Special Relativity as an Emergent Structure in a Timeless Euclidean Model

A. N. Smirnov

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Abstract

A model is considered in which the observable spacetime structure emerges from a real scalar field satisfying the Laplace equation in a four-dimensional Euclidean space without distinguished time or directions. The observer is described as a localized configuration of the same field on the hypersurfaces of a foliation; *events* are defined as local detector activations specified by a functional of a finite number of mode-decomposition coefficients of the field and the observer's parameters. It is shown that the choice of foliation gives rise to inertial reference frames, and that a consistent reconstruction under transitions between them is possible without introducing a global set of events—solely on the basis of the observer's operational description.

The model implies that the event structures of different inertial frames may differ, so that no global event space exists. It is proven that within the model it is impossible to transmit information about an event absent in a given frame but present in another. This leads to the distinction between two types of transformations. The first, *direct transformations*, describe the actual rearrangement of the event structure under a change of the inertial frame. The second, *observable transformations*, represent the operational re-description performed by an observer within their own frame, based on a hypothetical assumption of a global event set.

The invariance of all foliations, resulting from the full $O(4)$ symmetry of the Laplace equation, together with the justified existence of a finite maximal propagation speed $v_{\max}$, leads to observable transformations of Lorentz type with invariant $v_{\max}$. Thus, both postulates of special relativity are reproduced, and the causal structure emerges as a cone $\|\Delta\mathbf{r}\| = v_{\max}|\Delta t|$ within each frame. The results demonstrate that special relativity can emerge within a strictly Euclidean, timeless model.

1 Introduction

1.1 The Problem of Time and Causality

In modern fundamental physics, three interrelated questions remain open: (1) is it possible to formulate a physical theory in the complete *absence of time*; (2) how can causality and measurement be defined in such a context; (3) and what is the origin of the observable pseudo-Riemannian signature of spacetime $(-, +, +, +)$, which appears to emerge from an ostensibly more natural Euclidean symmetry. These questions become especially significant in the search for a unified theory combining quantum theory and gravitation, since many arguments [1] indicate that, in such a theory, time should not be a fundamental quantity but an emergent concept. Closely related to these issues is the question of the nature of the observer. In standard formulations, the observer is treated as an abstract external agent that does not affect the question of existence itself, merely measuring what objectively exists independently of observation.

Many existing approaches (causal sets [2, 3], loop quantum gravity [4], relational frameworks [5], and others) eliminate explicit time at the level of equations but retain it in a hidden form—through a partial order, an evolution parameter, a logical structure of "histories," or functional dependencies between states. Even in Euclidean formalisms such as the path integral, Wick rotation presupposes a return to time as a physical coordinate.

In contrast, the present model is formulated in a four-dimensional Euclidean space $\mathbb{E}^4$, where not only coordinate time but any fundamental structure defining order, evolution, or direction is absent. The field $\Phi(x)$, the only underlying field in the model, satisfies the Laplace equation and is specified as a single static configuration. All structures traditionally associated with time and causality in standard theories are not imposed a priori but are defined operationally through the interaction of the observer with the field. The observer here is not treated as an abstract external agent but as a physically realized configuration within the same field (see Section 3).

In standard quantum field theory, causality is postulated in terms of light cones and hypersurfaces of constant time. Such a structure requires the prior specification of time and the Minkowski metric and loses its justification if time is not fundamental but arises as an operational consequence. In problems related to gravitation, spacetime reconstruction, and timeless formulations of quantum theory, such postulates cease to be universally applicable.

In both special and general relativity, the Lorentzian structure of spacetime is taken as a starting assumption. In special relativity it follows from Einstein's two postulates—the equivalence of inertial frames and the existence of a finite invariant speed—which together lead to the Minkowski metric. In general relativity, the Lorentzian signature is assumed a priori in constructing the Einstein equations. However, the signature theorem [6] forbids a global transformation from a positive-definite to a pseudo-Riemannian form. Therefore, if a Lorentzian structure emerges within a Euclidean model, it can only have an *effective and local character*. This point is particularly important for approaches in which spacetime and dynamics are considered emergent phenomena.

Many existing frameworks likewise attempt to remove time from fundamental theory. In *causal set theory*, time persists as a partial order on events, effectively defining an oriented causal structure. In *loop quantum gravity* and the *spin foam* formalism, evolution is realized through transitions between boundary states, and the time axis is introduced only as an external interpretational parameter. In the *Page–Wootters mechanism* [7] and *relational quantum mechanics* [5], time is defined through quantum correlations between subsystems, though the existence of a pre-established Hilbert space of states and a measurement act is still assumed.

In *timeless* approaches, such as Barbour's model [8], the time parameter is abandoned, yet a configuration space or spatiotemporal relations are retained, allowing the recovery of dynamics. In *QBism* [9] and *observer-centric QFT* [10], the subject is introduced as an external interpretive structure rather than as a physically realized body within the same theory. In all these cases, some hidden form of time is retained, or the observer is treated as external; see also the comparative analysis in [11].

The proposed model differs radically: it eliminates not only coordinate time but any internal order or evolution parameter, while modeling the observer as a localized configuration of the same field. All causal relations, dynamics, and events arise *operationally*—from the interaction with the field on a chosen foliation—rather than being postulated from the outset.

In this work, operationality is understood as the definition of physical structures solely on the basis of the field's interactions with a localized observer, ***recorded by the observer themselves*** *(i.e., leaving a trace in their internal degrees of freedom), without invoking external time, predefined coordinate dynamics, or an a priori metric. This interpretation is close to the standard operational approach in physics [12, 13, 14], but it is radicalized by including the observer as an internal part of the model and by rejecting any presupposed time. This makes it possible to formalize causality, measurement, and observable transformations (see §3, §5) as internal structures of the model.*

In this sense, the model provides a rigorous realization of a timeless formalism with an internal observer, from which special relativity (SR) can be derived. Modeling the observer as a physical part of the field configuration makes it possible to reproduce phenomena inaccessible in frameworks with an external observer—from the reconstruction of causality to the emergence of special relativity.

1.2 Euclidean Models and the Role of the Observer

Euclidean methods have proven to be technically powerful tools, but in all known cases they are treated as auxiliary constructions—with an inevitable return to time after Wick rotation. Attempts to construct fully Euclidean models with direct physical meaning face several difficulties: there is no mechanism for the emergence of causality, the structure of events and its connection to observation remain unclear, and neither Lorentz transformations nor a finite invariant speed have been derived.

Moreover, in such approaches the observer is either absent or introduced *post factum* as an external agent. In the present work, we examine the possibility of describing the observer as a physical configuration arising within the same model—through a localized decomposition of the field (see Section 3). Such an observer interacts with the field $\Phi(x)$ and, by choosing a foliation, defines the quantities that acquire physical meaning, including the structure of events, their sequence, dynamics, and causal connectivity.

This approach allows both causality and measurement to be treated not as external postulates but as operational structures emerging within the model and dependent on the observer. At the same time, the model itself remains formally Euclidean, without introducing time or any predefined dynamics.

1.3 Purpose and Scope of the Work

The main objective of this study is to introduce *timeless minimal models* into the discourse of theoretical physics. We consider a model based on a four-dimensional Euclidean space endowed with a real scalar field that possesses neither preferred directions nor additional internal symmetries. The model is used as a minimally sufficient framework to demonstrate that a timeless construction can be consistent with the observable aspects of known physics. The goal is not to construct a complete physical theory; rather, we restrict ourselves to analyzing the key structural consequences of the model.

Scope and Limitations. We deliberately *do not consider* tools belonging to subsequent developments of the mathematical apparatus of special relativity and quantum field theory: Wick rotation, the Osterwalder–Schrader conditions, path integrals, the construction of a full unitary dynamics, and so on. Nor do we aim to derive the complete Lorentz group; for the purposes of this paper, it is sufficient to show that the *observable* transformations between inertial frames (IFRs) have the Lorentz form with an invariant $v_{\max}$, and that both postulates of special relativity arise operationally within the model.

By *observable transformations*, we intuitively refer to the reparametrization rules for operational coordinates and recorded observables constructed by an *observer-physicist* who remains within their own inertial frame (IFR). In contrast to *direct transformations*, which describe the actual re-expansion of the field and the observer's body when changing the foliation (and which may alter the set of reconstructible events), observable transformations reflect only how the observer-physicist *mentally reconstructs a description* in another foliation, based on the available information and on the hypothetical assumption of a global event space. A rigorous definition and derivation of their form are given in §5–§6.2.

To achieve the stated goal, we show that within a strictly Euclidean model governed by the Laplace equation it is possible to:

- formalize causality as a local operational structure, independently defined in each inertial frame (IFR);
- derive both postulates of special relativity;
- obtain Lorentz-type transformations as *observable* transformations between IFRs.

The construction is based on a scalar field $\Phi(x)$ in $\mathbb{E}^4$, satisfying

$$\Delta_{\mathbb{E}^4}\Phi(x) = 0.$$

The field $\Phi(x)$ is not treated as an object for which explicit solutions of the field equation are sought, but rather as a generalized configuration satisfying this equation in the weak (functional) sense. We are not concerned with explicit solutions per se, but with the physical consequences of the imposed operational constraints—specifically, the derivation of the transformations of special relativity and the structures that emerge from interaction with a localized observer.

The field contains no fundamental dynamics or temporal parameters and has no additional internal symmetries beyond the Euclidean group $O(4)$. Interaction with a localized observer (through foliation and modal reconstruction) allows one to operationally construct events, evolution, and the structure of inertial frames.

Main Results.

- Causality is formalized as a local operational structure independently defined in each IFR.
- It is proven that within an IFR it is impossible to transmit information about events absent from its own event structure; this establishes the informational isolation of IFRs.
- Two classes of transformations are distinguished: *direct* (re-expansion under foliation change without event bijection) and *observable* (conceptual reparametrization preserving the event structure).
- It is established that *observable transformations by construction preserve event structure* between IFRs, in contrast to direct transformations, which may alter the set of reconstructible events.
- It is shown that the observer's operational record (information) is not absolute: under a change of IFR, events may disappear or appear, while reconstruction remains consistent with the causal structure within each IFR (§5).
- Both postulates of special relativity are reproduced operationally for the observable transformations:
  - equivalence of all IFRs (invariance of the form of physical laws);
  - existence of a finite limiting speed of causal influence $v_{\max}$, common to all observers.
- *observable* transformations between IFRs are obtained in the Lorentz form with invariant $v_{\max}$.

Structure of the Paper. Sections 2–3 introduce the formulation of the model and the definition of the observer. Section 4 constructs inertial frames and defines relative velocity. Section 5 analyzes the types of transformations under changes of IFRs (direct and observable) and provides a kinematical derivation of the postulates of special relativity. Section 6 derives Lorentz-type transformations as observable transformations. Section 7 discusses the observer's operational information and its reconfiguration. Section 8 addresses the model's limitations, and Section 9 presents the conclusions and perspectives.

2 Fundamental Formulation

2.1 Euclidean Space $\mathbb{E}^4$

The model is based on a four-dimensional real Euclidean space $\mathbb{E}^4$, equipped with the standard metric δ_{AB} of signature $(+,+,+,+)$, where the Latin indices $A, B = 1, \ldots, 4$. This space contains no distinguished directions, coordinates, time axes, or causal structure. The geometry of $\mathbb{E}^4$ is invariant under the full orthogonal group $O(4)$, providing the maximal possible symmetry without introducing any additional structures.

The choice of four dimensions is motivated by the fact that the minimal dimension allowing for the reconstruction of the observable spacetime structure of special relativity coincides with the dimensionality of physical spacetime itself. Thus, $\mathbb{E}^4$ serves as the most natural candidate for reconstructing relativistic kinematics from a timeless model.

Any hyperplane in $\mathbb{E}^4$ is defined by the equation

$$n_A x^A = s,$$

where n_A is a unit normal vector and s is a real parameter. Such hyperplanes will play the role of foliations used by the observer for the operational definition of events. A formal introduction of the foliation and its properties will be given in the following section.

2.2 The Underlying Field of the Model and the Laplace Equation

We require a field equation that satisfies the following conditions: absence of preferred directions, Euclidean $O(4)$ symmetry, absence of internal symmetries, and smoothness of solutions. The Laplacian is the only second-order differential operator that is fully invariant under the group $O(4)$ and does not single out any direction. Therefore, the Laplace equation is not merely a convenient example but rather the unique choice that ensures maximal symmetry and minimal assumptions.

The subsequent constructions rely only on its structural properties—locality, linearity, and harmonicity—so that any equation sharing these features would lead to the same operational consequences.

On $\mathbb{E}^4$, we introduce a real scalar field $\Phi(x)$ satisfying

$$\Delta_{\mathbb{E}^4} \Phi(x) = 0, \tag{1}$$

where $\Delta_{\mathbb{E}^4} = \delta^{AB} \partial_A \partial_B$ is the Laplacian of the Euclidean space.

This equation contains no distinguished time, prescribes no internal dynamics, includes no internal symmetries or preferred directions, and involves no interactions—neither linear nor nonlinear. The solution $\Phi(x)$ is assumed to be fixed uniquely, including its boundary conditions. This reflects the fact that, in a timeless model, independent initial conditions cannot be specified: the entire content of the model is determined by a single field configuration, without invoking any notion of evolution.

2.3 Absence of Time and Causal Structure

No additional structures are introduced in the model that would define a direction of evolution, an order of events, or dynamical variables. This implies the complete absence of time—both as a coordinate and as an evolution parameter. The field $\Phi(x)$ is treated as a configuration on $\mathbb{E}^4$, determined by the Laplace equation and the boundary conditions, without invoking any internal dynamics.

Thus, neither the field nor the space $\mathbb{E}^4$ possesses causal relations at the fundamental level. The notions of causality and sequence of events will be introduced not as primitive entities but as emergent structures that arise only at the level of operational description. In particular, the role of the observer as the source of consistent causal reconstruction will be formalized in the next section.

2.4 Purpose of the Construction

The aim of this section is to formalize the minimal setting on which the subsequent operational description will be defined. At the fundamental level, the model does not incorporate any physical quantities, events, symmetries, or equations of motion. Everything that can be interpreted as spacetime, matter, or dynamics must emerge solely as a result of the interaction between a localized field structure and its global configuration. Such a localized structure will later be formalized as an *observer*.

3 Observer and the Operational Definition of Events

3.1 Foliation and Direction of Propagation

A foliation of the Euclidean space $\mathbb{E}^4$, defined by the hyperplanes $n_A x^A = s$ (see (2)), divides the space into a family of three-dimensional hypersurfaces

$\Sigma_s^{(\mathbf{n})}$, orthogonal to a chosen vector n_A and parameterized by the real scalar s. The choice of direction n_A fixes an inertial reference frame (IFR), and the parameter s in this frame plays the role of *operational time*—an internal parameter of evolution that arises only relative to the given foliation.

Each hyperplane is interpreted as a "moment of time" in the corresponding inertial reference frame. It will later be shown that different orientations of the normal vector $\mathbf{n}$ correspond to different IFRs. Here $\mathbf{n}$ is the vector, n_A its components, $n^A := \delta^{AB} n_B$, and $n_A n^A = 1$.

A fixed orientation of n_A determines the direction of *propagation* between slices, while the choice of foliation defines the structure of local temporal ordering. Thus, the direction of time in the model is not predefined but arises operationally: it is determined relative to the chosen orientation of hyperplanes, which is associated with the reference frame of the observer.

Definition (Causal Reconstruction). For a fixed observer O and a chosen foliation $\Sigma_s^{(\mathbf{n})}$ with normal $\mathbf{n}$, a *causal reconstruction* is defined as a procedure that, from the local information about the field Φ in a region $\Omega_O \subset \Sigma^3$, constructs a consistent description of the set of events E_O and their ordering $\preceq_O$ relative to the direction of propagation $\mathbf{n}$. The specific mechanism for identifying events will be given below (see Subsection 3.4).

Causal Reconstruction Requirement. Each operational IFR is associated with a choice of foliation $\Sigma_s^{(\mathbf{n})}$ and normal vector $\mathbf{n}$. For the reconstruction of events in a given IFR to be consistent with the principle of causality, the following conditions must hold:

- **(i)** The decomposition of the field $\Phi(x)$ into modes of the given foliation is constructed so that the individual modes u_α are localized within the hypersurface Σ^3 and admit local propagation along $\mathbf{n}$ with respect to the parameter s (a linear transformation of the coefficients $a_\alpha(s)$, see (8) below) with an effective speed v bounded above by a universal quantity $v_{\max}$[1].
- **(ii)** The equation satisfied by the field $\Phi(x)$ admits such local modes and preserves their evolutionary consistency along any direction $\mathbf{n}$.
- **(iii)** The transition between nearby IFRs (small rotations of the foliation) does not destroy the consistent event structure: as $\theta \to 0$, where $\theta = \arccos(\mathbf{n}\cdot\mathbf{n}')$, the sets of events in $\mathbf{n}$ and $\mathbf{n}'$ coincide identically.

[1] $v_{\max}$ denotes the limiting speed of interaction in the reconstructed spacetime; its universality for all foliations follows from consistency conditions, while its finiteness will be justified in Section 6.

These conditions are operational in nature: they follow from the requirement of reproducibility of the event structure and consistency between foliations. Causality is not postulated *a priori* but emerges as a condition for admissible reconstruction in the presence of a constraint on the speed of interaction transfer.

It should be noted that conditions (i)–(iii) are formulated for an idealized decomposition over the entire hypersurface Σ^3_s. In the operational sense, it is sufficient for an observer to perform a decomposition localized in a compact region $\Omega \subset \Sigma^3_s$, which will be introduced and justified in the following subsection 3.2.

Admissibility of Decompositions and Configurations. Not every solution of (1) allows for causal reconstruction. We consider the subset of solutions $\mathcal{S} \subset \ker \Delta$ that admit decomposition into the modes of the foliation Σ^3 and satisfy:

(a) localizability of the modes u_α within the region of the hypersurface Σ^3;

(b) the possibility of an event-based interpretation of the interactions between the field modes and the modes of the observer's body;

(c) preservation of consistent reconstruction under a small rotation of $\mathbf{n}$.

Thus, the admissibility of a field configuration is determined not only by the satisfaction of the equation itself but also by the operational realizability of a modal decomposition possessing a causal structure.

3.2 Localization of the Observer

The observer is not an external agent but is described as a localized structure in $\mathbb{E}^4$ within the field configuration $\Phi(x)$. A foliation is fixed as

$$\Sigma^3_s = \{\, x \in \mathbb{E}^4 \mid n_A x^A = s \,\}, \qquad s \in \mathbb{R}, \tag{2}$$

where $\mathbf{n}$ denotes the corresponding unit vector, n_A its components, and the measure d^3x on Σ^3_s is induced by the Euclidean metric. A local region $\Omega \subset \Sigma^3_s$, compact in the three spatial directions, is also fixed; this region defines the working domain for the modal decomposition of the field. It is within this region that the operationally accessible description of events is formed (in the physical sense, it may be regarded as an analogue of the observable part of the universe). For a particular observer's body, a smaller region $\Omega_O \subset \Omega$ is used.

Field Projections and Terminology. On Σ^3_s, an orthonormal set of functions $\{u_\alpha(x)\}_{\alpha\in\Lambda} \subset L^2(\Sigma^3_s)$ localized in Ω is chosen (for any α, β: $\int_{\Sigma^3_s} u_\alpha u_\beta \, d^3x = \delta_{\alpha\beta}$). The *field modes* on a slice are understood as the elements of this fixed basis. The local configuration of the field is decomposed over this basis:

$$\Phi(x)\big|_\Omega = \sum_{\alpha\in\Lambda} a_\alpha(s)\, u_\alpha(x), \tag{3}$$

where the coefficients are given by

$$a_\alpha(s) = \int_{\Sigma^3_s} u_\alpha(x)\, \Phi(x)\, d^3x = \int_\Omega u_\alpha(x)\, \Phi(x)\, d^3x, \tag{4}$$

since $\operatorname{supp} u_\alpha \subset \Omega$, and the measure d^3x is induced by the Euclidean metric on $\Sigma^{(\mathbf{n})}_s$. To make the orientation of the foliation explicit, we may write, when necessary, $a^{(\mathbf{n})}_\alpha(s)$. The vector $\mathbf{a}(s) = (a_\alpha(s))$ will be called the *local representation of the field in the modal basis*; it introduces no new degrees of freedom relative to Φ.

The Observer's Body and Internal Modes. The physical substrate of the observer O is defined by an *orthonormal*, localized set of its *internal modes* (sensitivity profiles) $\{\chi_\beta(x)\}_{\beta\in\Lambda_{\text{obs}}} \subset L^2(\Sigma^3_s)$, the subspace $\mathcal{H}_{\text{obs}} := \operatorname{span}\{\chi_\beta\}$, and the coordinates $\mathbf{b}(s) = (b_\beta(s))$ on the same slice. In general, the internal modes do not coincide with the field-mode basis but can be represented as local linear combinations of the field modes (see the next paragraph); this guarantees the comparability of descriptions by different observers. The detector sensitivity matrix $\rho^{(O)}$ relates $(\mathbf{a}, \mathbf{b})$ and is used below for the operational definition of an event (see (12)).

Common Reference Basis and Inter-Observer Consistency within an IFR. If two observers within the same foliation $\Sigma^{(\mathbf{n})}_s$ use incompatible bases, then the observation of one observer by another becomes ambiguous, and a direct comparison of results loses definiteness. Therefore, in each IFR a common *reference* orthonormal basis $\{u_\alpha\}_{\alpha\in\Lambda} \subset L^2(\Sigma^{(\mathbf{n})}_s)$ is fixed, identical for all observers on the given slice. The internal modes of any observer are expressed as local linear combinations of the elements of this basis:

$$\chi_\beta(x) = \sum_{\alpha\in\Lambda} C^{(O)}_{\beta\alpha}\, u_\alpha(x), \qquad b_\beta(s) = \sum_{\alpha\in\Lambda} C^{(O)}_{\beta\alpha}\, a_\alpha(s), \tag{5}$$

where the matrix $C^{(O)}$ has local support (within Ω_O). Thus, all operationally observable quantities are expressed through the same set of coefficients $a_\alpha(s)$, ensuring inter-observer consistency within a given IFR.

Operational Definition of the Body. The *body of the observer* is defined as the tuple $(\Omega_O, \{\chi_\beta\}_{\beta\in\Lambda_{\mathrm{obs}}}, \rho^{(O)}, I^{(O)}_{\mathrm{thr}})$, where $\operatorname{supp}\chi_\beta \subset \Omega_O$, and an event is registered according to the threshold criterion (11)–(12). Perturbations of the field that are nearly orthogonal to $\operatorname{span}\{\chi_\beta\}$ or produce a signal below $I^{(O)}_{\mathrm{thr}}$ are operationally not recorded.

Operational Records (Information). Records are implemented as distinguished registers $\mathbf{m}(s) \subset \mathbf{b}(s)$ of the internal state of the body. The operationally accessible information of the observer on a slice is the pair

$$\mathbf{I}^{(\mathbf{n})}(s) := \big(\mathbf{a}^{(\mathbf{n})}(s),\, \mathbf{m}(s)\big). \tag{6}$$

When the foliation changes, the observer's informational state and records may be reconfigured. The corresponding transformations are discussed in detail in Section 5.

3.3 Invariance of the Transfer Operator and the Emergence of Causality

For any foliation direction n_A, we introduce the coefficients

$$a^{(n)}_\alpha(s) = \int_\Omega u_\alpha(x)\, \Phi(x)\, d^3x, \qquad \alpha \in \Lambda, \tag{7}$$

where $\{u_\alpha\}$ is the reference basis of functions localized within the working region $\Omega \subset \Sigma^{(n)}_s$.

From the requirements of operational reconstruction within a given IFR, it follows that there exists a local linear transfer relation

$$a^{(n)}_\alpha(s{+}ds) = \sum_{\beta\in\Lambda} A^{(n)}_{\alpha\beta}[\Phi;\, s]\; a^{(n)}_\beta(s), \tag{8}$$

where the indices $\alpha, \beta \in \Lambda$ run over the set of modes of the reference basis $\{u_\alpha\}$. The matrix $A^{(n)}_{\alpha\beta}[\Phi;\, s]$ depends only on the *local* configuration of the field Φ in the neighborhood of the observation region on the slice $\Sigma^{(n)}_s$. For the transition to the internal modes of an observer O, one uses the coefficients $b_\beta(s)$ and the matrices $C^{(O)}$ that connect the subsets $\Lambda_O \subset \Lambda$ (see Section 3.2).

The consistency requirement of reconstruction under a small rotation of the foliation (see condition (iii) in the causal reconstruction requirements) implies that one and the same rule of basis construction is used for all hyperplanes, rather than introducing a new basis for each slice. That is, the basis

$\{u_\alpha\}$ is chosen from the admissible class of orthonormal functions that ensure localizability, event interpretability, and continuity under foliation rotation. Otherwise, as $\theta \to 0$, where $\theta = \arccos(\mathbf{n}\cdot\mathbf{n}')$, inconsistent transformations would appear, violating the continuity of event reconstruction.

Hence, the $O(4)$ rotational invariance reduces to the equality of the components of the transfer matrices:

$$A^{(\mathbf{n}')}_{\alpha\beta}[\Phi;\,\cdot] \;\equiv\; A^{(\mathbf{n})}_{\alpha\beta}[\Phi;\,\cdot]. \tag{9}$$

This condition is formulated *solely* for the coefficients $a^{(n)}_\alpha$ and does not assume the existence of a global correspondence of events between IFRs.

Remark 1 (On the Interpretation of the Invariance of the Transfer Operator)**.** *When changing foliations, the rotation* $\mathbf{n}\to\mathbf{n}'$ *is understood not as a coordinate transformation within a fixed basis, but as a replacement of the entire family of hyperplanes* $\Sigma^{(\mathbf{n})}_s \to \Sigma^{(\mathbf{n}')}_s$*. The basis* $\{u_\alpha\}$ *on each hyperplane is determined by the same construction rule* $\mathcal{U}[\Phi;\mathbf{n},s]$*, which depends only on the local properties of the field and the direction of the normal vector, and* does not transform by rotation. *A rotation of the foliation changes the arguments of the functions, but not the functions themselves, which are reconstructed anew for the new foliation by the same method. Therefore, the transfer operators* $A^{(\mathbf{n})}_{\alpha\beta}[\Phi;\cdot]$ *and* $A^{(\mathbf{n}')}_{\alpha\beta}[\Phi;\cdot]$ *coincide identically, expressing the* invariance*—rather than covariance—of the transfer law under a change of foliation.*

The invariance of the transfer law follows from the $O(4)$ symmetry of the field Φ in combination with the fixed rule for constructing the reference basis $\mathcal{U}[\Phi;\mathbf{n},s]$.

It should be noted that, for the purposes of this paper, it is sufficient to take any basis satisfying the above requirements. The construction of a complete theory may require imposing additional constraints.

Emergence of Causality. Thus, the principle of causality in each IFR is realized as a condition of operational consistency: the observer can reconstruct a causal structure if and only if there exists a corresponding local field decomposition and an invariant transfer relation (8)–(9). Despite the absence of fundamental time, the temporal direction and causal structure emerge as operational entities, determined by the choice of foliation and the admissibility of reconstruction.

3.4 Definition of an Event

Before defining what constitutes an event in the model, we first note several issues inherent in such a definition and list the assumptions needed for it.

Real detectors, spacecraft, etc., are confined to scales no larger than the Solar System—negligible compared to the size of the observable universe. Using this analogy, within a single IFR we may treat the working region Ω as the same for all observers at rest. Hence, in each IFR we assume that the observable region Ω—the region within which causal reconstruction is possible—is identical for all observers at rest with respect to that IFR.

In contemporary physical theory, and in contrast to classical physics, quantum physics lacks a single, widely accepted, and fully adequate definition of an *event* that is independent of the observer or the measurement process. Several leading interpretations of quantum mechanics exist, each defining "event" in its own way.

Within the present model it is clear that all observable phenomena depend on the observer and on measurement by the observer. Since our aim is not to construct a complete theory, we must adopt certain simplifications that preserve the properties of the model most relevant to our purposes.

In formulating the notion of an *event* we impose several requirements.

(i) Operational origin. An event should arise as a result of interaction between the observer and the field, rather than as a pre-given ontological entity: an event is defined as a configuration of interaction between the field modes and the observer's internal modes that leads to a discrete update of the observer's internal state (a record in the registers), as registered by the observer. The observer records only a local measurement outcome but, on that basis, reconstructs a network of causes and events that (according to the reconstruction) led to the measurement. This network is always a reconstruction, not something that exists independently. Thus, an event may be interpreted as having occurred far outside the observer's body. Analogously, when detecting a photon from a star, the observer operationally reconstructs a network of events that led to the emission of that photon.

(ii) Consistency across scales. Because the observer has finite extent and spectral limitations, interaction with the field is described by projection onto a finite-dimensional subspace. The definition of an event must be stable under coarse-graining/refinement of modal decompositions (i.e., independent of the chosen "resolution").

(iii) Sufficiency for the goals of the paper. The definition should be schematic (without constructing a full theory of measurement) yet retain the properties needed to derive a limiting speed and the Lorentz form of the observable transformations.

(iv) Classical regime. For simplicity we consider a regime in which the sets of events coincide (up to isomorphism of partially ordered sets) for all observers at rest in a given IFR (see below). This does not yield a global event space unifying different IFRs. We call this the classical regime because, in classical physics, event sets are the same for all observers.

After making an observation, the observer forms a hypothesis about what was observed. The primary open point in the model is what events are tied to. In quantum physics, events are associated with particles arising from gauge symmetries. In the present model gauge symmetries are not obtained and, given our goals, need not be. We shall therefore say that when the observer performs a measurement, they observe the consequences of certain events occurring within the region Ω—the region in which they can reconstruct causal relations. We now adopt the following assumption: the model contains some mathematical construction playing the role of an analogue of particles in quantum physics to which events are tied. The search for this construction is not the goal of this paper. Then one may say that a causal network arises upon observation. In view of the classical regime, we avoid observer-dependent events and pass to events common to observers within the same IFR. Thus, an analogue of a causal set arises, but specific to each IFR.

As will be shown later, the model admits a consistent reconstruction of event structure under transitions between IFRs (different foliation directions). Although events are defined relative to a specific observer and their IFR, agreement between IFRs is ensured by compatibility of reconstructions; the corresponding *observable* transformations turn out to be Lorentzian. As noted earlier, the model implies that a transition between IFRs is described not by one but by two types of transformations. The justification appears below.

While defining an event via equations in the model is a difficult task requiring a full theory, defining a measurement act is simpler. A measurement should lead to a change in the observer's information within their body, depending on the decompositions of the field and the observer's body.

Definition of a Measurement. Let observer O fix a foliation Σ_s and a subspace

$$\mathcal{H}^{(O)}_{\text{field}} = \text{span}\{u_\alpha : \alpha \in \Lambda_O\} \subset L^2(\Sigma_s),$$

where $\Lambda_O \subset \Lambda$, together with a set of their internal modes $\{\chi_\beta\}$. Their *detector (readout) functional* is a local scalar functional

$$\mathcal{R}_O(s) \;=\; F_O\big(\mathbf{a}(s), \mathbf{b}(s)\big), \tag{10}$$

where $\mathbf{a}(s) = (a_\alpha(s))_{\alpha\in\Lambda_O}$ and $\mathbf{b}(s) = (b_\beta(s))$ are the coefficients of the corresponding decompositions on Σ_s.

A *measurement act* $\mathsf{M}_O(s_0)$ is the instant s_0 at which

$$\mathcal{R}_O(s_0) \;\geq\; I_{\rm thr}^{(O)}, \tag{11}$$

where $I_{\rm thr}^{(O)} > 0$ is the sensitivity threshold. When this condition holds, one of the binary memory registers m_j flips discretely $0 \to 1$. The measurement act is localized within Ω_O.

Remark 2. *Refinements of the criterion are possible (e.g., extremum conditions or smoothing), but for the purposes of this paper the threshold condition* (11) *suffices.*

This functional can be regarded as analogous to a photodiode: if a combination of signals exceeds the threshold, the detector clicks, registering a one in memory.

Example (bilinear functional). As a special case of (10) one may use the bilinear form

$$\mathcal{M}_O(s) = \sum_{\alpha,\beta} \rho_{\alpha\beta}^{(O)}\, a_\alpha(s)\, b_\beta(s), \tag{12}$$

where $\rho^{(O)}$ is the sensitivity matrix; then $\mathcal{R}_O = \mathcal{M}_O$.

Stability under Coarse-Graining. Upon passing to a coarser description (grouping modes into effective combinations), the functional $\mathcal{R}_O$ is rewritten in terms of the new coefficients, while the criterion (11) is preserved. Thus, the definition of a measurement does not depend on the level of detail.

Separation of Notions. We distinguish between a *local measurement act* (a detector click within the observer's body Ω_O) and an *event* as an element of the causal network in the working region Ω ($\Omega_O \subset \Omega$).

Definition 1 (Event in the Model)**.** *An* event *is a vertex $E \in V_{\rm obs} \cup V_{\rm rec}$ of the causal network $\mathcal{C}_{\mathbf{n}} = (V, \prec)$ in the working region Ω, where:*

- $V_{\rm obs} = \{\mathsf{M}_O(s_k)\} \subset \Omega_O$ *are observable (local) vertices generated by measurement acts;*
- $V_{\rm rec} \subset \Omega$ *are reconstructed vertices for which there exists an operational causal relation to at least one $\mathsf{M}_O(s_k)$, consistent with the admissible action of the transfer operator* (8)*.*

The order $\prec$ is interpreted as "can influence," is defined within a given IFR, and does not require a global event set.

Note. The measurement act $\mathsf{M}_O(s)$ is a *local trigger* of recording (vertices V_{obs}), while an "event" in the broader sense is an element of the network $\mathcal{C}_{\mathbf{n}}$, including both local measurements and reconstructed vertices V_{rec}. Thus, events outside the observer's body are not identified with measurements but enter as reconstructed elements obtained from measurement data and reconstruction rules.

As an example: an observer may register a photon (V_{obs}) and then reconstruct the event of its emission by a distant star (V_{rec}).

Causal Network. The elements of $\mathcal{C}_{\mathbf{n}}$ are not confined to the observer's body: they are treated as consequences of interactions within the working region Ω. In each IFR there arises a *network of events*

$$\mathcal{C}_{\mathbf{n}} = \{\mathsf{E}_i\}, \tag{13}$$

equipped with a causal order $\mathsf{E}_i \prec \mathsf{E}_j$ if E_i can operationally influence E_j through the admissible action of the transfer operator (8) at fixed foliation $\Sigma_s^{(\mathbf{n})}$. Local measurement acts $\mathsf{M}_O(s)$ form a subset $V_{\text{obs}} \subset \mathcal{C}_{\mathbf{n}}$, and reconstructed vertices V_{rec} complete the network. The observer's memory is realized as an ordered subset $\{m_j\} \subset V_{\text{obs}}$.

Classical Regime. In the approximation we call the classical regime, we assume that for all observers at rest in a given IFR the event sets coincide: $\mathcal{C}_{\mathbf{n}}$ does not depend on the observer. This approximation allows us to ignore differences due to localization and spectral limitations and simplifies the derivation of Lorentz transformations. Under changes of IFR, the event sets may differ, and no global unified event space arises. In what follows we consider only the classical regime, except where explicitly stated otherwise.

Interpretation. Thus, an event is not reduced to an ontological "point in spacetime," but may be interpreted as an element of a discrete causal network arising from the interaction of the field with the observer. Unlike traditional *causal set* models, here each IFR has its own network $\mathcal{C}_{\mathbf{n}}$. Consistency between them is ensured by the compatibility of reconstructions under transitions between IFRs, as discussed later.

Remark 3 (Observer as Part of the Event Structure)**.** *Analogously to approaches used in causal-set-like theories, the observer may be described not* *only via a modal state but also as part of the event structure itself: their configuration selects the subset of events accessible for reconstruction in the given IFR. Unlike standard causal-set approaches, these events depend on the chosen foliation, and when the foliation changes, the subset is reorganized.*

4 Inertial Reference Frames and Relative Velocity

4.1 Foliations as Inertial Frames

In the absence of time and dynamics, each foliation direction in $\mathbb{E}^4$, specified by a unit vector n_A, defines a local event structure arising from the interaction of the observer with the field. Such a structure is completely determined by the choice of hyperplanes $n_A x^A = s$, orthogonal to n_A, and by the operational interpretation of s as an emergent operational time that appears through the observer's interaction with the field.

Within the present model, an *inertial reference frame* (IFR) is understood as a foliation direction n_A with respect to which events, causality, and observable quantities can be consistently defined through the localized decomposition of the field and its interaction with the observer's body. All IFRs in this work are treated in this operational sense.

No IFR is physically distinguished: the model is invariant under the full orthogonal group $O(4)$, and differences between IFRs arise solely from the choice of reconstruction direction. It will later be shown that transitions between foliation directions give rise to consistent transformations of observables that formally coincide with the Lorentz transformations.

Operational Principle of Inertia. Each body in the model is represented as a localized collection of modes of the underlying field, defined on the hyperplanes of a chosen foliation. Its *motion* in a fixed IFR is described as a sequence of events generated by the interaction of the body with the global configuration of the field $\Phi(x)$.

Let $\Sigma_s \equiv \Sigma_s^{(\mathbf{n})}$. If the geometry of the sequence of events along the parameter s changes within the hyperplanes Σ_s—for instance, if a displacement or curvature of the body's trajectory is observed—such behavior is interpreted as *acceleration*. According to the operational approach, acceleration requires a *cause*, i.e., an additional interaction of the body with field modes not belonging to its own subspace (the localized decomposition defining the body). Such an interaction is interpreted as *external* with respect to the body and leads to a deviation of its event trajectory from the inertial one.

Thus, if in an IFR a body maintains uniform and rectilinear motion (in terms of a consistent sequence of events), this signifies the absence of external influence and, therefore, the absence of a cause for change in its behavior. In this sense, causality in the model is realized through deviations from inertiality: every acceleration is operationally associated with an additional interaction.

Consequently, in the absence of external influence, the body's event trajectory remains rectilinear and uniform in the chosen IFR. This corresponds to the operational formulation of the principle of inertia: *if no cause acts upon a body, its reconstructed behavior in a given IFR remains unchanged.* In this way, inertiality is interpreted as the stability of the body's event structure under a fixed field configuration and a given foliation direction.

4.2 Transition Between Inertial Frames and the Definition of Relative Velocity

Let two inertial reference frames (IFRs) be given, corresponding to foliations along the directions n_A and n'_A. These directions are related by an orthogonal transformation of the Euclidean space:

$$n'_A = R_A{}^B n_B, \qquad R \in O(4). \tag{14}$$

It is convenient to introduce orthogonal projectors onto the hyperplanes of the slices:

$$P_{\mathbf{n}} := \nVdash - \mathbf{n} \otimes \mathbf{n}, \qquad P_{\mathbf{n}'} := \nVdash - \mathbf{n}' \otimes \mathbf{n}'.$$

Here $\nVdash$ denotes the identity operator on $\mathbb{E}^4$, and $\otimes$ the tensor product. We also use the notation $\Sigma_s^{(\mathbf{n})} := \{\, x \in \mathbb{E}^4 \mid \mathbf{n}\cdot x = s \,\}$.

For a small displacement $\Delta x = \mathbf{n}\, ds$ in the IFR $\mathbf{n}$, an observer in the IFR $\mathbf{n}'$ perceives a transverse shift in the hyperplane $\Sigma^{(\mathbf{n}')}$ equal to $\Delta \mathbf{r}' = P_{\mathbf{n}'} \Delta x$, with norm $\|\Delta \mathbf{r}'\| = ds\, \sin\theta$, while the increment of the parameter s' in the IFR $\mathbf{n}'$ is $ds' = \mathbf{n}'\cdot \Delta x = ds\, \cos\theta$, where $\cos\theta = \mathbf{n}\cdot\mathbf{n}'$.

Each foliation direction n_A defines a family of hyperplanes $n_A x^A = s$, interpreted as "moments of time" in the corresponding IFR. Events are defined as local interactions within these hyperplanes. However, the projections of the same points $x \in \mathbb{E}^4$ onto the hyperplanes of two different foliations—e.g., $n_A x^A = s$ and $n'_A x^A = s'$—differ. This leads to an operationally detectable displacement of events when performing successive transfers along the direction n_A, if n'_A is tilted with respect to n_A.

Such a discrepancy is naturally interpreted as an *observable relative velocity* between IFRs, defined as the ratio of the transverse displacement to the increment of emergent time in the compared IFR (with a dimensional scaling factor v_t):

$$v(\theta) \;=\; v_t \,\frac{\|\Delta \mathbf{r}'\|}{ds'} \;=\; v_t \,\tan\theta, \tag{15}$$

where v_t is a scaling constant that fixes the correspondence between the transfer parameter and the unit of observable time (to be specified in Section 6). In the limit $\theta \to 0$, we have $v \to 0$, whereas for $\theta \to \frac{\pi}{2}$ the velocity formally diverges, consistent with the absence of a global event space in the model.

Thus, the relative velocity between IFRs is determined solely by the angle between their foliations. In the limit $\theta \to 0$, the corresponding spacetimes become locally consistent.

4.3 Corollary: The Multiplicity of Spacetimes

Since the foliations n_A and n'_A lead to different decompositions of the field and therefore to different sets of events, each direction n_A defines its own *spacetime* with its own causal structure, understood in the model as the causal network $\mathcal{C}_{\mathbf{n}}$ with its order relation $\prec$. There exists no single consistent mapping between these spacetimes at the level of events. The model contains no global event space: only local projections exist, specific to each IFR.

Strictly speaking, there is no bijection between $\mathcal{C}_{\mathbf{n}}$ and $\mathcal{C}_{\mathbf{n}'}$ (see also Appendix A); only local event reconstructions that remain consistent in the limit $\theta \to 0$ are comparable.

The observable velocity arises as the relative discrepancy of events between these spacetimes under perpendicular displacements of the hyperplanes. In the limit $\theta \to 0$, the spacetimes become locally consistent, a property that will be used in deriving the observable transformations formally equivalent to the Lorentz transformations.

5 Observable Transformations and the Postulates of Special Relativity

5.1 Generalized Principle of Causality

In its classical formulation, causality relies on a global event space. In the present model, no such global space is postulated; causality is defined *operationally* within each individual IFR (see §3). As a result, the model implies a generalization of the principle of causality.

This generalization consists in abandoning the implicit postulate of a global event space and explicitly introducing an *internal* observer as part of the model. We use the notation of §3: $\mathcal{C}_{\mathbf{n}}$ denotes the operationally reconstructed set of events for the foliation with normal $\mathbf{n}$, and $\mathbf{a}^{(\mathbf{n})}(s) = (a_\alpha^{(\mathbf{n})}(s))$ represents the *local representation of the field in the modal basis*, evolving according to the transfer relation (8). The angle $\theta(\mathbf{n}, \mathbf{n}')$ denotes the angle between the normals, with $\cos\theta = \mathbf{n}\cdot\mathbf{n}'$.

Generalized Principle of Causality (as a Consequence of the Model).

(G1) **Locality by IFR.** Causality is defined and applied *independently* within each IFR $\mathbf{n}$, without appeal to a global event space.

(G2) **Consistency under small rotations.** As $\theta(\mathbf{n}, \mathbf{n}') \to 0$, the reconstructions of events become consistent: $\mathcal{C}_{\mathbf{n}} \triangle \mathcal{C}_{\mathbf{n}'} \to \varnothing$, where $A \triangle B$ denotes the symmetric difference of sets; see §3, condition (iii).

Remark. (G1) follows directly from the operational definition of events and the informational state on each foliation, while (G2) follows from the requirement of reconstruction consistency under small rotations (§3, condition (iii)) and from the use of the classical regime for events.

As follows from the above, the principle of causality applies separately and independently to each IFR, allowing for possible differences in causal relations between IFRs (see also Proposition 1 below).

5.2 Principle of Causality and the Observer

Recall that we consider events in the classical regime, invoking properties of the full model when necessary. As established earlier, each IFR corresponds to its own set of events (13) with causal relations among them. The observer, together with all the information available to them, forms a part of this set.

In the classical regime, the observer operates directly with the elements of $\mathcal{C}_{\mathbf{n}}$ corresponding to their foliation. In the general case, such events are specified through functionals of the local representation of the field in the modal basis $\mathbf{a}^{(\mathbf{n})}(s)$, which evolves according to (8). Any signal originating from systems that are external to the observer but internal to the model likewise manifests itself as a change in these functionals, provided that causality is preserved (see §5.1).

theorem 1 (Inaccessibility of Non-Reconstructible Events)**.** *Let an observer O be situated in an inertial reference frame (IFR) $\mathbf{n}$ and operate with the set of events $\mathcal{C}_{\mathbf{n}}$ reconstructed within their foliation $\{\Sigma_s^{(\mathbf{n})}\}$. Then no finite* *composition of admissible local operations defined within the IFR* **n** *can render operationally accessible any event that does not belong to the set* $\mathcal{C}_{\mathbf{n}}$.

Proof. We work in the classical regime, where the event set $\mathcal{C}_{\mathbf{n}}$ is fixed for all observers of a given IFR (see §3.4). Consider three classes of possible observer actions:

1. *Local transfer.* The evolution of the coefficients $a_\alpha^{(\mathbf{n})}(s)$ obeys the transfer law (8), which acts in the coefficient space and is local on $\Sigma_s^{(\mathbf{n})}$. It cannot generate an event outside the current network $\mathcal{C}_{\mathbf{n}}$, since its action is confined to the admissible reconstruction domain and does not alter the set of events but only reorganizes their internal relations.

2. *Composition of admissible local operations.* Any finite composition of such operations—functionals of $\mathbf{a}^{(\mathbf{n})}(s)$ and $\mathbf{b}^{(O,\mathbf{n})}(s)$, both local on $\Sigma_s^{(\mathbf{n})}$— remains within the algebra of observables in the IFR **n**. These operations do not extend the domain of definition of the network $\mathcal{C}_{\mathbf{n}}$ and do not create new vertices outside it.

3. *Rotation of the foliation.* The transition to another IFR $\mathbf{n}' = R\mathbf{n}$ is described by a direct transformation $D_{\mathbf{n}\to\mathbf{n}'}$ (see (16)), which replaces the entire network $\mathcal{C}_{\mathbf{n}}$ by a new one, $\mathcal{C}_{\mathbf{n}'}$. Since a change of foliation is not an operation admissible within the IFR **n**, and no bijection exists between $\mathcal{C}_{\mathbf{n}}$ and $\mathcal{C}_{\mathbf{n}'}$ (see Appendix A), an event $E' \in \mathcal{C}_{\mathbf{n}'} \backslash \mathcal{C}_{\mathbf{n}}$ cannot be mapped into $\mathcal{C}_{\mathbf{n}}$ by any finite sequence of operations within **n**.

Therefore, an event $E \notin \mathcal{C}_{\mathbf{n}}$ cannot be operationally reconstructed or registered by an observer in the IFR **n** through any finite sequence of local actions permitted by the transfer law and the conditions of causal reconstruction (§3). □

It follows that an observer cannot operationally confirm the existence of events absent from their current IFR—neither through signal exchange with observers in other IFRs, nor through subsequent transition to another IFR.

This forms the foundation for a new class of transformations—the *observable transformations*—which, as will be shown below, are Lorentz-like.

5.3 Two Types of Transformations

In the absence of a global event space, the transition between different inertial reference frames (IFRs), corresponding to the foliation directions n_A and n'_A, can be interpreted in two distinct ways.

Formal Definitions.

- **Direct transformations** describe the action of the Euclidean symmetry on the field configuration and the direction of foliation. The transition from IFR $\mathbf{n}$ to IFR $\mathbf{n}'$ entails replacing the slice $\Sigma_s^{(\mathbf{n})}$ with $\Sigma_{s'}^{(\mathbf{n}')}$ and, consequently, reconstructing a new set of events. Since no global event set exists, there is no bijection between $\mathcal{C}_\mathbf{n}$ and $\mathcal{C}_{\mathbf{n}'}$: direct transformations merely associate each slice with a new set of events in the rotated foliation.

  For direct transformations we write

  $$D_{\mathbf{n}\to\mathbf{n}'} : \mathcal{C}_\mathbf{n} \longmapsto \mathcal{C}_{\mathbf{n}'}, \tag{16}$$

  which corresponds to replacing the family of slices $\Sigma_s^{(\mathbf{n})} \mapsto \Sigma_{s'}^{(\mathbf{n}')}$ and subsequently reconstructing the event set $\mathcal{C}_{\mathbf{n}'}$.

- **Observable transformations** describe a hypothetical change of IFR from the perspective of a fixed observer who remains within their own system. The observer has no access to events absent from their own network $\mathcal{C}_\mathbf{n}$ (Proposition 1) and is therefore forced to proceed under the assumption of a global event space. This assumption allows them to construct transformations that, by definition, preserve eventhood. In the general case, without invoking the simplification of the classical regime, these transformations depend on the observer O and are represented by operators

  $$\mathcal{O}^{(O)}_{\mathbf{n}\to\mathbf{n}'} : \mathbf{b}^{(O,\mathbf{n})}(s) \mapsto \mathbf{b}^{(O,\mathbf{n}')}(s), \tag{17}$$

  acting on the observer's state subspace $\mathbb{B}_O$. In the *classical regime* (see §3.4), where the event set $\mathcal{C}_\mathbf{n}$ is identical for all observers within a given IFR, the dependence on O disappears, and the observable transformations reduce to a universal operator

  $$M_{\mathbf{n}\to\mathbf{n}'} : (t, \mathbf{r}) \mapsto (t', \mathbf{r}'). \tag{18}$$

  These transformations, by construction, preserve eventhood and are precisely those used below in deriving the Lorentz-like transformations (see §6.2).

Intuitive Description.

- **Observable transformations** are re-descriptions performed by an observer within their own IFR. Relying on the hypothetical assumption of a global event space, the observer interprets the results as if the same events were preserved when transitioning between IFRs. Observable transformations are not associated with an actual change of the observer's IFR.

- **Direct transformations**, in contrast, describe the action of the Euclidean symmetry on the field and the foliation. Upon transition to another IFR, the event set $\mathcal{C}_{\mathbf{n}}$ is replaced by $\mathcal{C}_{\mathbf{n}'}$. In this transition, some previously reconstructed events may leave the network, while new ones may appear, so that the observer's informational state changes.

Novelty of the Approach. The key element leading to the emergence of two types of transformations is the impossibility of transmitting within a given IFR information about an event that does not exist in this IFR but exists in another. In the standard formulations of special and general relativity, such a distinction is not made: coordinate transformations are simultaneously treated as both direct (actual) and observable. In the present model, the separation between *direct* and *observable* transformations constitutes a novel feature: it arises as a consequence of the informational isolation of IFRs and the absence of a global event space. In this sense, observable transformations can be viewed as the interpretation of a transition between IFRs from the observer's standpoint, whereas direct transformations describe the action of the symmetry on the field configuration.

5.4 Invariance of the Operational Law of Interaction

Earlier, the invariance of the local transfer law for the field coefficients $a_\alpha^{(\mathbf{n})}$ was established (see (9)), expressing the uniformity of the laws of physics across all IFRs. However, the observable quantities themselves depend on the choice of functionals and the parameters of the observer's body, and it is not *a priori* evident that their dynamics will also be invariant. It is therefore necessary to demonstrate that the invariance extends to the level of *operational* quantities—namely, to the observable functionals and operators $\mathbf{b}^{(O)}$. Thus, it is established that not only the equations for the fundamental field but also the laws of interaction, as registered by the observer, have the same form in all IFRs.

theorem 1 (Invariance of the Operational Law)**.** *Let* $A_{\alpha\beta}^{(\mathbf{n})}[\Phi; s]$ *be defined by the transfer relation* (8) *for an admissible basis satisfying condition (iii).*

Then, for any $\mathbf{n}, \mathbf{n}'$ *and all* s*, the following holds:*

$$A^{(\mathbf{n})}_{\alpha\beta}[\Phi; s] \equiv A^{(\mathbf{n}')}_{\alpha\beta}[\Phi; s],$$

that is, the local operational law of interaction/transfer is invariant under rotations of the foliation. If the detector is specified by a functional $F_O(\mathbf{a}, \mathbf{b})$ *that is local on* Σ_s*, then the observable operator*

$$\mathcal{O}^{(O)}_{\mathbf{n}\to\mathbf{n}'} : \ \mathbf{b}^{(O,\mathbf{n})}(s) \mapsto \mathbf{b}^{(O,\mathbf{n}')}(s) \tag{19}$$

is likewise defined invariantly, without reference to data in $\mathbf{n}'$*.*

Proof. The $O(4)$-invariance of the Laplace equation and condition (iii) ensure the same admissible class of bases under rotations, which yields (9). The locality of F_O and the invariance of (9) imply the well-definedness of the operator (19). □

This corresponds to the *first postulate of special relativity* in its operational formulation:

> The laws of physics have the same operational form in all inertial reference frames.

5.5 Constraint on the Limiting Velocity

Within the present model, causality is defined as the operationally consistent reconstruction of events within a single inertial reference frame (IFR). From this definition follows a constraint on the maximum speed at which an observable causal influence can propagate without violating the consistency of the reconstruction. This condition is not a direct consequence of the Laplace equation but is introduced as a necessary requirement for a self-consistent description of the observable history. It thereby excludes those formally admissible solutions of the Laplace equation that do not yield a coherent causal structure. The necessity of a limiting velocity may thus be regarded as a condition for the very possibility of an operational description of eventhood by the observer.

If such a maximal velocity $v_{\max}$ exists, then the $O(4)$ symmetry of the scalar field implies that its value is the same in all IFRs. It is important to emphasize that $v_{\max}$ constrains the speed of causal connections *only within a single IFR*. It does not impose any restriction on the relative velocities $v(\theta)$ between different IFRs, which are determined by the angle between the foliation directions. Values of $v(\theta)$ exceeding $v_{\max}$ do not lead to contradictions; they merely indicate that the event networks in the respective IFRs differ substantially and cannot be directly reconciled. For an observer, such velocities have no operational meaning, since they lie beyond the observer's own event structure. In other words, $v_{\max}$ must not be confused with the relative velocity $v(\theta)$ between IFRs introduced earlier.

For instance, when the foliations of two IFRs are oriented perpendicularly, the discrepancies that arise in their *direct* comparison of configurations are unbounded. However, in the *observable* description, events are by definition considered preserved under transitions between IFRs, while causal relations within each IFR are constrained by $v_{\max}$; it is this limitation that possesses operational significance.

The maximal velocity $v_{\max}$ is determined by the structure of the observer's modes and by the constraints on the consistent projection of the field configuration onto the chosen foliation. It is of a strictly operational nature: an observer cannot interpret two events as causally related if their reconstruction would require exceeding $v_{\max}$ within their own coordinate structure.

In principle, the quantity $v_{\max}$ can be computed from the field equation and the structure of the observer's admissible modes. However, doing so requires the construction of a complete theory, which lies beyond the scope of the present work.

In the following section (§6), $v_{\max}$ will be related to the scaling parameter of temporal normalization, which is determined by the consistency of reconstructions under small transitions between IFRs. Since such consistency is possible only for a finite value of this parameter, it will be shown that $v_{\max}$ must likewise be finite.

Thus, $v_{\max}$ represents the intrinsic limit of operational causality within a given IFR. It is independent of the choice of coordinates, registration procedures, or other observers, and it corresponds to the *second postulate of special relativity* in its operational formulation:

> There exists a limiting velocity $v_{\max}$, identical in all IFRs, which constrains causal connections within each IFR and ensures the consistency of the operational reconstruction of events.

6 Derivation of Lorentz Transformations from the Operational Structure

6.1 Constraints on the Class of Reconstructions

This section considers cases of operational reconstruction satisfying the following conditions:

- The direction of operational time is defined by a unit vector $\mathbf{n} \in \mathbb{E}^4$, $\|\mathbf{n}\| = 1$;
- The operational event space is identified with the hyperplane $\Sigma^{(\mathbf{n})} := \{ x \in \mathbb{E}^4 \mid \mathbf{n}{\cdot}x = \text{const} \}$ orthogonal to $\mathbf{n}$, with the actual reconstruction confined to the working region $\Omega \subset \Sigma^{(\mathbf{n})}$;
- The metric on $\Sigma^{(\mathbf{n})}$ is taken to be the induced Euclidean one; the distance in the slice between $\mathbf{r}_1, \mathbf{r}_2 \in \Sigma^{(\mathbf{n})}$ is $\lambda = \|\mathbf{r}_1 - \mathbf{r}_2\|$;
- The simplified classical regime from the definition of an event is considered, i.e., the event set $\mathcal{C}_\mathbf{n}$ in any IFR is the same for all observers at rest with respect to that IFR.

Remark on the Constraint of Causal Connections. Within the model, the reconstruction of events must preserve causal consistency. This requires that two spatially separated events at a distance λ in $\Sigma^{(\mathbf{n})}$ can be regarded as causally connected only if

$$t \;\geq\; \frac{\lambda}{v_{\max}}\,,$$

where t is the operational time between the events (see (20)), and $v_{\max}$ is the limiting velocity of causal interaction in the given IFR. This restriction is not postulated but arises as a consequence of the requirement of operational consistency of reconstruction (see also the consistency condition under small rotations, §5.1).

The invariance of the quantity $v_{\max}$ in all admissible reconstructions follows from the full $O(4)$ symmetry of the Laplace equation and from the *invariance of the coefficient transfer rule* (see (8)): since all directions in $\mathbb{E}^4$ are physically equivalent, the limiting velocity of interactions defined within a foliation cannot depend on the orientation of the hyperplane.

This fundamental property is complemented by the requirement of continuity for direct transformations: at $\theta = 0$, the event sets coincide identically, $\mathcal{C}_\mathbf{n} = \mathcal{C}_{\mathbf{n}'}$, and as $\theta \to 0$, the symmetric difference tends to the empty set,

$$\mathcal{C}_\mathbf{n} \triangle \mathcal{C}_{\mathbf{n}'} \;\to\; \varnothing.$$

In the observable description, this corresponds to the limit $v \to 0$ and the operational indistinguishability of reconstructions (see §5.1).

6.2 Operational Derivation of Lorentz Transformations

As shown earlier, the model gives rise to two types of transformations. The first type, *direct transformations*, describes how events transform "in reality" when transitioning between IFRs. In other words, direct transformations represent the actual restructuring of the event set when passing from one IFR to another. Information about which events exist in one IFR and which in another could, in principle, be obtained only if information about events absent in one IFR but present in another could be transmitted between them. However, as shown previously (1), such transmission is impossible. In this model, there thus exists a form of informational isolation between IFRs. All information accessible to an observer is confined to the information available within their own IFR—the IFR in which the observer is at rest.

The second type of transformations, *observable transformations*, describes a hypothetical change of IFR. Under such transformations, no actual change of IFR occurs. The observer-physicist, based on the information available within their IFR, constructs transformations that describe a *hypothetical* transition to another IFR—i.e., what, according to their data, would occur in other IFRs. Hence, by construction, observable transformations preserve eventhood under the hypothetical change of IFR. It should be emphasized that this is an *operational* preservation of eventhood (the observer assumes that events are the same), rather than an actual coincidence of event sets.

The distinction between direct and observable transformations is nontrivial. The key property of the model leading to this separation is precisely the impossibility of transmitting information about events that do not belong to the observer's IFR but exist in another.

Observable transformations between IFRs will be denoted as $M_{\mathbf{n}\to\mathbf{n}'} : (t, \mathbf{r}) \mapsto (t', \mathbf{r}')$ (see (18)). These operators describe a hypothetical change of foliation from the direction $\mathbf{n}$ to $\mathbf{n}'$ as interpreted by an observer who remains within their own IFR. By construction, $M_{\mathbf{n}\to\mathbf{n}'}$ acts linearly on the coordinates $(t, \mathbf{r})$ obtained from the normalization (20).

To construct the observable transformations, we explicitly state several properties that have already been established earlier.

(I) Preservation of Eventhood (by definition). Observable transformations are constructed such that, for a fixed observer in the IFR $\mathbf{n}$, they preserve the outcomes of all admissible registration and processing procedures. In other words, an event reconstructed in $\mathcal{C}_{\mathbf{n}}$ remains the same event under a hypothetical transition to $\mathbf{n}'$. This condition is not introduced as an additional postulate but reflects the operational construction of the observable transformations themselves.

(R) Regularity of the Transformation Family. For the observable transformations $M_{\mathbf{n}\to\mathbf{n}'}$, the following properties hold:

(i) **Identity at $v = 0$:** if the directions coincide, $\mathbf{n} = \mathbf{n}'$, then $M_{\mathbf{n}\to\mathbf{n}} = \mathbb{1}$;

(ii) $M_{\mathbf{n}\to\mathbf{n}'}$ depends continuously and differentiably on the transition parameter. In the model, this parameter is given by the angle between the directions $\mathbf{n}$ and $\mathbf{n}'$, and in the observable description it is equivalent to the relative velocity v. The derivative with respect to this parameter is also continuous;

(iii) **Compositionality:** $M_{\mathbf{n}\to\mathbf{n}''} = M_{\mathbf{n}'\to\mathbf{n}''} \circ M_{\mathbf{n}\to\mathbf{n}'}$.

These properties follow directly from the operational construction and from the linearity of the transfer rule; they are not introduced as independent postulates.

Operational Time Normalization. Consider two events $x_1, x_2 \in \mathbb{E}^4$ lying on the slices $\Sigma^{(\mathbf{n})}_{s_1}$ and $\Sigma^{(\mathbf{n})}_{s_2}$, respectively. Let $\Delta x := x_2 - x_1$ and define the projection onto the normal $\mathbf{n}$:

$$t \;:=\; \frac{\ell}{v_t}, \qquad \ell \;:=\; \mathbf{n}\cdot\Delta x = s_2 - s_1, \tag{20}$$

where $v_t > 0$ is a scaling parameter (later identified with $v_{\max}$), and the spatial component is given by $\mathbf{r} := P_{\mathbf{n}}\Delta x$, with $P_{\mathbf{n}} := \mathbb{1} - \mathbf{n}\otimes\mathbf{n}$.

Theorem 1 (Lorentz-Like Form of Observable Transformations)**.** *Let conditions (I) and (R) hold, and let there exist an invariant velocity $v_{\max} \in (0, \infty]$ identical in all IFRs. Then, for any fixed direction $\hat{\mathbf{u}} \subset \Sigma^{(\mathbf{n})}$, the family of observable transitions parameterized by the relative velocity v along $\hat{\mathbf{u}}$ forms a one-parameter group of linear transformations $(t, \mathbf{r}) \mapsto (t', \mathbf{r}')$, having one of the following forms:*

(i) If $v_{\max} < \infty$, the quadratic form

$$Q(t, \mathbf{r}) := v_{\max}^2\, t^2 - \|\mathbf{r}\|^2, \qquad Q(t', \mathbf{r}') = Q(t, \mathbf{r}), \tag{21}$$

is preserved, and the transformation takes the form

$$\gamma(v) = \frac{1}{\sqrt{1 - \frac{v^2}{v_{\max}^2}}}, \qquad \begin{aligned} t' &= \gamma\left(t - \frac{v}{v_{\max}^2}\, r_\parallel\right), \\ r'_\parallel &= \gamma\,(r_\parallel - vt), \\ \mathbf{r}'_\perp &= \mathbf{r}_\perp, \end{aligned} \tag{22}$$

that is, the observable transformations have a Lorentz form with invariant $v_{\max}$*;*

(ii) If $v_{\max} = \infty$*, the null cone degenerates, and the limiting form is the Galilean transformation:* $t' = t$*,* $r'_{\parallel} = r_{\parallel} - vt$*,* $\mathbf{r}'_{\perp} = \mathbf{r}_{\perp}$*. It will later be shown that this case is not realizable within the model.*

These conditions are standard for deriving the Lorentz (or limiting) transformations; therefore, we present only a sketch of the proof.

Sketch of the Proof. Conditions (R) ensure linearity and a block structure with respect to the decomposition $\mathbf{r} = r_{\parallel}\hat{\mathbf{u}} + \mathbf{r}_{\perp}$: mixing occurs only in the $(t, r_{\parallel})$ plane, while $\mathbf{r}_{\perp}$ transforms orthogonally (reflecting the homogeneity and isotropy of the slice).

Condition (I), together with the existence of an invariant velocity $v_{\max}$, implies the preservation of the null cone $\|\mathbf{r}\| = v_{\max}|t|$. For $v_{\max} < \infty$, this yields the preservation of two independent null directions $r_{\parallel} = \pm v_{\max} t$; the linear transformation preserving them has the Lorentz form (22). If $v_{\max} = \infty$, the cone degenerates, and the Galilean limit remains.

Compositionality and the condition $M(0) = \mathbb{1}$ (from (R)) exclude nontrivial multiplicative factors, so the form (21) is preserved exactly. □

6.3 Time Normalization and the Exclusion of the Galilean Branch

As established in Theorem 1, observable transformations admit two possible forms: the Lorentzian branch (i) for a finite invariant speed $v_{\max} < \infty$, and the Galilean branch (ii) for $v_{\max} = \infty$. It will now be shown that, within the present model, only the first of these is admissible.

Small-Velocity Limit. We employ the previously introduced normalization of the temporal coordinate through the scaling parameter v_t (see (20)). When comparing two foliations defined by the directions $\mathbf{n}$ and $\mathbf{n}'$, the angle θ between them is related to the observable relative velocity v by

$$\tan\theta = \frac{v}{v_t}. \tag{23}$$

For small angles, where $\tan\theta \approx \theta$, this yields

$$\theta \approx \frac{v}{v_t}. \tag{24}$$

Since all eventhood is formed in projections onto the chosen foliation, consistency requires that in the limit $v \to 0$ (i.e., for infinitesimal rotation) the observable outcomes of any fixed registration procedure coincide when described in IFRs $\mathbf{n}$ and $\mathbf{n}'$ (see condition (iii) in §5.1). This is possible if and only if the null cones in both systems are tangent to each other as $\theta \to 0$. If $v_t < v_{\max}$, then for small θ admissible causal links are excluded (some events incorrectly fall outside the cone); if $v_t > v_{\max}$, forbidden links are instead admitted. In both cases, an operationally observable inconsistency arises. The only consistent case is the coincidence of the temporal scaling parameter with the maximal admissible speed of causal interaction:

$$v_t = v_{\max}. \tag{25}$$

When considering an infinitesimal rotation, we rely on the properties of direct transformations following from the requirements of causal reconstruction; the obtained result refines the properties of observable transformations.

Physical Interpretation. If $v_t < v_{\max}$, time would be "stretched," and the observer would exclude admissible causal links. If $v_t > v_{\max}$, links incompatible with reconstructions in other IFRs would be admitted. In both cases, even an infinitesimal rotation of the foliation would lead to an operationally observable inconsistency.

Nondegeneracy of the Time Scale.

theorem 2 (Nondegeneracy $\Rightarrow v_t < \infty$)**.** *If $v_t = \infty$, the temporal scale becomes degenerate: for any finite shift along the normal $\ell = \mathbf{n}\cdot\Delta x$, one has $t = \ell/v_t \equiv 0$. Moreover, from* (23) *it follows that for any finite v the angle between the foliations is $\theta = \arctan(v/v_t) = 0$, so that distinct IFRs become indistinguishable. Finally, the invariant Q from* (21) *degenerates to $Q = -\|\mathbf{r}\|^2$: the null cone disappears, and causal classification of events becomes impossible. This contradicts the requirement of regularity at the identity transformation. Hence, $v_t < \infty$.*

Proof. By reduction to absurdity: for $v_t = \infty$ one simultaneously obtains $t \equiv 0$, $\theta = 0$, and degeneracy of Q; each of these consequences contradicts the regularity conditions and the existence of a nonzero causal cone. □

Corollary. From Proposition 2 it follows that $v_t < \infty$. Together with (25), this yields $v_{\max} = v_t < \infty$, so that in Theorem 1 only branch (i) is realized. Thus, the Galilean limit ($v_{\max} \to \infty$) is excluded, and the observable transformations take the Lorentz-like form (22).

6.4 Conclusion

Thus, within the considered class of reconstructions and under the conditions (I) and (R) (see Theorem 1), the observable transformations of Lorentz-like form are obtained. Both postulates of special relativity—the equivalence of inertial reference frames and the invariance of a finite limiting speed—arise here not as axioms but as consequences of operational reconstruction.

This reconstruction rests on three foundations:

- the full $O(4)$ symmetry of the underlying field (the Laplace equation);
- the operational definition of events through the observer and their informational state;
- the application of causality separately within each IFR and the impossibility of accessing events absent from the given IFR.

Once the equality $v_t = v_{\max}$ is established (see (25)), excluding the Galilean limit, the resulting observable transformations formally coincide with the Lorentz transformations under the substitution $c \mapsto v_{\max}$. They describe the observable event structure within each IFR as an *emergent* one, without introducing *a priori* a Minkowski metric, a fundamental temporal coordinate, or a global event space. In this way, special relativity is reproduced as an operationally consistent structure within a timeless model based on the Laplace equation (cf. [15]).

It should also be noted that the complete form of the direct transformations within this model cannot be obtained without constructing an extended theory; however, certain of their properties can be derived from the requirements of causal reconstruction.

7 Operational Information and Consistent Reconstruction Under a Change of IFR

In this section we consider the *classical regime* (see §3.4), in which the set of events $\mathcal{C}_{\mathbf{n}}$ in each inertial reference frame (IFR) is fixed and identical for all observers at rest with respect to that IFR. This simplification eliminates the dependence of the event structure on the observer's localization and the differences in working regions Ω, while retaining a sufficient operational foundation for analyzing reconstruction under a change of foliation.

7.1 Classical Regime and Event Structure

In the classical regime, the observer's information reduces to access to the set of events $\mathcal{C}_{\mathbf{n}}$ of their own IFR. Under a *direct* transition to another IFR, causal order remains consistent within each IFR, but identical identification of events between distinct IFRs is not required and occurs only in the limit of small foliation rotations.

7.2 Reconfiguration of the Event Structure Under Foliation Change

When transitioning from an IFR with normal $\mathbf{n}$ to one with normal $\mathbf{n}'$, the global field $\Phi(x)$ remains unchanged, but the set of events is reconfigured: some elements of $\mathcal{C}_{\mathbf{n}}$ disappear, while new ones appear in $\mathcal{C}_{\mathbf{n}'}$. This reconfiguration is described by a *direct* transformation (see (16)) and remains consistent with causality within each IFR (see §5.1). Intuitively:

- *Partial disappearance:* events that had operational meaning in $\mathbf{n}$ may no longer satisfy the reconstruction conditions in $\mathbf{n}'$;
- *Emergence:* new events may appear in $\mathbf{n}'$ that were absent in $\mathbf{n}$.

In contrast, the *observable* transformations $M_{\mathbf{n}\to\mathbf{n}'}$, constructed by the observer under a *hypothetical* change of IFR based on the assumption of a global event set, by definition preserve eventhood between IFRs and do not require a bijection between $\mathcal{C}_{\mathbf{n}}$ and $\mathcal{C}_{\mathbf{n}'}$ (cf. Proposition 1).

7.3 Conclusion

The model contains no global event set: in each IFR only its own structure $\mathcal{C}_{\mathbf{n}}$ is accessible, and no information can be obtained about events absent from that IFR (Proposition 1). The distinction between *direct* transformations $D_{\mathbf{n}\to\mathbf{n}'}$ (the reconfiguration of $\mathcal{C}$ under an actual change of IFR) and *observable* transformations $M_{\mathbf{n}\to\mathbf{n}'}$ (a hypothetical reinterpretation preserving eventhood) serves as the starting point for the derivation of Lorentz-like observable transformations in §6.

8 Limitations and Discussion

8.1 Finite Informational Capacity and Reconfiguration of Event Records

In the general case, beyond the approximation of the classical regime, the local representation of the field with respect to a chosen foliation is given by the coefficients $\mathbf{a}^{(\mathbf{n})}(s) = (a_\alpha^{(\mathbf{n})}(s))$ defined on the slices $\Sigma_s^{(\mathbf{n})}$ (see §3, (8), (9)). When transitioning between IFRs (i.e., changing the foliation $\mathbf{n} \to \mathbf{n}'$), the induced set of modes on the hyperplane $\{u_\alpha^{(\mathbf{n}')}\}$ as well as the corresponding coefficients $a_\alpha^{(\mathbf{n}')}$ change. Note that the global basis in $\mathbb{E}^4$ is fixed, and for different hyperplanes the sets $\{u_\alpha^{(\mathbf{n})}\}$ differ only by the rotation of the hyperplane. As a result, according to the direct transformation $D_{\mathbf{n}\to\mathbf{n}'}[\Phi]$ (16), the *operational record of events*—that is, the collection of detector readouts defined through functionals of these coefficients—is reconfigured as well.

In particular:

- **Partial disappearance:** combinations of modes that produced nonzero values of detector functionals in the $\mathbf{n}$-IFR may no longer satisfy the event conditions in the $\mathbf{n}'$-IFR;
- **Emergence:** new admissible events may arise in $\mathbf{n}'$ that were absent in $\mathbf{n}$.

These effects are a direct consequence of the finite spectral support of the observer (i.e., their finite informational capacity). When exchanging information, observers within the same IFR (using the common basis $\{u_\alpha^{(\mathbf{n})}\}$) synchronize their records and arrive at the same event structure $\mathcal{C}_\mathbf{n}$. Thus, operational consistency within an IFR is preserved despite possible differences in local records.

Remark on Different Bases. If one allows observers with distinct bases of modal decompositions, their event structures cannot be made fully consistent even through information exchange. Such a case lies beyond the scope of the present model and may be interpreted as the existence of several parallel reconstructions ("parallel universes"). In this paper, we restrict ourselves to the case of a single common basis, which ensures internal consistency within each IFR.

9 Conclusion and Outlook

In this work, we have examined a model in which the fundamental structure is a real scalar field $\Phi(x)$ satisfying the Laplace equation in four-dimensional Euclidean space $\mathbb{E}^4$, without time, privileged directions, or fundamental dynamics. The Laplace equation was employed in a functional sense: we did not seek explicit solutions but instead considered the subset of admissible configurations satisfying the imposed operational constraints. From the analysis of the observer's interaction with this field, the following main results were obtained:

- It was shown that the observer's foliation of space and the modal decomposition of the field generate a structure that can be interpreted as an inertial reference frame (IFR), possessing its own event structure, causality, and inertia (§3, §5).
- Two types of transformations between IFRs were identified: *direct* transformations (relating global field configurations) and *observable* transformations (constructed by the observer based on the assumption of a global event space).
- It was proven that an observer within a given IFR cannot obtain information about events absent from their own event structure; this establishes the distinction between direct and observable transformations (Proposition 1).
- Both postulates of special relativity were operationally reproduced:
  - the equivalence of all IFRs as observational foliations (invariance of the form of physical laws);
  - the existence of a finite limiting speed of causal interaction $v_{\max}$, identical for all observers.
- Observable transformations between IFRs were derived, which by construction preserve eventhood from the observer's perspective; it was shown that they take the Lorentz form with invariant $v_{\max}$ (Theorem 1, §6), while the Galilean limit is excluded as incompatible with the operational consistency of the temporal scale.
- It was established that the observer's information (the operational record of events) in this model is not absolute: when transitioning between IFRs, events may disappear or appear, while reconstruction remains consistent with the causal structure within each IFR (§7).

Thus, from purely Euclidean geometry and the Laplace equation, an observable spacetime structure of Minkowski type and a consistent causal order emerge. The results demonstrate that models lacking fundamental time can be made strictly consistent with the observable structures of spacetime and can be coherently incorporated into the modern theoretical-physics discourse as both non-contradictory and promising. This opens the possibility for further investigation of timeless models in a broader context, including the reconstruction of metric, dynamics, and interactions from geometric and operational foundations.

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A Absence of Bijection Between Single Slices

Consider two foliation directions in $\mathbb{E}^4$, defined by unit vectors $\mathbf{n}$ and $\mathbf{n}'$, and the corresponding level hyperplanes

$$\Sigma_s^{(\mathbf{n})} := \{x \in \mathbb{E}^4 \mid \mathbf{n}\cdot x = s\}, \qquad \Sigma_{s'}^{(\mathbf{n}')} := \{x \in \mathbb{E}^4 \mid \mathbf{n}'\cdot x = s'\}.$$

Let $\Phi(x)$ be a fixed solution of the Laplace equation $\Delta_{\mathbb{E}^4}\Phi = 0$. Its restrictions to $\Sigma_s^{(\mathbf{n})}$ and $\Sigma_{s'}^{(\mathbf{n}')}$ can be expanded over orthonormal bases defined on the respective slices (see (3)):

$$\Phi\big|_{\Sigma_s^{(\mathbf{n})}} = \sum_\alpha a_\alpha^{(\mathbf{n})}(s)\, u_\alpha^{(\mathbf{n})}, \qquad \Phi\big|_{\Sigma_{s'}^{(\mathbf{n}')}} = \sum_\beta a_\beta^{(\mathbf{n}')}(s')\, u_\beta^{(\mathbf{n}')}.$$

Here, $a_\alpha^{(\mathbf{n})}(s)$ and $a_\beta^{(\mathbf{n}')}(s')$ are the expansion coefficients, and $\{u_\alpha^{(\mathbf{n})}\}$, $\{u_\beta^{(\mathbf{n}')}\}$ are orthonormal bases in $L^2(\Sigma_s^{(\mathbf{n})})$ and $L^2(\Sigma_{s'}^{(\mathbf{n}')})$, respectively, induced by the Euclidean metric. Note that the basis functions are defined globally in $\mathbb{E}^4$ and coincide on all hyperplanes up to a rotation of the foliation; thus, the choice of foliation corresponds merely to a reorientation of the basis.

theorem 3 (Absence of Bijection Between Single Slices)**.** *If* $\mathbf{n} \neq \mathbf{n}'$*, then, in general, no bijective rule exists that expresses the set of coefficients* $\{a_\beta^{(\mathbf{n}')}(s')\}_\beta$

solely *in terms of the set* $\{a_\alpha^{(\mathbf{n})}(s)\}_\alpha$ *(for fixed* s, s'*) without knowledge of the full configuration* Φ *in the neighborhood of the corresponding layers. In particular, the mapping* $\{a_\alpha^{(\mathbf{n})}(s)\} \mapsto \{a_\beta^{(\mathbf{n}')}(s')\}$ *within natural classes of solutions is neither injective nor surjective.*

Proof. Non-injectivity. There exist at least two distinct harmonic functions that coincide on $\Sigma_s^{(\mathbf{n})}$ but yield different restrictions on $\Sigma_{s'}^{(\mathbf{n}')}$. For example, let $\Phi_1 \equiv 0$ and $\Phi_2(x) := \ell(x) := \mathbf{n}\cdot(x - x_0)$ with $\mathbf{n}\cdot x_0 = s$. Then ℓ is harmonic (being linear) and satisfies $\ell\big|_{\Sigma_s^{(\mathbf{n})}} = 0$, so the sets $\{a_\alpha^{(\mathbf{n})}(s)\}$ coincide for Φ_1 and Φ_2. However, on the rotated slice $\Sigma_{s'}^{(\mathbf{n}')}$, the trace of ℓ is generally nonzero, and $\{a_\beta^{(\mathbf{n}')}(s')\}$ differ.

Non-surjectivity. For a given set $\{a_\alpha^{(\mathbf{n})}(s)\}$, not every set $\{a_\beta^{(\mathbf{n}')}(s')\}$ can be realized by a harmonic Φ: it must belong to the image of the operator mapping "restriction to $\Sigma_s^{(\mathbf{n})}$" $\to$ "restriction to $\Sigma_{s'}^{(\mathbf{n}')}$". This operator is defined by solving an elliptic problem in the region between the slices and is inherently nonlocal. It imposes integral consistency conditions, meaning that arbitrary $\{a_\beta^{(\mathbf{n}')}(s')\}$ cannot be achieved for fixed $\{a_\alpha^{(\mathbf{n})}(s)\}$. □

Corollary. Between the descriptions on individual slices corresponding to different foliations, no bijective correspondence exists in terms of the coefficients of the instantaneous expansion. Therefore, *direct* transformations $D_{\mathbf{n}\to\mathbf{n}'}$, which relate the sets of events under a change of foliation, generally do not define a one-to-one correspondence between events. This emphasizes the necessity of distinguishing between the two types of transformations: direct transformations describe the reconfiguration of the event set under a change of IFR, whereas observable transformations $M_{\mathbf{n}\to\mathbf{n}'}$ act on the operationally accessible state and, by construction, preserve eventhood (see §5, (18)).