**The Analysis of Matter Accretion**

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**Abstract**

The concept of matter accretion is meant to theoretically explore the nature of a massive body growing from an initial state to a larger entity through accumulating material. This follows the assumption that the body growing from the accreted material is spherical. To examine this body, derivations using classical physics are made to describe its potential behavior. This led to a differential equation that shows the proportionality relationship between mass and potential energy. To specify this equation, it shows the $2^{nd}$ order time derivative of mass equaling to the negative Laplacian of the potential energy; this equation can be interpreted as a non-linear temporal growth of matter being equivalent to the opposite $2^{nd}$ order spatial derivative of its potential energy for somatic construction. Basically, the accretion of matter is affected by the loss of potential energy in space. In addition, following the derivation of the gravitational binding energy and incorporating it with a substitution into this equation, it led to a $2^{nd}$ order time dependent differential equation for the gravitational construction of matter. Having the derivation of this equation that describes accretion caused by gravity, it has a coefficient that matches with the Laplacian of the gravitational potential but by a factor of six. Following a hypothesis that this is a geometric factor instead of mere coincidence, this led to the derivation of a mathematic theorem to support its geometric property. Lastly, a query comes to mind if this concept of matter accretion could apply to quantum physics.

## *I. Introduction*

The inspiration of this paper arose from questioning an alternative history where Newton had devised the concept of mass-energy equivalence instead of Einstein. It is understandable that this query follows a modern day projection upon Newton and overrides his $17^{th}$ century environment; therefore, this paper is not meant to attempt to follow exactly how Newton could possibly arrive with this concept. Nevertheless, the idea is to explore a relationship between matter and energy with a pre-Einsteinian onlook. Ironically, a part of the inspiration comes from the Einsteinian concept itself that matter and energy are exchangeable. The question that came to mind is what if mass could be represented as a function of energy over how energy is usually represented conversely. There is kinetic energy and potential energy were they have mass playing a role to scale them. However, thinking about the mass and energy equivalence have me to question what if it was the other way around. Of course, kinetic energy is the energy of motion and having mass as some energetic function would mean that it has to be at rest. Plus, a simple way to easily answer this query is to use the rest mass energy to have mass equal to its energy over the speed of light squared. This is the reason to question what if this concept was never heard of and came to mind to start a fresh approach with their relationship. Plus, pondering further with this idea, what if there is a space and time element to structuralize the exchange?

## *II. The Physics of Somatic Accretion*

It is best to start with the fundamentals of physics, particularly with work and the potential energy relationship with force to initiate the derivation. The conceptualization that follows with the derivation is that matter will not be treated as a constant but grow in size from accumulating other matter. As there is a growth in mass, not counting the compression of the matter to either increase in density or turn into a blackhole, there is an increase in spatial occupation; in other words, let's treat the average density to be constant as the mass increases respectively through volume.

When a force causes a change in position, it induces work on an object and taking the gradient for the work should give you back the force. Now, the opposite change of potential energy will also render work. This should dictate that as the potential energy decreases, the work increases, hence work is also the positive change of kinetic energy. Mathematically, the representations can be seen below, where r is the variable for position:

Work-Force Relationship:

$$W = \int F \cdot dr$$

Work-Potential/Kinetic Energy Relationship:

$$W = -\Delta PE = \Delta KE$$

If the gradient of the work renders its net force, it will mean that the force is the opposite gradient of potential energy, considering that the force is conserved. However, imagine a diverging force expanding against a surface; we can even differentiate the surface as it may not be uniformly defined. The work of the expansion can be defined by the diverging net force against the surface. This can be represented quantifiably by the following while the potential energy is represented by this function PE = U(r) and S is the surface area:

Force-Potential Energy Relationship:

$$F = -\nabla U(r)$$

Work of Expansion:

$$W = \iint \nabla \cdot F \, dS$$

Basically, if something is expanding and something else is within the range of impact, it should feel the force of the impact causing it to move, hence work from the expansion. The diverging force can even be represented with a potential energy identity to conclude mathematically that it is the negative Laplacian of potential energy:

Diverging Force-Potential Energy Derivation:

$$\nabla \cdot F = \nabla \cdot \left(-\nabla U(r)\right) = -\nabla^2 U(r)$$

Diverging Force-Potential Energy Identity:

$$\nabla \cdot F = -\nabla^2 U(r)$$

Following Newton's $2^{nd}$ Law of motion, the force is defined by the mass multiplied by the acceleration, assuming the mass is constant. However, let's consider a nonlinear growth of mass within a rest frame as movement of the body is of no concern. The focus is to initialize the position of the body as it grows. The growth will be represented by the divergence of this force of a nonlinear time varying mass. Maybe, it is growing exponentially which is a nonlinear function to give a hypothetical example. Once taking the divergence of this force, it should leave a $2^{nd}$ order time variation of the mass for each axial component.

Newton's $2^{nd}$ Law of Motion:

$$F = m\frac{d^2x}{dt^2}$$

Expansion Force:

$$F = \frac{\partial^2 m_x}{\partial t^2}x\hat{x} + \frac{\partial^2 m_y}{\partial t^2}y\hat{y} + \frac{\partial^2 m_z}{\partial t^2}z\hat{z}$$

The Divergence of the Expansion Force:

$$\nabla \cdot F = \frac{\partial^2 m_x}{\partial t^2} + \frac{\partial^2 m_y}{\partial t^2} + \frac{\partial^2 m_z}{\partial t^2} = \frac{\partial^2 M(t)}{\partial t^2}$$

This can be represented not only in cartesian coordinates for the divergence but also in spherical and cylindrical coordinates. When doing this, assuming the body is a sphere, it already has uniformity with its growth. However, let's assume that this uniformity applies to the other coordinates as well:

Assumption:

$$\frac{\partial^2 m_r}{\partial t^2} = \frac{\partial^2 m_x}{\partial t^2} = \frac{\partial^2 m_y}{\partial t^2} = \frac{\partial^2 m_z}{\partial t^2}$$

Spherical Expansion Force:

$$F = \frac{\partial^2 m_r}{\partial t^2}r\hat{r}$$

Cylindrical Expansion Force:

$$F = \frac{\partial^2 m_r}{\partial t^2}r\hat{r} + \frac{\partial^2 m_z}{\partial t^2}z\hat{z}$$

Cartesian Divergence of the Expansion Force:

$$\nabla \cdot F = \frac{\partial^2 m_x}{\partial t^2} + \frac{\partial^2 m_y}{\partial t^2} + \frac{\partial^2 m_z}{\partial t^2} = 3\frac{\partial^2 m_r}{\partial t^2} = \frac{\partial^2 M(t)}{\partial t^2}$$

Spherical Divergence of the Expansion Force:

$$\nabla \cdot F = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left[\frac{\partial^2 m_r}{\partial t^2} r\right]\right)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial^2 m_r}{\partial t^2} r^3\right) = \left(\frac{\partial^2 m_r}{\partial t^2}\right)\frac{1}{r^2}\frac{\partial}{\partial r}(r^3)$$

$$= \left(\frac{1}{r^2}\right)\frac{\partial^2 m_r}{\partial t^2}(3r^2) = 3\left(\frac{r^2}{r^2}\right)\frac{\partial^2 m_r}{\partial t^2} = 3\frac{\partial^2 m_r}{\partial t^2} = \frac{\partial^2 M(t)}{\partial t^2}$$

Cylindrical Divergence of the Expansion Force:

$$\nabla \cdot F = \frac{1}{r}\frac{\partial}{\partial r}(rF) + \frac{\partial F}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\left[\frac{\partial^2 m_r}{\partial t^2} r\right]\right) + \frac{\partial}{\partial z}\left(\frac{\partial^2 m_z}{\partial t^2} z\right)$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r^2\left[\frac{\partial^2 m_r}{\partial t^2}\right]\right) + \frac{\partial}{\partial z}\left(\frac{\partial^2 m_z}{\partial t^2} z\right)$$

$$= \frac{2r}{r}\left(\frac{\partial^2 m_r}{\partial t^2}\right) + \frac{\partial^2 m_z}{\partial t^2} = 2\frac{\partial^2 m_r}{\partial t^2} + \frac{\partial^2 m_z}{\partial t^2} = 3\frac{\partial^2 m_r}{\partial t^2} = \frac{\partial^2 M(t)}{\partial t^2}$$

Due to the diverging force's potential energy identity, a relationship can be established between the growing, massive, spherical body and the potential energy by:

$$\frac{\partial^2 M(t)}{\partial t^2} = -\nabla^2 U(r)$$

Giving the equation above a nomenclature, it can be called the Accreted Matter Equation (A.M.E). It can have an accretion property related to gravity as its derivation follows a differential gravitational potential energy just as the gravitational binding energy. Starting with the derivation of the gravitational binding energy (Hannan, 2024), we can use Newton's Law of Universal Gravitation. Following this law, this defines the gravitational force. To get the gravitational potential energy (G.P.E) (van Biezen, 2013), the work of the gravitational effect has to be considered first, from which it defines the negative change of the potential energy. The gravitational force is attractive, denoted by its minus sign. Moreover, following the fact that the gravitational force weakens by distance, the initial distance for the potential energy can infinitely recede far off to space. The assumption that follows is that the angle for the work of the attraction force is zero between the force and its distance:

Attractive Gravitational Force:

$$F_g = -\frac{GMm}{l^2}$$

G.P.E Derivation:

(Work and Force)

$$W = \int F \cdot dl$$

(Work and Potential Energy)

$$W = -\Delta U = -(U - U_0) = -U + U_0$$

(Bounds)

$$-\infty \leq l \leq r$$

(Derivation Continues)

$$-U + U_0 = \int_{-\infty}^{r} F_g \cdot dl = \int_{-\infty}^{r} F_g \, dl \cos\theta = \int_{-\infty}^{r} F_g \, dl \cos 0 = \int_{-\infty}^{r} F_g \, dl(1) = \int_{-\infty}^{r} F_g \, dl$$

$$\int_{-\infty}^{r} F_g \, dl = \int_{-\infty}^{r} \left(-\frac{GMm}{l^2}\right) dl = -GMm \int_{-\infty}^{r} \frac{dl}{l^2}$$

$$-U + U_0 = -GMm\left(-\left[\frac{1}{r} - \left(-\frac{1}{\infty}\right)\right]\right) = -GMm\left(-\left[\frac{1}{r} + \frac{1}{\infty}\right]\right)$$

$$-U + U_0 = GMm\left[\frac{1}{r} + \frac{1}{\infty}\right] = \frac{GMm}{r} + \frac{GMm}{\infty}$$

$$-U = \frac{GMm}{r}$$

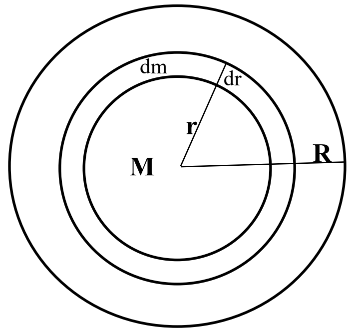
(Infinity as the denominator recedes to zero)

$$U_0 = \frac{GMm}{\infty} = 0$$

(Therefore, G.P.E):

$$U = -\frac{GMm}{r}$$

Now with the growth of a world through the gravitational potential energy, the energy has to be differentiable. Because of this fact, the potential energy is variating respectively to the differentiable mass as more material is accreted through gravity. Hence, the diagram shows the growth of a protoplanet where it grows from the differential mass *dm*, increasing in thickness of the differential radius *dr*:




Differentiable G.P.E:

$$dU = -\frac{GMdm}{r}$$

To start the derivation of the gravitational binding energy, the identities for the mass and differential mass needed to be defined respectively for their density and volume. Consider the density of both mass *M* and differential mass *dm* as they are the same, following the growth of the same planet. Therefore, these masses can be defined as:

$$M = \rho V = \rho\left(\frac{4}{3}\pi r^3\right)$$

$$dm = \rho dV = \rho(4\pi r^2 dr)$$

These properties can now be substituted into the differential gravitational potential energy, and once integrated with radius bounds $0 \leq r \leq R$ to get the total potential energy, it will lead to the gravitational binding energy:

Substitution:

$$dU = -\frac{GMdm}{r} = -\frac{G}{r}\left(\rho\left(\frac{4}{3}\pi r^3\right)\right)(\rho(4\pi r^2 dr))$$

Simplification:

$$dU = -\frac{16}{3}\frac{G}{r}\rho^2\pi^2 r^5 dr = -\frac{16}{3}G\rho^2\pi^2 r^4 dr$$

Integration:

$$U = -\frac{16}{3}G\rho^2\pi^2\int_0^R r^4\,dr = -\frac{16}{15}G\rho^2\pi^2 R^5$$

Let the radius for the density be r = R as:

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3}$$

Substitution:

$$U = -\frac{16}{15}G\rho^2\pi^2R^5 = -\frac{16}{15}G\left(\frac{M}{\frac{4}{3}\pi R^3}\right)^2\pi^2R^5 = -\left(\frac{9}{16}\right)\left(\frac{16}{15}\right)\left(\frac{\pi^2}{\pi^2}\right)\left(\frac{R^5}{R^6}\right)GM^2$$

Gravitational Binding Energy:

$$U = -\frac{3}{5}\frac{GM^2}{R}$$

The gravitational binding energy determines the strength of what can keep material together by the gravitational force. Note in the Accreted Matter Equation, there is the negative Laplacian of the potential energy. As we are speaking of the growth of a world, which is roughly a sphere, spherical coordinates are best to undergo a further derivation. The Accreted Matter Equation shows the time variation of matter increasing while accumulating energy. Now as the sphere that has mass only expanding in position, there is no need to consider the changes with the polar or azimuthal angles. Similar to the derivation of the gravitational binding energy, the gravitational potential energy has to be differentiated; however, since we are planning to use the Accreted Matter Equation, the potential energy will use a partial derivative along with the substitution of the mass and differential mass:

Partial Differential G.P.E:

$$\partial U(r) = -\frac{GM\partial m}{r} = -\frac{G}{r}\left[\rho\left(\frac{4}{3}\pi r^3\right)\right][\rho(4\pi r^2\partial r)] = -\frac{16}{3}G\rho^2\pi^2r^4\partial r$$

Note, that having the potential energy partially derived respect to the radius can be immediately substituted into the Accreted Matter Equation. It will handle the first derivative of the Laplacian of the potential energy:

Partial Derivative G.P.E

Respect to Radius r:

$$\frac{\partial U(r)}{\partial r} = -\frac{16}{3}G\rho^2\pi^2r^4$$

Remember that the radius will only be differentiated so that the Laplacian of the potential energy will be:

$$\nabla^2U(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial U(r)}{\partial r}\right)$$

The substitution can now be seen within the Accreted Matter Equation as the first derivative is already defined:

$$\frac{\partial^2M(t)}{\partial t^2} = -\nabla^2U(r) = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial U(r)}{\partial r}\right) = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left[-\frac{16}{3}G\rho^2\pi^2r^4\right]\right)$$

Next is to finish working out the derivation which would lead to something intriguing following the behavior of mass:

$$-\nabla^2 U(r) = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left[-\frac{16}{3}G\rho^2\pi^2 r^4\right]\right) = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(-\frac{16}{3}G\rho^2\pi^2 r^6\right)$$

$$-\nabla^2 U(r) = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(-\frac{16}{3}G\rho^2\pi^2 r^6\right) = -\frac{1}{r^2}(32G\rho^2\pi^2 r^5)$$

$$-\nabla^2 U(r) = -\frac{1}{r^2}(-32G\rho^2\pi^2 r^5) = 32G\rho^2\pi^2 r^3$$

Even though the conclusion of the derivation may appear nonsensical, the mass of a sphere can be factored out from the expression to the right of the derived equation:

$$-\nabla^2 U(r) = 32G\rho^2\pi^2 r^3 = (24\pi\rho G)\left[\rho\left(\frac{4}{3}\pi r^3\right)\right] = 24\pi\rho GM$$

Because of the Accreted Matter Equation, a second-order time variant differential equation can be made to study the behavior of material accretion from gravity:

Accreted Matter Equation:

$$\frac{\partial^2 M(t)}{\partial t^2} = -\nabla^2 U(r)$$

Gravitational A.M.E:

$$\frac{\partial^2 M(t)}{\partial t^2} = 24\pi\rho GM(t)$$

Notice the coefficient of the mass in this gravitational differential equation as it appears to be the Laplacian of the gravitational potential by a multiple of six. Maybe the equation can be arranged for the potential energy identity in space while accounting for the average mass contributing to the construction. Let Φ be the gravitational potential and m be the average mass accreted as the equation is arranged to this:

Gravitational Potential A.M.E:

$$\nabla^2 U(r) = -6m\nabla^2\Phi$$

Laplacian of the Gravitational Potential:

$$\nabla^2\Phi = 4\pi G\rho$$

The A.M.E is not meant to serve as a contrarian position to Einstein's derivation of the mass-energy equivalence. Even more advantageous, it can be incorporated into the A.M.E, which results in an intriguing derivation if having the force being a gradient of the total work (energy) from the accretion. Doing this will render an energy wave equation of the rest mass energy where the speed of light in a vacuum is the speed of the wave:

Total Energy A.M.E Version:

$$\frac{\partial^2 M(t)}{\partial t^2} = \nabla \cdot F = \nabla^2 E$$

Rest Mass Energy:

$$M(t) = \frac{E}{c^2}$$

Substitution:

$$\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \nabla^2 E$$

Rest Mass Energy Wave Equation:

$$\frac{\partial^2 E}{\partial t^2} = c^2 \nabla^2 E$$

## *III. Mathematical Framework*

The query to follow is to ponder why the coefficient of the Gravitational A.M.E is a multiple of six for the Laplacian of the Gravitational Potential. The question is whether it is merely a mathematical coincidence or some consequential factor? Let's explore the hypothesis that it could possibly be a geometric factor. The reason for this inference is that when taking the Laplacian of the gravitational potential energy, the derivation led to the factor of six. Therefore, it may not be so coincidental that the factor of six is important for the accretion of a spherical mass. Let's derive a new mathematical theorem to explore this factor. We will start with a cube where the origin of a three-dimensional axial spread centers inside it. With each extension of an axial line out of the cube, it represents an eye centering each square face. Let a sphere be inside this cube starting from an infinitesimally small size but let it grow while stopping where its edges touch each face of the cube. It should be a multiple of six, from which the gradient lines enter each face of the cube, drawing to the center to feed and grow the sphere until its edges reach them. The idea is that the surface area of a sphere is affected while the sphere grows in size, and the six faces of the cubes are the "windows" accounting the growth until the spherical far edges reach the "eyes" of the cube. Let's call this the Six-Eye Theorem. Exploring this theorem, mathematically, it appears that considering all natural numbers, the number of eyes affected will match the inverse factor of $\pi$, being its angular increment to map the surface area of a sphere:

Surface Element of a Sphere:

$$dS = r^2 \sin\theta \, d\theta d\phi$$

$$S = n \iint dS = n \int_{-\phi}^{\phi} \int_{0}^{\pi} r^2 \sin\theta \, d\theta \, d\phi$$

$$= 2nr^2 \int_{-\phi}^{\phi} d\phi = 4n\phi r^2, 0 < \phi \leq \pi$$

$$S = 4n\phi r^2, \qquad \phi = \left|\pm\frac{\pi}{n}\right|, \qquad -\frac{\pi}{n} \leq \phi \leq \frac{\pi}{n}, \qquad n \in \mathbb{N}$$

Notice the bounds of the polar angle $0 \leq \theta \leq \pi$ to account each face of the cube as the bounds for the azimuthal angle $-\frac{\pi}{n} \leq \phi \leq \frac{\pi}{n}$ sets the symmetry for the angular increments. Now, let $n = 6$, and see that the coefficient is 24. However, later we must explore the case where $n = 5$ and the Laplacian derivation using cylindrical coordinates to show its significance.

Six-Eye Theorem ($n = 6$):

$$S = 24\phi r^2 = 4\pi r^2, \qquad \phi = \left|\pm\frac{\pi}{6}\right|, \qquad -\frac{\pi}{6} \leq \phi \leq \frac{\pi}{6}$$

Let us now imagine the matter accretion happening across an infinitesimally thick disk using cylindrical coordinates. The complete derivation with cylindrical coordinates follows where the z-axis is accounted for as we can explore two derivations, one where the z-axis is radius dependent and the other where the variable z is independent. However, take notice that the coefficient where $n = 5$ is 20, and see the coefficient for the accretion across the disk. Later exploring the z-axis accretion, the factor should be a single multiple as $n = 1$, accounting the height of the sphere or the maximum length of the base conjoined hemispheres:

Six-Eye Theorem ($n = 5$):

$$S = 20\phi r^2 = 4\pi r^2, \qquad \phi = \left|\pm\frac{\pi}{5}\right|, \qquad -\frac{\pi}{5} \leq \phi \leq \frac{\pi}{5}$$

Six-Eye Theorem ($n = 1$):

$$S = 4\phi r^2 = 4\pi r^2, \qquad \phi = |\pm\pi|, \qquad -\pi \leq \phi \leq \pi$$

Accretion Disk Derivation:

$$\frac{\partial^2 M(t)}{\partial t^2} = -\nabla^2 U(r) = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U(r)}{\partial r}\right) = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\left[-\frac{16}{3}G\rho^2\pi^2 r^4\right]\right)$$

$$-\nabla^2 U(r) = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\left[-\frac{16}{3}G\rho^2\pi^2 r^4\right]\right) = -\frac{1}{r}\frac{\partial}{\partial r}\left(-\frac{16}{3}G\rho^2\pi^2 r^5\right)$$

$$-\nabla^2 U(r) = -\frac{1}{r}\frac{\partial}{\partial r}\left(-\frac{16}{3}G\rho^2\pi^2 r^5\right) = -\frac{1}{r}\left(-\frac{80}{3}G\rho^2\pi^2 r^4\right)$$

$$-\nabla^2 U(r) = -\frac{1}{r}\left(-\frac{80}{3}G\rho^2\pi^2 r^4\right) = \frac{80}{3}G\rho^2\pi^2 r^3$$

$$-\nabla^2 U(r) = \frac{80}{3}G\rho^2\pi^2 r^3 = (20\pi\rho G)\left[\rho\left(\frac{4}{3}\pi r^3\right)\right] = 20\pi\rho G M$$

$$\frac{\partial^2 M(t)}{\partial t^2} = 20\pi\rho G M(t)$$

or

$$\nabla^2 U(r) = -5m\nabla^2\Phi$$

Both complete derivations of the A.M.E, using the cylindrical coordinates, involve the z-axis having a symmetrical axial spread. We can start with the radius dependent z-axis, z(r), accounting the height of the cylinder. However, we are dealing with a sphere as it needs to be represented in cylindrical coordinates. Let's first follow a lesson from Archimedes to account the surface area of a cylinder but omit the two circular bases of it. We can do a proof of this using a limit. Let the circular bases recede to zero as the surface area of the cylinder morphs into the surface area of a sphere, given that the height of the cylinder is twice the magnitude of the radius of the bases. Now, let the radius of the cylindrical bases be $r$ as $R = r$. The value $r$ is the initial value letting the variable $R$ recede to zero through the limit:

Total Surface Area of Cylinder with variable $R$:

$$S = 2\pi Rh + 2\pi R^2$$

Receding Bases with Height being ($h = 2r$):

$$\lim_{R \to 0} 2\pi rh + 2\pi R^2 = 2\pi rh = 2\pi r(2r) = 2(2)\pi r^2 = 4\pi r^2$$

Starting the derivation of the A.M.E using cylindrical coordinates, we will start by reintroducing the differential gravitational potential energy; however, we will take a superposition of it where one accounts the mass spread across the infinitesimally thin disk and the other for the height of the sphere across the z-axis. When showing the math, we can define the differential potential energy to two parts with subscripts 1 and 2 respectively for the superposition.

Differential Potential Energy

$$dU = -\frac{GMdm}{r}$$

Superposition:

$$dU = dU_1 + dU_2$$

The spatial dimension in the cylindrical coordinate is only accounted for, which should be the radius of the cylindrical bases and the z-axis. Now, let's reflect back to Archimedes' proof for the surface area of a sphere where the length in the z-axis is the same as twice the radius of the disk. The disk has already been defined and stored into $dU_1$, being the first potential part of the superposition; however, for the second potential part $dU_2$, the differential volume for the mass will not be for a sphere but the differential volume of a cylinder. Using the cylinder's differential volume, the height is the variable that is being differentiated, which is twice the size of the radius. Moreover, instead of using radius substitution like $dU_1$, let's keep the mass integrand as itself since it has the potential of being a radius independent function. Take note at the end that both parts of the superposition are potential energy functions of the radius, from which the z axis does not have to be used for the differentiation.

Initial Mass Built:

$$M = M(t) = \rho\left(\frac{4}{3}\pi r^3\right)$$

First Potential Part:

$$dU_1 = -\frac{GMdm_1}{r} = -\frac{16}{3}G\rho^2\pi^2 r^4 dr$$

Second Potential Part:

$$dU_2 = -\frac{GMdm_2}{r} = -\frac{GM(t)}{r}\left(\rho dV_{Cylinder}\right)$$

$$= -\frac{GM(t)}{r}(\rho\pi r^2 dh) = -\frac{GM(t)}{r}\left(\rho\pi r^2(2dr)\right) = -\frac{GM(t)}{r}(2\rho\pi r^2 dr) = -2\pi\rho r GM(t)dr$$

Total Differential Potential Energy:

$$dU = -\frac{16}{3}G\rho^2\pi^2 r^4 dr - 2\pi\rho r GM(t)dr$$

Total Derivative of Potential Energy respect to the Radius:

$$\frac{dU(r)}{dr} = -\frac{16}{3}G\rho^2\pi^2 r^4 - 2\pi\rho r GM(t)$$

Cylindrical Coordinates of the Radius:

$$\nabla^2 U(r) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U(r)}{\partial r}\right)$$

A.M. Equation:

$$\frac{\partial^2 M(t)}{\partial t^2} = -\nabla^2 U(r)$$

Solve:

$$-\nabla^2 U(r) = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U(r)}{\partial r}\right) = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\left[-\frac{16}{3}G\rho^2\pi^2 r^4 - 2\pi\rho r GM(t)\right]\right)$$

$$= -\frac{1}{r}\frac{\partial}{\partial r}\left(\left[-\frac{16}{3}G\rho^2\pi^2 r^5 - 2\pi\rho r^2 GM(t)\right]\right)$$

$$= -\frac{1}{r}\left(-\frac{80}{3}G\rho^2\pi^2 r^4 - 4\pi\rho r GM(t)\right)$$

$$= \frac{80}{3} G\rho^2\pi^2 r^3 + 4\pi\rho GM(t)$$

$$= 20\pi\rho G\left(\frac{4}{3}\pi r^3\right) + 4\pi\rho GM(t)$$

$$= 20\pi\rho GM(t) + 4\pi\rho GM(t)$$

$$= 24\pi\rho GM(t)$$

Conclusion:

$$\frac{\partial^2 M(t)}{\partial t^2} = 24\pi\rho GM(t)$$

The next proof is to have a z-axis independent derivation from the radius as it may be best to approach this one in a slightly different perspective. Let's keep the disk A.M.E portion only for the radius differentiation. However, for the independent z-axis differentiation part, let's consider the curved side of the two hemispheres, ascending together to make a full sphere with the disk mass. Being z-axis independent, the z variable will be used assuming that it shares the same magnitude of the radius, by which the height should be twice its length. This time, the entire spatial dimensions will be accounted for using cylindrical coordinates as the derivation should come as follows:

Spatial Cylindrical Coordinates:

$$\nabla^2 U(r,z) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_r}{\partial r}\right) + \frac{\partial^2 U_z}{\partial z^2}$$

Differential Height:

$$dh = 2dz$$

Radius Derivative of Potential Energy:

$$\frac{dU(r)}{dr} = -\frac{16}{3} G\rho^2\pi^2 r^4$$

Z-axis Potential Energy:

$$dU_z = -\frac{GM(t)dm_z}{z}$$

Double Hemisphere Height Adjustment for Differential Mass:

$$dm_z = \rho 2\pi z^2 dh = \rho 2\pi z^2(2dz) = 4\pi\rho z^2 dz$$

Z-axis Derivative of Potential Energy:

$$\frac{dU_z}{dz} = -\frac{GM(t)}{z}(4\pi\rho z^2) = -4\pi\rho z GM(t)$$

A.M. Equation:

$$\frac{\partial^2 M(t)}{\partial t^2} = -\nabla^2 U(r)$$

Solve:

$$-\nabla^2 U(r,z) = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right) - \frac{\partial^2 U}{\partial z^2} = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\left[-\frac{16}{3}G\rho^2\pi^2 r^4\right]\right) - \frac{\partial}{\partial z}\left(-4\pi\rho z G M(t)\right)$$

$$-\nabla^2 U(r,z) = 20\pi\rho G M(t) - \frac{\partial}{\partial z}\left(-4\pi\rho z G M(t)\right)$$

$$-\nabla^2 U(r,z) = 20\pi\rho G M(t) - 4\pi\rho G M(t) = 20\pi\rho G M(t) + 4\pi\rho G M(t) = 24\pi\rho G M(t)$$

Conclusion:

$$\frac{\partial^2 M(t)}{\partial t^2} = 24\pi\rho G M(t)$$

This should provide support for the significance of the mathematical theorem, showing that the factor of five represents a flat, radial spread. However, the full spherical mass accounted for is at a factor of six. There could be a real numerical value in between the two to represent a near spherical accretion, maybe in some sort of an ellipsoidal shape. However, to account for three spatial dimensions, the factor should be above five but can be less than or equal to six.

Potential Geometric Bounds of 3D Space:

$$\varphi \in \mathbb{R}^+, \qquad 5 < \varphi \leq 6$$

Both the spherical coordinates and cylindrical coordinates for the A.M.E are accounted for to show the same result. This should be true as the coordinates shift should not affect the magnitude of the A.M.E but would affect how to approach deriving it. Nevertheless, a derivation following the cartesian coordinates can be done to confirm that each three coordinate system renders the same results, hopefully strengthening the validity of the A.M.E. Following the cartesian coordinates, we can reflect back to the Six-Eye Theorem to note what angular incrementation to use. Following the math, it seems that for each axis, the portion of the sphere follows twice the length per dimension to account for the negative and positive portion of them, where $n = 2$. Therefore, the cartesian coordinate derivation for the gravitational A.M.E should follow this wise. Let's follow the superposition of the differential potential energy; however, instead of having two spatial dimension parts, there will be three to account for the three dimensions of the cartesian coordinate system. The next step is to account for the growth from both positive and negative directions of each axis, which should be twice the spread length for each spherical differential volume. Each of the spatial dimensions are independent from the radius, being the variable for the x, y, and z-axis. Therefore, the cartesian form of the Laplacian can be used to derive them.

Six-Eye Theorem ($n = 2$):

$$S = 8\phi r^2 = 4\pi r^2, \qquad \phi = \left|\pm\frac{\pi}{2}\right|, \qquad -\frac{\pi}{2} \le \phi \le \frac{\pi}{2}$$

Axial Superposition:

$$dU = dU_x + dU_y + dU_z = -\frac{GM(t)dm_x}{x} - \frac{GM(t)dm_y}{y} - \frac{GM(t)dm_z}{z}$$

Axial Differential Mass Definitions:

$$dm_x = \rho 4\pi x^2 dl_x, dm_y = \rho 4\pi y^2 dl_y, dm_x = \rho 4\pi z^2 dl_z$$

Double Length of each Intercepting Axial Spread:

$$dl_x = 2dx, dl_y = 2dy, dl_z = 2dz$$

Simplify the Axial Potential Energies:

$$dU_x = -\frac{GM(t)(\rho 4\pi x^2 dl_x)}{x} = -\frac{GM(t)\big(\rho 4\pi x^2(2dx)\big)}{x} = -\frac{GM(t)(\rho 8\pi x^2 dx)}{x}$$

$$dU_y = -\frac{GM(t)\big(\rho 4\pi y^2 dl_y\big)}{y} = -\frac{GM(t)\big(\rho 4\pi y^2(2dy)\big)}{y} = -\frac{GM(t)(\rho 8\pi y^2 dy)}{y}$$

$$dU_z = -\frac{GM(t)(\rho 4\pi z^2 dl_z)}{z} = -\frac{GM(t)\big(\rho 4\pi z^2(2dz)\big)}{z} = -\frac{GM(t)(\rho 8\pi z^2 dz)}{z}$$

Differential Potential Energies Respect to their Axial Lengths:

$$\frac{\partial U_x}{\partial x} = -\frac{8\pi x^2 \rho GM(t)}{x} = -8\pi x \rho GM(t)$$

$$\frac{\partial U_y}{\partial y} = -\frac{8\pi y^2 \rho GM(t)}{y} = -8\pi y \rho GM(t)$$

$$\frac{\partial U_z}{\partial z} = -\frac{8\pi z^2 \rho GM(t)}{z} = -8\pi z \rho GM(t)$$

Solve:

$$-\nabla^2 U = -\frac{\partial^2 U_x}{\partial x^2} - \frac{\partial^2 U_y}{\partial y^2} - \frac{\partial^2 U_z}{\partial z^2}$$

$$-\nabla^2 U = -\frac{\partial}{\partial x}\big(-8\pi x \rho GM(t)\big) - \frac{\partial}{\partial y}\big(-8\pi y \rho GM(t)\big) - \frac{\partial}{\partial z}\big(-8\pi z \rho GM(t)\big)$$

$$-\nabla^2 U = -\big(-8\pi \rho GM(t)\big) - \big(-8\pi \rho GM(t)\big) - \big(-8\pi \rho GM(t)\big)$$

$$-\nabla^2 U = 8\pi \rho GM(t) + 8\pi \rho GM(t) + 8\pi \rho GM(t) = 3\big(8\pi \rho GM(t)\big)$$

$$-\nabla^2 U = 3\big(8\pi\rho G M(t)\big) = 24\pi\rho G M(t)$$

Conclusion:

$$\frac{\partial^2 M(t)}{\partial t^2} = 24\pi\rho G M(t)$$

This confirms that each coordinate system renders the same result for the gravitational A.M.E. Now, it seems unconventional that the conclusion for the gravitational A.M.E happens to have a coefficient that is a multiple of six for the Laplacian of the gravitational potential. This could raise the question of its relationship with the original and its interpretation. Exploring this query, the conclusion leads to the fact that the reason is due to the three dimensional accretion while the Laplacian of the gravitational potential accounts for a single spatial dimension. This can be shown mathematically following the divergence of the force for accretion. However, while using the spherical coordinate system for the divergence, the length to consider is the diameter instead of just the radius as shown previously. Let the diverging force account for the diameter spread for symmetry while it equals to the gravitational accretion.

Accretion Force with the Diameter Spread:

$$F = \frac{\partial^2 m_r}{\partial t^2}(2r)\hat{r}$$

Divergence of the Force:

$$\nabla \cdot F = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left[\frac{\partial^2 m_r}{\partial t^2}(2r)\right]\right)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial^2 m_r}{\partial t^2}(2r^3)\right) = \left(\frac{\partial^2 m_r}{\partial t^2}\right)\frac{1}{r^2}\frac{\partial}{\partial r}(2r^3)$$

$$= \left(\frac{1}{r^2}\right)\frac{\partial^2 m_r}{\partial t^2}(6r^2) = 6\left(\frac{r^2}{r^2}\right)\frac{\partial^2 m_r}{\partial t^2} = 6\frac{\partial^2 m_r}{\partial t^2} = \frac{\partial^2 M(t)}{\partial t^2}$$

A.M.E Substitution:

$$\frac{\partial^2 M(t)}{\partial t^2} = \nabla \cdot F = 6\frac{\partial^2 m_r}{\partial t^2} = 24\pi\rho G M(t)$$

Conclusion regarding Single Dimensional Accretion:

$$6\frac{\partial^2 m_r}{\partial t^2} = 24\pi\rho G M(t)$$

$$\therefore \frac{\partial^2 m_r}{\partial t^2} = 4\pi\rho G M(t)$$

Laplacian of the Gravitational Potential:

$$\nabla^2 \Phi = 4\pi G\rho$$

From this finding, maybe the diverging force shown earlier can be corrected for accounting twice the axial length for each coordinate system. Therefore, each of them should have a factor of six to be accounted for the total matter accretion. This still follows the assumption that the axial lengths share the same magnitude as well as each mass component:

Assumption:

$$\frac{\partial^2 m_r}{\partial t^2} = \frac{\partial^2 m_x}{\partial t^2} = \frac{\partial^2 m_y}{\partial t^2} = \frac{\partial^2 m_z}{\partial t^2}$$

Cartesian Accretion Force:

$$F = \frac{\partial^2 m_x}{\partial t^2}(2x)\hat{x} + \frac{\partial^2 m_y}{\partial t^2}(2y)\hat{y} + \frac{\partial^2 m_z}{\partial t^2}(2z)\hat{z}$$

Spherical Accretion Force:

$$F = \frac{\partial^2 m_r}{\partial t^2}(2r)\hat{r}$$

Cylindrical Accretion Force:

$$F = \frac{\partial^2 m_r}{\partial t^2}(2r)\hat{r} + \frac{\partial^2 m_z}{\partial t^2}(2z)\hat{z}$$

Divergence of Cartesian Accretion Force:

$$\nabla \cdot F = 2\frac{\partial^2 m_x}{\partial t^2} + 2\frac{\partial^2 m_y}{\partial t^2} + 2\frac{\partial^2 m_z}{\partial t^2} = 6\frac{\partial^2 m_r}{\partial t^2} = \frac{\partial^2 M(t)}{\partial t^2}$$

Spherical Divergence of the Accretion Force:

$$\nabla \cdot F = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left[\frac{\partial^2 m_r}{\partial t^2}(2r)\right]\right)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial^2 m_r}{\partial t^2}(2r^3)\right) = \left(\frac{\partial^2 m_r}{\partial t^2}\right)\frac{1}{r^2}\frac{\partial}{\partial r}(2r^3)$$

$$= \left(\frac{1}{r^2}\right)\frac{\partial^2 m_r}{\partial t^2}(6r^2) = 6\left(\frac{r^2}{r^2}\right)\frac{\partial^2 m_r}{\partial t^2} = 6\frac{\partial^2 m_r}{\partial t^2} = \frac{\partial^2 M(t)}{\partial t^2}$$

Cylindrical Divergence of the Accretion Force:

$$\nabla \cdot F = \frac{1}{r}\frac{\partial}{\partial r}(rF) + \frac{\partial F}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\left[\frac{\partial^2 m_r}{\partial t^2}(2r)\right]\right) + \frac{\partial}{\partial z}\left(\frac{\partial^2 m_z}{\partial t^2}(2z)\right)$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(2r^2\left[\frac{\partial^2 m_r}{\partial t^2}\right]\right) + \frac{\partial}{\partial z}\left(\frac{\partial^2 m_z}{\partial t^2}(2z)\right)$$

$$= \frac{4r}{r}\left(\frac{\partial^2 m_r}{\partial t^2}\right) + 2\frac{\partial^2 m_z}{\partial t^2} = 4\frac{\partial^2 m_r}{\partial t^2} + 2\frac{\partial^2 m_z}{\partial t^2}$$

$$= 4\frac{\partial^2 m_r}{\partial t^2} + 2\frac{\partial^2 m_r}{\partial t^2} = 6\frac{\partial^2 m_r}{\partial t^2} = \frac{\partial^2 M(t)}{\partial t^2}$$

This should provide more support for the Six-Eye Theorem as the factor of six for each of them sums up the total accretion of the spherical matter in three dimensions. There is, however, something peculiar with the cylindrical coordinate system derivation. Notice that when doubling each length coefficient to account for the diameter of the sphere, the other coordinate systems correspond to the Theorem with the differential volume of each dimensional part of the potential energy. The diverging force for the cartesian coordinate system shows twice the differential mass components for each three axis; this follows the derivation for the gravitational A.M.E where each differential mass doubled, accounting for each end of the axes from their intersection. As for the spherical coordinate system, just considering the diameter itself is enough to show the property of the Six-Eye Theorem; it is the same when deriving the gravitational A.M.E. However, the cylindrical coordinates show a factor of four for the radius spatial portion of the force and two for the z-axis. The derivation of the gravitational A.M.E shows something different where the factor is five for the radius and a factor of one for the z-axis. Now, each coordinate system adds up to a factor of six, including the cylindrical coordinates to confirm the theorem; however, the diverging force and the derivation of the gravitational A.M.E have undergone different division of factors to reach the factor of six. Apparently, the diverging force somehow loses a factor from the z-axis and is given to the radius portion of the force. The doubling of the cartesian and cylindrical coordinate system is merely a conjecture since the reasoning to double the spherical coordinate system is supported by factoring the diameter of the sphere instead of the radius. While the diverging force of the cartesian coordinate system follows the doubling to reach the factor of six which corresponds with its gravitational A.M.E derivation, maybe another conjecture can be made for the cylindrical coordinate system; however, the reasoning is not so clear for interpretation of why it works physically. Nonetheless, due to the gravitational A.M.E derivation having a factor of five and one, maybe instead of purely doubling each spatial coordinate, the factor for the radius will be doubled with a halved factor while the z-axis is left as is. This should follow the factor of five and one which are found when deriving the gravitational A.M.E using cylindrical coordinates. This still follows the assumption that the mass components are equally distributed such as in a spherical body during accretion.

Assumption:

$$\frac{\partial^2 m_r}{\partial t^2} = \frac{\partial^2 m_x}{\partial t^2} = \frac{\partial^2 m_y}{\partial t^2} = \frac{\partial^2 m_z}{\partial t^2}$$

Cylindrical Divergence of the Accretion Force (Modified):

$$\nabla \cdot F = \frac{1}{r}\frac{\partial}{\partial r}(rF) + \frac{\partial F}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\left[\frac{\partial^2 m_r}{\partial t^2}(2.5r)\right]\right) + \frac{\partial}{\partial z}\left(\frac{\partial^2 m_z}{\partial t^2}(z)\right)$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(2.5r^2\left[\frac{\partial^2 m_r}{\partial t^2}\right]\right) + \frac{\partial}{\partial z}\left(\frac{\partial^2 m_z}{\partial t^2}z\right)$$

$$= \frac{5r}{r}\left(\frac{\partial^2 m_r}{\partial t^2}\right) + \frac{\partial^2 m_z}{\partial t^2} = 5\frac{\partial^2 m_r}{\partial t^2} + \frac{\partial^2 m_z}{\partial t^2}$$

$$= 5\frac{\partial^2 m_r}{\partial t^2} + \frac{\partial^2 m_r}{\partial t^2} = 6\frac{\partial^2 m_r}{\partial t^2} = \frac{\partial^2 M(t)}{\partial t^2}$$

Confirmation of the divergence of the accretion force itself can be examined to see that it follows the gravitational A.M.E. However, note that the accretion force accounts all three dimensions while the gravitational force will account a single spatial dimension. This is to claim that the accretion force will be three times the gravitational force to account for each axis. Following this proof, the mass of attraction will be the same as itself while using the gravitational force for the derivation. Under these conditions, the calculation should be as follows:

Masses Attracted Equivalence:

$$M = m = \rho\left[\frac{4}{3}\pi r^3\right]$$

Scaled Relationship with Net Force and 1D Gravitational Force:

$$F_{net} = 3F_G \rightarrow \nabla \cdot F_{net} = \nabla \cdot (3F_G) = 3\nabla \cdot F_G$$

Divergence of the Net Accretion Force:

$$\nabla \cdot F_{net} = \frac{3}{r^2}\frac{\partial}{\partial r}(r^2 F_G) = \frac{3}{r^2}\frac{\partial}{\partial r}\left(r^2\left[\frac{GMm}{r^2}\right]\right) = \frac{3}{r^2}\frac{\partial}{\partial r}(GMm) = \frac{3}{r^2}\frac{\partial}{\partial r}(GMm)$$

$$= \frac{3G}{r^2}\frac{\partial}{\partial r}\left(\left[\rho\left(\frac{4}{3}\pi r^3\right)\right]^2\right) = \frac{3G}{r^2}\frac{\partial}{\partial r}\left(\frac{16}{9}\rho^2\pi^2 r^6\right)$$

$$= \frac{3G}{r^2}\left(\frac{96}{9}\rho^2\pi^2 r^5\right) = \frac{96}{3}\frac{G}{r^2}(\rho^2\pi^2 r^5) = 32G\rho^2\pi^2 r^3 = (24\pi\rho G)\left(\rho\left[\frac{4}{3}\pi r^3\right]\right) = 24\pi\rho GM(t)$$

Conclusion:

$$\nabla \cdot F_{net} = 24\pi\rho GM(t)$$

$$-\nabla^2 U(r) = \nabla \cdot F_{net} = \frac{\partial^2 M(t)}{\partial t^2}$$

Gravitational A.M.E:

$$\frac{\partial^2 M(t)}{\partial t^2} = 24\pi\rho G M(t)$$

## IV. First Order A.M.E and Potential Quantum Physics Application

Lastly, there is a question that comes to mind whether this concept of matter accretion could possibly reach to quantum physics. Before continuing following with this query, let's acknowledge the first order time derivative of matter accretion. However, instead of the first order time derivative of mass following the divergence of an accreted force, it follows the divergence of an accretion impulse for the expanding body.

A.M.E:

$$\frac{\partial^2 M(t)}{\partial t^2} = -\nabla^2 U(r)$$

A.M.E Accreted Force Identity:

$$\frac{\partial^2 M(t)}{\partial t^2} = -\nabla^2 U(r) = \nabla \cdot F$$

A.M.E Impulse Identity:

$$\frac{\partial^2 M(t)}{\partial t^2} = \nabla \cdot F = \frac{\partial}{\partial t} \nabla \cdot \vec{p}$$

First Order A.M.E:

$$\frac{\partial M(t)}{\partial t} = \nabla \cdot \vec{p}$$

Therefore, when the accretion of matter has a constant growth, there may not be an accretion force but there could still be an accretion impulse of the expansion. The query involving quantum physics starts with questioning the nature of the Higgs Field. An elementary particle that interacts with it starts out massless and goes at the speed of light; however, it is impeded by the Higgs field from which it gains mass following symmetry breaking (Higgs, 1964). The question is when the momentum operator is used to study the motion of a particle, could the First Order A.M.E be quantized to play a role in studying the motion as the particle gains mass from the Higgs Field? What if the Hamiltonian operator accounts the total energy of the rest mass of the particle? Maybe the impulse of the First Order A.M.E can be quantized and integrated respecting to time to measure its increase.

Momentum of Particle Wave Function:

$$\hat{p}\Psi = -i\hbar\nabla\Psi$$

Quantized Matter Accretion:

$$\nabla \cdot \hat{p}\Psi = -i\hbar\nabla^2\Psi$$

Rest Mass Energy:

$$M = \frac{E}{c^2}$$

$$\frac{\partial M(t)}{\partial t} \longrightarrow \nabla \cdot \hat{p}\Psi$$

$$\frac{1}{c^2}\frac{\partial \hat{E}}{\partial t}\Psi = -i\hbar\nabla^2\Psi$$

$$\hat{E}\Psi = \hat{H}\Psi$$

Total Energy of Mass Acquired:

$$\hat{H}\Psi = -\mathrm{c}^2 i\hbar \int_{t_0}^{t} \nabla^2\Psi \, dt$$

It will be best to admit my ignorance over this complex subject regarding the surface level understanding I have; however, I care to share this thought regarding the depths of examining the possible nature of matter accretion. I believe there is a Lagrangian density function to study the elementary particle that gains mass from the Higgs Field (Higgs, 1964). Moreover, there is a conversion from the Hamiltonian function, being the total energy, to the Lagrangian function, which is the difference between the kinetic energy and potential energy. Due to this being a query there is no further analysis of this conceptualization. Nevertheless, I wish to yield towards my lack of understanding and hopefully capture the attention of those who have a professional background in the field to answer or at least examine this question.

## *V. Closure*

The main premise of this paper is to explore the application of matter accretion with a different perspective of defining the force of growth. The assumption that follows with this concept is that the accreting body is a spherical body. Moreover, the validity of the gravitational A.M.E is tested by the calculations through different coordinate systems to see if they render the same results; the reason being is that the magnitude of the gravitational A.M.E should not alter under a different coordinate system while the calculation approach will. Therefore, while doing the calculus to examine a spherical growth of matter, it shows the same quantity for each derivation. Following the Six-Eye Theorem, it confirms the relationship between the gravitational A.M.E's coefficient and the Laplacian of the gravitational potential. Due to the coefficient being a multiple of six for the Laplacian of the gravitational potential, the interpretation is that the factor of six represents the three dimensional accretion; however, using the Laplacian of the gravitational potential will just measure a single spatial dimension and direction during the matter accretion. The diameter spread within the accretion force accounts for the factor of six and is confirmed from the derivations of each coordinate system. When accounting the gravitational force for accretion, it will only represent a third of the accretion force's strength because it measures a single spatial dimension. Lastly, the concluding query is meant to express the potential relationship with the concept of matter accretion and quantum physics. It is understandable if the reasoning for this relationship may be misguided by surface level understanding; however, I believe it will be best to mention this in the paper as a query to at least either get insight or discuss over it. The reason it is in this paper is that is a thought to share to explore whether it has the potential to imply a scientific phenomenon or could be corrected for the misguided reasoning. While the level my confidence is greater for the derivations of the matter accretion than of the quantum physics query, I care not to present this paper in a definitive stance as I acknowledge the entire work is theoretical; there also could be something that is unacknowledged regarding my reasoning. The hope I have is to offer the concept of this paper and its applications up for discussion with those of a professional background in physics. The conceptualization of this paper does touch on classical physics; however, the topic may fall into the field of astrophysics and at the end quantum physics exploring matter accretion. In conclusion, the sole agenda of this paper is to ignite a conversation over the concept it presents. I believe what is mentioned is not definitive and conclusive in itself but welcomes a discussion of the topic to explore a new truth. I have the desire to study general relativity, giving me insight over Einstein's Field Equations. I believe in doing so, it can assist me exploring to see if there can be a relationship established with the gravitational A.M.E, accounting for the curvature of spacetime. The goal of this paper is to set up a foundation for exploring the depths of the relationship between matter and energy. The A.M.E is meant to be, at least to my hope, the next chapter for Einstein's famous equation, $E = mc^2$ or better known as the rest mass energy. It is meant to be the mass-energy equivalence with dimensions of space and time.

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