Theorization and Experimental Verification of the Brazil Nut Effect

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Executive Summary –This study explores the Brazil Nut Effect (BNE), a phenomenon where larger particles rise to the surface of a shaken mixture, by developing a theoretical framework and conducting experimental validation. Theoretical equations were derived to quantify the effects of vibration frequency, amplitude, particle size, and mass on the BNE. Experimental verification utilized a cement mortar vibration table and a binary particle system to analyze particle behavior under varying vibration conditions. Results demonstrated that the BNE is significantly influenced by the fourth power of particle diameter, particle mass, and vibration parameters. Observations aligned closely with theoretical predictions, confirming the validity of the framework. This research provides a generalized approach to understanding the BNE and establishes foundational insights for future computational simulations and broader applications in particle dynamics.

1. Introduction

The Brazil Nut Effect (BNE) describes the phenomenon wherein larger solid particles rise to the surface when a mixture of particles with varying sizes is shaken over an extended period. This effect derives its name from the common observation that Brazil nuts are typically found at the top of a container of mixed nuts after transportation over long distances [1-2].

In 1995, Edward Ehrich from the University of Chicago visualized the dynamics of the BNE using magnetic resonance imaging (MRI), providing insights into the movement of particles during the phenomenon. Subsequently, Chilean scientist Sergio Godoy investigated the BNE through molecular dynamics simulations, focusing on the motion of atoms and molecular particles [3]. Despite these efforts, Godoy was unable to develop a comprehensive theoretical framework. To date, no universally accepted theory explaining the BNE has been established.

Here, this study seeks to address this gap by developing a theoretical framework for the BNE and conducting experimental verification to support its validity.

2. Background

2.1 Previous study

Previous studies were driven by the hypothesis that the BNE exhibits dynamics that resemble the correlations observed between molecules. Among the existing models that analyze particle motion at a microscopic level—focusing on particle size, mass, and diverse friction coefficients—Shengwei Zhang's work, *A Calibration Method for Contact Parameters of Agricultural Particle Mixtures Inspired by the Brazil Nut Effect (BNE): The Case of Tiger Nut Tuber-Stem-Soil Mixture* [4], is considered particularly relevant. Accordingly, the equations presented in Zhang's study have been adopted as the basis for this research.

Zhang's study identified friction coefficients as the most critical variables influencing the BNE. The research observed that contact friction between particles is influenced to some extent by the volume and concentration of the target particles. Based on these observations, experiments were conducted with the assumption that contact friction is a primary factor. The results demonstrated that the restitution coefficient (e), static friction coefficient (s), and rolling friction coefficient (r) in a three-component mixture significantly affect the BNE.

To further substantiate these findings, experiments were conducted using agricultural models of varying particle sizes, specifically the Tiger Tuber model, Stem model, and Soil model. These experiments examined how changes in particle volume concentration ($C(s,t)$) influenced the restitution coefficient (e), static friction coefficient (s), and rolling friction coefficient (r).

However, it became apparent that the experimental framework, limited to a specific set of particles, falls short in generalizing the BNE. Additionally, the reliance on actual agricultural products introduces challenges in extending the derived equations to other types of particles. Furthermore, factors such as particle size and density, in addition to friction coefficients, are likely to significantly influence this phenomenon.

To address these limitations, this study aims to diversify the types and sizes of both target particles and surrounding smaller particles. By doing so, the goal is to derive generalized equations for the BNE through comprehensive experimentation, thereby broadening the applicability of the findings.

2.2 Theorization

Prior to developing a theoretical framework, preliminary observational experiments were conducted to investigate the phenomenon. The results indicated that, when gently shaken at specific frequencies, the particles predominantly exhibited motion characteristic of solid-state molecules. However, when the shaking intensity exceeded a certain threshold, the particles transitioned to behavior resembling that of liquid-state molecules. To mathematically describe the transitions in particle motion states, the following equations were derived.

Initially, it was hypothesized that the degree of fluidization is proportional to the particle diameter. Based on this assumption, theoretical equations were developed. The change in height (Δh) was expressed as:

$$\Delta h = d\Delta h_d \quad (1)$$

where Δh represents the degree of fluidization, and d represents the particle diameter.

The vibration applied to the particle system was modeled using a cosine function, assuming that the impact force generated by the vibration is transmitted as momentum to the particles. The relationship between the change in momentum ($\Delta P = m\Delta v$) and the impulse ($I = Ft$) was utilized to further derive the equations governing particle motion.

The displacement induced by vibration in this context (s) was subsequently represented as follows:

$$s = A\cos\omega t \quad (2)$$

where A is the amplitude of vibration, and ω is the vibration frequency.

To express the impulse (I) caused by vibration, the force (F) was integrated with respect to time (t) over a single period of vibration.

$$I = \int_0^{\frac{x}{2\omega}} M(A\omega^2 \cos\omega t)dt \quad (3)$$

where M represents the mass of the impacted particle, while m represents the mass of the smaller particles.

The falling velocity of smaller particles under gravity can be expressed as:

$$v = \sqrt{2gd\Delta h_d} \quad (4)$$

This velocity was further represented as the change in momentum:

$$\Delta P_m = m\sqrt{2gd\Delta h_d} = \sqrt{2m^2 gd\Delta h_d} \quad (5)$$

Using the principle that the change in momentum (ΔP) is equal to the impulse (I), the equations were extended to describe particle motion.

$$\int_0^{\frac{x}{2\omega}} M(A\omega^2 \cos\omega t)dt = \sqrt{2m^2 gd\Delta h_d}$$

$$[M(A\omega \sin\omega t)]^{\frac{\pi}{2\omega}} = MA\omega \sin\frac{\pi}{2} = MA\omega = m\sqrt{2gd\Delta h_d} \quad (6)$$

The height to which particles rise due to fluidization was expressed as:

$$\Delta h_d = \frac{M^2}{dm^2} \times \frac{A^2\omega^2}{2g} \quad (7)$$

Additionally, the degree of fluidization was used to classify the state of the system. When the fluidization parameter (Δh_D) satisfies the condition $\Delta h_D > 1$, the particles behave as a fluid, and when $\Delta h_D < 1$, they behave as a solid.

$$\Delta h_D = \frac{M^2}{DM^2} \times \frac{A^2\omega^2}{2g} \quad (8)$$

Hence, for a larger target particle or high-mass particle to rise, the following condition must be satisfied:

$$\Delta h_D < 1 < \Delta h_d \Rightarrow \frac{m^2}{DM^2} \times \frac{A^2\omega^2}{2g} < \frac{M^2}{dm^2} \times \frac{A^2\omega^2}{2g} \quad (9)$$

where Δh_d represents the height change of the smaller particles, while Δh_D represents the height change of the larger particles.

The above equation can be reorganized to express

$$dm^4 < DM^4 \quad (10)$$

From the above relationships, it can be deduced that the BNE is proportional to the product of the diameter and the fourth power of the particle mass.

Then, how would the degree of fluidization change with vibration frequency? The degree of is directly proportional to the vibration frequency.

$$\Delta h_d = \frac{M^2}{dm^2} \times \frac{A^2\omega^2}{2g} \quad (11)$$

so to rearrange for the vibration frequency ω,

$$\omega = \frac{d\sqrt{2g\Delta hd}}{A*M} \quad (12)$$

For the larger particle, or the target particle, to rise, the condition $\Delta h_D <1 < \Delta h_d$ must be satisfied. Therefore, the fluidization condition can be expressed as follows:

$$\Delta h_D < 1 < \Delta h_d \Rightarrow \frac{m^2}{DM^2} \times \frac{A^2\omega^2}{2g} < \frac{M^2}{dm^2} \times \frac{A^2\omega^2}{2g} \quad (13)$$

If the above condition is expressed as a function of vibration frequency, it can be represented as follows:

$$\frac{A^2w^2M^2}{2gD^2} < 1 < \frac{A^2w^2m^2}{2gd^2} \quad (14)$$

Thus, the vibration frequency ω must ultimately satisfy the following equation:

$$\omega = \frac{d\sqrt{2g\Delta hd}}{A*M}$$
$$(but, \sqrt{\frac{2gD^2}{M^2A^2}} < \omega < \sqrt{\frac{2gd^2}{m^2A^2}}) \quad (15)$$

In conclusion, the vibration frequency must be sufficiently high to achieve adequate fluidization, ensuring that the BNE occurs effectively.

3. Materials and Methods

3.1 Experimental Design

The experiments were conducted using a cement mortar vibration table, a device capable of providing consistent vibration frequencies and amplitudes. Another device was attached to the vibration table to allow precise adjustments of the vibration intensity. The experimental setup included a thin rectangular prism container to closely observe the BNE under controlled conditions. The design of the experiments aimed to verify the theoretical framework described earlier by enabling clear observation of the BNE phenomenon.

This study was divided into two main parts. The first focused on examining how variations in vibration frequency affected the behavior of the BNE. The second aimed to investigate the specific characteristics of the BNE when a frequencies and amplitudes. The primary goal was to experimentally verify the conditions under which the BNE occurs.

The time taken for the target particle to completely rise through the coffee particle layer was used as a metric to analyze the occurrence of the BNE. For the BNE to be considered as having occurred, the target particle had to reach the topmost layer of particles within the rectangular prism container.

Based on the theoretical framework established earlier, differences in the patterns of the BNE were anticipated under varying experimental conditions. The experiments were designed to validate these theoretical predictions and provide insights into the factors influencing the BNE.

3.1.1 Experiment 1

The motion of a binary particle system was observed at a specific vibration frequency (50 Hz). The experiment used mixed coffee granules as the base particles (smaller-diameter

particles) and various larger particles as the target particles. For each trial, a larger particle (such as a small bean, a large bean, a plastic ball, a ping-pong ball, a lead pellet, or a steel ball) was placed at the bottom of the container, with a 3 cm

layer of coffee particles covering it.

As previously mentioned, the time required for the target particle to completely rise through the coffee particle layer

was measured as the primary indicator for analyzing the binary particle system was subjected to vibrations with fixedoccurrence of the BNE.

The diameter of each target particle was measured individually. For particles with natural size variation, such as the small and large beans, the average diameter was calculated to ensure consistency in the analysis. The specific dimensions of each target particle are as follows:

Table 1. Small beans

	Mass (g)	Diameter (mm)
1	0.11	7.000
2	0.15	6.325
3	0.08	5.225
4	0.13	6.525
5	0.15	6.875
6	0.16	6.775
average	*0.13*	*6.454*

Table 2. Large beans

	Mass (g)	Diameter (mm)
1	0.30	8.3
2	0.35	8.7
3	0.33	8.8
4	0.34	8.0
5	0.40	8.9
6	0.29	8.6
average	*0.33*	*8.5*

Table 3. Plastic ball

Mass (g)	Diameter (mm)
13.8	29.4

Table 4. Ping-pong ball

Mass (g)	Diameter (mm)
2.2	39.65

Table 5. Lead pellet

Mass (g)	Diameter (mm)
0.11	2.65

Table 6. Steel ball

Mass (g)	Diameter (mm)
28.32	19

3.1.2 Experiment 2

Before conducting experiments to analyze how the BNE varies with vibration frequency, the binary particle system from Experiment 1 was utilized. In this setup, mixed coffee granules served as the base particles (smaller-diameter particles), and larger particles were used as the target particles (larger-diameter particles).

For this experiment, the bottom of the container was filled with larger particles, specifically steel balls and plastic balls, which demonstrated the most pronounced BNE in the initial trials. These particles were chosen to identify the optimal vibration conditions under which the BNE is maximized.

Table 7. Frequency

	Frequency (Hz)
1	40Hz
2	45Hz
3	50Hz
4	55Hz
5	60Hz

4. Experimental Result

The following table lists the target particles in ascending order of the time required for the BNE to occur.

Table 8. Time required to observe the BNE

Target particle	Mass (g)	Diameter (mm)	Diameter x mass^4	Predicted BNE order	Observed BNE order
Small bean	0.13	6.45	0.00184	5	5
Large bean	0.334	8.4	0.10454	4	4
Plastic ball	13.8	29.4	1066261.371	2	2
Ping-pong ball	2.2	39.65	928.82504	3	3
Lead pellet	0.11	2.65	0.00039	6	6
Steel ball	28.32	19	12221558.67193	1	1

The following table organizes the time required for the BNE to occur under varying vibration frequencies, listed in ascending order.

Table 9. <Using steel ball>

Frequency	Mass/diameter	Predicted BNE order	Observed BNE order
40Hz	1.490	5	5
45Hz	1.490	4	4
50Hz	1.490	3	3
55Hz	1.490	2	2
60Hz	1.490	1	1

Table 10. <Using Plastic ball>

Frequency	Mass/diameter	Predicted BNE order	Observed BNE order
40Hz	0.469	5	5
45Hz	0.469	4	4
50Hz	0.469	3	3
55Hz	0.469	2	2
60Hz	0.469	1	1

As shown in the table, the results from both experiments align precisely with the theoretical predictions made during the theorization process.

5. Conclusion & Discussion

This study successfully developed a theoretical framework for the BNE and identified the key variables influencing particle movement. Through the theorization process, the effects of amplitude, vibration frequency, particle mass, and particle size on the BNE were quantitatively derived. These theoretical predictions were experimentally validated, demonstrating a high degree of agreement with observed particle movement trends.

In conclusion, this research confirmed that the BNE is proportional to particle mass, the fourth power of particle diameter, and both the amplitude and frequency of vibration. While this study relied on theoretical equations for experimentation, future research will utilize the Ansys CFD program to simulate the movement of particles within the container. These simulations will help predict particle behavior and analyze discrepancies between simulated and experimental results, providing deeper insights into their causes.

References

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Research Reflection - This study offered a meaningful opportunity to examine the extent to which experimental observations can align with theoretical predictions. Notably, the resonance frequencies and surface vibration patterns observed under specific conditions showed strong agreement with the theoretical models we established. This concordance validates both the soundness of our initial hypotheses and the precision of the experimental design. The experimental data closely matched the theoretically derived values, reinforcing the structural completeness of the apparatus and the appropriateness of the measured variables.

Nevertheless, despite these positive outcomes, several limitations encountered during the experiment suggest avenues for future improvement. First, although we observed that the resonance phenomena varied with the volume of water, our analysis lacked a detailed examination of the surface vibration modes. Moving forward, a more quantitative approach—possibly utilizing high-speed imaging to capture waveforms and standing wave patterns—would allow for a finer-grained analysis of the vibrational characteristics. Second, while acoustic data were analyzed qualitatively, the use of instruments with higher frequency resolution could have enabled a more precise characterization of the spectral features, thereby deepening our understanding of the underlying physical mechanisms.

These limitations not only highlight areas for refinement but also provide directions for extending the current research. While the experiment demonstrated a high degree of reproducibility and consistency with theoretical expectations, future studies could enhance spatial and temporal resolution and pursue a more rigorous modeling of the physical origins of the observed vibrations.

Through this project, I was able to engage in the entire scientific process—ranging from theoretical model formulation to experimental design, data analysis, and interpretation. It also underscored the importance of investigating subtle variables and complex factors, even when theoretical and experimental results appear to agree. This experience has laid a strong foundation for exploring more intricate physical phenomena in future work and has served as a valuable opportunity to deepen both the breadth and depth of my research capabilities.