QUANTITATIVE THEORETICAL EVALUATION OF THE TIME QUANTUM VALUE BASED ON THE HEIZENBERG UNCERTAINTY PRINCIPLE AND THE ANALYTIC MODEL OF NONSTATIONARY STOCHASTIC PROCESSES

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Annotation. *Based on the clear physically interpreted variant of the Heizenberg uncertainty relation connecting the uncertainties for impulse and a space coordinate of any material object, the other variant related to the uncertainties of time and energy for such an object was analytically derived based on the classical mechanics. Considering a measuring procedure as the combination of minimum two non stationary stochastic processes a formula to define the time –quantum was derived using the early developed analytic model. Numerical value of the time – quantum then evaluated was shown to be in a good accordance with the known characteristics of our Universe. Dependence of the time-quantum value from the properties of material objects comprised and the evolution time for the Universe was shown.*
Key words: *Uninterpretable for macro-objects uncertainty relation, the analytic model for non stationary stochastic processes, evaluating time –quantum analytic formula, University evolution time effect on the time-quantum value*

As it well known[1-3], quantum mechanics and the related models are the basis of modern science. In its turn, the Heizenberg uncertainty relation is one of the foundation of the quantum mechanics [2,4]. The relation has reliable theoretical and experimental foundation, particularly for the case of a wave packet independently of its scale. However, the variant of the relation connecting uncertainties of energy and time has only the quantum mechanical justification which is very complex [4] and uninterpretable for macro-scale objects. Meantime, there are rigorously detected general analogy between the quantum mechanical scale and macro- scale objects behavior [5-8].

Besides, high actuality is conserved for the unresolved by now the problem of space –time quantizing. Big progress in the field was reached for last decades [9-11] by application of the corresponding notions to unification of quantum mechanics and general relativity. However, a reliable theoretical justification for the phenomenon existence is absent so far.

So, a theoretical analysis is actual for the basics of the uncertainty relation within the frame of classical mechanics together with theoretical quantitative evaluations of the space-time quantum value.

1. CLASSICAL MECHANICS BASED DEDUCING THE VARIANT OF THE HEIZENBERG UNCERTAINTY RELATION

The variant which is of interest is written as:

$$\Delta E \cdot \Delta t \geq h \tag{1}$$

where: ΔE – is uncertainty in measuring an energy of an object;
Δt - is uncertainty in determination of time for measuring an energy of an object.
h – is the Planck constant.

The above variant of the uncertainty relation is connected with the other one, which has the form:

$$\Delta p \cdot \Delta x \geq h \tag{2}$$

where: $\Delta p = m \cdot \Delta v$ – is uncertainty in measuring an impulse p or velocity v of an object with a mass m;
Δx - is uncertainty in determination of location of an object been under the measuring.

To demonstrate the equivalence of the (1) and (2) variants lets to conduct the following algebraic transformations:

$$m \cdot a = F;\ m \cdot (v_2 - v_1) \approx F(t_2 - t_1);\ \Delta p \approx F \cdot \Delta t;\ F \approx \Delta E/\Delta x;\ \Delta p \approx \Delta E/\Delta x/\Delta t;$$
$$\Delta p \cdot \Delta x \approx \Delta E \cdot \Delta t \tag{3}$$

where F – is a force, acting on a material object during the measuring its characteristics;
$a = (v_2 - v_1)/(t_2 - t_1)$ – is an acceleration of a material object due to the measuring;

(t_2-t_1) is the time interval of measuring or acting the force during the measuring.
Further, using the modified Planck formula: $\Delta E = h\cdot\Delta\omega$, where $\Delta\omega = 1/\Delta t$, we shall have the full identity from the last relation:

$$h\cdot\Delta\omega\cdot\Delta t = h$$

Thus, contrary to the well established quantum character of an energy changes in a tiny object, it is impossible to make a decision about possibility for the quantum space or time to exist within the quantum mechanics: it may be as permitted or prohibited.

2. THEORETICAL EVALUATION A POSSIBILITY OF THE TIME-QUANTUM EXISTENCE

It is well known that quantum effects appear first at all during measuring spacial and energetic characteristics of tiny material objects in the Universe. In its turn, a goal of a measuring procedure is determination of a state of a material object there. A general requirement to a measuring procedure is its minimal influence on an object of the procedure. It is well known [12] that the "measuring" effect is inevitable in the quantum mechanics and cannot be less than an energy quantum. Besides, providing the minimal perturbation effect on an object of a measuring requires strong restriction on the interacting time of the contacting "object" and "subject" of a measuring procedure to prohibit exchanges of more than one energy quantum.

So, based on above, we should consider a "measuring procedure" (MP) in the Universe as an interaction of minimum three material objects: 1) a measured object the characteristics of which are measured, 2) an object which provides energy exchanges during the measurement from the object to the measuring subject, 3) a measuring subject registering the measurement results. It may be seen also that a typical measuring procedure includes minimum two interacting processes which have to be considered as nonstationary stochastic processes (NSPs) due to their restrictions in time. In other words, a description of MP has to take into account inevitable development of corresponding NSPs. Such opportunity is provided by the early developed analytic model [13-16]. According to [13], we may write the following relations:

$$\sigma \approx \Delta E/(\Delta x\cdot S) \text{ or } \Delta E \approx \sigma\cdot(\Delta x\cdot S);\quad \Delta x = \bar{v}\cdot\Delta t;\qquad \bar{v} = P_2\cdot v_{max} + P_1\cdot v_{min}$$

$$\Delta E \approx \sigma\cdot\Delta t\cdot P_2\cdot const_1,$$
$$\Delta E\cdot\Delta t \approx \sigma\cdot\Delta t^2\cdot P_2\cdot const_1$$

where: σ - is a stress or external energy gradient acting to a unit square S of a measured object;

$const_1 = S\cdot v_{max}$;

v_{max} – is the maximum speed of the energy transfer during a measuring;

v_{min} – is the minimum speed of the energy transfer during a measuring ($v_{min} = 0$);

P_1 - is a probability for an energy transfer object to be in an immobile state ($v_{min} = 0$) during measuring procedure

P_2 - is a probability for an energy transfer object to be in a state of moving with the maximum velocity v_{max}

Taking into account, additionally, the minimal possible energy quantity (energy quantum value) which has to be transferred during a measuring process in the Universe, we shall have:

$$\Delta E \approx \sigma\cdot\Delta t\cdot P_2\cdot const_1 = h\cdot\Delta\omega\,;\ \Delta\omega = 1/\Delta t$$

$$\Delta t^2 \approx h/(\sigma\cdot P_2\cdot const_1)$$

Proceeding from the Heizenberg uncertainty relation variant, we shall also have:

$$\Delta E\cdot\Delta t \approx \sigma\cdot\Delta t^2\cdot P_2\cdot const_1 \geq h;$$

$$\Delta t^2 \geq h/(\sigma\cdot P_2\cdot const_1)$$

The last relation shows that the quantity Δt should be considered as the minimal time interval of doing measuring procedure in the Universe, exceeding which satisfies the uncertainty relation and vice versa. Hence, the quantity Δt should be the time quantum and may be evaluated from the formula:

$$\delta t = \sqrt{h/\,(\sigma \cdot P_2 \cdot const_1} \quad (4)$$

Additionally, it is necessary to take into account the following features of the measuring procedure in the Universe, which are flow from the analytic model application. As it was shown in [...] there should be two different regimes of a NSP development. So, the value of the time quantum has not to depend on the regimes type. Hence, we may write for the 1-st and 2-nd regimes:

$$\delta t_1 = \sqrt{h/\,(\sigma_1 \cdot P_{21} \cdot const_1} \qquad \delta t_2 = \sqrt{h/\,(\sigma_2 \cdot P_{22} \cdot const_1}$$

$$\sigma_1 \cdot P_{21} = \sigma_2 \cdot P_{22} \quad (5)$$

where, $\sigma_1$and $\sigma_2 -$ is external force gradient per unit square or stress value providing development respectively of the 1-st and 2-nd regime of a NSP development;

$P_{21}$and P_{22}- is the probabilities for an energy carrier to be in the moving state (moving with the velocity $v = v_{max}$) during respectively the 1-st and 2-nd regime of NSP development.

The above relation was verified numerically using the early developed analytic model (See Fig.1). As it follows from the graph, the equation (5) is correct on the early stage of a NSP energy transferring in the Universe, namely within the time interval: 0…~ 500 arbitrary units at various the model parameters values. It should be noted that at a time exceeding ~ 500 arb. units the relation is incorrect, hence the time quantum does not exist.

An additional verification of the time-quantum existence may be obtained from the followings.

Lets consider a NSP in general form or for the case of 1-st or 2-nd NSP regimes development. Then, we may write, for each NSP regimes:

$$\Delta p_{21(2)} \approx (\Delta E/\Delta x)\cdot \Delta t_{1(2)}\,; \quad \Delta p_{21(2)}/S \approx \sigma_{1(2)}\cdot \Delta t_{1(2};\quad \Delta p_{21(2)} = \Delta\bar{v}_{21(2)}\cdot m; \qquad \Delta\bar{v}_{21(2)} = \Delta P_{21(2)}\cdot v_{max};$$
$$(m\cdot c/S)\cdot \Delta P_{21(2)} = \sigma_{1(2)}\cdot \Delta t_{1(2};$$

Taking into account the relation (5), we shall have:

$$(m\cdot c/S)\cdot \Delta P_{21} = \sigma_1\cdot \Delta t_1$$
$$(m\cdot c/S)\cdot \Delta P_{22} = \sigma_2\cdot \Delta t_2$$

Assuming, as we did it above that $\Delta t_1 = \Delta t_2$ as a common time of a measurement for each NSP regime, it is easy to see, that:

$$\Delta P_{21}/\Delta P_{22} = \sigma_1/\,\sigma_2$$
$$P_{21}/\,P_{22} = \sigma_1/\,\sigma_2$$

Considering the above changes in the physical characteristics of a measured object as indefinitely small ones, we may write:

$$dP_{21}/P_{21} \approx dP_{22}/P_{22}$$

Hence, after the standard integrating procedure, we shall have:

$$lnP_{21} + C = lnP_{22} \quad (6)$$

Graphic representation of the just obtained relation calculated using the analytic model is given on Fig2.

As it seen from the graphs, very close coincidence of the characteristics is observed at various values of the model constants and the integration constant *C*. It should be emphasized also that the relation is accomplished below the time ~ 500 arb.units and is not accomplished above the time value. This result is in accordance with the above calculated data, that may be considered as a confirmation of the existence of the time-quantum and the condition of its appearance.

So, proceeding from the above analysis, minimum three particular conclusions may be formulated:

- The notion of time quantum existence is in accordance with the early developed analytic model for a NSP development, that provides energy transferring between material objects in the Universe;
- The time quantum exists at beginning stages of a NSP development providing energy transferring between material objects but does not exist at a finishing stages of the NSP energy transferring in the Universe;
- The structure of real space-time: discrete or continuous, depends on a stage of energy transfer process in the Universe.

Further, we shall evaluate a numerical value of the time-quantum. Let consider for that a beginning of a NSP for transferring an exceeding energy in the Universe. Appearance of an exceeding energy in a point of space-time, in general case, is equivalent to appearing some material objects there. Due to absence of any relevant specific data, it is natural to suppose the equivalence of the objects characteristics, particularly, masses, volumes etc. Assuming a size in a one dimension for such an object as a , we may write:

$$\delta t = \sqrt{1/\,(\Delta E/a)} \cdot \sqrt{h/\,(P_2 \cdot\ v_{max})}$$

Assuming for simplicity that $P_2 = 1$ and $v_{max} = c$, e.g. that material object transferring energy in the Universe constantly been in a moving state with the maximum speed which is equal to the speed of light. This assumption agrees with the notions about oscillating elementary particles [17]. Proceeding from the above, we may write:

$$\delta t \approx 10^{-21} \cdot \sqrt{(\Delta E/a)}\ \text{sec}$$

As it flows from the last relation, the time-quantum is dependent on the energy gradient acting on an energy transferring material object. Such a result is in accordance with the modern notions about relative character of the space-time e.g. about dependence of the space-time characteristics on the properties of material objects in the Universe [18].

Assuming that the time – quantum may be equal to the Planck-time, e.g. $\delta t \approx t_p \approx 10^{-43}$ seconds [19] and respectively, $a \approx c \cdot \delta t \approx 10^{-35} m$, we shall have the maximum value of the energy gradient: $(\Delta E/a) \approx 10^{44} J/m$, that corresponds to the force level according to the Planck scale: $F_p = 1.2103 \cdot 10^{44} N$. Besides, considering $a \approx l_p \approx 10^{-35}\ m$, we shall have $\Delta E \approx 10^9\ J$ that corresponds to the Plank energy: $E_p \approx 10^9\ J$ and is in accordance with the results of the quantum electrodynamics $\sim 10^{19}\ GeV$ for the physical vacuum base energy.

Additionally it is necessary to note that according to the above developed analytic model [13,17], the 2-nd regime of a NSP development inevitably leads to formation of the periodic spatial configurations of energy transferring objects. In the case of the plastic deformation of FCC poly-crystals such configurations are groups of dislocations, their tangles and walls. As it was shown recently [10, 21], the analogous configurations are observed in the Universe: groups of galaxies, their clusters, voids and networks.

CONCLUSIONS

1. The equivalence of the two known variants of the Heisenberg uncertainty relation was shown based on the classical mechanics
2. Existence of time-quantum in our Universe was theoretically shown and grounded within the frame of the early developed analytic model providing good, quantitative description for a nonstationary stochastic process of macroplastic deformation in FCC polycrystalline metals
3. A formula for the time – quatum numerical evaluation was derived based on the early developed analytic model for NSPs.
4. The dependence of the structure of the space-time in our Universe on material objects characteristics was shown, that agreed with the modern physical notions.
5. The numerical value of the specified time-quantum was evaluated based on the well known values of some basic relevant parameters. Good accordance is reached for the quantity to be applied to define some well known related characteristics.
6. Existence of the time –quantum on initial stages of the energy transferring process in the Universe and its absence on the finishing stages were both shown

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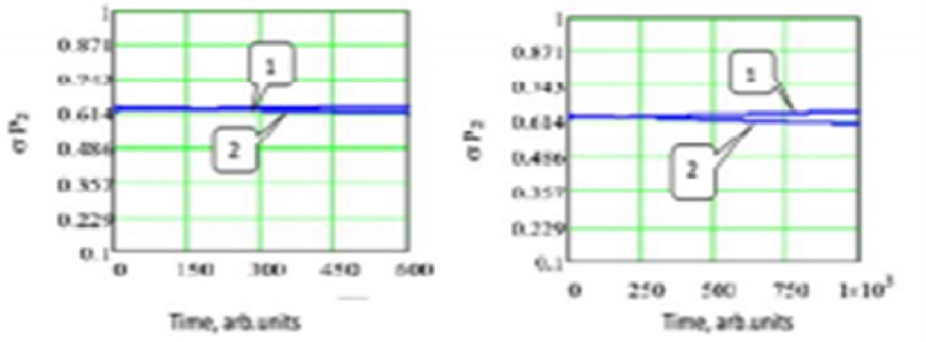
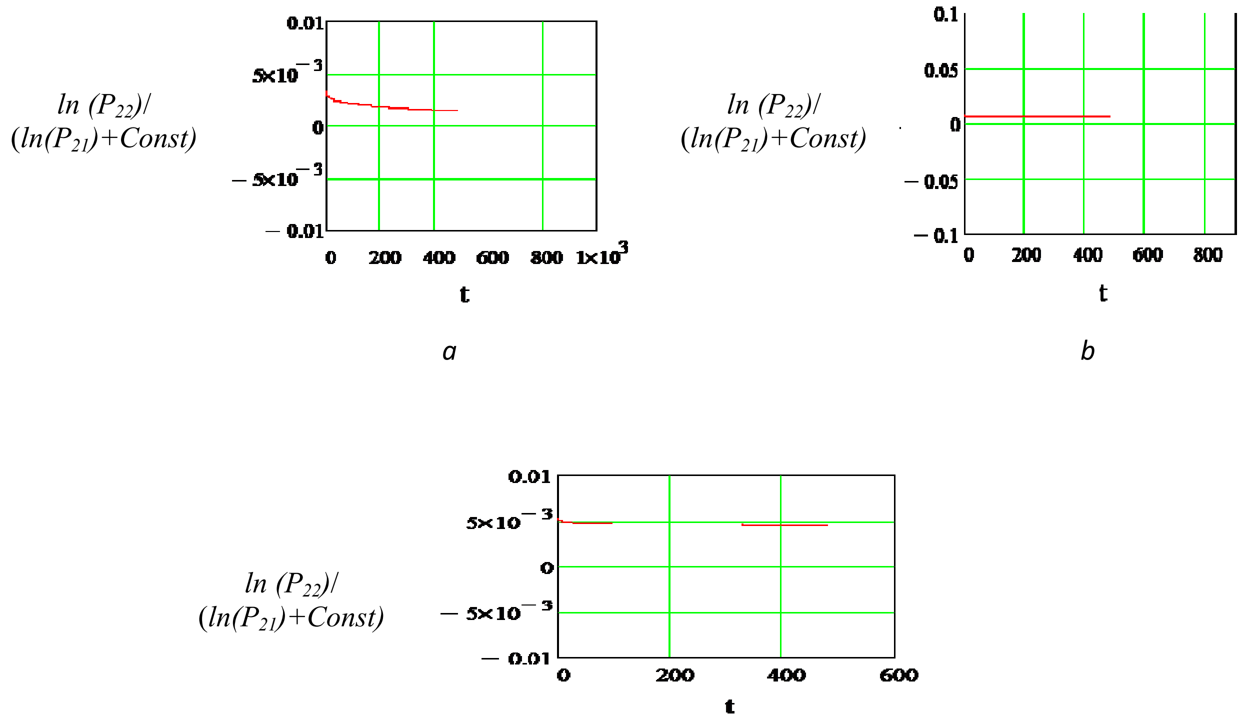


a *b*

Fig. 1. Calculated time dependencies of the quantities: $\sigma_1 P_{21}$ (curves 1) and $\sigma_2 P_{22}$ (curves 2) on the graphs *(a)* and *(b)* at various analytic model constants: $A_1 = 2.1, t^*_1 = 8{\cdot}10^4$ *arb.units,* $\sigma^*_1 = 1.2$ *MPa;* $A_2 = 10^{4.9}$ *,* $t^*_2 = 10^4$ *arb.units,* $\sigma^*_2 = 9.5{*}10^4$ *MPa;*



c

Fig.2. Calculated time dependencies of the quantities *ln* (P_{22}) and *ln*(P_{21})+*Const* in the form of their relation *ln* $(P_{22})/(ln(P_{21})+Const)$ at various the analytic model constants: *a)* $A_2 = 10^9$, $t^*_2 = 0.09$ arb.units, $A_1 = 3$, $t^*_1 = 200$ arb.units, *Const = 900; b)* $A_2 = 10^{18}$, $t^*_2 = 0.03$ arb.units, $A_1 = 3$, $t^*_1 = 200$ arb.units, *Const = 500;c)* $A_2 = 10^{18}$, $t^*_2 = 0.09$ arb.units, $A_1 = 5$, $t^*_1 = 200$ arb.units, *Const = 700*