$ and $\mathbf{r}_{\mathrm{CC}}$ may be different from R_0 due to the relativistic length contraction in the direction of $\mathbf{v}_{\mathrm{CM}}$. Thus, instead of assuming $|\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}| \approx R_0$, the "uncontracted" length (the distance in the rest system of the electron) should be assumed to be approximately equal to R_0.

To this end, the "contracted" distance $\mathbf{d} \stackrel{\mathrm{def}}{=} \mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}$ in the observer's frame is split into a part $\mathbf{d}_\parallel$ parallel to $\mathbf{v}_{\mathrm{CM}}$ and a part $\mathbf{d}_\perp$ orthogonal to this direction with $|\mathbf{d}|^2 = |\mathbf{d}_\parallel|^2 + |\mathbf{d}_\perp|^2$:

$$\begin{aligned}
\mathbf{d}_\parallel &\stackrel{\mathrm{def}}{=} \frac{\mathbf{d}\cdot\mathbf{v}_{\mathrm{CM}}}{|\mathbf{v}_{\mathrm{CM}}|^2}\mathbf{v}_{\mathrm{CM}} && (15)\\
\mathbf{d}_\perp &\stackrel{\mathrm{def}}{=} \mathbf{d} - \mathbf{d}_\parallel && (16)
\end{aligned}$$

$|\mathbf{d}_\parallel|$ may be considered the result of a length contraction with the inverse Lorentz factor $1/\gamma_{\mathrm{CM}} = \sqrt{1 - |\mathbf{v}_{\mathrm{CM}}|^2/c^2}$. Thus, the "uncontracted" length in the direction of $\mathbf{v}_{\mathrm{CM}}$ is $\gamma_{\mathrm{CM}}|\mathbf{d}_\parallel|$. On the other hand, $|\mathbf{d}_\perp|$ is not affected by the length contraction since it is orthogonal to $\mathbf{v}_{\mathrm{CM}}$. Thus, the "uncontracted" distance between $\mathbf{r}_{\mathrm{CM}}$ and $\mathbf{r}_{\mathrm{CC}}$ in the electron's rest frame depends on the direction of $\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}$ relative to $\mathbf{v}_{\mathrm{CM}}$ and is given by $\sqrt{\gamma_{\mathrm{CM}}^2|\mathbf{d}_\parallel|^2 + |\mathbf{d}_\perp|^2}$.

Due to this dependency on the direction of $\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}$, replacing $|\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}|$ by R_0 (see the previous section) is not possible in the relativistic case. Instead, the length of the vector $\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}$ should be replaced by the length of a vector, whose "uncontracted" length is R_0. Fortunately, it is easy to construct such a vector by scaling the vector $\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}$ with the factor $R_0/\sqrt{\gamma_{\mathrm{CM}}^2|\mathbf{d}_\parallel|^2 + |\mathbf{d}_\perp|^2}$. The length of the scaled vector is then $|\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}|\, R_0/\sqrt{\gamma_{\mathrm{CM}}^2|\mathbf{d}_\parallel|^2 + |\mathbf{d}_\perp|^2}$.

Thus, Eq. (7) for the relativistic spin motion may be rewritten as:

$$\begin{aligned}
\frac{\mathrm{d}^2\mathbf{r}_{\mathrm{CC}}}{\mathrm{d}t^2} &= \frac{c^2 - \mathbf{v}_{\mathrm{CM}}\cdot\mathbf{v}_{\mathrm{CC}}}{|\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}|^2}(\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}) && (17)\\
&= \frac{c^2 - \mathbf{v}_{\mathrm{CM}}\cdot\mathbf{v}_{\mathrm{CC}}}{|\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}|}\frac{\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}}{|\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}|} && (18)\\
&\approx \frac{c^2 - \mathbf{v}_{\mathrm{CM}}\cdot\mathbf{v}_{\mathrm{CC}}}{|\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}|\frac{R_0}{\sqrt{\gamma_{\mathrm{CM}}^2|\mathbf{d}_\parallel|^2 + |\mathbf{d}_\perp|^2}}}\frac{\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}}{|\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}|} && (19)
\end{aligned}$$

It might be worthwhile to check the plausibility of the right-hand side of the last equation by considering two special cases: $\mathbf{d}_\parallel = 0$ and $\mathbf{d}_\perp = 0$. For $\mathbf{d}_\parallel = 0$, we have $\sqrt{\gamma_{\mathrm{CM}}^2|\mathbf{d}_\parallel|^2 + |\mathbf{d}_\perp|^2} = |\mathbf{d}_\perp| = |\mathbf{d}| = |\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}|$; and, therefore, the denominator of the first fraction on the right-hand side of the equation reduces to R_0, which is plausible since there should be no effect of length contraction in this case. On the other hand, for $\mathbf{d}_\perp = 0$, we have $\sqrt{\gamma_{\mathrm{CM}}^2 |\mathbf{d}_\parallel|^2 + |\mathbf{d}_\perp|^2} = \gamma_{\mathrm{CM}} |\mathbf{d}_\parallel| = \gamma_{\mathrm{CM}} |\mathbf{d}| = \gamma_{\mathrm{CM}} |\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}}|$; and, therefore, the denominator reduces to the fully length-contracted R_0/γ_{CM}, which is plausible since the full effect of length contraction should apply in this case.

5 Precession of Spin Motion in a Uniform Magnetic Field

Based on Eqs. (2) and (3), Rivas computed the precession of the spin motion of a non-relativistic classical electron in a uniform magnetic field $\mathbf{B}$ [Riv01, Ch. 2, Sec. 4.3][Riv24, Sec. 2.2.8]. His result for the angular frequency of the spin precession is $|q_{\mathrm{CM}} \mathbf{B}|/(2\, m_{\mathrm{CM}})$ [Riv01, page 83][Riv24, page 97], i.e., Larmor's angular frequency for a particle with $g = 1$. Rivas attributed the value $g = 1$ of the model to a missing spin contribution [Riv01, page 69][Riv24, page 90], and argued that a deeper analysis of the gyromagnetic ratio of electrons leads to $g \approx 2$ [Riv01, Ch. 6, Sec. 1.2][Riv24, Sec. 6.1]. Regardless of the gyromagnetic ratio, however, it remains unclear how the classical version of Rivas's electron model could describe a motion that matches the experimentally measured spin precession of electrons.

After numerically confirming Rivas's result for the frequency of the spin precession, the present author included an ad-hoc term in the equation of motion for $\mathbf{r}_{\mathrm{CC}}$, namely an acceleration of a parameterized mass m_{CC} at $\mathbf{r}_{\mathrm{CC}}$ by the external electromagnetic force. Preliminary numerical simulations showed that the parameter m_{CC} in the new force term may be tuned to model Larmor's angular frequency for a range of values of the electron's gyromagnetic ratio, specifically $m_{\mathrm{CC}} \approx 0.3\, m_{\mathrm{CM}}$ for $g \approx 2$. At the time of writing, the only other motivation for the additional force term is that it allows for a convenient interpretation as presented below.

More specifically, the additional force term is inserted in Eq. (3) while Eq. (2) remains unchanged. Thus, the new equations of motion are:

$$\frac{\mathrm{d}^2 \mathbf{r}_{\mathrm{CM}}}{\mathrm{d}t^2} = \frac{q_{\mathrm{CC}}}{m_{\mathrm{CM}}} \left(\mathbf{E} + \mathbf{v}_{\mathrm{CC}} \times \mathbf{B} \right) \tag{20}$$

$$\frac{\mathrm{d}^2 \mathbf{r}_{\mathrm{CC}}}{\mathrm{d}t^2} = \omega^2 \left(\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}} \right) + \frac{q_{\mathrm{CC}}}{m_{\mathrm{CC}}} \left(\mathbf{E} + \mathbf{v}_{\mathrm{CC}} \times \mathbf{B} \right) \tag{21}$$

where the electromagnetic fields $\mathbf{E} = \mathbf{E}(t, \mathbf{r}_{\mathrm{CC}})$ and $\mathbf{B} = \mathbf{B}(t, \mathbf{r}_{\mathrm{CC}})$ are always evaluated at $\mathbf{r}_{\mathrm{CC}}$, and $\mathbf{v}_{\mathrm{CC}}$ is constrained by $|\mathbf{v}_{\mathrm{CC}}| = c$.

The relativistic Eqs. (6) and (7) become:

$$\mathbf{F}_{\mathrm{ex}} \stackrel{\mathrm{def}}{=} q_{\mathrm{CC}} \left(\mathbf{E} + \mathbf{v}_{\mathrm{CC}} \times \mathbf{B} - \frac{1}{c^2} \mathbf{v}_{\mathrm{CM}} \left(\left(\mathbf{E} + \mathbf{v}_{\mathrm{CC}} \times \mathbf{B} \right) \cdot \mathbf{v}_{\mathrm{CM}} \right) \right) \tag{22}$$

$$\frac{\mathrm{d}^2 \mathbf{r}_{\mathrm{CM}}}{\mathrm{d}t^2} = \frac{1}{\gamma_{\mathrm{CM}} m_{\mathrm{CM}}} \mathbf{F}_{\mathrm{ex}} \tag{23}$$

$$\frac{\mathrm{d}^2 \mathbf{r}_{\mathrm{CC}}}{\mathrm{d}t^2} = \frac{c^2 - \mathbf{v}_{\mathrm{CM}} \cdot \mathbf{v}_{\mathrm{CC}}}{\left| \mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}} \right|^2} \left(\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}} \right) + \frac{1}{\gamma_{\mathrm{CM}} m_{\mathrm{CC}}} \mathbf{F}_{\mathrm{ex}} \tag{24}$$

where $\mathbf{v}_{\mathrm{CC}}$ is constrained by $|\mathbf{v}_{\mathrm{CC}}| = c$ as in the non-relativistic case.

As mentioned, preliminary simulations of an electron in a uniform magnetic field with the setting $m_{\mathrm{CC}} \approx 0.3\, m_{\mathrm{CM}}$ resulted in a spin precession at Larmor's angular frequency for $g \approx 2$. The fact that m_{CC} had to be less than m_{CM} for this result encourages the following interpretation of the proposed equations of motion: The terms $q_{\mathrm{CC}} \left(\mathbf{E} + \mathbf{v}_{\mathrm{CC}} \times \mathbf{B} \right)$ in Eq. (21) and $\mathbf{F}_{\mathrm{ex}}$ in Eq. (24) might be interpreted as the direct effect of an external electromagnetic force on the electron's charge at $\mathbf{r}_{\mathrm{CC}}$. The terms $\omega^2 \left(\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}} \right)$ in Eq. (21) and $\left(\mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}} \right) \left(c^2 - \mathbf{v}_{\mathrm{CM}} \cdot \mathbf{v}_{\mathrm{CC}} \right) / \left| \mathbf{r}_{\mathrm{CM}} - \mathbf{r}_{\mathrm{CC}} \right|^2$ in Eq. (24) could then represent an additive, internal force on the electron's charge at $\mathbf{r}_{\mathrm{CC}}$ that keeps it on an orbit around $\mathbf{r}_{\mathrm{CM}}$. In this interpretation, the terms $q_{\mathrm{CC}} \left(\mathbf{E} + \mathbf{v}_{\mathrm{CC}} \times \mathbf{B} \right)$ in Eq. (20) and $\mathbf{F}_{\mathrm{ex}}$ in Eq. (23) could describe an indirect, time-averaged effect of an external electromagnetic force since it might not be the external force itself that accelerates $\mathbf{r}_{\mathrm{CM}}$ but the motion of $\mathbf{r}_{\mathrm{CC}}$ that accelerates the whole mass m_{CM} of the electron, which is represented by the center of mass at $\mathbf{r}_{\mathrm{CM}}$. $m_{\mathrm{CC}} < m_{\mathrm{CM}}$ supports this interpretation since this relation means that the center of charge is more easily accelerated by an external force than the center of mass, and, therefore, the center of charge might tend to lead ahead and pull the center of mass with it.

It should be mentioned that this interpretation might help explaining why the external electromagnetic force in Rivas's theory has to be evaluated for the electromagnetic field at $\mathbf{r}_{\mathrm{CC}}$ but accelerates the center of mass at $\mathbf{r}_{\mathrm{CM}}$. While this feature might be considered a strange curiosity in a theory of point-like electrons, it is an essential characteristic of non-linear field-theoretical models of electrons [BIF34, Kra23], where only a very small (in good approximation point-like) region of the field solution reaches large enough field strengths to interact with external fields.

6 Conclusion and Future Work

As mentioned in Section 1, the present remarks are based on the present author's ongoing efforts to implement a somewhat general numerical integration of Rivas's equations of motion of spinning electrons. This is not a trivial task since finding convergent numerical routines for these equations can be difficult—as Rivas mentioned already in the epilogue of his book [Riv01, page 314]. The present author plans to address specific physical scenarios in future work.

With more evidence for the usefulness of Rivas's model, interest in a plausible interpretation of this model is likely to grow. Based on the discussion in Section 5, the present author hopes that a slightly modified version of Rivas's model could be interpreted as an approximation to an underlying classical field-theoretical model of electrons—similarly to previously proposed modifications of Beck's neo-classical model of electrons [Kra24a].

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A Revisions

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