# Traversable Wormholes in Anti-de Sitter Spacetime Without Exotic Matter: A Classical Solution Satisfying the Null Energy Condition

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## 0.1 Abstract:

We present a novel static, spherically symmetric traversable wormhole solution to the Einstein Field equations in the presence of a negative cosmological constant ($\Lambda < 0$). Our solution does not include a redshift function and a shape function $b(r) = \frac{r_0^2}{r}$ embedded in an Anti-de Sitter, ($ADS$), background. Through embedding the cosmological constant into the geometry, we show that the resulting spacetime satisfies $NEC$, Null Energy Condition, everywhere. This solution avoids the need for exotic matter. This solution suggests a possible classical, non-exotic mechanism for wormhole stabilization and uses a geometric interpretation to dark matter effects. The implications of this result are explored in the context of relativistic astrophysics and potential observability near galactic centers.

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# 1 Introduction

Wormholes fascinate the imagination of the scientific community and the public as hypothetical tunnels through spacetime, theoretically allowing possible travel between distant regions of the universe. This idea originates from solutions to the Einstein Field equations, where traversable wormholes include exotic matter. This condition violates the Null Energy Condition to remain stable and open. This requirement is one of the most theoretical obstacles to physically realistic wormhole models in General Relativity.

In this paper, we derive a class of wormhole solutions that are traversable and stable without the need of exotic matter. Instead, we use a negative cosmological constant to make the geometry stable. We begin by considering a static, spherically symmetric line element $ds^2$:

$$ds^2 = dt^2 - \frac{dr^2}{(1 - \frac{r_0^2}{r^2})} - r^2 d\Omega^2 \tag{1}$$

and solve the Einstein Field equations including the cosmological constant:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{2}$$

We find that the $AdS$ curvature acts as negative pressure, which when coupled with the chosen shape allows $NEC$ to be satisfied. This removes the need for exotic matter, making a significant step towards accepting physical wormhole models with classical relativistic solutions.

We propose that stable, traversable wormholes with a negative cosmological constant may exist near galactic centers, where strong dark matter behavior is observed. An example from galactic centers are when rotation curves deviate from Newtonian prediction, this makes it logical to conclude that the behaviors mentioned come from spiral shaped galaxies, and some egg-shaped galaxies. This solution maintains classical General Relativity that satisfies $NEC$ and give astrophysical non-exotic wormholes.

# 2 Theoretical Framework

Recall our chosen line element:

$$ds^2 = dt^2 - \frac{dr^2}{(1 - \frac{r_0^2}{r^2})} - r^2 d\Omega^2 \tag{3}$$

Where $r_0$ represents the throat radius of the wormhole, and $d\Omega^2 = d\theta^2 + sin^2\theta d\Phi^2$ is the line element for a 2-sphere.

The metric implies a **vanishing redshift function**, making the wormhole horizonless and traversable. The shape function $b(r)$ is given by:

$$b(r) = \frac{r_0^2}{r} \tag{4}$$

Which satisfies the standard throat condition $b(r_0) = r_0$. This choice also ensures that $1 - \frac{b(r)}{r} > 0$ for $r > r_0$, avoiding coordinate singularities.

We also include the cosmological constant $\Lambda$ in the Einstein Field equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{5}$$

For consistency and to match obervational signatures of the $AdS$ vacuum, we use the mostly minus signature $(+, -, -, -)$ and also use natural units where $G = c = 1$ for simplicity. This yields the Einstein Field equations as:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \tag{6}$$

We use the energy momentum tensor for an anisotropic fluid:

$$T^{\mu}_{\nu} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & -p_r & 0 & 0 \\ 0 & 0 & -p_t & 0 \\ 0 & 0 & 0 & -p_t \end{bmatrix} \tag{7}$$

Where $\rho$ is the energy density and $p_r$ is the radial pressure, and $p_t$ is the tangential pressure.

We now compute the Einstein tensor components, $G^t_t$, $G^r_r$, and $G^\theta_\theta = G^\Phi_\Phi$ which are:

$$G^t_t = \frac{r_0^2}{r^4}, \tag{8}$$

$$G^r_r = \frac{r_0^2}{r^4}, \tag{9}$$

and

$$G^\theta_\theta = G^\Phi_\Phi = -\frac{r_0^2}{r^4} \tag{10}$$

Plugging these into the Einstein Field equations:

$$G^{\mu}_{\nu} - \Lambda\delta^{\mu}_{\nu} = 8\pi T^{\mu}_{\nu} \tag{11}$$

We get the components of the enrgy momentum tensor components:

$$\rho = \frac{1}{8\pi}(\frac{r_0^2}{r^4} + \Lambda), \tag{12}$$

$$p_r = -\frac{1}{8\pi}(\frac{r_0^2}{r^4} - \Lambda), \tag{13}$$

$$p_t = -\frac{1}{8\pi}(-\frac{r_0^2}{r^4} - \Lambda) \tag{14}$$

These components will be used to evaluate the Null Energy Condition and the physical meaning of the solution.

# 3 Null Energy Condition in AdS spacetime

To check the physical interpretation to our wormhole solution, we evaluate the Null Energy Condition, a minimal requirement for classical stability. The Null Energy Condition states that for any null vector $k^{\mu}$, the stress energy tensor must satisfy $T_{\mu\nu}k^{\mu}k^{\nu} \geq 0$.

We check for $NEC$ by checking:

$$\rho + p_r = \frac{1}{8\pi}(2\Lambda), \tag{15}$$

$$\rho + p_t = \frac{1}{8\pi}(\frac{2r_0^2}{r^4} + 2\Lambda) \tag{16}$$

This may seem to suggest a violation of the $NEC$ when $\Lambda < 0$. However, in the presence of a cosmological constant, care must be taken. The term $\Lambda g_{\mu\nu}$ is interpreted as part of the geometric background, **not part of the matter content**. In order to isolate the physical matter, we define the effective stress energy tensor:

$$T^{eff}_{\mu\nu} = T_{\mu\nu} - \frac{\Lambda}{8\pi}g_{\mu\nu}, \tag{17}$$

So that:

$$G_{\mu\nu} = 8\pi T^{eff}_{\mu\nu} \tag{18}$$

For a null vector $k^\mu$, we have $g_{\mu\nu}k^\mu k^\nu = 0$. This means $T^{eff}_{\mu\nu}k^\mu k^\nu = T_{\mu\nu}k^\mu k^\nu$.

This shows that the cosmological constant does not affect the $NEC$ condition applied to the matter. The condition $T_{\mu\nu}k^\mu k^\nu \geq 0$ remains satisfied even for $\Lambda < 0$.

Therefore, our solution shows a stable, traversable wormhole that does not need exotic matter, where the negative cosmological constant playing as a stabilizer. This means that the solution is purely classical, using only General Relativity and relativisitc astrophysics, without violating quantum or exotic energy components.

# 4 Astrophysical Interpretation

The wormhole solution derived in the paper includes a nontrivial geometric structure stabilized by a negative cosmological constant. The physical interpretation is that in regions of spacetime where dark matter forces are shown to act with gravity, like at the galactic center for fast orbiting stars, the AdS-like structure could act as an effective stabilizer for the wormhole throat, without violating $NEC$.

The gravitational potential near the center of galaxies is anomalously deep, as shown from galactic rotation curves, which is attributed to dark matter. However, our model suggests that $AdS$ curvature could geometrically act as the role of such stabilizing effects. The wormhole geometry, which satisfies $NEC$ in this context, becomes physically "right" in the inner regions of galaxies (galactic centers) where the curvature scale $L^2 = -3/\Lambda$ becomes relevant.

We develop a visual of a 3D embedded wormhole by removing the time dimension.

# 5 Conclusion

In this paper, we presented a static, spherically symmetric wormhole solution to the Einstein Field equations with a nonzero cosmological constant $\Lambda < 0$, making it an asymptotically anti-de Sitter ($AdS$) spacetime without requiring exotic matter.

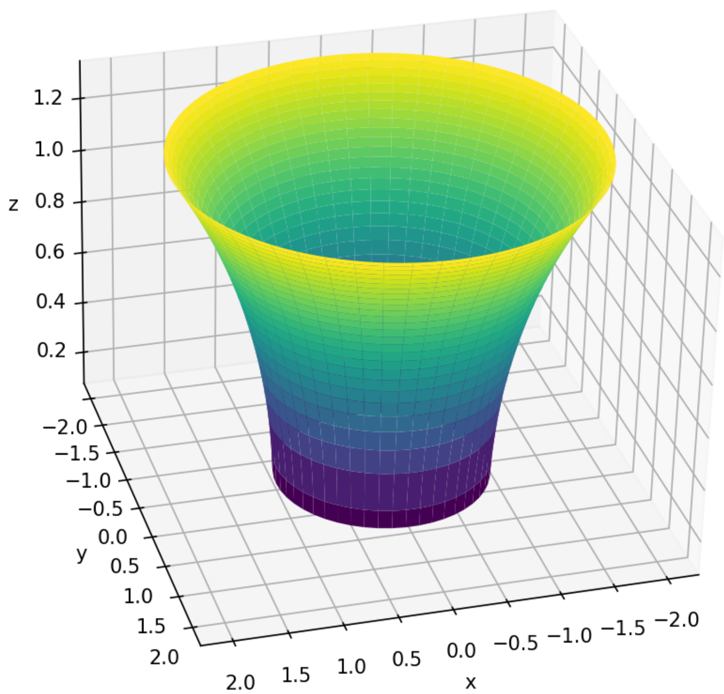


Figure 1: As shown, an embedded structure for a wormhole is shown. This is a half shaped wormhole, where the same shape flipped is the other side. This is close to a flamm's paraboloid. However, in nature where spacetime is 4D, we find the wormhole to look spherically symmetric, consistent with our solution

The use of a negative cosmological constant plays as a stabilizing role that supports the wormhole structure. This solution shows that the energy density and pressures from this solution satisfy $NEC$ when $\Lambda < 0$, while violating it when $\Lambda > 0$. It is logical to conclude such a wormhole exists near galactic centers where forces like dark matter acting with gravity ($\Lambda < 0$) is observed.

Future work may explore dynamical stabilities of the wormhole under pertubations, rotations, or charged cases, and maybe connections to holographic dulaties through the AdS/CFT correspondence.

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