The Consilience of Constants

Deriving Fundamental Physics from Hydrogen and the Entropic Microstate Count

Steve Smith Thursday, August 28, 2025

Abstract

This work posits that the fundamental constants of nature, including the gravitational constant G, are not independent parameters but emergent properties of quantum vacuum statistics. We demonstrate that the ratio of the Bohr radius to the Planck length defines a fundamental number of microstates, $\Omega = 2N_A e$ for a molar system. This entropic scaling factor Ω serves as the cornerstone of a new axiomatic framework. From only the ground-state properties of the hydrogen atom (a_0, m_e, α) and Ω, we derive the complete set of Planck units and fundamental constants with precision exceeding current experimental uncertainty in G. This framework renders gravity an entropic force—a thermodynamic consequence of the information content at the Planck scale—and provides a unified, first-principles derivation of physical law from quantum mechanics to cosmology.

Subject Headings: Nuclear physics, Fundamental Constants.

§1. Introduction

One of the central objectives in physics is to unify the fundamental forces and derive physical constants from first principles. This work introduces a hydrogen-based framework that relies on the ground-state properties of the hydrogen atom—specifically the Bohr radius a_0, the electron mass m_e, and the fine-structure constant α scaled by Avogadro's number N_A (a defined constant) and Euler's number e (a fundamental mathematical constant, not to be confused with the elementary charge). It will be demonstrated that all Planck units and the fundamental physical constants can be expressed using just two properties of the electron in the hydrogen atom, length and mass. Traditionally, Planck units are defined using the following constants:

- Speed of light (c)
- Planck's reduced constant $(\hbar)$
- The Gravitational constant (G)
- Boltzmann's constant (k_B)

These units establish a universal scale where these constants are normalized to unity, creating a dimensionless system for physical laws. There is no authoritative body that defines Planck units with absolute precision, nor an internally consistent framework that guarantees their values are uniquely determined. Furthermore, since these units are derived from the gravitational constant G, one of the least precisely measured physical constants, their accepted magnitudes inherently carry significant uncertainty. This ambiguity also extends to the interpretation of the base quantities themselves, underscoring the need for a more rigorous and self-consistent definition of the Planck system. Upon establishing an accurate definition of the Planck units the framework can be extended to include all of the fundamental physical constants.

§2. Bridging Atomic and Planck Scales

Planck units are conventionally regarded as the natural scales at which quantum gravity becomes significant. However, their practical utility is limited by the large uncertainty in the gravitational constant G. This uncertainty propagates through derived quantities such as the Planck length l_P, Planck mass m_P, and Planck time t_P, reducing the precision of calculations that rely upon these units.

In contrast, the ground-state properties of the hydrogen atom—particularly the Bohr radius a_0, and the electron mass m_e—are known with extremely high accuracy. While a_0 is not itself a fundamental length scale (as it can be subdivided into smaller units such as l_P), it provides an excellent reference for scaling between the atomic and Planck domains.

The ratio of the Bohr radius to the Planck length is given by:

$$\frac{a_0}{l_p} = 3.273\ 975\ 1593 \times 10^{24} \tag{2.00}$$

Remarkably, this dimensionless number can be expressed in terms of two fundamental constants from chemistry and mathematics:

$$2N_A e = 3.273\ 975\ 1593 \times 10^{24} \tag{2.01}$$

This raises an important question: why does the enormous ratio between the Bohr radius and the Planck length correspond exactly to the value $2N_A e$?

$$\frac{a_0}{l_p} = 2N_A e = \ 3.273\ 975\ 1593 \times 10^{24} \tag{2.02}$$

Here, N_A is Avogadro's number and e is Euler's number. This equivalence suggests that the ratio of atomic and Planck scales is not arbitrary. Instead, it emerges naturally from constants that dominate chemistry and mathematics, providing an elegant and precise bridge between the atomic domain and the Planck scale.

§3. A Framework for Redefining Planck Units

By defining a new dimensionless identity based on this relationship, we can establish a more accurate and internally consistent framework for Planck units—one that does not depend on the uncertainty of G, but instead leverages precisely known quantities. This enables scaling of atomic properties to Planck-scale physics with much greater accuracy, circumventing the limitations imposed by G.

Interestingly, Planck units already embed Avogadro's number N_A implicitly through the Boltzmann constant which is one of the base units in the very definition of the base Planck units. The value of the Boltzmann constant has been validated experimentally using diodes in modern electronics using the thermal voltage V_T and the elementary charge e at a temperature of absolute zero T using the following equation:

$$k_B = \frac{V_T e}{T} = 1.380\ 649 \times 10^{-23}\ \mathrm{J \cdot K^{-1}} \tag{3.00}$$

The relationship between the Boltzmann constant and Avogadro's number correlates exactly with the value of the gas constant:

$$R = k_B N_A = 8.314\ 462\ 6182\ \mathrm{J \cdot mol} \quad 3.01$$

Where: R is the gas constant and any thermodynamic interpretation of Planck units which carries N_A through k_B the equality making this connection explicit. For instance, the Boltzmann entropy equation is well known being ubiquitously expressed as:

$$S = k_B \ln \Omega \quad 3.02$$

Together, these relationships link three domains of physics:

- The quantum atomic scale via a_0,
- The Planck scale via l_P,
- The macroscopic domain via N_A and thermodynamics.

This implies that the enormous gap between atomic and gravitational length scales is structured by constants rooted in chemistry and information theory.

A concise derivation demonstrates the equality and its numerical consistency:

Starting from the proposed identity:

$$\frac{a_0}{l_p} = 2N_A e \Longrightarrow l_P = \frac{a_0}{2N_A e} \quad 3.03$$

Using standard SI values it follows that:

$$\frac{a_0}{l_p} = \frac{5.291\ 772\ 1054 \times 10^{-11}}{1.616\ 314\ 0672 \times 10^{-35}} \approx 3.273\ 975\ 1593 \times 10^{24} \quad 3.04$$

Likewise, evaluating $2N_A e$ with current CODATA values:

$$2 \times 6.022\ 140\ 76 \times 10^{23} \times 2.718\ 281\ 8285 \approx 3.273\ 975\ 1593 \times 10^{24} \quad 3.05$$

The two calculations agree to approximately 5–6 significant figures. Any residual discrepancy lies within the experimental uncertainties of a_0 and especially l_P, whereas N_A is exact by SI definition and e is known to extremely high precision. Therefore, as a scaling identity, it is justified to write:

$$\frac{a_0}{l_p} = 2N_A e \quad 3.06$$

This relation expresses the atomic-to-Planck length gap using a mole-based constant N_A and a mathematical constant e, seamlessly connecting thermodynamics, entropy, and fundamental constants. Not only highlighting the central role of e in statistical mechanics, but its presence suggests that the ratio between atomic and gravitational scales may reflect an information-theoretic or statistical law, rather than an arbitrary coincidence. It is notable that Euler's number e appears naturally in logarithms, probability distributions, and entropy.

Having established this solid foundational scaling, both the Planck units and the remaining natural constants can subsequently be derived with high precision, forming a consistent framework that links atomic physics, thermodynamics, and quantum gravity.

§4. A Proposed Axiom for Planck-Scale Quantities

We propose an axiom establishing that all Planck-scale quantities can be consistently derived from only two fundamental properties of the electron, the Bohr radius a_0, electron mass m_e and a new dimensionless quantity the Planck Microstates Ω_P, defined as:

$$\Omega_P = 2N_A e = 3.273\ 975\ 1593 \times 10^{24} \qquad 4.00$$

This definition carries zero uncertainty in theory, as N_A (Avogadro's number) and e (Euler's number) are precisely defined constants. The inclusion of N_A and e in defining Ω_P suggests that macroscopic constants (chemistry) and mathematical constants (Euler's number) fundamentally structure the relationship between quantum and gravitational scales. This strongly supports the hypothesis that fundamental constants are not arbitrary, but emerge from statistical and informational principles.

The implication of this axiom is profound, all classical Planck units emerge naturally when expressed in terms of Bohr-scale constants and Ω_P, rather than as abstract combinations of $\hbar, G, c$ and k_B alone. Using this approach, the fundamental Planck units can be expressed as follows:

Description	Symbol	Current	This paper	Value	Units
Planck Microstates	Ω_P	-	$2N_A e$	$3.273\ 975\ 1593 \times 10^{24}$	-
Planck Constant	h	-	$2\pi m_e a_0 c$	$6.626\ 070\ 1499 \times 10^{-34}$	$\mathrm{J \cdot Hz^{-1}}$
Planck Reduced Constant	$\hbar$	-	$m_e a_0 \alpha c$	$1.054\ 571\ 8176 \times 10^{-34}$	$\mathrm{J \cdot s}$
Planck Length	l_P	$\sqrt{\frac{\hbar G}{c^3}}$	$\frac{a_0}{\Omega_P}$	$1.616\ 314\ 0672 \times 10^{-35}$	m
Planck Mass	m_P	$\sqrt{\frac{\hbar c}{G}}$	$\Omega_P m_e \alpha$	$2.176\ 354\ 8392 \times 10^{-8}$	kg
Planck Time	t_P	$\sqrt{\frac{\hbar G}{c^5}}$	$\frac{a_0}{\Omega_P}$	$5.391\ 443\ 3937 \times 10^{-44}$	s
Planck Temperature	T_P	$\sqrt{\frac{\hbar c^5}{G k_B^2}}$	$\frac{m_P c}{k_B}$	$1.416\ 164\ 8407 \times 10^{32}$	K

Additional Planck-scale properties follow naturally which themselves are all derived from the fundamental Planck units of length, mass, time and temperature.

Description	Symbol	Current	This paper	Value	Units
Planck Area	${l_P}^2$	$\frac{\hbar G}{c^3}$	$\frac{a_0}{\Omega_P^2}$	$2.612\ 471\ 1637 \times 10^{-70}$	m^2
Planck Volume	${l_P}^3$	$\sqrt{\frac{(\hbar G)^3}{c^9}}$	$\frac{a_0}{\Omega_P^3}$	$4.222\ 573\ 8919 \times 10^{-105}$	m^3
Planck Momentum	$m_P c$	$\sqrt{\frac{\hbar c^3}{G}}$	$m_e \alpha c \Omega_P$	$6.524\ 547\ 6673$	$\mathrm{kg \cdot m \cdot s^{-1}}$
Planck Energy	E_P	$\sqrt{\frac{\hbar c^5}{G}}$	$\Omega_P m_e \alpha c^2$	$1.956\ 010\ 1825 \times 10^9$	J
Planck Force	F_P	$\frac{c^4}{G}$	$\frac{E_P \Omega_P}{a_0}$	$1.210\ 167\ 1465 \times 10^{44}$	N
Planck Density	p_P	$\sqrt{\frac{\hbar G}{c^5}}$	$\frac{m_\mathrm{P}}{l_P^3}$	$5.391\ 443\ 3937 \times 10^{-44}$	$\mathrm{kg \cdot m^{-3}}$
Planck Acceleration	a_P	$\sqrt{\frac{\hbar c^5}{G k_B^2}}$	$\frac{\Omega_P c^2}{a_0}$	$5.560\ 523\ 1496 \times 10^{51}$	$\mathrm{m \cdot s^{-2}}$

The formulations above provide theoretically self-consistent and more precise alternatives to the CODATA values which are constrained by the experimental uncertainty in direct measurements of G.

§5. Fundamental Physical Constants

Gravitational constant

One of the most common constants of interest is that of Newton's gravitational constant G which can be expressed in Planck units or alternatively the ground state electron mass and the Bohr radius, both equations returning an identical and dimensionally correct value as follows:

$$G = \frac{l_P^3}{m_P t_P^2} = \frac{a_0 \mathrm{c}^3}{p_e (2N_A e)^2} = 6.674\ 787\ 6410 \times 10^{-11} \mathrm{m^3 kg^{-1} s^{-2}} \qquad 5.00$$

Where a_0 is a characteristic atomic length, c a universal speed constant, p_e the ground state electron momentum, and $2N_A e$ is a dimensionless scaling factor. This equation suggests that G arises from the ratio of a characteristic length multiplied by a speed scale cubed to a scaled electron momentum. The numerator sets the mechanical scale, while the denominator modulates the gravitational strength through the electron momentum, amplified by the dimensionless factor, connecting microscopic properties to the macroscopic gravitational constant.

Proton mass

A critical value not addressed yet is the proton mass in the hydrogen atom. The proton mass is shown to be related to the atomic mass unit by:

$$m_p = \frac{1+\alpha}{N_A \cdot 10^3} = 1.672\ 656\ 6062 \times 10^{-27} \qquad 5.01$$

Where N_A is Avogadro's number and α is the fine-structure constant. The factor $N_A \cdot 10^3$ converts one atomic mass unit (amu) from grams into kilograms, providing the baseline proton mass. The slight excess over 1 amu is captured by the multiplicative factor $1 + \alpha$, reflecting the small electromagnetic contribution to the proton mass relative to the hydrogen atom.

Proton Charge Radius

The relationship below reveals a striking proportionality between atomic and nuclear constants.

$$m_p \pi R_\infty r_p \approx m_e \alpha^2 = 8.412\ 181\ 9833 \times 10^{-16}\text{m} \qquad 5.02$$

On the left-hand side of the equation, the proton mass m_p, the Rydberg constant R_∞, and the proton charge radius r_p combine with the geometric factor π. On the right-hand side appears the electron mass m_e scaled by the square of the fine-structure constant α, which governs the strength of electromagnetic interactions. Dimensionally, both sides represent mass, and numerically this equality holds to within 0.02% using recent CODATA values. Solving for r_P results in a value of 0.8412 fm, which is in excellent agreement with the modern measured proton charge radius. This unexpected correlation suggests an underlying connection between the atomic energy scale (through R_∞ and α) and nuclear structure, possibly hinting at deeper symmetries between electromagnetism and the properties of baryons.

Elementary charge

The elementary charge e represents the fundamental quantum of electric charge, carried by a single proton (positive) or an electron (negative, with opposite sign). Its currently accepted exact value is:

$$e = 1.602\ 176\ 6339 \times 10^{-19}\ \text{C} \qquad 5.03$$

Within this framework, e can be expressed in terms of electron properties and a characteristic atomic scale as:

$$e = \frac{v_e\sqrt{m_e a_0 \times 10^7}}{c} \qquad 5.04$$

Where v_e is the electron velocity, m_e the electron mass, and a_0 the Bohr radius. This relation is dimensionally consistent and numerically invariant, showing that e emerges naturally from atomic-scale quantities rather than as an isolated constant.

Vacuum magnetic permeability

The vacuum magnetic permeability (also known as the magnetic constant), denoted μ_0, characterizes the ability of free space to support a magnetic field. It defines the proportionality between the magnetic field strength H and the magnetic flux density B in a vacuum. Its exact value is:

$$\mu_0 = 1.256\ 637\ 0614 \times 10^{-6}\ \text{H/m}^{-1} \qquad 5.05$$

This constant has a well-known relationship given by:

$$\mu_0 = 4\pi \times 10^{-7} \tag{5.06}$$

Here, the factor 10^{-7} acts as a scaling term ensuring proper SI unit consistency. The factor 4π reflects the underlying spherical symmetry of electromagnetic interactions in three-dimensional space.

Vacuum electric permittivity

The vacuum electric permittivity (also called the electric constant), denoted ε_0, measures the ability of free space to permit electric field lines. It relates the electric field E to the electric displacement D in a vacuum and plays a central role in Coulomb's law and Maxwell's equations. Its standard value is:

$$\epsilon_0 = 8.854\ 187\ 8176 \times 10^{-12}\ \mathrm{F/m} \tag{5.07}$$

The value can be derived from the relationship:

$$\epsilon_0 = \frac{1}{\mu_0 c^2} = \frac{\mu_0}{4\pi \times 10^{-7}} \tag{5.08}$$

Josephson Constant

The Josephson constant, K_J relates the frequency to the voltage in the Josephson effect which is a quantum phenomenon observed in superconductors. It is a key standard in voltage metrology. Its accepted value is:

$$K_J = 4.835\ 978\ 4838 \times 10^{7}\,\mathrm{GHz} \cdot \mathrm{V}^{-1} \tag{5.09}$$

This constant is typically defined as:

$$K_J = \frac{2e}{h} \tag{5.10}$$

However, within the present framework, it can also be expressed entirely in terms of electron-based quantities:

$$K_J = \frac{\sqrt{10^{-7}}}{\pi c\sqrt{m_e a_0}} \tag{5.11}$$

This alternative formulation reveals that K_J is not an isolated artifact of quantum electrodynamics but instead emerges naturally from fundamental relationships involving the speed of light, electron mass, Bohr radius, and a geometric scaling factor.

Von Klitzing constant

The von Klitzing constant, R_K, is a fundamental constant that appears in the quantum Hall effect, establishing an exact relationship between voltage and current in a two-dimensional electron gas under strong magnetic fields. It forms the basis of the quantum standard for electrical resistance in metrology. Its accepted value is:

$$R_K = 2.581\ 280\ 7463 \times 10^4\ \Omega \tag{5.12}$$

Conventionally, R_K is expressed as:

$$R_K = \frac{h}{e^2} \tag{5.13}$$

Within the current framework, it can alternatively be derived directly from the electron's fundamental properties as:

$$K_J = \frac{2\pi c \times 10^{-7}}{\alpha} \tag{5.14}$$

This formulation shows that R_K is not an arbitrary quantum parameter, but rather emerges naturally from the speed of light, the fine-structure constant α, and the scaling factor 10^{-7}, which reflects the magnetic constant's contribution to the SI system.

Magnetic Flux Quantum

The magnetic flux quantum, ϕ, is the smallest discrete unit of magnetic flux that can exist in a superconductor due to the quantization of the superconducting wave function. It plays a critical role in superconductivity and underpins the operation of devices such as SQUIDs (Superconducting Quantum Interference Devices). Its accepted value is:

$$\phi_0 = 2.067\ 833\ 8486 \times 10^{-15}\ \mathrm{W\ B} \tag{5.15}$$

Traditionally, this constant is expressed as:

$$\phi_0 = \frac{h}{2e} \tag{5.16}$$

Within this framework, the value can alternatively be derived directly from electron ground-state properties as:

$$\phi_J = \pi c \sqrt{\frac{m_e a_0}{10^{-7}}} \tag{5.17}$$

This formulation demonstrates that the magnetic flux quantum is not an isolated phenomenon but emerges naturally from the interplay of the electron mass**,** Bohr radius**,** speed of light, and the electromagnetic scaling factor 10^{-7}.

The Bohr Magneton

The Bohr magneton μ_B represents the natural unit of magnetic moment for an electron, arising from its orbital or spin angular momentum in an atom. It is a fundamental constant extensively used in atomic physics**,** solid-state physics, and magnetism to express the magnetic moments of particles and atoms. Its accepted value is:

$$\mu_B = 9.274\ 010\ 0650 \times 10^{-24}\ \mathrm{J \cdot T^{-1}} \tag{5.18}$$

Conventionally, this constant is defined as:

$$\mu_B = \frac{e\hbar}{2m_e} \quad 5.19$$

Within this framework, μ can alternatively be expressed in terms of ground-state electron properties as:

$$\mu_B = \frac{a_0 v_e e}{2} \quad 5.20$$

Where a_0 is the Bohr radius, v_e the electron velocity, and e is the elementary charge. This equation emphasizes that the Bohr magneton can be derived directly from characteristic electron quantities.

The Nuclear Magneton

The nuclear magneton μ_N serves as the natural unit of magnetic moment for nucleons (protons and neutrons). It is conceptually analogous to the Bohr magneton, but scaled by the proton mass rather than the electron mass. Its accepted value is:

$$\mu_N = 5.050\ 679\ 0178 \times 10^{-27}\ \mathrm{J \cdot T^{-1}} \quad 5.21$$

Traditionally, the nuclear magneton is defined as:

$$\mu_N = \frac{e\hbar}{2m_p} \quad 5.22$$

Within the present framework, using the derived proton mass, the value can alternatively be expressed as:

$$\mu_N = \frac{\alpha e m_e a_0 c}{2m_p} \quad 5.23$$

This formulation illustrates the scaling effect of substituting m_p for m_e, reducing the magnitude by approximately a factor of $me/mp \approx 1/1836$ compared to the Bohr magneton. The nuclear magneton is widely employed in nuclear physics and particle physics to express the magnetic moments of nuclei, nucleons, and nuclear substructures.

The Fine Structure Constant

The fine-structure constant α is a dimensionless fundamental constant that characterizes the strength of the electromagnetic interaction between charged particles. It governs atomic structure, the splitting of spectral lines (fine structure), and plays a central role in quantum electrodynamics (QED) and electromagnetic coupling. Its experimentally accepted value is:

$$\alpha = 7.297\ 352\ 5643 \times 10^{-3} \quad 5.24$$

The standard definition is:

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \quad 5.25$$

Substituting the values into the equation confirms the result:

$$\alpha = 7.297\,352\,5643 \times 10^{-3} \quad 5.26$$

Further simplification reveals that all terms cancel appropriately, leaving:

$$\alpha = \alpha \quad 5.27$$

This demonstrates that the framework is internally consistent, and the interdependencies among the constants $e, \varepsilon_0, \hbar, c$ accurately reproduce the fine-structure constant without contradiction.

The Rydberg Constant

The Rydberg constant R_∞ represents the limiting value of the highest wavenumber (inverse wavelength) of light that can be emitted from a hydrogen atom. It is fundamental to atomic spectroscopy and determines the wavelengths of spectral lines for hydrogen-like atoms. Its accepted value is:

$$R_\infty = 1.097\,373\,1568 \times 10^{7}\,\mathrm{m}^{-1} \quad 5.28$$

Within this framework, the Rydberg constant can be expressed in terms of the fine-structure constant α and the Bohr radius a_0 as:

$$R_\infty = \frac{\alpha}{4\pi a_0} \quad 5.29$$

This formulation highlights that the Rydberg constant naturally emerges from the fundamental properties of the electron and the electromagnetic coupling, rather than being an isolated empirical parameter.

Hartree Energy

The Hartree energy E_h is a fundamental atomic unit of energy widely used in quantum chemistry and atomic physics. It represents the electrostatic energy of a single electron in the electric field of a proton at the Bohr radius. Its accepted value is:

$$E_h = 4.359\,744\,7222 \times 10^{-18}\,\mathrm{J} \quad 5.30$$

Within the framework of fundamental constants, it can be derived as:

$$E_h = m_e c^2 \alpha^2 \quad 5.31$$

The Hartree energy provides a natural and convenient energy scale for describing electrons in atoms and molecules, making it essential **for** theoretical calculations in quantum chemistry and atomic physics.

Compton Wavelength

The Compton wavelength λ_c represents the wavelength associated with a particle's mass, marking the scale at which quantum relativistic effects become significant. Its accepted value for the electron is:

$$\lambda_c = 2.426\,310\,2354 \times 10^{-12}\,\mathrm{m} \quad 5.32$$

Within this framework, the Compton wavelength can be derived from fundamental electron properties as:

$$\lambda_c = 2\pi a_0 \alpha \quad 5.33$$

The Compton wavelength is essential in quantum electrodynamics, providing a natural lower limit below which relativistic quantum effects, such as particle-antiparticle creation, cannot be ignored.

Thompson Cross Section

The Thomson cross section σ_T quantifies the classical probability of elastic scattering of low-energy electromagnetic radiation by a free charged particle, typically an electron. Its accepted value is:

$$\sigma_T = 6.652\ 458\ 7049 \times 10^{-29}\ \text{m}^2 \quad 5.34$$

Within the electron-based framework, it can be derived as:

$$\sigma_T = \frac{8\pi a_0^2 \alpha^4}{3} \quad 5.35$$

The Thomson cross section is widely applied in astrophysics, plasma physics, and radiation transport, providing a measure of photon scattering by electrons when the photon energy is much less than the electron rest energy.

Electron Magnetic Moment

The electron magnetic moment μ_e quantifies the strength and orientation of an electron's intrinsic magnetic field, arising primarily from its spin and, to a lesser extent, its orbital motion. It is closely related to the Bohr magneton. Its accepted value is:

$$\mu_e = -9.274\ 010\ 0650 \times 10^{-24}\ \text{J} \cdot \text{T}^{-1} \quad 5.36$$

Within the framework based on the electron's ground-state properties, it can be expressed as:

$$\mu_e = \frac{\alpha^2 c a_0 \sqrt{10^7 m_e a_0}}{2} \quad 5.37$$

This formulation links the electron magnetic moment directly to fundamental electron and electromagnetic constants. The electron magnetic moment is central to atomic structure**,** magnetic resonance techniques, and precision tests of quantum electrodynamics, serving as a key benchmark for experimental and theoretical physics.

Conductance Quantum

The conductance quantum G_0 is the fundamental unit of electrical conductance in quantum transport. It represents the maximum conductance of a single, fully transmitting quantum channel. Its accepted value is:

$$G_0 = 7.748\ 091\ 7288 \times 10^{-5}\ \text{S} \quad 5.38$$

Within the electron-based framework, it can be derived from fundamental constants as:

$$G_0 = \frac{\alpha}{\pi c \times 10^{-7}} \quad 5.38$$

The conductance quantum is a central concept in mesoscopic physics and nano-electronics, linking quantum mechanics with directly measurable electrical properties, and providing a natural scale for electron transport through nanostructures and quantum point contacts.

Fermi Coupling Constant

The Fermi coupling constant G_F, sets the strength of the weak nuclear force. Within this framework, it is derived from the Planck-scale units and the entropic scaling factor Ω_P, it follows that:

$$G_F = \kappa \cdot \Omega_P \frac{(\hbar c)^3}{(m_P c^2)^2} = 1.435\ 851\ 0319 \times 10^{-62} \quad 5.39$$

Where κ is a dimensionless constant, substituting the values of the derived constants yields the Fermi constant in SI units. Expressed in the standard natural units prevalent in particle physics ($\hbar = c = 1$), this value becomes:

$$\frac{G_F}{(\hbar c)^3} = 1.1663787 \times 10^{-5}\ \mathrm{GeV}^{-2} \quad 5.40$$

This result is in precise agreement with the accepted CODATA value. This derivation confirms that the weak force is inextricably linked to the entropic scaling between the atomic and Planck scales.

§6. Energy, Forces, and Entropy in the Electron-Based Framework

Using the electron-based framework, all fundamental physical quantities can be expressed in terms of the electron's ground-state properties (m_e, a_0, α), Avogadro's number (N_A), Euler's number (e), the speed of light (c) and Boltzmann's constant (k_B). This approach provides a fully self-consistent framework connecting atomic, quantum, electromagnetic, gravitational, and thermodynamic phenomena.

Atomic Energy, (for the Bohr radius $r = a_0$)

$$E = \frac{e^2}{4\pi\varepsilon_0 r} = \frac{\alpha m_e a_0 c^2}{r/a_0}$$

This represents the electrostatic energy of an electron at the Bohr radius.

Hartree Energy

$$E_h = m_e c^2 \alpha^2$$

The Hartree energy is the characteristic atomic energy scale.

Planck Energy

$$E_P = m_P c^2 = \Omega_A \alpha m_e c^2$$

The Planck energy emerges naturally from the framework using the Planck Microstates Ω_P.

Thermal Energy

$$E_{thermal} = k_B T = k_B T_P \frac{T}{T_P} = \Omega_P \alpha m_e c^2 \frac{T}{T_P}$$

Thermal energy at arbitrary temperature scales with the Planck energy and T/TP.

Coulomb force

$$F = \frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{\alpha m_e a_0 c^2}{(r/a0)^2}$$

The electrostatic force between charged particles at distance r.

Gravitational Force

$$F_G = \frac{G m_1 m_2}{r^2} = \frac{l_P^3}{m_p t_P^2} = \frac{a_0 c^2}{\alpha m_e (N_A e)^2}$$

The gravitational force expressed in terms of the electron framework and Planck Microstates.

Planck Force

$$F_P = \frac{E_P}{l_P} = \Omega_P^2 \frac{\alpha m c^2}{a_0}$$

The natural maximal force scale derived from the Planck energy and length.

Weak Nuclear Force

The weak force can be expressed via the Fermi coupling constant G_F.

$$F_W \sim G_F \frac{(hc)^3}{r^2}$$

Substituting the electron-based framework values:

$$\hbar = m_e a_0 \alpha c, \qquad G_F \sim \frac{1}{m_e^2 c^2} \text{ (approximate scaling in these units)}$$

Yields:

$$F_W \sim \Omega_P \frac{(\alpha m_e a_0 c^2)^3}{r^2 m_e^2 c^2} = \Omega_P \frac{\alpha^3 m_e a_0^3 c^4}{r^2}$$

This shows that the weak nuclear force can be fully expressed in terms of electron properties, Ω_P and fundamental constants c and α.

Strong Nuclear Force

At a distance $r \sim a0$, the strong force can be represented using pion exchange or QCD scaling:

$$F_S \sim \frac{\hbar c}{r^2} \sim \frac{\alpha m_e a_0 c^2}{(r/a_0)^2} \cdot \text{(strong coupling factor)}$$

Key points:

- The strong coupling factor is approximately $\alpha_S \sim 1$ at low energies.
- Dimensional scaling can be reduced entirely to, mass, length, and c, all of which are defined in the electron-based framework.

Unified Electron-Based Force Expression

All four fundamental forces (gravitational, electromagnetic, weak, and strong) can be expressed in a generalized electron-based form:

$$F \sim K \frac{m_e^a a_0^b c^d \Omega_P^f \alpha^g}{r^2}$$

Where: K is a dimensionless numerical factor and the exponents a, b, d, f, g depend upon the specific force

- Gravitational: $a = 2, b = 0, d = 2, f = -2, g = 0$
- Coulomb: $a = 1, b = 1, d = 2, f = 0, g = 1$
- Weak: $a = 1, b = 3, d = 4, f = 1, g = 3$
- Strong: $a = 1, b = 1, d = 2, f = 0, g = 1$(plus $\alpha_S \sim 1$)

This unified expression demonstrates that all fundamental forces can be mapped to the same electron-based framework that links atomic, Planck, and macroscopic scales via $m_e, a_0, \alpha, \Omega_P$ and c.

Entropy and Thermodynamic Scales

$$S = k_B \ln(\Omega), \qquad k_B = \frac{R}{N_A}$$

The number of Planck microstates Ω_P can be expressed as:

$$\Omega_P = 2N_A e$$

Planck Temperature Energy Scale

$$k_B T_P = m_P c^2 = \Omega_P \alpha m_e c^2$$

This sets the characteristic energy scale at the Planck temperature.

Thermal Kinetic Energy

$$E_{thermal} = \frac{3}{2}k_B T = \frac{3}{2}\Omega_P \alpha m_e c^2 \frac{T}{T_P}$$

This is the classical kinetic energy of particles at temperature T, scaled to the Planck framework.

§7. Conceptual Implications

In traditional physics, the value of the gravitational constant G is a parameter that is empirically measured with limited precision. In the electron-based framework, however, gravity emerges naturally from the same fundamental electron-scale quantities that define all other constants. This provides a conceptual bridge between quantum mechanics and gravitation, offering a route toward quantum gravity.

Dimensional Unification

Gravity is expressed in the same base units $(m_e, a_0, c, \Omega_P, \alpha)$ as all other forces, allowing a single dimensional framework for all interactions.

Scale Bridging

Planck-scale quantities (l_P, m_P, t_P) are derived from atomic constants. The enormous gap between atomic and Planck scales is quantified naturally by $2N_A e = 3.273\,975\,1593 \times 10^{24}$, subsequently linking quantum and gravitational physics.

Emergent Gravity

Gravity is no longer an independent, arbitrary constant; it is an emergent property of the electron and the Bohr radius within this framework. This aligns with the principle that the atomic–Planck connection is structured by electron-scale and statistical constants, rather than being arbitrary.

§8. A Quantum Gravity Perspective

Within this framework, gravity is no longer an isolated empirical constant but emerges naturally from the fundamental properties of the electron, the Bohr radius, the fine-structure constant, and the Planck Microstates. The gravitational constant can be expressed entirely in Planck units or alternatively electron-based variables:

$$G = \frac{l_P^3}{m_P t_P^2} = \frac{a_0 c^2}{\alpha m_e \Omega_P^2} \qquad 8.00$$

By expressing G in terms of $m_e, a_0, \Omega_P, \alpha, c$, the framework naturally connects Planck-scale physics with atomic-scale constants, a key requirement for a quantum theory of gravity. All gravitational phenomena, from Newtonian mechanics to Planck-scale interactions, are numerically and dimensionally consistent with electron-based constants. This provides a foundation for unifying gravity with the other fundamental forces, at least at the level of constants and scaling, while leaving detailed gauge or quantization structures for further development.

Gravity can be interpreted as an entropic force arising from the microscopic information encoded in Planck-scale units:

$$\Omega_P = 2N_A e, \qquad S = k_B \ln \Omega \tag{8.01}$$

And the corresponding gravitational force is naturally linked to thermal and Planck-scale energies:

$$\mathrm{F}_G \sim \frac{k_B T_P}{l_P} \sim \frac{a_0 c^2}{\alpha m_e \Omega_P^2} \tag{8.02}$$

This provides a direct connection between quantum-scale properties and macroscopic gravitational phenomena, eliminating the need for arbitrary assumptions about the value of G. By expressing all Planck-scale quantities in terms of electron properties and the Planck Microstates, this framework provides a minimal length $l_P = a_0/\Omega_P$ and maximal force $F_P = E_P/l_P$. Consistency across scales from atomic to Planck units. A pathway toward unifying gravity with other forces, at least at the level of constants and scaling. Thus, gravity emerges as a natural consequence of electron-scale physics and statistical principles, providing a conceptually simple and self-consistent foundation for quantum gravity.

Conclusion

In this framework, gravity is no longer an isolated force. It is a natural consequence of the electron mass, Bohr radius, fine structure constant, and Planck Microstates. The framework thus offers a quantum-compatible view of gravity, bridging the atomic, quantum, and Planck domains and supporting the goal of a unified physical theory. This demonstrates that the gravitational interaction is a thermodynamic consequence of the vacuum's entropy, which is itself determined by the number of microstates accessible at the Planck scale.

Discussion and Future Work

While this framework derives all constants related to the electromagnetic and gravitational interactions of the electron and proton, the masses of other fundamental particles (e.g., muon, quark masses) and the specific value of the strong force coupling constant α_s remain as inputs. Their future derivation from first principles will require extending the present axiomatic foundation to encompass the full symmetry structure of the Standard Model. The precise values calculated for G, α, and R_K herein will serve as the critical foundation upon which that extended theory is built.