Φ geometry in physics: a single angle for SR, energy–momentum, and numerical stability

Meta

Φ geometry is a visual re-parameterization of standard relativistic relations via one angle φ on a unit (or c-scaled) circle. One diagram unifies β, γ, E, p and velocity addition. Complementarity (sin ↔ cos) removes "bad denominators" and makes edge cases numerically stable.

Keywords: φ-geometry; special relativity; energy–momentum; visual pedagogy; complementary branches; numerical stability; electromagnetism; de Broglie; string theory.

TL;DR — in two lines

Define $\beta = \sin\varphi$, $\gamma^{-1} = \cos\varphi$, $\gamma\beta = \tan\varphi \Rightarrow \gamma = \sec\varphi$; $E/(mc^2) = \sec\varphi$; $p/(mc) = \tan\varphi$.
Invariant: $\sec^2\varphi - \tan^2\varphi = 1 \Leftrightarrow E^2 - (pc)^2 = (mc^2)^2$. Complementarity ($\varphi \leftrightarrow 90° - \varphi$) avoids division by small numbers.

1. The idea in one picture

Take a unit (or c-scaled) circle and inscribe a right triangle with angle φ at O. Then
$\beta = v/c = \sin\varphi$; $\gamma^{-1} = \cos\varphi$; $\gamma\beta = \tan\varphi$.
From the picture you read common SR effects: time dilation $\Delta t = \Delta\tau / \cos\varphi$; length contraction $L = L_0 \cos\varphi$; velocity addition stays inside the circle ($|\beta| < 1$ is "seen from the arc").

2. Definitions and identities (short)

- $\gamma = 1/\sqrt{(1 - \beta^2)} = \sec\varphi$.
- Energy projections: $E\cos\varphi = mc^2$, $E\sin\varphi = pc \Rightarrow E^2 = (mc^2)^2 + (pc)^2$.
- Energy–momentum invariant: $\sec^2\varphi - \tan^2\varphi = 1 \Leftrightarrow (E/(mc^2))^2 - (p/(mc))^2 = 1 \Leftrightarrow E^2 - (pc)^2 = (mc^2)^2$.

3. Why this helps

Visual pedagogy. "One angle — one language": β, γ, E, p, Δt, L are read off the same triangle.
Numerical stability. If cos φ or sin φ is small, switch to the complementary angle $\Phi = 90° - \varphi$ (swap sin ↔ cos) to avoid small denominators and loss of significance.
Single parameter φ. Independent measurements of the "light" and "matter" shares of a scene return the same φ:

$$\sin^2\varphi = \frac{E_{light}}{E_{light} + E_{matter}}, \cos^2\varphi = \frac{E_{matter}}{E_{light} + E_{matter}}.$$

4. Extensions and adaptations (different tasks)

This geometry can be **developed and adapted** to specific domains:

- **Gravitational estimates (outside the horizon):** use $\cos\varphi_g ® = \sqrt{1-\frac{R_S}{r}}, \sin\varphi_g ® = \sqrt{\frac{R_S}{r}}$, for $r > R_S = \frac{2GM}{c^2}$ — a quick "gravity scale" for order-of-magnitude timing/frequency checks.
- **Electromagnetism:** compact sin/cos forms for basic field transforms.
- **Quantum / de Broglie:** quick atomic estimates from $v/c \approx Z\alpha/n$ → a simple $\varphi(Z,n)$ "atomic scale".
- **Engineering edges:** whenever a variable is near 0 or 1, swapping to the complementary branch improves numerical robustness.

5. Quick sanity-check numbers

- Electron, K = 5 MeV: $\beta \approx 0.9957$, $\gamma^{-1} \approx 0.0927 \Rightarrow \sin^2\varphi + \cos^2\varphi \approx 1$.
- Muon, E ≈ 3 GeV: $\beta \approx 0.99938$, $\gamma^{-1} \approx 0.0352 \Rightarrow \sin^2\varphi + \cos^2\varphi \approx 1$.
- Au-197 at T ~ 300 K (estimate): $\beta \sim 6.5\times10^{-7}$, $\gamma^{-1} \approx 1 \Rightarrow \sin^2\varphi + \cos^2\varphi \approx 1$.

  (Use $\sin\varphi = (pc)/E$, $\cos\varphi = (mc^2)/E$; the squares sum to ~1.)

6. Desk test: LED (≈1 hour on a standard bench)

Measure: $P_{elec} = I \cdot V$; P_{opt} (integrating sphere/radiometer); P_heat (calorimetry/thermal balance).
Balance: $P_{elec} \approx P_{opt} + P_{heat}$.
Shares: $f_{light} = \frac{P_{opt}}{P_{opt} + P_{heat}}$; $f_{matter} = 1 - f_{light}$.
Angle: $\sin\varphi = \sqrt{f_{light}}$; $\cos\varphi = \sqrt{f_{matter}} \Rightarrow \varphi = \arcsin\left(\sqrt{f_{light}}\right) = \arccos\left(\sqrt{f_{matter}}\right)$.
Criteria: (i) $f_{light} + f_{matter} \approx 1$; (ii) φ from sin and from cos agree within errors; (iii) near edges (f → 0 or 1) the estimate stays stable by switching branch.

7. Where else it helps

Ultra-relativistic regimes ($\cos\varphi \ll 1$), quasi-static regimes ($\sin\varphi \ll 1$), teaching, fast back-of-the-envelope checks — and, importantly, it often helps **to imagine and anticipate** the character of a phenomenon before detailed calculation thanks to a single clear diagram.

8. Illustration (gravity "scale" on the φ-circle)

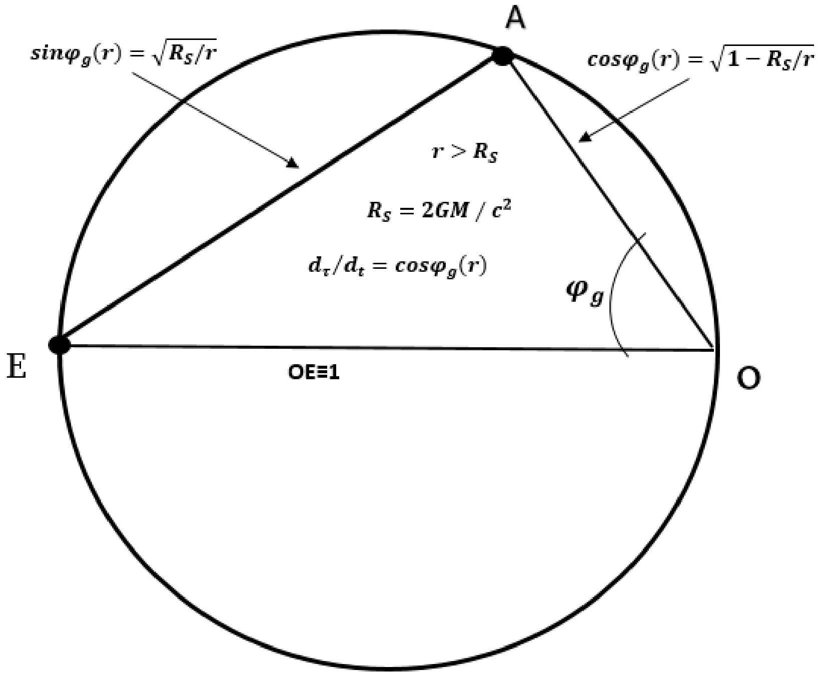

Figure caption. Fig. 1 — *φ-triangle with OE ≡ 1 and a gravity scale outside the horizon:* $cos\,\varphi_{g(r)} = \sqrt{1 - \frac{R_S}{r}}, sin\,\varphi_{g(r)} = \sqrt{\frac{R_S}{r}}, with\ R_S = \frac{2GM}{c^2 and} r > R_S$

The picture lets you read quick estimates "off the arc" and switch branches (sin↔cos) at extreme r without numerical issues.

ALT text (for SEO/accessibility): "φ-triangle on a unit circle; gravity scale with $\cos\varphi_g ® = \sqrt{1 - \frac{R_S}{r}}, \sin\varphi_g ® = \sqrt{\frac{R_S}{r}}, R_S = \frac{2GM}{c^2}$."

9. FAQ

Can boosts reach φ = 90°? No. For massive particles φ < 90°. The 90° point is the massless (photon) boundary.

Is this new dynamics? No. It's a re-parameterization of standard SR/EM with gains in clarity and numerical stability.

10. Contact

Mikheili Mindiashvili — Independent Researcher, Georgia.

ORCID: 0009-0008-1924-8264 • Email: mindia-m@mail.ru