Gravity as Toroidal Geometry of Time: Reformulating Einstein with Angular Tension

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Abstract

We propose a reformulation of Einstein's field equations based on the concept of a time-tensional toroidal geometry. The unification constant $\alpha_U = A_p K_e$ replaces Newtonian mass as the source of curvature. We define a tension-based vector field U^μ, from which we construct a geometric energy-momentum tensor $\mathcal{T}^{\mu\nu}$, and derive a new class of field equations. We show that in the weak-field limit, this formulation recovers Newtonian gravity, while offering a deeper ontological interpretation of space, time, and matter.

1 Introduction

Einstein's general relativity ties the curvature of space-time to the presence of energy and momentum via the Einstein field equations. However, mass and energy are not fundamental in this framework — they are inserted as external properties. We propose an internal origin of curvature: a tension field emerging from toroidal oscillation of time itself.

2 The Tensional Field U^μ

We define a vector field U^μ encoding the angular oscillation of time in a toroidal geometry:

$$U^\mu = \begin{pmatrix} \omega_t \\ \omega_r \cos(\theta) \\ \omega_\theta \sin(\theta) \\ \omega_\phi \end{pmatrix}$$

This field describes the helicoidal projection of time along a stable minimal area A_p, modulated by an electrostatic tension K_e.

3 Geometric-Tensional Field Equations

We replace the traditional stress-energy tensor $T_{\mu\nu}$ by a tension-based tensor:

$$\mathcal{T}^{\mu\nu} = \sigma \left(U^\mu U^\nu - \frac{1}{2} g^{\mu\nu} U^\lambda U_\lambda \right)$$

where σ is a tension density scalar, and $\alpha_U = A_p K_e$ is the universal coupling constant.

The modified field equation becomes:

$$G_{\mu\nu} = \alpha_U \left(U^\mu U^\nu - \frac{1}{2} g_{\mu\nu} U^\lambda U_\lambda \right)$$

4 Toroidal Metric with Oscillating Time

We define a minimal toroidal metric incorporating oscillating time:

$$ds^2 = -c^2(dt + \epsilon \sin(\omega\theta) d\theta)^2 + (R + r\cos\theta)^2 d\phi^2 + r^2 d\theta^2 + dr^2$$

The oscillatory term in time defines the curvature source.

5 Recovery of Newtonian Gravity

In the weak-field limit $r \ll R, \epsilon \ll 1$, we evaluate radial geodesics and recover the classical Newtonian form:

$$F_r = -r \left(\frac{d\theta}{d\tau} \right)^2 \quad \Rightarrow \quad F = \frac{G m_1 m_2}{r^2}$$

Rather than postulating Newton's constant, we derive it from the angular-tensional geometry. Given:

$$\alpha_U = A_p \cdot K_e$$

and knowing that the gravitational constant is:

$$G = \frac{A_p \cdot c^3}{\hbar} = \frac{\alpha_U \cdot c^3}{\hbar K_e}$$

This links gravitation directly to Planck area geometry and electrostatic tension, replacing mass with angular persistence as the source of curvature.

6 Tensorial Properties and Action Variation

In this technical section, we explore the variational basis of the proposed field equations, derived from a geometric-tensional action, in analogy with the Hilbert-Einstein action. We consider a pure tensional formulation, where the source of curvature is the field U^μ, associated with the angular tension of time in toroidal geometry.

6.1 Geometric-Tensional Action

We define the total action as:

$$S = \int \left[\frac{1}{2\alpha_U} R + \mathcal{L}_U \right] \sqrt{-g}\, d^4x$$

Where: - R is the Ricci scalar, representing spacetime curvature - $\mathcal{L}_U$ is the Lagrangian density of the tensional field - $\alpha_U = A_p \cdot K_e$ is the unifying constant of the model

The Lagrangian for the field U^μ is given by:

$$\mathcal{L}_U = -\frac{1}{2}\sigma\left(U^\mu U^\nu g_{\mu\nu}\right)$$

The variation of the action with respect to the metric $g^{\mu\nu}$ yields the energy-tension tensor:

$$\delta S = \int \left(\frac{1}{2\alpha_U}\delta R + \delta\mathcal{L}_U\right)\sqrt{-g}\,d^4x$$

Using the standard variation of the Hilbert term, we obtain:

$$G_{\mu\nu} = \alpha_U\left(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U^\lambda U_\lambda\right)$$

6.2 Invariants and Conservation

By construction, the tensor $\mathcal{T}^{\mu\nu}$ is symmetric, and local conservation is ensured by:

$$\nabla_\mu \mathcal{T}^{\mu\nu} = 0$$

This result reinforces the physical consistency of the model with the fundamental principles of general relativity.

6.3 Coupling with Fields and Extensions

The field U^μ may couple to other physical fields such as electromagnetism, through interaction terms like:

$$\mathcal{L}_{\text{int}} = \lambda F_{\mu\nu}U^\mu U^\nu$$

Or scalar fields:

$$\mathcal{L}_{\text{int}} = \eta\phi\,\nabla_\mu U^\mu$$

This opens a pathway for a unified tensional description of all interactions.

6.4 Final Comment

This formulation completes the dynamic structure of the toroidal-tensional model, connecting geometry with oscillating time via a variational action, and paving the way for quantum and cosmological extensions.

7 Outlook: Toward a Quantized Tensional Geometry

The geometric-tensional formulation introduced here suggests a natural pathway to quantization. The field U^μ, being vectorial and oscillatory, may admit canonical quantization or path integral formulation, particularly in the context of time-frequency duality.

We conjecture that spacetime itself behaves as a coherent system with minimal quantized area A_p, and that curvature is a macroscopic projection of underlying oscillations. This points toward a discretized curvature operator and energy levels associated with toroidal tension modes.

Future work will aim to express U^μ in terms of creation and annihilation operators, study its propagators, and evaluate its behavior under local gauge transformations.

We expect that the toroidal time model can be extended to reproduce not only gravity, but also the structure of quantum fields over tension-based geometry.

8 Conclusion

Gravity is not a force between masses, but a curvature caused by time tension in a toroidal structure. The reformulated field equations unify geometric dynamics and replace the arbitrary notion of mass with a measurable property: angular persistence.

References

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