Three fatal errors in the Relativity Theory

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Abstract: Though the Relativity Theory is over 100-years old, some questions still remain. Or this is the current accepted view of the Relativity Theory and this is how I was adviced to start this article. I was to pose some questions, not to state that there are fatal errors in the theory, to camouflage this article as aiming to fix some small problems in the theory, not refuting it. Supposedly it would help to get the article pass the editor to a review. But I could not bend the truth so much, not as a retired professor. The truth must finally be told: the article proves that the Relativity Theory is completely wrong, all of it. It is extremely strange that this theory was ever accepted and it is scandalous that it has been considered correct and verified for so long.

1. Three fatal errors in the Relativity Theory

This section shows fatal errors in the Special Relativity Theory, in the relativistic mass formula (invalidating Einstein's proof of $E = mc^2$), and in the General Relativity Theory. For a fatal error in Einstein's geodesic metric, see [3].

1.1. Error in the Lorentz transform, projection to time axis forgotten

This section shows that the Lorentz transform does not make the speed of light constant in all inertial reference frames. Einstein incorrectly calculated the speed of light in the moving coordinate system in the Lorentz transform, he did not take the projection of the points to the time axis in the moving coordinate system. When this projection is correctly done, the speed of light for the moving reference frame is in conflict with the measurements of the speed of light showing that the Lorentz transform is wrong in an unrecoverable way and the whole Special Relativity Theory is false.

The Lorentz Transform in two dimensions is

$$x' = \gamma(x - vt), \ t' = \gamma\left(t - \frac{v}{c^2}x\right), \qquad \text{where} \quad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}. \tag{1}$$

Solving x' from the first equation gives $x = \gamma^{-1}x' + vt$, inserting it to the second equation gives

$$t' = \gamma\left(t - \frac{v}{c^2}vt - \frac{v}{c^2}\gamma^{-1}x'\right) = \gamma^{-1}t - \frac{v}{c^2}x'. \tag{2}$$

When $x' = 0$, equation t' gives the time in the t'-axis, let us write $t'_c = \gamma^{-1}t$, thus

$$t' = t'_c - \frac{v}{c^2}x'. \tag{3}$$

1

This equation shows that t' is a local time, not the coordinate time. Let us take a projection of (x', t') on the x'-axis. The projection is done by keeping x' constant and letting the time on the t'-coordinate axis vary, thus t'_c varies from minus infinity to plus infinity giving a line. This line intersects the x'-axis and the intersection point gives the projection on the x'-axis.

In order for x' to be constant, we must have $x = \gamma^{-1}x' + vt$. The equation for x' in the Lorentz transform yields

$$x' = \gamma \left(\gamma^{-1}x' + vt - vt \right) = x' \tag{4}$$

showing that x' does not depend on t'_c. The line we obtain by varying t'_c is a vertical line and the projection $Pr_1(x', t')$ on the x'-axis is x'.

The projection $Pr_2(x', t')$ on the t'-axis is obtained by keeping t'_c constant and varying x' from minus infinity to plus infinity. This gives a line and the intersection of this line with the t'-axis is the projection on the t'-axis. From (3) we see that this line is a straight line sloping down when x' grows and the intersection with the t'-axis is t'_c. Thus, $Pr_2(x', t') = t'_c$.

Let the speed of light in the coordinate system (x, t) be c. Let light be sent from the origin to the positive x-axis at the moment $t = 0$. The receiver for the light is at $(x, 0)$ at the time $t = 0$ and let t be the time when light arrives to the receiver. Light travels the distance ct and the receiver travels the distance vt, thus

$$ct = x + vt, \qquad \frac{x}{t} = c - v \tag{5}.$$

If light is sent to the negative x-direction, then

$$\frac{x}{t} = c + v \tag{6}.$$

The speed of light in the coordinate system (x', t') is

$$c' = \frac{Pr_1(x', t')}{Pr_2(x', t')} = \gamma^2 \frac{x}{t} = \gamma^2(c - v) \tag{7}$$

if light is sent to the positive x-direction and $c' = \gamma^2(c + v)$ if light is sent to the negative x-direction. The speed of light is not constant in all inertial frames of reference.

The error Einstein made in the Special Relativity Theory is that he did not take the projections on the x' and t'-axes but calculated the speed of light in (x', t') as

$$c'' = \frac{x'}{t'} = c. \tag{8}$$

The error is the same as if in our time-zone time system one would calculate the flight time of an airplane crossing time zones by subtracting the local time of the origin from the local time of the destination.

2

It can be easily shown that there is no possible way to insist that we can use the coordinates (x', t') in the way they are used in (8), that is, pretend that they are ordinary two-dimensional coordinates of a 2-dimensional plane and that t' is not a local time but the time in the t'-coordinate. It merely requires to think of the Earth as the moving inertial frame and to imagine the non-moving frame. Let us show this.

On the Earth the speed of light in vacuum is measured to be c. In order to measure this speed we do not use a local time as in (3) but we use an ordinary two-dimensional coordinate system where the projection of (x, t) on the t-axis is t. Such ordinary coordinates we have in the moving inertial frame by taking the coordinates (x', t'_c) and not the coordinates (x', t'). The coordinates x' and t'_c are orthogonal as the transform

$$x' = \gamma(x - vt), \ t'_c = \gamma^{-1}t \tag{9}$$

is the Galileo transform where the axes x' and t'_c are multiplied by a constant. Let us imagine another inertial frame that is moving with the speed $-v$ with respect to our Earth. Let us assume this frame is just like our Earth and consequently the speed of light is c in that frame. We take this other frame as the (x, t) coordinate system and our Earth as the (x', t') coordinate system in the Lorentz transform. The Lorentz transform shows that c'' in (8) is c, but the coordinates that we use on the Earth to measure the speed of light are not (x', t'), they are (x', t'_c) and the speed of light on the Earth is $c' = \gamma^{-1}(c - v)$ when light is sent to the positive x-direction. Let $v = c/2$ and we notice that if the Lorentz transform is correct, then the speed of light on the Earth must be

$$c' = \frac{4}{3}\frac{c}{2} = \frac{2}{3}c \tag{10}$$

but the speed of light on the Earth is measured as c. This shows that we cannot define that in the moving frame we must use the coordinates (x', t'). In the moving frame ordinary 2-dimensional coordinates are (x', t'_c) and if we use coordinates (x', t'), then we have to take a projection on the t'-axis.

This example also shows that the Lorentz transform is completely wrong: it gives a contradiction: we have measured that the speed of light on vacuum on the Earth is c, therefore it must be c also on an imaginary planet that is just like the Earth, but if the Lorentz transform is correct, then the speed of light is not c on the Earth. This is a contradiction. There is no Lorentz transform, there is no time dilation, there is no length contraction, the Special Relativity Theory is false in an unrecoverable way, it cannot be fixed, it must be totally discarded. There is no reason to demand that equations of movement are Lorentz covariant.

1.2. If mass depends on velocity, kinetic energy does not equal work it makes

Einstein proposed that mass depends on the velocity. He did not check if mass that depends on velocity is a possible assumption or does it lead to a conflict. It

is not a possible assumption: mass cannot depend on velocity because it conflicts with work done by kinetic energy equalling kinetic energy in a situation where all of the kinetic energy is used to do this work. As mass cannot depend on velocity, Einstein's proof of Olinto de Pretto's formula $E = mc^2$ is wrong, the proof uses the relativistic mass formula. Let us show why mass cannot depend on velocity.

Work W done by force $F(s)$ is

$$W = \int F ds \qquad \text{thus} \quad F = \frac{d}{ds}W. \tag{11}$$

Force coming from kinetic energy is obtained from the Euler-Lagrange equation for a one-dimensional system with energy $E = E_p + E_k$, where E_p is potential energy and E_k is kinetic energy:

$$\frac{\partial}{\partial x}E - \frac{d}{dt}\frac{\partial}{\partial \dot{x}}E = 0 \tag{12}$$

simplifying to

$$\frac{\partial}{\partial x}E_p(x) - \frac{d}{dt}\frac{\partial}{\partial \dot{x}}E_k(\dot{x}) = 0. \tag{13}$$

The first term is clearly force coming from potential energy, therefore the second term is also force: it is force coming from kinetic energy:

$$F = \frac{d}{dt}\frac{\partial E_k(\dot{x})}{\partial \dot{x}}. \tag{14}$$

Einstein's relativistic mass $m(\dot{x})$ is a function of velocity only, therefore kinetic energy $E_k(t, x, \dot{x})$ in (14) is in this case a function of velocity only $E_k = E_k(\dot{x})$ and the partial derivative in (13) is a derivative of $v = \dot{x}$. Simplifying and writing $v = \dot{x}$ gives

$$F = \frac{d}{dt}\frac{d}{dv}E_k(v). \tag{15}$$

Let the force F in (15) make the work W, this means that F in (11) is the same as F in (15):

$$\frac{d}{ds}W = \frac{d}{dt}\frac{d}{dv}E_k(v). \tag{16}$$

Because all kinetic energy E_k is used to make the work W in (16), it follows that $E_k = W$ must hold. Denoting

$$y(v) = \frac{d}{dv}E_k(v) \tag{17}$$

and inserting $W = E_k$ into (16) yields

$$\frac{d}{ds}E_k(v) = \frac{d}{dt}y(v). \tag{18}$$

4

Equation (18) can be written as

$$\frac{dt}{ds}\frac{d}{dt}E_k(v) = \frac{dv}{dt}\frac{d}{dv}y(v) \tag{19}$$

$$\frac{1}{v}\frac{dv}{dt}\frac{d}{dv}E_k(v) = \frac{dv}{dt}\frac{d}{dv}y(v) \tag{20}$$

$$\frac{1}{v}y(v) = \frac{d}{dv}y(v) \tag{21}$$

$$\frac{1}{v}dv = \frac{1}{y}dy \tag{22}$$

$$y(v) = cv \tag{23}$$

where c is constant. Then

$$E_k(v) = \int y\,dv = \frac{1}{2}cv^2 \tag{24}$$

and as (24) holds in the limit when v approaches zero, we can identify c as constant mass m. Thus, for work to equal energy that can do this work (as it must equal), mass cannot depend on velocity.

Let us add a physical experiment that refutes the relativistic mass formula. Let us put a package of one kilo of flour on a table and see for a while if it turns into a black hole and sucks everything, including the observer, into it. Nothing happens, the observer sitting on a chair next to the table is still alive. This refutes the relativistic mass formula. This is because there exists a coordinate system that is moving with speed v that is extremely close to c with respect to the rest frame of the table where the flour is placed. It is not even necessary for the observer to think of this moving frame, the moving coordinate system does exist as a coordinate system, no mass needs to be at rest in this fast moving coordinate system. Notice that this is not a thought experiment, this is a real physical experiment where the finding is that nothing happens.

Taking this moving inertial reference frame as the (x, t) coordinate system and the rest frame of the table as the (x', t') coordinate system, this kilo of flour moves extremely fast, very close to the speed of light. Assuming that v is sufficiently close to c and that the relativistic mass formula holds, this kilo of flour grows so large that it turns into a black hole in (x, t) coordinates. But notice: if it turns into a black hole in one inertial reference frame, it turns into a black hole in every inertial reference frame because moving coordinate systems are only different views to the same one and real physical world: what happens in one of them must happen in all of them. If the black hole ate the observer in one reference frame, there is no observer in any reference frame. There are no parallel worlds in the Relativity Theory so that the observer could be alive and well in one world and sucked into a black hole in another.

1.3. The General Relativity Theory does not approximate Newtonian gravitation

In a situation where there is a point mass in otherwise empty space, the Einstein equations in spherical coordinates can be easily solved, the equations give the Schwarzschild solution. This solution approximates Newtonian gravitation field in large distances. Yet, the Schwarzschild solution is not a correct solution to the Einstein equations because the speed of light is not constant in this solution.

This is so because if the speed of light is the same to each space direction in every point (in the rest frame), then the metric must be induced by a scalar field ψ. The field inducing the Scwarzschild solution is not a scalar field, it is a tensor field. The reason why the field must be scalar is explained by a short argument:

In Cartesian coordinates a metric is given in signs (-,+,+,+) (as Einstein used) as
$$ds^2 = -g_{00}c^2dt^2 + g_{11}dx_1^2 + g_{22}dx_2^2 + g_{33}dx_3^2$$

Light-like worldpaths have $ds = 0$. A worldpath through spacepoint (x_1, x_2, x_3) is in the direction of the x_i-coordinate if $dx_j = 0$ for every other space infinitesimal dx_j, $j \neq i$. A light-like worldpath in the direction x_i gives the equation $g_{00}c^2dt^2 = g_{ii}dx_i^2$ and from this we get the speed of light in the direction of x_i: $c = \sqrt{g_{ii}/g_{00}}dx_i/dt = \sqrt{g_{ii}/g_{00}}$ as dx_i and dt are Euclidean infinitesimals. The requirement that light has the same speed c to each direction implies that in any place (x, y, z) and at any time t holds $g_{11} = g_{22} = g_{33}$ and the metric is induced by a scalar field: $g_{11} = g_{22} = g_{33} = \psi^2$, $g_{00} = \psi^2/c^2$.

The Schwarzschild metric (in signs (+,-,-,-)) is

$$ds^2 = -B(r)c^2dt^2 + B(r)^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2$$

where $B(r) = \left(1 - \frac{r_s}{r}\right)$. It is not induced by a scalar field and consequently the speed of light is not the same to all three space directions. We can express the metric in Cartesian coordinates where x and y are on the plane orthogonal to the radius and z is in the direction of the radius: $ds^2 = -B(r)c^2dt^2 + B(r)^{-1}dz^2 + dx^2 + dy^2$. The speed of light is different along the z-axis than in the (x, y)-plane. Measurements show that the speed of light in vacuum, when measured on the Earth, is the same c, known to many decimals, both in the horizontal plane as in the vertical direction. The Schwarzschild solution does not explain Newtonian gravity on the Earth.

To verify that there are no solutions to the Einstein equations that approximate the Newtonian gravity in the situation of a point mass in otherwise empty space, and the speed of light is constant, it is enough to notice that nobody has given a solution for this problem that has the metric induced by a scalar field in 100 years and the solution that is offered to students is the Schwarzschild solution. Nobody found a scalar field solution because there is no such solution. This is proven in Section 2.

When the metric is induced by a scalar field the Einstein equations in Cartesian coordinates the Einstein equations in this situation lead to the following equations: (the equation number given here is the same as in Section 2 so that it is easier to find the equation)

$$\frac{\partial}{\partial x_i}\left(\frac{\partial \psi}{\partial x_i}\psi^{-2}\right) = \frac{\partial}{\partial x_j}\left(\frac{\partial \psi}{\partial x_j}\psi^{-2}\right). \tag{56}$$

In a spherically symmetric situation the solution must be spherically symmetric and ψ in (56) is a scalar function. From (56) the problem is immediately seen: if $\psi = \psi(r)$ and $x_i = x$ and $x_j = y$, then we get powers x^2 to the left side and y^2 to the right side. There is no possible way to sum these terms into a function of r because there is missing z^2. The problem is that Einstein's equations have several equations. In the Gauss equation (equation (26) in Section 2, see also (45), a scalar equation) there is only one equation and the terms x^2, y^2 and z^2 sum into r^2.

Section 2 derives the formula (56) and proves that these equations have no solutions that approximate Newtonian gravity. The material in the section may initially look like background material that can be found from some standard textbook. This is not the case: standard textbooks do not understand that the field can only be a scalar field if the speed of light is to be the same to each direction in each point in the rest frame. All material in Section 2 is given in order to 1) prove that the metric must be induced from a scalar field, a fact that is not understood in standard textbooks, 2) derive (56), and 3) prove that (56) have no solutions that approximate Newtonian gravity in large distances.

Experiments have shown that the Newtonian field is a very good approximation to the real field in large distances, therefore the results means that the General Relativity Theory (GRT) is fundamentally flawned. In the beginning of Section 2 there is text describing how to solve Ricci tensors and that the Newtonian gravitation potential does not give Ricci tensors that approximately solve Einstein equations, see (46) in Section 2, which is alone rather strange and points to the Einstein equations being wrong, which intuition the rest of Section 2 proves to be correct: the Einstein equations are a wrong way to split the field equation into several equations.

2. Derivation and solution of (56)

The gravitation field on the Earth and close to the Earth appears to be very close to the Newtonian gravitation potential. We will first show that the Newtonian potential does not nearly satisfy the field equation of General Relativity Theory (GRT) as one might expect if Newtonian gravity is an approximation of the more correct theory GRT. The Newtonian potential is

$$\varphi = \varphi(r) = -\frac{G\rho}{r}. \tag{25}$$

7

It is the solution to the Newtonian field equation

$$\Delta\varphi = -4\pi G\rho. \tag{26}$$

Let us assume that the mass is a spherical mass with a finite radius. In the empty space outside this radius ρ is constant and we have

$$\frac{\partial^2}{\partial x^2}\varphi = G\rho\frac{1}{r^5}(r^2 - 3x^2). \tag{27}$$

By symmetry

$$\Delta\varphi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\varphi = G\rho\frac{1}{r^5}(3r^2 - 3x^2 - 3y^2 - 3z^2) = 0. \tag{28}$$

Thus, in the empty space outside the mass the solution fulfills the equation

$$\Delta\varphi = 0. \tag{29}$$

In the same way, if we solve the field equation of GRT

$$R_{\mathrm{ab}} - \frac{1}{2}Rg_{\mathrm{ab}} = k_0 T_{\mathrm{ab}} - \lambda g_{\mathrm{ab}} \tag{30}$$

in the empty 4-space outside a spherical mass. The equation reduces to

$$R_{\mathrm{ab}} = \left(\frac{1}{2}R - \lambda\right)g_{\mathrm{ab}}. \tag{31}$$

The Newtonian gravitational potential (25) yields in orthogonal local coordinates the following metric

$$ds^2 = c^{-2}g_{00}\phi^2 - g_{11}\phi^2 - g_{22}\phi^2 - g_{33}\phi^2 \tag{32}$$

For easier notations in equations of GRT c is usually set to one by rescaling seconds and meters. In (30) and (31) $c = 1$. We will follow this convention, but in (40) c is shown as (40) is needed later. For any scalar potential field ψ, the metric g_{ab} corresponding to the field ψ in Cartesian coordinates (t, x, y, z) where $x^0 = t$, $x^1 = x$ $x^2 = y$, $x^3 = z$ and the signs are (+,-,-,-), is given by

$$g_{00} = \psi^2,\ g_{11} = -\psi^2,\ g_{22} = -\psi^2,\ g_{33} = -\psi^2,$$

8

$$\text{and} \quad g_{ab} = 0 \text{ if } a \neq b. \tag{33}$$

In spherical coordinates (t, r, θ, ϕ), where $x^0 = t$, $x\acute{z} = r$, $x^2 = \theta$, $x^3 = \phi$ and the signs are $(+,-,-,-)$, the metric is given by

$$g_{00} = \psi^2, \ g_{11} = -\psi^2, \ g_{22} = -\psi^2 r^2, \ g_{33} = -\psi^2 r^2 \sin^2\theta,$$

$$\text{and} \quad g_{ab} = 0 \text{ if } a \neq b. \tag{34}$$

For any orthogonal metric (i.e., $g_{ab} = 0$ if $a \neq b$) holds

$$g^{aa} = \frac{1}{g_{aa}} \tag{35}$$

and the Christoffel symbols satisfy (the notation $g_{ab,c} = \partial_c g_{ab}$) the following:

$$\Gamma^a_{aa} = \tfrac{1}{2} g^{aa} g_{aa,a}, \tag{36}$$

$$\Gamma^a_{ba} = \tfrac{1}{2} g^{aa} g_{aa,b} \ , \text{ if } a \neq b$$

$$\Gamma^a_{bb} = -\tfrac{1}{2} g^{aa} g_{bb,a} \ , \text{ if } a \neq b$$

$$\Gamma^a_{bc} = 0 \ , \text{ if } a \neq b,\ a \neq c \text{ and } c \neq b.$$

In order to get the Einstein equations we calculate the Christoffel symbols for the metric in both coordinate systems. Then we calculate the Ricci entries

$$R_{bd} = R^a_{bad} = \Gamma^a_{bd,a} - \Gamma^a_{ba,d} + \Gamma^e_{bd}\Gamma^a_{ae} - \Gamma^e_{ba}\Gamma^a_{ed}. \tag{37}$$

All ways to do the calculation are tedious. One way to calculate is the following, it may be easier to program as an algorithm than some others. In an orthogonal metric

$$R_{jj} = \sum_{\substack{i=0 \\ i \neq j}}^{4} \left\{ \frac{1}{4} g^{ii} g_{ii,j} \left(g^{jj} g_{jj,j} - g^{ii} g_{ii,j} \right) - \frac{1}{2} \partial_j \left(g^{ii} g_{ii,j} \right) \right\} \tag{38}$$

$$- \sum_{\substack{i=0 \\ i \neq j}}^{4} \left\{ \frac{1}{4} g^{ii} g_{jj,i} \left(\sum_{\substack{k=0 \\ k \neq j}}^{4} g^{kk} g_{kk,i} - g^{jj} g_{jj,i} \right) + \frac{1}{2} \partial_i \left(g^{ii} g_{jj,i} \right) \right\}, \ j = 0,1,2,3$$

In an orthogonal metric the off-diagonal Ricci entries have the equation

$$R_{ij} = \frac{1}{4} \sum_{\substack{k=0 \\ k \neq i,\, k \neq j}}^{4} g^{kk} g_{kk,i} \left(g^{ii} g_{ii,j} - g^{kk} g_{kk,j} \right) + \frac{1}{2} \partial_i \left(g^{ii} g_{ii,j} \right)$$

$$- \frac{1}{2} \partial_j \left(\sum_{\substack{k=0 \\ k \neq j}}^{4} g^{kk} g_{kk,i} \right) + \frac{1}{4} g^{jj} g_{jj,i} \sum_{\substack{k=0 \\ k \neq i,\, k \neq j}}^{4} g^{kk} g_{kk,j}. \tag{39}$$

The off-diagonal Ricci entries are zero both in Cartesian and in spherical coordinates. After a fairly long calculation the result is as follows.

For Cartesian coordinates the nonzero Ricci entries are:

$$R_{00} = -\frac{1}{c^2}\psi^{-1}\Box\psi + 3\psi^{-2}\left(\frac{\partial\psi}{\partial t}\right)^2 + \frac{1}{c^2}\psi^{-2}\sum_{i=1}^{3}\left(\frac{\partial\psi}{\partial x_i}\right)^2 - 2\psi^{-1}\frac{\partial^2\psi}{\partial t^2} \quad (40)$$

$$R_{\mathrm{ii}} = \psi^{-1}\Box\psi + c^2\psi^{-2}\left(\frac{\partial\psi}{\partial t}\right)^2 - \psi^{-2}\sum_{i=1}^{3}\left(\frac{\partial\psi}{\partial x_i}\right)^2 - 2\psi^{-1}\frac{\partial^2\psi}{\partial x_i^2} + 4\psi^{-2}\left(\frac{\partial\psi}{\partial x_i}\right)^2$$
$$\text{for}\quad i = 1, 2, 3$$

where we wrote x^i as x_i in order not to confuse an index with a power. The box $\Box$ is the D'Alembertian and the signs (i.e., η^{ab}) are (+,-,-,-). In Cartesian coordinates

$$\Box = c^2\partial_0^2 - \partial_1^2 - \partial_2^2 - \partial_3^2. \quad (41)$$

The Ricci scalar

$$R = g^{\mathrm{aa}}R_{\mathrm{aa}} \quad (42)$$

for the metric given by a scalar field is

$$R = c^2\psi^{-2}R_{00} - \psi^{-2}R_{11} - \psi^{-2}R_{22} - \psi^{-2}R_{33} = -6\psi^{-3}\Box\psi. \quad (43)$$

For spherical coordinates the nonzero Ricci entries are (when $c = 1$)

$$R_{00} = \frac{1}{2}\psi^{-2}\left\{\frac{\partial^2\psi^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2\psi^2}{\partial\theta^2} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi^2}{\partial\phi^2} - 3\frac{\partial^2\psi^2}{\partial t^2}\right\} \quad (44)$$

$$+\frac{1}{2}\psi^{-2}\left\{\frac{2}{r}\frac{\partial\psi^2}{\partial r} + \frac{1}{r^2}\cot\theta\frac{\partial\psi^2}{\partial\theta}\right\} + \frac{3}{2}\psi^{-4}\left(\frac{\partial\psi^2}{\partial t}\right)^2$$

$$R_{11} = \frac{1}{2}\psi^{-2}\left\{-3\frac{\partial^2\psi^2}{\partial r^2} - \frac{1}{r^2}\frac{\partial^2\psi^2}{\partial\theta^2} - \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi^2}{\partial\phi^2} + \frac{\partial^2\psi^2}{\partial t^2}\right\}$$

$$+\frac{1}{2}\psi^{-2}\left\{-\frac{2}{r}\frac{\partial\psi^2}{\partial r} - \frac{1}{r^2}\cot\theta\frac{\partial\psi^2}{\partial\theta}\right\} + \frac{3}{2}\psi^{-4}\left(\frac{\partial\psi^2}{\partial r}\right)^2$$

$$R_{22} = \frac{1}{2}\psi^{-2}\left\{-r^2\frac{\partial^2\psi^2}{\partial r^2} - 3\frac{\partial^2\psi^2}{\partial\theta^2} - \frac{1}{\sin^2\theta}\frac{\partial^2\psi^2}{\partial\phi^2} + r^2\frac{\partial^2\psi^2}{\partial t^2}\right\}$$

$$+\frac{1}{2}\psi^{-2}\left\{-4r\frac{\partial\psi^2}{\partial r}-\cot\theta\frac{\partial\psi^2}{\partial\theta}\right\}+\frac{3}{2}\psi^{-4}\left(\frac{\partial\psi^2}{\partial\theta}\right)^2$$

$$R_{33}=\frac{1}{2}\psi^{-2}\left\{-r^2\sin^2\theta\frac{\partial^2\psi^2}{\partial r^2}-\sin^2\theta\frac{\partial^2\psi^2}{\partial\theta^2}-3\frac{\partial^2\psi^2}{\partial\phi^2}+r^2\sin^2\theta\frac{\partial^2\psi^2}{\partial t^2}\right\}$$

$$+\frac{1}{2}\psi^{-2}\left\{-4r\sin^2\theta\frac{\partial\psi^2}{\partial r}+3\sin\theta\cos\theta\frac{\partial\psi^2}{\partial\theta}\right\}+\frac{3}{2}\psi^{-4}\left(\frac{\partial\psi^2}{\partial\phi}\right)^2.$$

The Ricci scalar in sperical coordinates gives the same equation also in spherical coordinates

$$R=g^{\mathrm{ab}}R_{\mathrm{ab}}=\psi^{-2}R_{00}-\psi^{-2}R_{11}-\psi^{-2}r^{-2}R_{22}-\psi^{-2}(r^{-2}\sin^{-2}\theta)R_{33}$$
$$=-6\psi^{-3}\Box\psi \tag{45}$$

when the D'Alembertian is expressed in spherical coordinates and the signs are set to (+,-,-,-). Ricci entries in Cartesian coordinates are easier to work with, and in a small area in the empty space outside the mass we certainly can use Cartesian coordinates.

The Newtonian gravitational potential (25) is a solution to the case of empty space with a point mass at the origin. The Ricci tensor entries for the metric are obtained from the potential field ϕ in (25) in Cartesian coordinates from (40). $\partial_0\phi=0$ and $\Box\phi=-\Delta\phi=0$, thus

$$R_{00}=\frac{1}{c^2}\sum_{j=1}^{3}(\partial_j\phi)^2=\frac{1}{c^2}\left(\left(\frac{\partial r^{-1}}{\partial x}\right)^2+\left(\frac{\partial r^{-1}}{\partial y}\right)^2+\left(\frac{\partial r^{-1}}{\partial z}\right)^2\right)=\frac{1}{c^2}\frac{1}{r^2}$$

$$R_{\mathrm{ii}}=\frac{1}{r^2}+2\frac{x_i^2}{r^4}\quad\text{for}\quad i=1,2,3. \tag{46}$$

We notice that these entries are not zero, but the Ricci scalar is zero. In the classical limit $R_{00}=0$ but $R_{\mathrm{ii}}\neq 0$ for $i=1,2,3$. Especially $R_{\mathrm{ii}}-c^{-2}R_{00}$ is not zero and it does not tend to any small number in the classical limit $c\rightarrow\infty$.

In the case of a scalar field ψ and Cartesian local coordinates as in (33) and a point mass in an empty space as in (31) any solution of the Einstein equations (30) satisfies

$$R_{00}-c^{-2}R_{\mathrm{ii}}=\left(\frac{1}{2}R-\lambda\right)\left(c^2g_{00}-g_{\mathrm{ii}}\right)=\left(\frac{1}{2}R-\lambda\right)\left(\psi^2-\psi^2\right)=0. \tag{47}$$

11

Next we show that if the speed of light is constant, then the metric must necessarily be induced by a scalar field. We assume that light travels along geodesics of the gravitational field. From the place (x, y, z) and the time t light can move an infinitesimally small distance to any direction. We can consider the 3-space and the time as a four-dimensional Euclidian space. In the Euclidean 4-space the infinitesimal movements would be (dx, dy, dz, dt), but we want to give this 4-space a different metric. Therefore let the infinitesimal movements at the place (x, y, z) and the time t be (Adx, Bdy, Cdz, Ddt) where A, B, C, D are functions of the place (x, y, z) and the time t. The 3-dimensional sphere of the metric at the place (x, y, z) and the time t is

$$dr^2 = (Adx)^2 + (Bdy)^2 + (Cdz)^2. \tag{48}$$

The requirement that light has the same speed c to each direction implies that in any place (x, y, z) and at any time t holds

$$c = \frac{Adx}{Ddt} = \frac{Bdy}{Ddt} = \frac{Cdz}{Ddt}. \tag{49}$$

Thus $Adx = Bdy = Cdz$. That is, in light-like worldpaths $ds = 0$, we take a worldpath that is in the direction of one of the space coordinates and solve it: it gives the speed of light to that direction. In the Euclian metric the infinitesimals in each direction are equally long, i.e., $|dx| = |dy| = |dz|$. It means that $A = B = C$, that is, the infinitesimal spheres of our 3-dimensional metric are round:

$$dr^2 = A^2dx^2 + A^2dy^2 + A^2dz^2 \quad \text{and} \quad D = \frac{1}{c}A. \tag{50}$$

Let us write $A = -\psi$, where $\psi < 0$ is a scalar gravitational field. The location of our light spot in an infinitesimal sphere of the 4-dimensional space in the geometry for gravitation field metric is

$(-\psi dx, -\psi dy, -\psi dz, -c^{-1}\psi dt)$ and the line element is

$$dr^2 = \psi^2 dx^2 + \psi^2 dy^2 + \psi^2 dz^2 + c^{-2}\psi^2 dt. \tag{51}$$

In the Minkowski space there is a pseudometric where $\eta_{aa} = (+, -, -, -)$ the line element in Cartesian local coordinates is

$$ds^2 = -\psi^2 dx^2 - \psi^2 dy^2 - \psi^2 dz^2 + c^{-2}\psi^2 dt^2. \tag{52}$$

Identifying g_{ii} for a general metric in Cartesian local coordinates in a Minkowski space

$$\mathrm{ds}^2 = -g_{11}\mathrm{dx}_1^2 - g_{22}\mathrm{dx}_2^2 - g_{33}\mathrm{dx}_3^2 + g_{00}\mathrm{dx}_0^2. \tag{53}$$

we see that $g_{00} = c^{-2}\psi^2$ and $g_{\mathrm{ii}} = \psi^2$, $i = 1, 2, 3$. There is no other metric that can yield the velocity of light as a scalar constant in the local environment. If the speed of light is not a scalar constant, then it becomes a vector function $c_i(x_0, x_1, x_2, x_3)$, $i = 1, 2, 3$. It is necessary that the speed of light is locally constant in Einsteinian relativity theory as c appears in many equations in that theory.

We notice that the metric that gives equal speed c of light to each direction in any place (x, y, z) and any time t is necessarily created by a scalar field ψ and the geometry is conformal: the space can expand and contract in any place, but the infinitesimal sphere, if taken as a sphere of a 4-dimensional manifold, is always perfectly round. This condition implies that angles are preserved.

Let us proceed to solve the Einstein equations in the case of an empty space with a point mass at the origin. As the metric is induced by a scalar field, the case with a point mass in the origin gives $\Box\psi = 0$. Therefore also holds $R = -6\psi^{-3}\Box\psi = 0$. The Einstein equations reduce to (47), which can be written as

$$c^2 R_{00} = R_{\mathrm{ii}}. \tag{53}$$

For every $i \in \{1, 2, 3\}$ in the expression of R_{ii} in (40) there are only two terms that are different for different i. We move all other terms in the expression (40) for R_{ii} to the left side of (53). Then the left side is the same for every i:

$$c^2 R_{00} - \psi^{-1}\Box\psi - c^2\psi^{-2}\left(\frac{\partial\psi}{\partial t}\right)^2 + \psi^{-2}\sum_{i=1}^{3}\left(\frac{\partial\psi}{\partial x_i}\right)^2$$

$$= -2\psi^{-1}\frac{\partial^2\psi}{\partial x_i^2} + 4\psi^{-2}\left(\frac{\partial\psi}{\partial x_i}\right)^2 \tag{54}$$

and the right side can be written more compactly by using

$$\frac{\partial}{\partial x_i}\left(\frac{\partial\psi}{\partial x_i}\psi^{-2}\right) = \psi^{-2}\left(\frac{\partial^2\psi}{\partial x_i^2} - 2\left(\frac{\partial\psi}{\partial x_i}\right)^2\psi^{-1}\right) \tag{55}$$

The right side of (54) for $i, j \in \{1, 2, 3\}$ gives the equations which lead to the contradiction in finding an approximation for the Newtonian gravitation potential:

$$\frac{\partial}{\partial x_i}\left(\frac{\partial\psi}{\partial x_i}\psi^{-2}\right) = \frac{\partial}{\partial x_j}\left(\frac{\partial\psi}{\partial x_j}\psi^{-2}\right). \tag{56}$$

13

Equations (56) are solved by any function the form $\psi = \psi(\rho)$ where $\rho = \sum x_j$, but the solution we need is close to the radially symmetric Newtonian gravitation field in this special case of a single point mass in the origin. We will first show that there is no solution $\psi = \psi(r)$. Inserting $\psi(r)$ to the left side of (56) gives

$$\frac{\partial}{\partial x_i}\left(\frac{\partial \psi}{\partial x_i}\psi^{-2}\right) = r^{-3}\psi'\psi^{-3}(-1 + x_i^2 r^{-3}(3r - f + f'/f)) \tag{57}$$

where $\psi' = \psi'(r) = d\psi(r)/dr$ and $f = f(r) = \psi(r)'/\psi(r)$. The only way (56) can hold is that the coefficient of x_i^2 is zero. Thus

$$3r = f - f'/f = \psi'/\psi - f'/f. \tag{58}$$

Integrating gives

$$\frac{3}{2}r^2 + c = \ln \psi - \ln f = -\ln(\psi'/\psi^2) \tag{59}$$

i.e.,

$$\psi'(r) = \psi(r)^2 C e^{-\frac{3}{2}r} \tag{60}$$

where c and $C = \exp(-c)$ are intergration constants.

Equation (60) is not possible. As ψ approximates the Newtonian gravitation potential $-GM/r$, it grows as r^{-1}. Therefore ψ' grows as r^{-2} and not with exponential dumping as in(60). Next, let usconsider if the solution ψ can be something else than $\psi = \psi(r)$. If the solution has cylinder symmetry, the proof above still works as we only need a plane (x_i, x_j) where the solution depends on the radius on that plane $r = (x_i^2 + x_j^2)^{-1/2}$. Let us consider a more general solution $\psi = a/r$ where a is a function that is close to $-GM$. Inserting $\psi = r^{-1} + a$ to the left side of (56) gives

$$\frac{\partial}{\partial x_i}\left(\frac{\partial \psi}{\partial x_i}\psi^{-2}\right)$$

$$= r^{-1}\psi^{-1}\left(-1 + \frac{x_i^2}{r^2} + \frac{\partial a}{\partial x_i}\frac{x}{a}(1 - r^{-2}) - \frac{2}{a^2}\left(\frac{\partial a}{\partial x_i}\right)^2 + \frac{1}{a}\frac{\partial^2 a}{\partial x_i^2}\right) \tag{61}$$

The term that should be cancelled is x_j^2/r^2. Keeping other x_j constant, a is a function only of x_i.

If the term x_j^2/r^2 is obtained by the $\partial a/\partial x_i$ term, then a as a function of x_i grows as

$$\frac{x}{a}\frac{da}{dx_i} \sim -\frac{x_i^2}{r^2} \tag{62}$$

giving for the leading term of a

$$\frac{1}{a}\frac{da}{dx_i} \sim -\frac{x_i}{r^2}. \tag{63}$$

14

Integrating yields for the leading term of a

$$\ln(a) = -\ln(r^2) \qquad \text{i.e.,} \qquad a = Cr^{-2} \tag{64}$$

where C is an integration constant. This is impossible as a is close to $-GM$.

If the x^2/r^5 term is obtained from the $(\partial a/\partial x_i)^2$, then a as a function of x_i grows as

$$\frac{2}{a^2}\left(\frac{da}{dx_i}\right)^2 \sim \frac{x_i^2}{r^2} \tag{65}$$

giving for the leading term of a

$$\frac{\sqrt{2}}{a}\left(\frac{da}{dx_i}\right) = -\frac{x_i}{r}. \tag{66}$$

Integrating yields the leading term of a

$$\ln(a) = -2^{-1/2}\ln(r) \quad \text{i.e.,} \quad a = Cr^{-\sqrt{1/2}} \tag{67}$$

where C is an integration constant. This is impossible as a is close to $-GM$.

If the x^2/r^5 term is obtained from the $\partial^2 a/\partial x_i^2$ term, then a as a function of x_i grows as

$$\frac{1}{a}\frac{d^2a}{dx_i^2} \sim \frac{x_i^2}{r^2}. \tag{68}$$

Writing

$$\frac{1}{a}\frac{d^2a}{dx_i^2} = \frac{1}{a^2}\frac{da}{dx_i} + \frac{d}{dx_i}\left(\frac{1}{a}\frac{da}{dx_i}\right) \tag{69}$$

$$\int -\frac{x_i^2}{r^2}dx_i = -\frac{x_i^2}{r^2} + \int 2\ln(r)dx_i$$

and integrating gives the leading term of a

$$-\frac{1}{a} + \frac{1}{a}\frac{da}{dx_i} = -\frac{x_i^2}{r^2} + \int 2\ln(r)dx_i. \tag{70}$$

We get

$$a = \exp\left(\int\left(\frac{1}{a} - \frac{x_i^2}{r^2} + C\right)dx_i\right)\exp\left(2\int\ln(r)dx_i\right). \tag{54}$$

Here C is an integration constant. The function a is almost constant. The first integral can stay near a constant value when we let x_i grow to large values where x_i/r is nearly one, or the first integral may grow or decrease as $\exp(C_1x_i)$, but the second integral grows with a smaller speed than $\int C_1 x_i$ for any C_1. It cannot be compensated by the first integral, threfore a cannot stay close to $-GM$ when x_i grows. Either a goes to zero, or a grows without limit.

All three cases fail to cancel the x^2 term while keeping a close to $-GM$. This means that ψ is not an approximation of the Newtonian gravitation field. This

result is natural. In Cartesian coordinates the terms R_{ii}, $i = 1, 2, 3$, are all symmetric but they depend on x_i. If the solution is a function of r, we get the term x_i^2 from R_{ii}. Summing all terms R_{ii}, $i = 1, 2, 3$, we get a function of r^2. This happens in the Ricci scalar. But in Einstein's equations there is an equation for each i. We can form the equation (57) relating i and j. The x_i^2 term cannot be cancelled with the term x_j^2 and they cannot make r^2. Thus, (57) cannot be satisfied.

We conclude that there are no solutions to the Einstein equations (30) that approximate the Newtonian gravitation potential in the single point mass situation and that have locally constant speed of light in vacuum. Locally constant speed of light means here that the speed of light is the same to each direction in each point in the rest frame. The speed of light can only be locally constant if the field is a scalar field. In the spherically symmetric situation the field should depend only on r, but to make a complete proof, we also checked all other possibilities. Nothing gives a field that approximates Newtonian gravity.

3. About experiments that verify the Relativity Theory

There are many experiments that claim to verify the Special Relativity Theory, less of those that claim to prove the General Relativity Theory. There is nuclear power that convincingly verifies that Olinto de Pretto's formula $E = mc^2$ does work, but experiments do not show that relativistic mass exists at all.

There is no need to go through these experiments in this article. No finite number of experiments where a theory predicts correctly can ever prove a theory correct. There can be an alternative explanation that also satisfies all these experiments. We can never know all possible alternative theories, they need not have been found yet.

A single experiment where a theory predicts incorrectly refutes the theory. In the end of each section 1.1, 1.2 and 1.3 there is described an experiments that refutes the discussed part of the relativity theory.

A note about references: Relativity Theory is explained in Einstein's original articles 1. and 2. No other references are needed in this article. Reference 3. is my paper, it shows a fourth fatal error in the Relativity Theory.

4. References

1. Einstein A. Zur Elektrodynamik bewegten Körper. *Annalen der Physik*, 1905.

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