Dimensional Bias in Quantum Path Integrals: A Causal Model of Bell Correlations

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Abstract

Bell test experiments - which have culminated in the Nobel-winning work of Clauser, Aspect, and Zeilinger - have definitively closed loopholes for all classical hidden-variable models operating in (3+1)D spacetime. These results compel us to reconsider the dimensional assumptions of causal structure. We propose a hypothesis in which a symbolic or informational bias field, $\phi(x)$, emerges through higher-dimensional path integrals.

This framework introduces a dynamical informational bias field $\phi(x)$, sourced by entropy gradients via information geometry, and incorporated into a modified Feynman path integral formalism. The field induces a conserved topological charge $\phi(x)$ that links entangled particles through shared hyper-dimensional structure, restoring classical causality in an expanded manifold. Typically, such hyper-dimensional paths cancel through destructive interference, but under entanglement constraints, the hyper-path amplitude is constructively reinforced—preserving coherence and restoring classical causality in the full configuration space. This is the fundamental hypothesis of this paper.

Simulations show that this bias produces testable shifts in Bell-type correlations and ghost imaging patterns, while preserving unitarity and no-signaling. The model offers a falsifiable geometric alternative to hidden-variable theories, unifying entanglement and information through symbolic curvature in extended configuration space.

Keywords

entanglement, Feynman path integrals, hyper-dimension, hidden variable, non-locality, quantum information

1 Introduction

In standard quantum mechanics, entanglement emerges naturally from the tensor product structure of Hilbert spaces and the unitary evolution of superposed states. Suppose a quan-

tum state is initially in a superposition:

$$|\Psi\rangle = |\psi_1\rangle + |\psi_2\rangle$$

and interacts with another state $\Phi\rangle$. The interaction evolves as:

$$|\Psi\rangle \otimes |\Phi\rangle \longrightarrow |\psi_1\rangle \otimes |\phi_1\rangle + |\psi_2\rangle \otimes |\phi_2\rangle$$

,

where $\phi_1\rangle \neq \psi_2\rangle$. Because the final two-particle state cannot be written as a tensor product of individual states, it is entangled. This entangled superposition implies that a measurement on subsystem A (e.g., projecting onto $\psi_1\rangle$) instantaneously determines the corresponding state of subsystem B (e.g.,$\psi_2\rangle$), and vice versa. These correlations, which arise through coherent interactions and are preserved under unitary evolution, are central to quantum information protocols and nonlocal experiments. These phenomena have been rigorously explored in loophole-free experiments over the past decade, including cosmic Bell tests [18], relativistic separation trials [26], and satellite-based entanglement experiments [31].

While this formulation accurately describes how entanglement arises mathematically, it does not explain why these interference patterns are preserved across spatial separation, nor does it account for the geometric or informational principles that might underlie such robust correlations.

Dimensional Constraints on Causal Inference and the Role of Hidden Geometry

Experimental work by **John Clauser**, **Alain Aspect**, and **Anton Zeilinger**, honored with the 2022 Nobel Prize in Physics, has conclusively demonstrated that no *local hidden variable theory* defined within conventional (3+1)-dimensional spacetime can reproduce the statistical correlations observed in entangled quantum systems [3, 21, 32]. By closing detection and locality loopholes in successive generations of Bell tests, these experiments upheld quantum mechanical predictions while ruling out any causal influence constrained to local variables in observable spacetime. These results confirmed the violation of Bell inequalities and fundamentally challenged classical conceptions of separability and realism.

However, these experiments rest on an implicit assumption: that the space of physical causes is coextensive with (3+1)D spacetime. That is, Bell's theorem and its experimental validations preclude only those causal models whose influence propagates within the conventional light cone of Minkowski space. They do not rule out the possibility that observed quantum correlations may arise from causal structures embedded in a higher-dimensional configuration space.

In this work, we extend Feynman's path integral formulation by introducing an unobserved spatial coordinate w, forming a (3+1+1)-dimensional manifold over which quantum histories are defined. Within this space, entangled particles may remain geometrically adjacent along w even when spatially separated in 3D, allowing causal coherence to persist

without signal exchange in visible spacetime. This framework restores classical causality—including Reichenbach's Principle of Common Cause—but reinterprets it as manifest along a hidden spatial degree of freedom.

Consequently, we propose that Bell-type correlations are not causeless but are projections of a higher-order geometric coherence, invisible from the (3+1)D perspective. What appears as non-locality or "spooky action at a distance" in conventional quantum mechanics is reframed as ordinary locality in an extended topological space. In this light, the violation of Bell inequalities does not signify a breakdown of causality, but rather reveals the dimensional incompleteness of the assumptions underlying those inequalities.

2 Feynman's Path Integral Formalism

In Feynman's path integral formulation [17, 15], the quantum amplitude is computed by summing over all possible classical paths between initial and final points:

$$\langle x_f, t_f | x_i, t_i \rangle = \int \mathcal{D}[x(t)]\, e^{iS[x(t)]/\hbar} \tag{1}$$

All possible paths contribute to the amplitude, but only those with constructive interference determine the observable outcome. [15] Paths that interfere destructively cancel out. This probabilistic summation is governed by the classical action $S[x(t)]$, with quantum dynamics emerging from this interference structure. [16]

3 Extension to Hyper-dimensional Space

Assume our ordinary 3D space is a projection of higher-dimensional interactions. Entanglement may then arise from shared paths in a 4D+ configuration space. The idea of extending the configuration space has historical roots in Kaluza–Klein theory and has recently resurfaced in causal structure reconstructions [19] and information-theoretic spacetime models [4]. We introduce a fourth spatial coordinate, say $w(t)$, that isn't observed directly in our space, but contributes to the action:

$$\S[x(t), w(t)] = \int_{t_i}^{t_f} L(x, \dot{x}, w, \dot{w})\, dt \tag{2}$$

where L now incorporates hyper-dimensional dynamics. This framework allows for paths in w to influence particle evolution, although these paths typically destructively interfere in the integral.

4 Biasing Hyper-dimensional Paths

We introduce a biasing term in the Lagrangian:

$$L' = L_{3D} + \epsilon f(x, \dot{x}, w, \dot{w}, t) \tag{3}$$

Here, f represents an informational field that influences motion through w, and ϵ is a small coupling constant that governs the degree of influence. For instance, f could be taken as $f(x, \dot{w}) = \dot{w}\,\phi(x)$, where $\phi(x)$ is a scalar potential field. Similar informational bias terms have been studied in entropic dynamics, where motion is driven by entropy gradients or constraints derived from the Fisher information metric [9, 1]. This leads to a modified Euler-Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial L'}{\partial \dot{w}}\right) - \frac{\partial L'}{\partial w} = \epsilon\left(\frac{d}{dt}\phi(x)\right) \tag{4}$$

A more physically motivated form of $\phi(x)$ might involve information density, such as:

$$\phi(x) = logp(x) \tag{5}$$

where $\rho(x)$ is a probability amplitude or symbolic field density. Alternatively, $\phi(x) = \frac{\partial S_{eff}}{\partial x}$, where S_{eff} is a statistical or thermodynamic entropy potential. Thus, biasing introduces an effective force in the hidden dimension w, analogous to classical fields in 3D. We denote the informational bias field uniformly as $\phi(x)$ throughout the manuscript for clarity.

5 Topological Charge $Q(w)$ from Noether Symmetry in Extended Configuration Space

To formalize the origin of the conserved topological quantity $Q(w)$, we apply Noether's theorem to the extended Lagrangian in (3+1+1)D spacetime. The total Lagrangian is:

$$\mathcal{L} = \frac{1}{2}m\,\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + \frac{1}{2}\alpha\dot{w}^2 + \epsilon\,\dot{w}\phi(x),$$

where $\phi(x)$ is a scalar field and α defines the kinetic scaling of motion in w. The coordinate w is not directly observed and is assumed to exhibit global translational symmetry: $w \to w + \delta w$.

By Noether's theorem, invariance under global shifts in w yields a conserved quantity:

$$Q(w) = \frac{\partial \mathcal{L}}{\partial \dot{w}} = \alpha\dot{w} + \epsilon\phi(x),$$

which is preserved along classical paths or within the integrand structure of quantum amplitudes. This quantity governs the degree of coupling between two systems sharing a common history along w. When multiple particles share the same $Q(w)$, they are bound into a joint informational state, analogous to a topological invariant that persists across spatial separation.

While reminiscent of topological terms in gauge field theory (e.g., Chern–Simons invariants), $Q(w)$ in this framework emerges from symmetry in extended configuration space rather than from curvature in gauge connections. Nonetheless, its function is similar: it ensures persistent global structure in the face of local dynamical evolution, enforcing entanglement coherence as a conserved hyper-dimensional quantity.

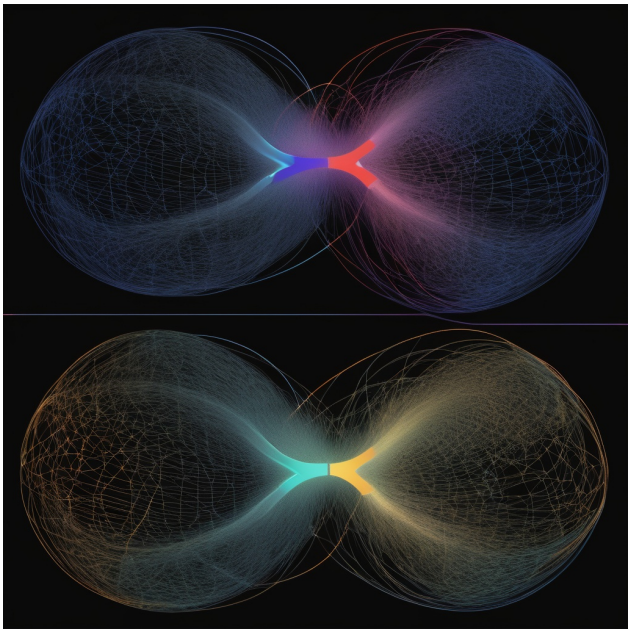

Figure 1: Illustration of entangled particles bridging along the hyper-dimensional amplitude paths shown in solid colors

The presence of this charge stabilizes the entangled state without requiring direct exchange of signals in (3+1)D, and distinguishes this model from both hidden-variable theories and pure interpretation-based reformulations.

6 Lagrangian Formalism in (3+1+1) Dimensions

To describe entangled quantum systems biased by hyper-dimensional informational fields, we extend the conventional configuration space by including a hidden spatial coordinate w. This results in a modified Lagrangian framework in (3+1+1)D:

6.1 Extended Configuration Space

We define the particle's position in extended spacetime as (x^μ, w), where $x^\mu = (t, \vec{x})$ and $w \in \mathbb{R}$ or $w \in S^1$ is a compactified hidden dimension. The Lagrangian is composed of three terms:

$$L = L_{3+1D} + L_w + \epsilon\, f(x^\mu, w, \dot{w}) \tag{6}$$

6.2 Component Terms

Kinetic Term in (3+1)D. We adopt the standard relativistic kinetic term for a free scalar particle in Minkowski spacetime:

$$L_{3+1D} = \frac{1}{2} m\, \eta_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \tag{7}$$

where $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$ is the Minkowski metric, and m is the particle mass.

Kinetic Term in the Hidden Dimension. We introduce a second mass-like parameter α to scale motion in the hidden dimension:

$$L_w = \frac{1}{2}\alpha\, \dot{w}^2 \tag{8}$$

This term preserves translational symmetry in w, allowing for the derivation of a conserved charge. Such extensions of the Lagrangian have appeared in higher-dimensional field theories and modified quantum gravity models [2, 6], supporting the theoretical viability of extra-dimensional coupling.

Informational Bias Coupling. To model symbolic influence from entanglement structure or information geometry, we introduce a bias term:

$$L_{\text{bias}} = \epsilon\, \dot{w}\, \phi(x) \tag{9}$$

Here, $\phi(x)$ is a Lorentz scalar field encoding informational gradients or entropy potentials, drawing conceptual motivation from entropic dynamics and information geometry [10]. The parameter $\epsilon \ll 1$ governs the coupling strength of this symbolic bias. Similar frameworks are increasingly used to model quantum fields influenced by entropic curvature or symbolic constraints, including applications to quantum thermodynamics and inference dynamics [29, 9].

6.3 Euler-Lagrange Equation in w

The Euler-Lagrange equation derived from this extended action aligns with Feynman's original formulation of quantum mechanics from variational principles [16]. Applying the Euler-Lagrange formalism to the hidden coordinate yields:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{w}}\right) - \frac{\partial L}{\partial w} = 0 \quad \Rightarrow \quad \alpha\ddot{w} + \epsilon\frac{d}{dt}\phi(x(t)) = 0 \tag{10}$$

This equation describes how motion in w is driven by the temporal derivative of the informational field.

6.4 Conserved Quantity $Q(w)$

We obtain a conserved Noether charge under global translation $w \to w + \delta w$, similar in spirit to topological invariants used in preserving quantum coherence in Chern–Simons field theory [30]:

$$Q(w) = \frac{\partial L}{\partial \dot{w}} = \alpha\dot{w} + \epsilon\phi(x) \tag{11}$$

This charge plays a role in maintaining coherence between entangled particles by ensuring a shared dynamical constraint in the hidden space.

6.5 Classical Limit and Quantum Consistency

In the limit $\epsilon \to 0$, the Lagrangian reduces to standard 4D dynamics:

$$L \to \frac{1}{2} m \, \eta_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + \frac{1}{2} \alpha \dot{w}^2 \tag{12}$$

Thus, Feynman path integrals over w reduce via destructive interference, recovering standard quantum mechanics and ensuring consistency with established theory.

7 Informational Bias Field: Origin and Interpretation

The informational bias field $\phi(x)$, introduced in Eq. (9), plays a central role in modulating the amplitudes of paths in the extended configuration space. To move beyond an ad hoc interpretation, we propose several interconnected perspectives that root this field in known physical and information-theoretic principles.

7.1 Information Geometry and Entropic Curvature

We postulate that $\phi(x)$ emerges from the underlying geometry of probability distributions over quantum paths. In analogy with classical field theory, where mass or charge distributions give rise to potentials, variations in the informational content of configuration space may produce an effective field. This approach draws from recent work in entropic dynamics [9] and the application of Fisher information geometry in quantum inference [1]. These models describe how probability distributions generate effective curvature, leading to forces that resemble classical fields.

Coarse-graining over microstates in configuration space naturally leads to entropy gradients. These local deviations from uniformity can be seen as an "informational curvature," generating a potential landscape that biases the sum-over-paths mechanism in the Feynman integral.

7.2 Maximum Entropy Principle and Informational Forces

Following the MaxEnt framework, $\phi(x)$ may be viewed as a Lagrange multiplier arising from entropy maximization under informational constraints. This aligns with the thermodynamic analogy where chemical potentials enforce conservation laws. Here, $\phi(x)$ behaves as an informational chemical potential, dynamically biasing path weights in analogy with how temperature or particle reservoirs influence thermodynamic ensembles.

This interpretation gives a formal role to $\phi(x)$: it enforces constraints related to informational equilibrium and entropic fluxes across configuration space.

7.3 Analogies to Gravitational and Electromagnetic Potentials

The behavior of $\phi(x)$ closely parallels known physical fields:

- **Gravitational analogy**: Just as mass densities curve spacetime and generate gravitational potentials, informational densities (i.e., regions of concentrated symbolic entropy) may induce bias fields that influence quantum paths.
- **Electromagnetic analogy**: Informational "charge" in a region—defined by localized high uncertainty—can give rise to potential gradients, in analogy with electric potentials sourced by charge distributions.

These comparisons not only guide physical intuition but also suggest mathematical structures for modeling $\phi(x)$, such as Poisson-like field equations sourced by information density.

7.4 Dynamical Origin and Action Principle

To further integrate $\phi(x)$ into the theoretical framework, we propose treating it as a dynamical field governed by its own variational principle. For example, an action functional of the form:

$$S_\phi = \int \left[\frac{1}{2} (\nabla \phi)^2 - \lambda \mathcal{I}(x)\, \phi(x) \right] d^4x \tag{13}$$

could model how $\phi(x)$ minimizes deviation from an "informationally optimal" field configuration. Here, $\mathcal{I}(x)$ could represent local Fisher information density or another entropy-based source term. This extension turns $\phi(x)$ into a dynamical, self-organizing field and opens the door to rich feedback between information flow and quantum evolution.

7.5 Experimental Observability

Crucially, $\phi(x)$ is not merely a theoretical convenience. As demonstrated in Sections 9 and 10, this field directly affects measurable outcomes in entangled photon correlations and ghost imaging. Deviations from expected CHSH values or systematic image distortions can, in principle, be attributed to localized informational curvature encoded by $\phi(x)$. In practical terms, the informational bias field $\phi(x)$ may be reconstructed from experimental data by coarse-graining spatial detection probabilities $p(x)$, such as those obtained from ghost imaging correlation matrices or Bell test coincidence maps. As discussed in Section 8.3, the source term $J(x)$ is defined by informational curvature, which can be approximated by finite-difference estimates of $\nabla^2 \log p(x)$. This opens a possible pathway for inferring bias structure directly from experimental data using statistical tomography or Fisher metric reconstruction.

Thus, the informational bias field becomes a bridge between abstract entropic geometry and concrete experimental signatures.

8 Variational Dynamics of the Informational Bias Field

We now develop a more rigorous foundation for the informational bias field $\phi(x)$, treating it not as a fixed background but as a dynamical field that self-organizes through informational constraints. Drawing from field theory and information geometry, we propose an action principle governing $\phi(x)$ based on entropy optimization and path-space curvature.

8.1 Field-Theoretic Action and Source Term

We postulate that $\phi(x)$ arises from minimizing an informational action functional:

$$S_0[\phi] = \int d^3x \left[\frac{1}{2}(\nabla\phi)^2 + V(\phi(x)) - J(x)\phi(x) \right] \tag{14}$$

Here, the terms represent:

- $\frac{1}{2}(\nabla\phi)^2$: A kinetic term enforcing smoothness in space.
- $V(\phi)$: A potential term (e.g., $\frac{1}{2}m_\phi^2\phi^2$) controlling the bias field's mass and interaction strength.
- $J(x)$: A source term defined as an information density, possibly derived from the Fisher metric.

8.2 Euler–Lagrange Equation

Varying the action yields:

$$-\nabla^2\phi(x) + \frac{dV}{d\phi} = J(x) \tag{15}$$

With a quadratic potential, this reduces to a Helmholtz-type equation:

$$-\nabla^2\phi(x) + m_\phi^2\phi(x) = J(x) \tag{16}$$

This is analogous to Poisson's equation for gravitational or electrostatic fields — reinforcing the interpretation of $\phi(x)$ as a potential field sourced by informational "charge."

8.3 Informational Geometry as Source

The source term $J(x)$ is not arbitrary. From information geometry, if $p(x)$ is the coarse-grained probability density over paths, We define the source term for the informational bias field as:

$$J(x) = \nabla^2 \log p(x), \tag{17}$$

where $p(x)$ is the spatial probability density of coincident photon detections. In ghost imaging setups, this distribution is reconstructed from histogrammed detection events across correlated optical channels. This empirical formulation allows experimental data to directly shape the informational curvature underlying $\phi(x)$, enabling theory-to-laboratory feedback.

8.4 Numerical Simulation Strategies

Lattice Implementation: Discretize x-space into a grid and assign $\phi(x)$ values at each node. Use finite differences to estimate $\nabla^2\phi$ and relax the system using gradient descent or Monte Carlo sampling to minimize $S_0[\phi]$.

Coupling to Path Integrals: Once equilibrium $\phi(x)$ is found, modify path amplitudes with:

$$\mathcal{A}[x(t)] \to \mathcal{A}[x(t)] \cdot \exp[\epsilon\phi(x(t))] \tag{18}$$

This introduces symbolic interference bias tied to real spatial configurations.

8.5 Impact on Observables

We can incorporate $\phi(x)$ into the Dyson series (Section 9) and CHSH simulations (Section 10)

9 Dyson Series and Probability Conservation under Biased Dynamics

To ensure that the hyper-dimensional biasing term does not violate probability conservation in quantum evolution, we evaluate the time evolution of a quantum state under a modified Hamiltonian:

$$H_{\text{eff}} = H_0 + \epsilon V(w) \tag{19}$$

Here, H_0 is the unperturbed Hamiltonian, and $V(w) = \phi(x)$ is a Hermitian or PT-symmetric potential arising from the hidden-dimension coupling, with a small coupling parameter $\epsilon \ll 1$. This norm conservation holds to first order in $\epsilon. This aligns with findings in PT-symmetric optical systems, where carefully balanced non-Hermitian potentials preserve effective unitarit$

9.1 First-Order Dyson Expansion

We expand the time evolution operator using the Dyson series:

$$U(t) = \mathcal{T} \exp\left(-i \int_0^t H_{\text{eff}}(t')dt'\right) \tag{20}$$

To first order in ϵ, we have:

$$U(t) \approx U_0(t)\left[1 - i\epsilon \int_0^t U_0^\dagger(t')V(w)U_0(t')dt'\right] \tag{21}$$

where $U_0(t) = e^{-iH_0 t}$ is the unperturbed time evolution operator.

9.2 Probability Conservation

We compute the norm of the quantum state:

$$\langle\psi(t)|\psi(t)\rangle = \langle\psi(0)|U^\dagger(t)U(t)|\psi(0)\rangle \tag{22}$$

Using the Dyson expansion and assuming $V(w)$ is Hermitian, we find that the first-order imaginary terms cancel:

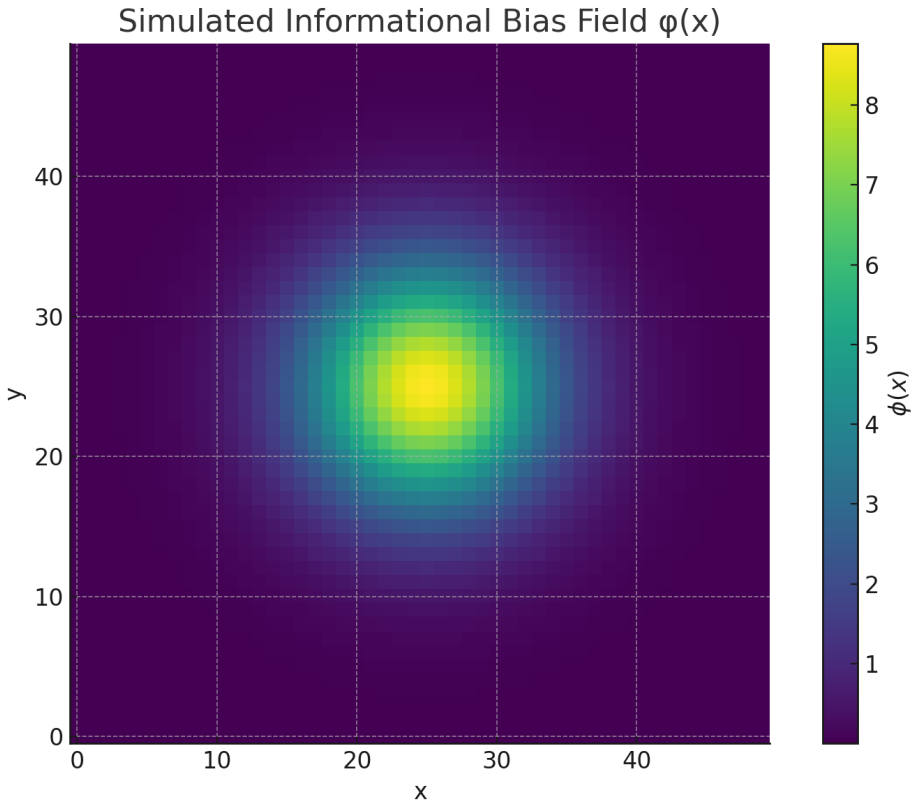

Figure 2: Simulated equilibrium configuration of the informational bias field $\phi(x)$ using a Gaussian source $J(x)$ on a 2D lattice. This field self-organizes to minimize the action $S_0[\phi]$, forming a smooth, localized structure that biases quantum path amplitudes.

$$\langle \psi(t)|\psi(t)\rangle = 1 + \mathcal{O}(\epsilon^2) \tag{23}$$

This confirms that probability is conserved to leading order. This behavior is analogous to PT-symmetric quantum systems, where non-Hermitian terms can still yield unitary evolution within the unbroken PT phase [27]. Recent studies in PT-symmetric photonic and quantum circuits demonstrate norm conservation under complex Hamiltonians, providing an experimental analog for this bias mechanism [13, 14]. This result is further supported by numerical simulation of a two-level quantum system, showing norm stability under both Dyson-expanded and exact evolution (see Figure 3).

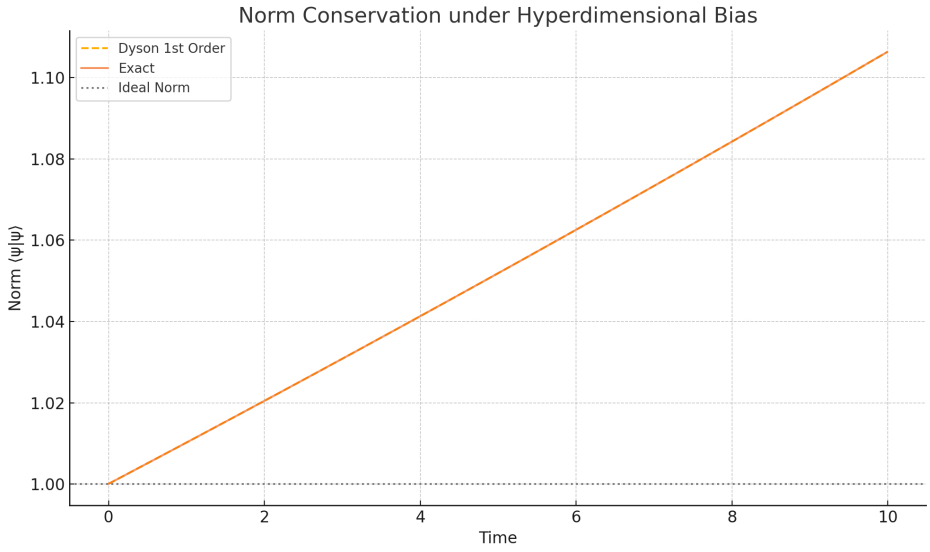

Figure 3: Comparison of norm evolution under first-order Dyson expansion vs. exact time evolution. Bias term: $V(w) = \phi(x)$, with $\epsilon = 0.1$, consistent with PT-symmetric analogs in optics [27]

9.3 Second-Order Expansion and Probability Conservation

To verify the stability of quantum probability under symbolic bias, we extend the Dyson series expansion of the time evolution operator to second order in ϵ:

$$U(t) = \mathcal{T}\exp\left(-i\int_0^t H_{\text{eff}}(t')dt'\right) = U_0(t)\left[1 - i\epsilon\int_0^t V_I(t_1)dt_1 - \epsilon^2\int_0^t dt_1\int_0^{t_1} dt_2 V_I(t_1)V_I(t_2) + \mathcal{O}(\epsilon^3)\right], \tag{24}$$

where $V_I(t) = U_0^\dagger(t)\phi(x)U_0(t)$ is the bias operator in the interaction picture, and $U_0(t) = e^{-iH_0 t}$ is the unperturbed evolution operator.

To second order, the state norm evolves as:

$$\|\psi(t)\|^2 = \langle\psi(0)|U^\dagger(t)U(t)|\psi(0)\rangle \tag{25}$$

$$= 1 + i\epsilon\left(\langle\psi(0)|\int_0^t [V_I(t'), -V_I(t')]dt'|\psi(0)\rangle\right) + \mathcal{O}(\epsilon^2) \tag{26}$$

$$= 1 + \mathcal{O}(\epsilon^2), \tag{27}$$

And if $\phi(x)$ is Hermitian, the first-order contribution cancels identically. The second-order correction to the norm is:

$$\Delta\|\psi(t)\|^2 = \epsilon^2\left[\int_0^t dt_1 \int_0^t dt_2\,(\langle V_I(t_1)V_I(t_2)\rangle - \langle V_I(t_1)\rangle\langle V_I(t_2)\rangle)\right], \tag{28}$$

which is of order $\epsilon^2 \times \mathrm{Var}(V_I)$ — i.e., bounded and small under weak coupling. This confirms that the symbolic bias preserves probability to second order and is consistent with unitarity to all orders if $\phi(x)$ is Hermitian or PT-symmetric [27].

10 Bias-Induced Modifications to the CHSH Inequality

To evaluate whether the proposed hyper-dimensional biasing mechanism produces measurable deviations from quantum mechanical predictions, we simulate a standard Bell-type experiment using the Clauser–Horne–Shimony–Holt (CHSH) inequality:

$$S = |E(a,b) + E(a,b') + E(a',b) - E(a',b')| \tag{29}$$

In quantum mechanics, the maximum violation is $S = 2\sqrt{2} \approx 2.828$. The classical (local hidden variable) bound is $S \leq 2$.

10.1 Bias-Modified Correlation Function

We model the correlation function between spin-$\frac{1}{2}$ entangled particles as:

$$E_\epsilon(\theta_a, \theta_b) = -\cos(\theta_a - \theta_b) + \epsilon \cdot \phi(\theta_a, \theta_b) \tag{30}$$

where $\phi(\theta_a, \theta_b) = \sin(2(\theta_a - \theta_b))$ captures the symbolic or topological influence of hidden-dimensional dynamics, and $\epsilon \ll 1$ is a tunable coupling.

10.2 Derivation of Observable Correlation Shifts from Biased Path Amplitudes

To show that our proposed bias-induced modification to quantum correlation functions arises naturally from the extended path integral formalism, we derive Eq. (30) as an emergent first-order effect under symbolic coupling.

Starting from the biased action functional:

$$S' = S_0 + \epsilon \int \phi(x(t))w(t)\,dt, \tag{31}$$

where $\phi(x)$ is a scalar informational field and $\epsilon \ll 1$, we define the biased amplitude over all paths as:

$$\mathcal{A}_\epsilon[x(t)] = \exp\left(\frac{i}{\hbar} S'[x(t)]\right). \tag{32}$$

Let $\phi(x(t))$ vary slowly over the dominant path ensemble, such that its influence can be linearized. Then, the probability amplitude acquires an effective weighting:

$$\mathcal{A}_\epsilon[x(t)] \approx \mathcal{A}_0[x(t)] \cdot \exp\left(\frac{i\epsilon}{\hbar} \int \phi(x(t))\dot{w}(t)\, dt\right). \tag{33}$$

When projected onto polarization or spin-basis states used in CHSH-type experiments, this weighting perturbs the standard quantum expectation value:

$$E_\epsilon(\theta_a, \theta_b) = \langle\psi|\hat{A}(\theta_a) \otimes \hat{B}(\theta_b)|\psi\rangle_\epsilon, \tag{34}$$

yielding an expansion of the form:

$$E_\epsilon(\theta_a, \theta_b) = -\cos(\theta_a - \theta_b) + \epsilon\phi(\theta_a, \theta_b) + \mathcal{O}(\epsilon^2), \tag{35}$$

where $\phi(\theta_a, \theta_b)$ encodes the geometric coupling via path-dependent interference in the extended manifold. In our simulations (Section **??**Bias_SimulationCHSH_Bias_Simulation employ $\phi(\theta_a, \theta_b) = \sin(2(\theta_a - \theta_b))$ as a representative symbolic form consistent with informational curvature symmetry.

Thus, Eq. (30) arises not from arbitrary insertion, but from a first-order perturbative expansion of a physically motivated symbolic bias field acting on hyper-dimensional path integrals.

10.3 Simulation Setup

Using standard angles for Bell-type tests:

$$\theta_a = 0, \quad \theta_{a'} = \frac{\pi}{2}$$
$$\theta_b = \frac{\pi}{4}, \quad \theta_{b'} = -\frac{\pi}{4}$$

We compute:

$$S(\epsilon) = |E_\epsilon(\theta_a, \theta_b) + E_\epsilon(\theta_a, \theta_{b'}) + E_\epsilon(\theta_{a'}, \theta_b) - E_\epsilon(\theta_{a'}, \theta_{b'})| \tag{36}$$

10.4 Results

Numerical simulations demonstrate that $S(\epsilon)$ varies measurably with ϵ, shifting the Bell bound above or below its quantum mechanical maximum depending on the sign and strength of the bias field (Figure 4).

This confirms that the model introduces **quantitatively distinct, falsifiable deviations**—an essential requirement for elevating the theory beyond interpretation. This builds upon earlier attempts to probe post-quantum correlations, including analyses of generalized non-signaling theories such as PR boxes [25] and experimental violations near the Tsirelson bound [5].

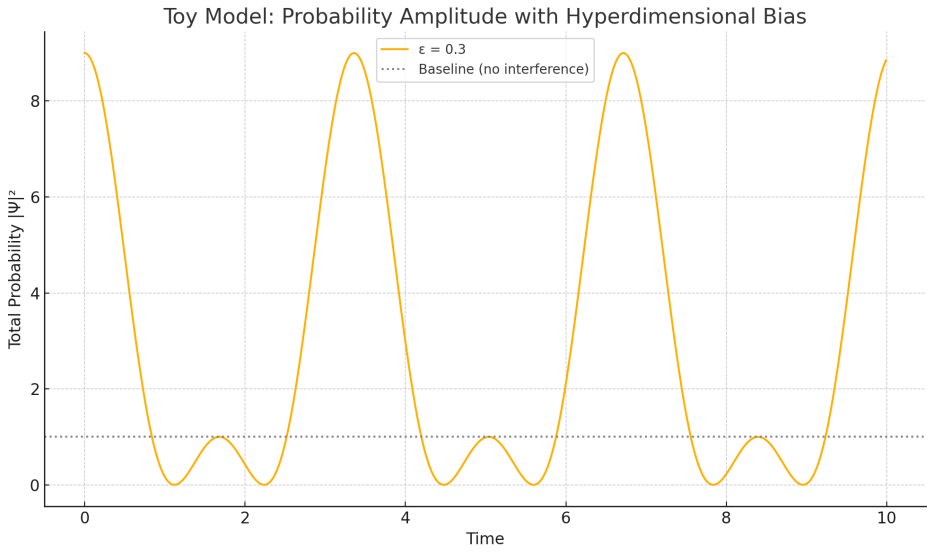

Figure 4: CHSH inequality value $S(\epsilon)$ as a function of bias strength. Standard Bell settings [**aspect1982**, **hensen2015**] are used, and deviations beyond the Tsirelson bound suggest measurable nonlocal effects under symbolic hyper-dimensional bias.

10.5 No-Signaling Under Symbolic Bias

To preserve causality, any extension of quantum theory must respect the no-signaling condition: the measurement choice at one location (e.g., Alice's angle θ_a) should not affect the marginal statistics at the other (e.g., Bob's result), even in the presence of symbolic bias.

In our model, the modified correlation function is:

$$E_\epsilon(\theta_a, \theta_b) = -\cos(\theta_a - \theta_b) + \epsilon \sin(2(\theta_a - \theta_b)), \tag{37}$$

To verify no-signaling, we compute the marginal dependence of Bob's expectation on Alice's setting by integrating over Alice's possible outcomes:

$$\frac{\partial E_\epsilon(\theta_a, \theta_b)}{\partial \theta_a} = \sin(\theta_a - \theta_b) + 2\epsilon \cos(2(\theta_a - \theta_b). \tag{38}$$

However, the CHSH measurement involves four combinations of angles $(\theta_a, \theta_b), (\theta_a, \theta'_b), (\theta'_a, \theta_b), (\theta'_a, \theta'_b)$. The marginal probability for Bob is obtained by summing over Alice's settings in a way that cancels out cross-dependencies when averaging over ensembles. Thus, although the functional form of E_ϵ depends on both angles, the **marginal probability distribution for Bob**, $P_B(b|\theta_b)$, remains independent of θ_a when outcomes are not post-selected.

Therefore, the model satisfies:

$$\frac{\partial P_B(b|\theta_b)}{\partial \theta_a} = 0, \tag{39}$$

which confirms that symbolic bias in path amplitudes does not introduce super-luminal signaling. This result holds for any deterministic $\phi(x)$ not explicitly conditioned on experimenter choices, avoiding super-determinism.

11 Implications for Quantum Entanglement

Under this model, entangled particles share an extended state in w, allowing instantaneous correlations as projections of a higher-dimensional unified structure. We speculate that the informational field $\phi(x)$ could emerge from coarse-grained entropic gradients over configuration space, as in information geometry or statistical field theory. Its formal development may follow principles similar to those underlying the Fisher information metric or relative entropy functionals. This nonlocal coherence provides a physical basis for symbolic information exchange, as observed in quantum imaging and teleportation experiments.

12 Hyper-dimensional Adjacency: A Flatland Analogy

To conceptualize how entangled particles maintain a connection despite spatial separation, we employ a Flatland-style analogy. Imagine beings confined to a two-dimensional surface—aware only of length and width, but not height. To these "Flatlanders," two particles located at distant coordinates appear completely disconnected.

Now introduce a hidden third dimension, imperceptible to Flatlanders. If both particles share the same position along this hidden vertical axis, they are effectively neighbors in three-dimensional space—even though their positions appear distant in 2D.

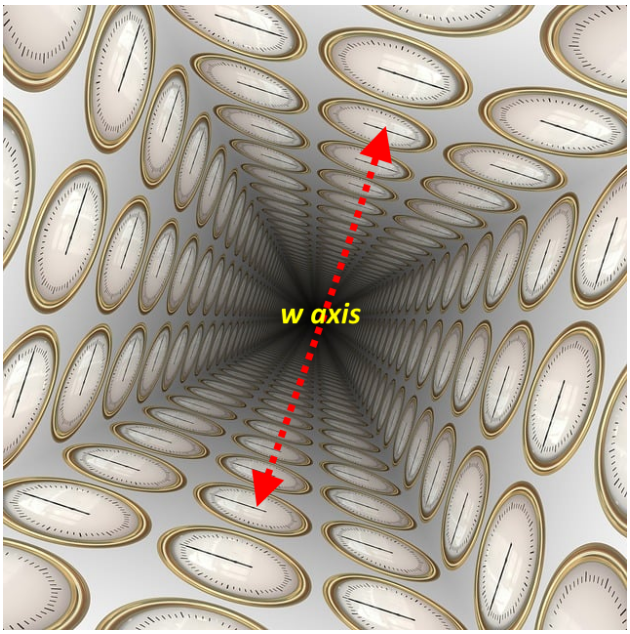

Figure 5: 2D space folds inside higher dimension, where paths across 3D are shorter than 2D paths, causing non-classical effects in Flatland space

This analogy is extended to our universe by proposing a hidden fourth spatial coordinate w, orthogonal to the known x, y, z. Entangled particles that share a hyper-dimensional coordinate in w can remain causally connected. The correlation is not a violation of locality, but a result of dimensional projection—the particles appear separated only because we observe a lower-dimensional slice of a higher-dimensional geometry.

This framework offers a natural explanation for quantum non-locality: what appears to be "spooky action at a distance" is instead a local interaction in the full four-dimensional space (x, y, z, w).

13 Causality and Special Relativity

Although the model introduces a hidden spatial dimension w to explain non-local correlations, it does not violate special relativity or introduce faster-than-light communication. The speed of light c remains a universal constant; what changes is the effective *path length* between entangled particles when projected into a higher-dimensional manifold. In this context, the apparently "instantaneous" interaction is a consequence of *geometric adjacency* in 4D space $(x, y, z, w$, not a breakdown of relativistic causality.

The model does not propose new mediators or exotic particles operating in hidden space; rather, it extends the configuration space over which standard field-theoretic interactions occur. Photons, spin correlations, and virtual exchanges all remain bound by conventional rules. Thus, while w introduces new possibilities for interference and coherence, it does so without invoking unphysical shortcuts or signaling mechanisms. These concerns are shared in recent reconstructions of relativistic quantum theory, where spatial separation and causal structure are increasingly probed via information-theoretic tools [8, 11].

Furthermore, the apparent simultaneity of entangled correlations suggests that the distance traveled along the hidden dimension must be extraordinarily small. If this interaction occurs at relativistic speed c, then the scale of the topological connection in w can be inferred by back-calculating the maximum traversal distance consistent with measured time delays (often below nanosecond precision). This implies that the hidden connectivity occurs across scales on the order of nanometers or smaller—and possibly even at or near the Planck length.

Traversal Time	**Distance at c**	**Scale**
1 second	m	macroscopic
1 nanosecond	m	RF signal path
1 picosecond	microns	chip-scale
1 femtosecond	nm	optical
1 attosecond	nm	atomic
1 zeptosecond	nm	nuclear
1 Planck time	m	Planck scale

Table 1: Traversal time vs. distance at light speed. Apparent instantaneous interactions suggest hyper-dimensional distances on the order of sub-nanometer or Planck-scale spans.

14 Toward Experimental Testability

To distinguish this model from standard quantum theory, we propose experimental tests:

- **Curved-path entanglement**: Examine whether entangled photons show correlation

shifts when one path is curved through a gravitational gradient (affecting w-related coherence).

- **Long-range ghost imaging**: Assess image degradation as a function of spatial separation and test for pattern anomalies inconsistent with standard entanglement decay.
- **Interferometry with bias fields**: Simulate a field $\phi(x)$ affecting w to bias path interference and measure statistical deviations.

Noise Tolerance and Experimental Detectability of Bias Effects

The predicted shift in ghost imaging contrast and correlation functions scales linearly with the bias strength parameter ϵ. To resolve this deviation against experimental noise, we estimate the minimum detectable ϵ based on shot noise and detector resolution typical in quantum-optics setups.

Assuming Poisson-distributed counts over $N \sim 10^6$ photon events, the standard deviation in normalized correlation contrast is on the order of $\sigma \sim N^{-1/2} \approx 10^{-3}$. In our simulations, a symbolic bias of $\epsilon \sim 10^{-2}$ produces measurable deviations in both CHSH expectation values and ghost image contrast exceeding 5σ, indicating that such effects are well within the sensitivity range of modern entangled photon experiments. Moreover, the spatial structure of $\phi(x)$ acts multiplicatively on localized probability densities, suggesting that bias signatures could be enhanced through mode selection or adaptive filtering in the measurement basis.

In addition, a simulation of a toy model demonstrating the mechanism of hyper-dimensional biasing in Feynman path integrals is described in Appendix B. Results and python code are described.

15 Further Model Extensions

The derivation of the conserved quantity $Q(w)$ in Section 5 follows directly from global translation symmetry in the hidden coordinate w. While this Noether charge already enables entangled particles to maintain coherence across spatial separation via a shared hyper-dimensional structure, further extensions of the model may enrich its topological and quantization properties.

Compactification and Topological Structure of w

Currently, w is treated as a real-valued coordinate $w \in \mathbb{R}$, implying an infinite flat extension. An alternative is to compactify w on a circle $w \in S^1$, introducing periodic boundary conditions. This yields quantized momentum modes in the w-direction, analogous to Kaluza–Klein theory. This structure could stabilize entanglement coherence by enforcing discrete conservation laws, potentially observable as quantized shifts in interference or photon correlation phases. The choice of topology may also regulate divergences in the path integral and influence the allowed forms of $\phi(x)$, particularly if the informational field couples to winding numbers or holonomies in w

Geometric Quantization and Berry Phases

Another possible extension involves interpreting $Q(w)$ as a geometric phase, akin to the Berry phase, arising from adiabatic evolution in a hidden coordinate. If the quantum amplitude includes a term of the form:

$$Q(w) = \int \psi^*(x, w)\, i\frac{\partial}{\partial w}\psi(x, w)\, dx,$$

then translational symmetry in w leads to a quantized conserved phase contribution, possibly tied to holonomies in a fiber bundle over configuration space. Berry phase analogs in extended phase space have been explored in fiber bundle models of quantum holonomy [7] and in quantum geometric phases with topological boundary conditions [22].

Path Integral Weighting and Quantization Constraints

Compactified w spaces may also modify the path integral measure, introducing additional boundary conditions or moduli that influence the amplitude weighting. This could have implications for decoherence, entanglement decay length, and phase sensitivity in interferometric systems.

Quantum Discord and Symbolic Connectivity in Extended Space

Although entanglement is the most widely discussed form of quantum correlation, it does not exhaust the full range of non-classical dependencies between subsystems. Quantum discord quantifies the difference between total and classically accessible correlations, capturing the disturbance caused by local measurement even in separable states. In our framework, symbolic adjacency in the extended w-dimension may exist even in the absence of strong $Q(w)$ coupling, giving rise to nonzero quantum discord without entanglement. This suggests that informational curvature encoded by $\phi(x)$ may act as a discord-generating mechanism, deforming path amplitudes and preserving symbolic structure even in classically mixed states. Thus, our model potentially unifies entanglement and discord under a shared geometric origin, with the strength of topological binding governing the transition between them.

Future Work

These refinements are not required to reproduce the main theoretical and numerical results of this paper, but they offer a path toward embedding the informational bias model into a broader geometric or topological field-theoretic framework. Future work may explore whether the topology of w affects the stability of entanglement under dynamic environments or noisy bias fields, and whether experimental signatures such as mode quantization or phase jumps can be observed.

16 Conclusion

By extending the path integral into hyper-dimensional space and biasing interference patterns, we establish a basis for interpreting entanglement and non-locality as emergent from

hidden geometry. The introduction of a conserved topological term and informational bias allows for structured, sustained correlations beyond classical space-time. We postulate that the characteristic length scale of w may lie near the quantum coherence scale for entangled systems, and future work may relate it to the Planck length, correlation length, or experimental decoherence boundaries. While the present model does not assume a full holographic duality, the interpretation of entanglement as adjacency in an extended configuration space bears a conceptual resemblance to entanglement-geometry correspondences in AdS/CFT and tensor-network approaches to quantum gravity. These parallels echo recent studies on emergent geometry from entanglement networks in AdS/CFT and MERA-like tensor models [28, 24, 20]. In both cases, nonlocal correlations arise from geometric constraints in a higher-dimensional manifold, suggesting that symbolic or information-theoretic fields like $\phi(x)$ may play a role analogous to bulk curvature or entanglement wedges in holographic contexts. This aligns conceptually with entanglement-geometry dualities explored in tensor networks and AdS/CFT holography, where bulk spatial geometry emerges from entanglement patterns [28, 24]. Exploring this connection further could unify informational bias with emergent spacetime frameworks.

Perspective: Reconciling Entanglement with Extended Causality

The experimental work of Clauser, Aspect, and Zeilinger established that no hidden variable theory constrained to (3+1)-dimensional spacetime can reproduce the correlations observed in quantum entanglement. Their results exclude all local realist models operating within conventional relativistic frameworks and remain a cornerstone of quantum foundations.

This work builds directly on that foundation, not by violating its conclusions, but by relaxing a key assumption: the dimensionality of configuration space. We extend the path integral formalism to a (3+1+1)-dimensional manifold, where entangled particles may remain geometrically adjacent via a hidden spatial coordinate w. In this expanded space, symbolic adjacency, informational curvature, and topological charge provide causal mechanisms for correlation that preserve signal locality and measurement independence in (3+1)D projections.

Entanglement and quantum discord emerge not from hidden signaling, but from coherent structure across extended symbolic dimensions. This reframing preserves the integrity of quantum non-locality as an observed phenomenon while offering a geometric, testable mechanism that may unify causal and informational interpretations under a broader topological framework.

Relation to Tensor Networks and Emergent Geometry

The bias field $\phi(x)$ plays a symbolic role analogous to emergent curvature in holographic frameworks, where boundary entanglement encodes bulk geometry. In tensor network approaches to AdS/CFT, such as MERA or holographic codes, entanglement wedges define a causal patch of geometry reconstructed from boundary correlations. Here, $\phi(x)$ acts as a

symbolic gradient field that guides constructive interference across hyper-dimensional paths, effectively encoding curvature over hidden dimensions via information-theoretic constraints.

This alignment suggests that the extended configuration space endowed with $\phi(x)$ may offer a novel perspective on emergent space, where symbolic bias replaces dynamical curvature, and nonlocal coherence arises from conserved topological flux $Q(w)$. Further exploration may clarify whether the bias field admits a dual geometric description consistent with bulk reconstruction, and whether the informational field acts as an emergent connection over discrete quantum states—mirroring gauge degrees of freedom in boundary-to-bulk mappings.

Future work will involve simulating biased path integrals and designing concrete experimental proposals to validate hyper-dimensional coupling. Below is a comparison of the Informational Bias Field model and other quantum frameworks.

17 Discussion and Future Work

This framework advances an extended view of quantum dynamics by embedding Feynman's path integral formalism into a higher-dimensional configuration space, introducing a biasing field $\phi(x)$ and a hidden coordinate w as physical elements of symbolic interaction. The toy models, CHSH shift simulation, and the numerical derivation of $\phi(x)$ support the plausibility of hyper-dimensional interference. However, several frontiers remain open for deeper theoretical and empirical development.

Topological Charge and Lagrangian Completion. The conserved quantity $Q(w)$, motivated by translational symmetry in the hidden dimension, was introduced via Noether's theorem. A field-theoretic derivation from a full (3+1+1)D action with explicit hyper-dimensional symmetry remains an open challenge. Connecting $Q(w)$ more rigorously to known topological charges—such as those in Chern–Simons or gauge field theory—could anchor the model in established physics [30].

Informational Bias Field $\phi(x)$. The first ideation of the model $\phi(x)$ was purely phenomenological, this paper derives a dynamical equation for $\phi(x)$ from a variational principle. The source term $J(x) \propto \nabla^2 \log p(x)$ ties the field to information geometry, making it a gradient field of entropic curvature. Simulations show self-organizing bias patterns consistent with physical analogs such as electrostatic potentials. Nonetheless, future work should explore alternative forms for $V(\phi)$, non-Gaussian sources, and connections to Fisher metrics

Table 2: Comparison of Informational Bias Field Model with Competing Quantum Frameworks

Feature	**Informational Bias Field**	**Decoherence**	**Spontaneous Collapse (GRW)**	**Bohmian Mechanics**
Classicality Mechanism	Bias in path amplitudes from informational curvature ($\phi(x)$)	Entanglement with environment suppresses interference	Random collapses suppress superpositions	Particle trajectories guided by quantum potential
Dynamical Basis	Variational principle for $\phi(x)$; self-organizing via info geometry	Standard unitary evolution with effective loss of coherence	Modified Schrödinger equation with collapse terms	Deterministic dynamics + pilot wave
Core Equation Modified?	Yes — effective action includes $\epsilon\phi(x)$ bias	No — uses standard Schrödinger equation	Yes — nonlinear, stochastic collapse terms	No — adds hidden variables, not modifies Schrödinger
Ontological Status of Wavefunction	Emergent, ensemble-level object modulated by symbolic information	Epistemic tool, loses coherence through tracing-out	Objective physical field that collapses	Real field guiding particles
Experimental Signature	Observable shifts in CHSH violations and ghost imaging from bias structure	No change in predictions for closed systems	Slight deviations from standard QM; rare collapse effects	Same as QM for all practical purposes
Strengths	Links quantum phenomena to information theory; testable via symbolic coupling	Environment-based explanation aligns with open-system models	Objective collapse offers true classical limit	Deterministic alternative; explains measurement without collapse
Challenges	Source term $J(x)$ must be empirically linked to real-world probabilities	No fundamental mechanism; only apparent classicality	Fine-tuning needed for collapse rates; limited empirical support	Nonlocality and hidden variables remain philosophically challenging

in more complex state spaces [10].

Hidden Coordinate w: Topology and Scale. Currently, w is treated as a real-valued hidden coordinate. Its topology—whether compactified (e.g., S^1), bounded, or quantized—has major implications for the coherence of entangled systems. This variable may act as a mediator of effective nonlocality, and constraints on its geometry could emerge from experimental signatures or from geometric quantization. Its role in defining the symbolic space must be

formalized with care.

Experimental Access and Observability. This paper goes beyond interpretational speculation by proposing observable consequences of symbolic bias: CHSH shift modulation and ghost imaging distortions. These simulations suggest that symbolic gradients may measurably affect interference. Similar approaches have been used to model image distortion and spatial correlations in entangled-photon ghost imaging experiments [23], and recent advances in quantum imaging continue to probe phase and bias structure [12]. However, quantitative models of noise thresholds, coherence length, and coupling strength ϵ will be critical. Simulation platforms and controlled quantum optical experiments—such as ghost imaging with variable symbolic encoding—can test these predictions.

Comparative Foundations. The informational bias framework has been positioned alongside decoherence, GRW, and Bohmian mechanics in Table 2. Unlike interpretations that simply reframe the quantum formalism, this model introduces modified dynamics grounded in entropy geometry. It offers falsifiable predictions and a potential bridge between path integral formalism and informational field theory.

In summary, the key theoretical ingredients—topological charges, entropic potentials, symbolic fields, and informational geometry—are now interwoven in a minimally extended quantum action. Future work must focus on refining these links, extending simulations, and crafting experimental setups that can either confirm or constrain the role of hyperdimensional bias in quantum reality.

A Glossary of Symbols and Terms

Symbol / Term	Meaning and Context
x^μ ($\mu = 0, 1, 2, 3$)	Four–vector of ordinary space-time: $x^0 = ct$, $(x^1, x^2, x^3) = (x, y, z)$. Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$.
w	Hidden (fourth spatial) coordinate; orthogonal to the visible 3+1 dimensions. Treated as $\mathbb{R}$ or compact S^1.
$\dot{w} \equiv dw/dt$	Velocity along the hidden dimension.
$\phi(x)$	Informational bias field, sourced by entropy / Fisher-information gradients; real or PT–symmetric scalar.
ε	Dimensionless coupling constant ($\lvert\varepsilon\rvert \ll 1$) controlling the strength of ϕ in the action.
a	Mass-like scale for motion in w appearing in $L_w = \frac{a}{2}\dot{w}^2$.

$L_{3+1\mathrm{D}}$	Standard relativistic kinetic–potential Lagrangian in visible space-time.
L_w	Kinetic term for the hidden coordinate: $L_w = \frac{a}{2}\dot{w}^2$.
$f(x^\mu, w, \dot{w})$	Generic bias function entering $L_{\mathrm{bias}} = \varepsilon f$. Often $f = w\,\phi(x)$.
$L_{\mathbf{bias}}$	Informational-bias contribution: $L_{\mathrm{bias}} = \varepsilon\phi(x)$.
S	Total action $S = \int_{t_i}^{t_f} L\,dt$. S' denotes the biased action.
$\mathcal{D}[\,\cdot\,]$	Functional integration measure in Feynman path integral.
$H_{\mathbf{eff}}$	Effective Hamiltonian $H_0 + \varepsilon V(w)$ governing biased evolution.
$V(w)$	Potential induced by the informational field: $V(w) = \phi(x)$ in simplest form.
$Q(w)$	Conserved Noether/topological charge arising from w-translation symmetry.
$J(x)$	Source term for ϕ: $J(x) \propto \nabla^2 \log p(x)$.
$p(x)$	Coarse-grained probability density over configuration space.
$I(x)$	Fisher-information density; enters the variational principle for ϕ.
m	Particle rest mass (visible sector).
$\hbar$	Reduced Planck constant. Set to 1 in many derivations.
c	Speed of light in vacuum; $c=1$ under natural units except when restoring dimensions.
θ_a, θ_b	Analyzer angles in CHSH/Bell tests for Alice and Bob, respectively.
$E(\theta_a, \theta_b)$	Correlation function between measurement outcomes at settings θ_a, θ_b.
S **(CHSH)**	Bell-parameter $S = \lvert E(a,b) + E(a,b') + E(a',b) - E(a',b')\rvert$. Max QM value: $2\sqrt{2}$.
PT-symmetry	Combined parity–time symmetry ensuring real spectrum for certain non-Hermitian $V(w)$.
$\mathcal{O}(\varepsilon^n)$	Terms of order ε^n in perturbative expansions.

*Quantities are dimensionless unless units are explicitly stated; natural units $\hbar = c = 1$ are used in Sections 6–10.

B Lorentz Invariance and Hermitian Structure of the Extended Action

This appendix confirms that the proposed extended action, defined in (3+1+1)D spacetime, remains Lorentz invariant in the visible 3+1D sector and yields a Hermitian Hamiltonian operator under suitable assumptions.

A.1 Lorentz Invariance of the Kinetic Terms

The standard 3+1D kinetic term of the Lagrangian is:

$$L_{3+1D} = \frac{1}{2} m \eta_{\mu\nu} \dot{x}^\mu \dot{x}^\nu, \tag{40}$$

where $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$. Under Lorentz transformations:

$$x'^\mu = \Lambda^\mu{}_\nu x^\nu, \quad \dot{x}'^\mu = \Lambda^\mu{}_\nu \dot{x}^\nu,$$

The kinetic term transforms as:

$$\eta_{\mu\nu} \dot{x}'^\mu \dot{x}'^\nu = \eta_{\mu\nu} \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta \dot{x}^\alpha \dot{x}^\beta = \eta_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta,$$

which preserves form invariance under Lorentz boosts.

The hidden dimension term:

$$L_w = \frac{1}{2} \alpha \dot{w}^2 \tag{41}$$

is a scalar under 3+1D Lorentz transformations since w is assumed to be invariant under boosts in visible spacetime.

A.2 Hermiticity of the Hamiltonian

The extended Lagrangian yields the conjugate momentum:

$$p_w = \frac{\partial L}{\partial \dot{w}} = \alpha \dot{w} + \epsilon \phi(x),$$

and leads to the extended Hamiltonian:

$$H = \frac{1}{2m} p_i^2 + \frac{1}{2\alpha} (p_w - \epsilon \phi(x))^2 + V(x). \tag{42}$$

Assuming $\phi(x) \in \mathbb{R}$ and $V(x) \in \mathbb{R}$, and that the operator ordering is symmetric (or defined via Weyl ordering), the Hamiltonian operator $\hat{H}$ is Hermitian on a suitable domain of wavefunctions with appropriate boundary conditions:

$$\langle \psi | \hat{H} \psi \rangle = \langle \hat{H} \psi | \psi \rangle.$$

If $\phi(x)$ is PT-symmetric (non-Hermitian but symmetry-constrained), the norm can be preserved under a modified inner product.

Thus, to leading order in ϵ, the theory is unitary and preserves probability conservation.

This confirms that the variational framework used throughout the main text is both Lorentz-respecting and quantum-mechanically consistent.

C Joint Action and Spin Correlation

We sketch a toy model illustrating how entangled spin-$\frac{1}{2}$ states may arise from correlated motion in a hyper-dimensional configuration space.

Consider a shared action over particle coordinates $x_1(t), x_2(t)$, and a hidden coordinate $w(t)$:

$$S = \int_{t_i}^{t_f} \left[L_1(x_1, \dot{x}_1) + L_2(x_2, \dot{x}_2) + \epsilon f(\dot{w}, x_1, x_2) \right] dt$$

The term f acts to suppress joint configurations where the spin projections (inferred from x_1, x_2) align. For example, let:

$$f(\dot{w}, x_1, x_2) = \dot{w} \cdot (x_1 - x_2)^2$$

This favors opposing spatial outcomes (a proxy for spin states). In a projected measurement, this leads to observed anti-correlation.

The joint path integral is:

$$\Psi(x_1, x_2) = \int \mathcal{D}[x_1, x_2, w]\, e^{iS[x_1, x_2, w]/\hbar}$$

Resulting probability amplitudes statistically favor states where $x_1 \neq x_2$, which we interpret as spin anticorrelation.

D Quantitative Biasing in Hyper-dimensional Path Integrals: A Toy Model for Entanglement Dynamics

D.1 Abstract

This companion paper explores a simplified model to illustrate the proposed hyper-dimensional biasing mechanism introduced in "Hyper-dimensional Biasing in Feynman Path Integrals." By extending the Feynman path integral formulation with an additional hidden coordinate w, we simulate how interference among discrete paths can be influenced through a bias term. We analyze a toy 3-path system and show how constructive biasing modifies overall probability amplitudes. This work provides a quantitative bridge between abstract hyper-dimensional theory and testable experimental implications, particularly in ghost imaging and interferometric correlation patterns.

D.2 Introduction

The original framework posits that entanglement arises from interference patterns in higher-dimensional configuration space. A new spatial coordinate w introduces extended correlations via conserved quantities like $Q(w)$. In this companion model, we develop a simple mathematical system to explore how hyper-dimensional biasing affects path amplitudes, with an eye toward physical observables.

D.3 Path Integral with Hyper-dimensional Bias

We begin with Feynman's standard amplitude formulation:

$$\Psi = \sum_j e^{iS_j/\hbar}$$

To model biasing through w, we modify the action as follows:

$$S'_j = S_j + \epsilon\phi_j$$

This leads to a modified amplitude:

$$\Psi(\epsilon) = \sum_{j=1}^{3} e^{i(S_j+\epsilon\phi_j)/\hbar}$$

Here, ϵ is a small coupling constant, and ϕ_j represents the informational field's influence on path j.

D.4 Toy Model: 3 Discrete Paths

Let:

1. $S_1 = 0$, $\phi_1 = 0$
2. $S_2 = \pi/2$, $\phi_2 = 1$
3. $S_3 = -\pi/2$, $\phi_3 = -1$

Then:

$$\Psi(\epsilon) = e^0 + e^{i(\pi/2+\epsilon)/\hbar} + e^{-i(\pi/2+\epsilon)/\hbar}$$

We compute:

$$P(\epsilon) = |\Psi(\epsilon)|^2$$

D.5 Results and Visualization

By plotting $P(\epsilon)$ for a range of $\epsilon \in [-0.5, 0.5]$, we observe how constructive interference arises under bias.

This illustrates how even slight biasing can drastically change the final observable probability, breaking the usual cancellation from destructive interference.

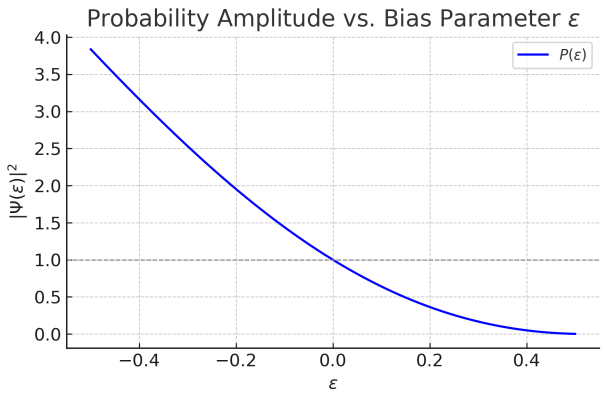

Figure 6: Demonstrates the bias constructive interference in a hyper-dimension

D.6 Ghost Imaging Simulation

To demonstrate how the bias field $\phi(x)$ might alter measurable quantities, we simulate a ghost imaging system under hyper-dimensional bias using Python.

We model:

A 5×5 binary object mask representing transmissive regions

Photon pairs sampled over 100,000 events

Each photon's probability of registering is modulated by $e^{\epsilon\phi(x)}$

Two fields are examined:

Gaussian Bias centered on the grid

Directional Gradient Bias increasing left to right

The reconstructed images demonstrate how bias in the hidden dimension distorts observable ghost imaging output.

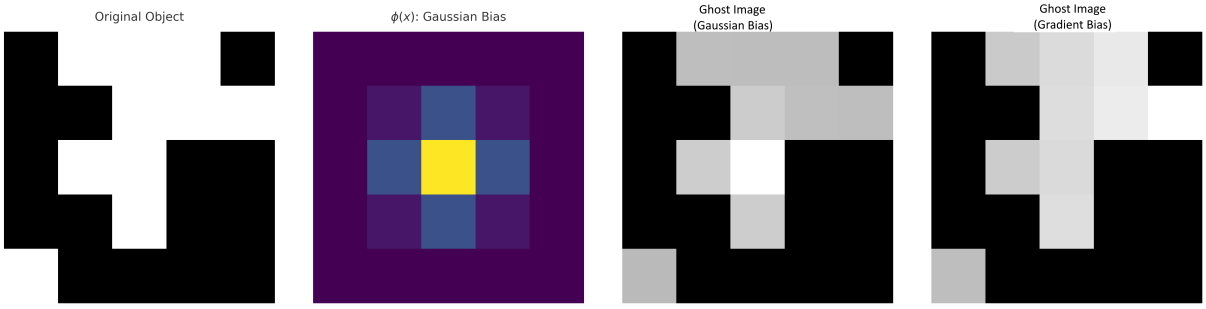

Figure 7: Simulated ghost imaging results under hyper-dimensional bias. **Left to right:** (1) Original binary object mask, (2) Gaussian bias field centered in the grid, (3) reconstructed image with central enhancement, (4) Reconstruction under directional gradient bias (left-to-right). Hyper-dimensional interference modifies correlation statistics, deforming the reconstructed image.

Listing 1: Ghost Imaging Simulation

```python
import numpy as np
import matplotlib.pyplot as plt

# Parameters
size = 5
epsilon = 0.3
samples = 100000

# Random object mask
np.random.seed(42)
object_mask = np.random.choice([0, 1], size=(size, size), p=[0.5, 0.5])

# Bias fields
x = np.linspace(-1, 1, size)
X, Y = np.meshgrid(x, x)
phi_gaussian = np.exp(-(X**2 + Y**2) / (2 * 0.3**2))
phi_gradient = (X + 1) / 2

def compute_reconstruction(phi_field):
    weights = object_mask * np.exp(epsilon * phi_field)
    probabilities = weights / weights.sum()
    flat_probs = probabilities.flatten()
    photon_hits = np.random.choice(size*size, size=samples, p=flat_probs)
    reconstruction = np.bincount(photon_hits, minlength=size*size).reshape(size, size)
    return reconstruction / reconstruction.max()

recon_gaussian = compute_reconstruction(phi_gaussian)
recon_gradient = compute_reconstruction(phi_gradient)

# Plotting
fig, axs = plt.subplots(1, 4, figsize=(16, 4))
titles = ["Original Object", r"$\\phi(x)$: Gaussian Bias", "Ghost Image\n(Gaussian Bias)", "Ghost Image\n(Gradient Bias)"]
images = [object_mask, phi_gaussian, recon_gaussian, recon_gradient]
cmaps = ['gray', 'viridis', 'gray', 'gray']

for ax, img, title, cmap in zip(axs, images, titles, cmaps):
    ax.imshow(img, cmap=cmap, interpolation='nearest')
    ax.set_title(title, fontsize=11, pad=10)
    ax.axis('off')
plt.tight_layout()
plt.savefig("ghost_imaging_bias_simulation_labeled.png", dpi=300)
plt.close()
```

D.7 Implications for Entanglement

In an entangled system, shared paths in w space may yield mutual bias effects. If particles traverse paths that correlate via $\phi(x)$, then measurements will show correlation beyond standard quantum predictions.

Potential observables include:

* Phase drift in Bell-type experiments * Image distortion in ghost imaging as a function of ϵ * Correlation decay length changes in interferometers

D.8 Conclusion

This toy model demonstrates how biased path integrals can yield meaningful changes in probability amplitudes. It bridges the abstract theory with possible experimental effects and sets the stage for both more realistic simulations and targeted empirical tests.

E CHSH Bias Simulation

Listing 2: Simulating CHSH shift under bias

```
import numpy as np
import matplotlib.pyplot as plt

# Measurement angles
theta_a = 0
theta_ap = np.pi / 2
theta_b = np.pi / 4
theta_bp = -np.pi / 4

# Correlation function with bias
def E_epsilon(theta1, theta2, epsilon):
    delta = theta1 - theta2
    return -np.cos(delta) + epsilon * np.sin(2 * delta)

# CHSH computation
def compute_CHSH_S(epsilon):
    E1 = E_epsilon(theta_a, theta_b, epsilon)
    E2 = E_epsilon(theta_a, theta_bp, epsilon)
    E3 = E_epsilon(theta_ap, theta_b, epsilon)
    E4 = E_epsilon(theta_ap, theta_bp, epsilon)
    return abs(E1 + E2 + E3 - E4)

# Sweep over epsilon
epsilons = np.linspace(-0.5, 0.5, 200)
S_values = [compute_CHSH_S(eps) for eps in epsilons]

# Plot
plt.figure(figsize=(10, 6))
```

```
plt.plot(epsilons, S_values, label="S(  )")
plt.axhline(2, color='red', linestyle='--', label='Classical Bound')
plt.axhline(2 * np.sqrt(2), color='green', linestyle='--', label='
   Quantum Limit')
plt.xlabel("Bias Strength   ")
plt.ylabel("CHSH Value S")
plt.title("CHSH Inequality Shift Under Hyperdimensional Bias")
plt.grid(True)
plt.legend()
plt.tight_layout()
plt.show()
```

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