

# **Title: Revisiting the Mass Radius of the Proton in light of Controverted Equation**

**AUTHOR:** Ikechukwu Iloh Udemá<sup>1</sup>

**Affiliation:** <sup>1</sup>Department of Chemistry and Biochemistry, Research Division, Ude International Concepts Ltd., 862217, B. B. Agbor, Delta State, Nigeria.

**ORCID:** <https://orcid.org/0000-0001-5662-4232>

**Email address:** [udema\\_ikechukwu99@yahoo.com](mailto:udema_ikechukwu99@yahoo.com)

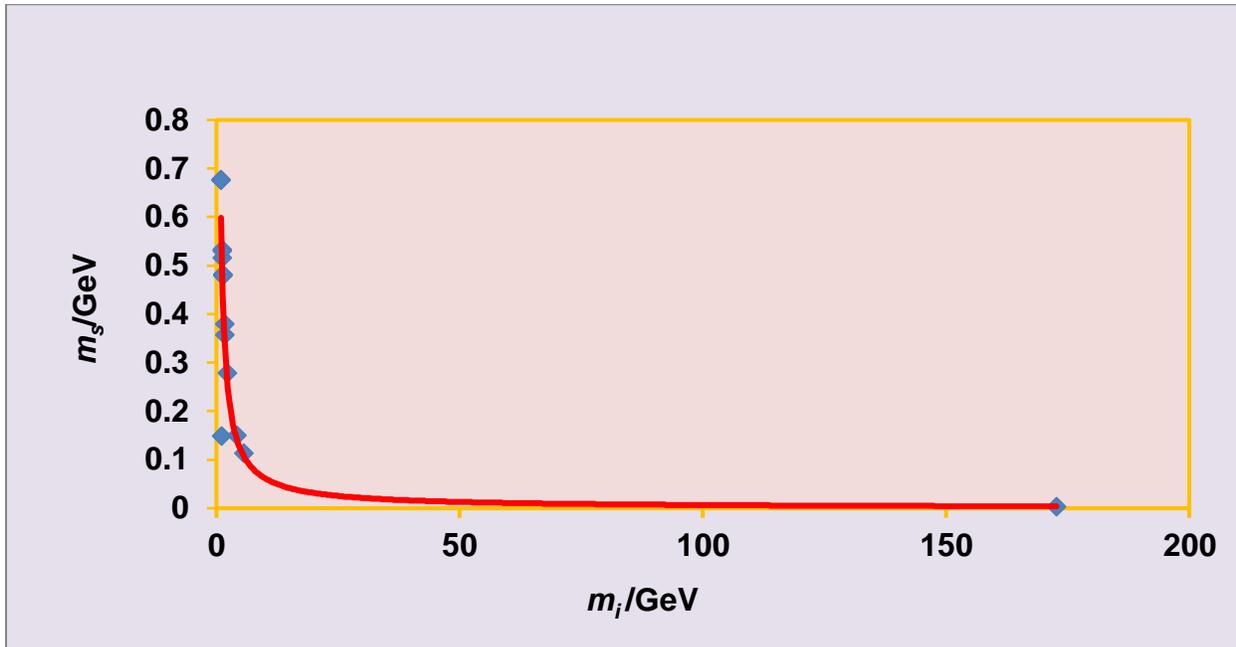
## ABSTRACT

The root mean square mass radii of the proton are produced by means of the extraction of the effective mass from different kinds of photoproduction and, strangely, center of mass energy without overt exploration of the equation ( $\langle R_m^2 \rangle = 12/m_s^2$ ) that ought to contain the missing fundamental constant. Thus, by exploring theoretical and computational means, the study's goal is to address frameworks' and related methods' congruence with the outcomes that could be obtained from them with the following two of four objectives. 1) To derive an equation defining the fundamental constant missing from the controverted equation in the literature; 2) to justify the argument that the missing fundamental constant obscured the relationship, if at all, between QCD and the derived equation, *etc.* Some of the computed values of root mean square mass radius ( $R_m$ ) of the proton based on  $1.19518 \exp. (-25)/m_s$  (From Eq. (13) are 0.644869 fm (its effective mass ( $m_s$ ) is equal to 1.06 GeV); 0.83361 fm (its  $m_s$  is equal 0.82 GeV); 0.551246 fm (its  $m_s$  is equal to 1.24 GeV). If the derived equation ( $\langle R_m^2 \rangle = 12\hbar^2 c^2/m_s^2$ ) bears any iota of relevance to the QCD framework, then there was no basis whatsoever for the omission of the fundamental constant,  $\hbar c$  or  $\hbar^2 c^2$ . Thus, the omission, until proved otherwise, only served to obscure the fact that the equation does not have any bearing on the QCD framework. A definite value of  $R_m$  is 1.10168 fm. While  $m_s$  showed an inverse relationship based on a power law with rest mass, future studies may focus on the theoretical determination of the former.

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**Keywords:** Classical framework, QCD framework, mass-energy equivalence, root mean square mass radius, mass, proton.

## GRAPHICAL ABSTRACT



Subatomic particles with masses exceeding the mass of the nucleon have a power law relationship between their effective mass and rest mass; the lower limit of their masses is outside the purview of this investigation.

## 1. INTRODUCTION

This section highlights the acts of prominent members of the scientific community that give the impression that they are indifferent to the contributions of others. This notwithstanding, this study, adopted alternative and corrective techniques. Because some literature knowledge is reserved, some questions are also directed to artificial intelligence (AI) on the internet, which provides an objective response that either supports or refutes the reservation. This in no way implies that the concept and theoretical background generated in this study are AI-assisted. It only compares slightly with AI-based Video Assistant Referee (VAR) analysis and reports in soccer. In line with normal practices, an overview critique is made of the literature, which focuses on quantum chromodynamics and associated form factors without giving the impression that one is anywhere close to the basic minimum of intelligibility. Concern about how knowledge expands and is hindered is also expressed.

Virtually nothing cannot be revisited, if doing so, is necessary otherwise innovations in history could have been impossible. There were theories before the Chemiosmotic theory, which very good universities make a part of their curriculum. What is, however, important in any theory, suitable or not, is that it must have a focus, which is to be reached as the ultimate goal (s) with rational and systematic methodology (ies) exhibiting a sequence characterized by connectivity. Simply put, the result (equation or value-based result) and conclusion(s) must have bearing with conceptual frameworks producing such result or that lead to any conclusion(s). Given that learning is continuous in life, it will be uncalled for to suggest that the teacher has nothing to learn from those who learn from him. If not, such behavior is a Hitler-like adherence to a superiority complex that does not give credence to the axiom that no child without exception (or any person of whatever nonterminal age) anywhere is necessarily a *tabula rasa*. Where did Alfred Mendel learn the mechanism of biological inheritance? Discovery is a very difficult means of learning. Alert: it can be relatively less difficult. On the contrary it is not unusual to find mathematical treatise in high energy physics that is difficult and made even more difficult because hardly are steps leading directly or indirectly to each other found. The present study has no justification for such an approach because each equation has all the parameters for the computation of the dependent variables, given the evidential suggestion that fundamental parameters can be linked to each other.

The theoretical and conceptual backgrounds are inalienable aspects that aid the creation of models (equations); this is why in an essay examination, in which a solution is expected for a question in any aspect of the natural sciences (*e.g.*, physics), a candidate is expected to state steps leading to the derivations or value-based result such that even if the final equation or value-based solution is wrong due to, perhaps, a mistake, the candidate can still score a grade A mark because each logical step has its score as long as all preceding steps are correct. If it is pertinent and expedient at elementary and university levels, it must be exceedingly more pertinent at doctoral and postdoctoral levels. In light of this, the experts (preferably from highly advanced nuclearized nations) in high energy physics, where quantum chromodynamics (QCD) and perhaps quantum electrodynamics (QED) are crucial, should make time to explain how  $\langle R_m^2 \rangle = 12/m_s^2$

(Kharzeev, 2021, Wang *et al.*, 2021, Wang *et al.*, 2023) is derivable on the basis of the QCD framework.

The GlueX Collaboration data was purported to be used to extract the root mean square mass radius ( $R_m$ ) of the proton, defined as:  $R_m = 0.55 \pm 0.03$  fm. The extracted mass radius is significantly smaller than the root mean square (rms) charge radius of the proton  $R_c = 0.8409 \pm 0.0004$  fm (Kharzeev, 2021). Besides, 0.55 fm was purported to be obtained from the square root of the mean square mass radius of the proton equal to 12 (12 has no unit to it) divided by the square of  $m_s$  where  $m_s = 1.24 \pm 0.07$  GeV. The question as to whether or not 0.55 fm is obtainable by taking the square root of the solution of 12 divided by the square of 1.24 GeV was answered by AI as follows: “No, 0.55 femtometers (fm) cannot be obtained by taking the square root of the solution of 12 divided by the square of 1.24 GeV. The calculation  $12/(1.24 \text{ GeV})^2$  results in a value with units of inverse energy squared ( $1/\text{GeV}^2$ ), which cannot be converted to length (fm) without additional constants. The units are incompatible, and the calculation leads to a value much smaller than 0.55 fm.” There was a three-step breakdown of the answer, which is omitted for the sake of brevity. The same article was first endorsed before being posted at the arXiv preprint server (arXiv: 2102.00110 [hep-ph]). Personal experience has shown that authors approached for endorsements never replied, showing disapproval or otherwise. The question is, what is the motivation driving the researchers’ behaviour? If they want the highest international prize or award in cash, let the awarding bodies continue, but it must not be based on covert and overt falsity or deliberate omission. See the article titled “Methodological fallacy in proton mass radius determination: Alternative classical approach based on Einstein’s mass-energy equivalence principle” (Udema, 2025).

### 1.1 Alternative perspective to AI’s answer to similar question

**Question:** Is the claim that 0.55 fm is equal to the square root of 12 divided by the square root of the square of 1.24 GeV published in Physical Review D correct?

**Answer:** “No, the claim that 0.55 fm is equal to the square root of 12 divided by the square root of the square of 1.24 GeV is not correct and is unlikely to be published in Physical Review D. The units do not match, and the numerical value is off.

Here's why:

**Units:**

Fermis (fm) are a unit of length ( $1 \text{ fm} = 10^{-15}$  meters), while GeV is a unit of energy. Length and energy are fundamentally different dimensions and cannot be directly equated. The square root of 12 divided by the square root of the square of 1.24 GeV would still result in an energy unit ( $\text{GeV}^{-1}$ ), not a length unit (fm).

**Numerical Discrepancy:**

The calculation would be approximately:  $\sqrt{12} / \sqrt{(1.24 \text{ GeV})^2} = \sqrt{12}/1.24 \text{ GeV} \approx 2.77 / 1.24 \text{ GeV} = 2.23 \text{ GeV}^{-1}$ . This is not equal to 0.55 fm.

**Physical Review D:**

Physical Review D is a highly respected physics journal that adheres to rigorous standards of mathematical and physical correctness. Such a blatant error in units and calculation would not be published.”

The framework by which the mass radius of the proton is determined is essentially quantum chromodynamics (QCD) characterized by exploration of form factors. In one instance the form factor of the traceless gluon term  $T_{00}$  in the energy-momentum tensor (EMT) has been recently evaluated in lattice QCD (Shanahan & Detmold, 2019). There is indeed the difficulty of comprehending the authors' claim of "fitting this form factor by the dipole form (to what equation is it connected to and has it unambiguous connection to QCD framework?)" and extracting the effective mass of  $1.13 \pm 0.06$  GeV, whose mass equivalence is  $(2.01441 \pm 0.10696)$  *exp.*  $(-27)$  kg, considering the need for the knowledgeability of how the experimental procedure aids in accomplishing such extraction. The only takeaway is that the gluon, considered massless and yet contributing to the overall rest mass of the proton, is still relevant in light of the mass-energy equivalence principle in a purely classical context. The compositeness scale of the scalar gravitational form factor  $m_s^2 \equiv G(0)/(dG/dt|_{t=0})$  is purported to have been verified (Kharzeem, 2021); "but the interest in such claim is that it appears that the square of effective mass is also referred to as compositeness scale which gives a quantitative characterization to scalar gravitational form factor." The observation in these contributions is that the authors ignore pedagogical principles, which call for "speaking at the level of the learners" much of the time for greater comprehension. According to Wang *et al.* (2021), in the nonrelativistic limit, the mass distribution can be deduced using the scalar gravitational form factor instead of the form factor of mass density  $T_{00}$  attributed to (Kharzeev, 2021). Explicitly, the definition of the mass radius is given by  $\langle R_m^2 \rangle = 12/m_s^2$ . How this equation can be derived based on QCD framework is neither known nor clear if based on equations that are not logically interconnected.

In research, congruence between method and outcome refers to how the study's procedures—such as the collection and analysis of data—align with its underlying theoretical and philosophical assumptions and how the results make sense in relation to the selected methodology. For the sake of a wider audience—not only renowned scholars who are arXiv—endorsable—this is crucial to emphasize. Therefore, it is not out of the question to revisit the mass radius of the proton in light of the controverted equation in the literature. The goal is to address the issue of the frameworks' and related methods' congruence with the outcomes that could be obtained from them. Thus, the objectives of the study are as follows: 1) To derive an equation defining the fundamental constant absent from the controverted equation in the literature; 2) To show that nearly all mass radii of the proton in the literature were calculated using the derived equation containing the missing fundamental constant; 3) To derive additional variations of the mass-energy-equivalence-based equation for calculating the proton's mass radius; and 4) justify the argument that the missing fundamental constant obscured the relationship, if at all, between QCD and the derived equation.

## 2.0 THEORY

This theory is simply anchored on classical framework relying on rest mass of subatomic particles. This is important because the Einstein's mass-energy equivalence calculations also rely on the rest mass of the particle. Various form factors had always been explored pursuant to the determination of the rest mass of the proton. The form factor of the energy momentum tensor (EMT) had been compared with the scalar gravitational form. In this regard Kharzeev (2021) with reference to the literature gave equation of the form factor of the EMT for a proton (spin 1/2 particle of mass,  $M$ ); here again the unit of  $M$  cannot be anything other than kg or in terms of

mass-energy equivalence, GeV or lower. Additionally, is the equation (Kharzeev, 2021) expressing the mass of a particle given as:

$$M = \int \mu dV, \quad (1)$$

where if,  $\mu$  is the mass density in contrast to number density, then “ $V$  should be the volume”. Furthermore, given that subatomic particles are unlikely to have the same density, it is unclear how Eq. (1) seems to describe the masses of various particles given the same density and different volumes. Additionally, it is improbable that the same subatomic particle could have varied volumes, which raises questions about the general function of Eq. (1) in QCD formalism, where effective mass appears to be the norm. Given the mass-energy equivalence principle, the rest mass is still of interest in the equation regardless of its intended use.

Then, as long as the rest mass of elementary subatomic particles is an integral part of all frameworks, QCD in particular, it goes to show that there should be an explanation as to how the key parameter, the dipole parameter, also regarded as an adjustable parameter and effective mass, is influenced by the rest mass, which, on the basis of strictly classical context aligned with the mass-energy equivalence principle, is explored for the computation of the mass radius. Let’s put this classical and/or deterministic framework in context by asking the question, can credit be given to the highly nuclearized North American and Western European scientists if they object to the fact that the size of a confinement cannot be smaller than that of the matter that is completely confined or accommodated in it? Since quantum chromodynamics is a confining theory, quarks are found in hadronic bound states rather than as free, solitary particles in nature. Calculating the quark masses using the same methods as those for determining the masses of non-confined particles is impossible due to quark and gluon confinement. As a result, their masses are calculated theoretically at some energy scale using QCD frameworks (Mes. 2019).

It is obvious that the rest mass is the key to the determination of the mass radius. Therefore, the views about the source of the mass of the nucleon are relevant. To this end the view held by Kharzeev (2021) is that the chiral limit of massless quarks provides an accurate approximation to the physical world. Within such limits, the trace of EMT contains only the gluon term. Based on the methodological question of forward matrix elements, the mass of the nucleon is entirely due to the gluons (Kharzeev, 2021). The implication is that the mass radius of the proton is mainly due to the mass of the gluons on the basis of the mass-energy equivalence principle. While one may be comfortable with the claim that the charge radius is extracted from the coupling of the photon to quarks, whereas the mass radius results from the coupling to gluons (Kharzeev, 2021), it should be apparently contradictory to opine that the gluon radius of the proton is significantly smaller than its quark radius (Kharzeev, 2021) given an earlier assertion that the mass of the proton is due mainly to the gluon. Within the confines of the nucleon, quarks and gluons contribute to the nucleon's total mass. However, the radii of the charge and mass distributions are thought to be determined by the masses of the lightest physical states excited from the vacuum by the vector quark and scalar gluon currents, respectively (Kharzeem, 2021). "A layman would want to know if an elephant should be housed in a smaller space in nuclearized states."

## 2.1 Reexamination and correction of the controverted equation

The belief that the smallness of the mass radius is the outcome of the interplay of scale anomaly and spontaneously broken chiral symmetry (Kharzeem, 2021) does not go well with the mass-energy equivalence principle. This is on top of the evidence that suggests the  $\rho$ -meson, with a mass ( $m_\rho$ ) of about 770 MeV (1.37265 exp. (-27) kg), is the lightest physical state in the vector quark current, while the scalar glueball, which is the result of interacting gluons, has a much larger mass ( $m_G$ ) of about 1600 MeV (2.85226 exp. (-27) kg) in the scalar gluon current (Kharzeem, 2021). The interest in the masses is that they reaffirm the fact that the mass of the gluon is greater than the quark; otherwise, there is no known subatomic particle whose mass is 2.85226 exp. (-27) kg in nature.

With reference to the work of Kharzeem (2021), Wang *et al.* (2021) opined that in the nonrelativistic limit, the mass distribution can be deduced using the scalar gravitational form factor instead of the form factor of mass density,  $T_{00}$  (Kharzeem, 2021). Explicitly, the definition of the mass radius is given by the immediate equation (Eq. (2)) below. How such equation leads to  $\langle R_m^2 \rangle = 12/m_s^2$  is the bone of contention even if result such as “0.67 +/- 0.03 fm, with the dipole cutoff  $m_s = 1.01 +/- 0.04$  GeV” (Wang *et al.*, 2021)) was produced.

$$\langle R_m^2 \rangle \equiv \frac{6}{M} \frac{dG(t)}{dt} \Big|_{t=0}, \quad (2)$$

Although Eq. (2) shows the symbol of “equivalent to,” it calls for interpretation that could enable comprehension by all rather than a few but significant members of a team of collaborators that can always be endorsed. Citing the works in the literature (Ji, 2021; Ji & Liu, 2021; Kharzeev, 2021), Wang *et al.* (2021) opined that the scalar gravitational form factor ( $G(t)$ ) is defined as the matrix element of the trace of the energy momentum tensor (EMT), whose equation is omitted because it cannot aid discussion in addition to having undefined terms that may be known only to collaborators. However, since the momentum transfer ( $t$ ) term has the square of energy as a unit, the unit of  $M$  (or  $G(0)$ ), which is kg, can cancel the unit off. Hence, it could have been most appropriate to state Eq. (2) in the following way.

$$\langle R_m^2 \rangle \propto \frac{6}{M} \frac{dG(t)}{dt} \Big|_{t=0}, \quad (3)$$

Hence,

$$\langle R_m^2 \rangle = \varphi \frac{6}{M} \frac{dG(t)}{dt} \Big|_{t=0}, \quad (4)$$

where  $\varphi$  a unit defined as  $J^2 m^2$  is a fundamental constant leaving one with the question as to whether  $G(t)$  and  $t$  terms are experimentally determined. Meanwhile, the gravitational form factors (GFFs) are, according to Wang *et al.* (2021) usually parameterized as the dipole form, but it is not clear if GFFs refer to scalar GFFs mentioned earlier. Thus (Wang *et al.*, 2021),

$$G(t) = \frac{M}{\left(1 - \frac{t}{m_s^2}\right)^2}, \quad (5)$$

It can be clearly seen that in Eqs (4) and (5)  $G(t)$  has the unit of mass leaving one with the assumption that form factors such as scalar GFF in line with context may be with or without unit

(dimensionless). It seems that if  $M$  remains the rest mass, of a proton,  $G(t)$  and  $t$  variables are probably the experimental variables by which the effective mass is calculated or using regular term, extracted even though, it is regarded as a free parameter to be determined experimentally (Wang *et al.*, 2021). According to the definition (not clearly specified), the mass radius is connected to the dipole parameter  $m_s$  as

$$\langle R_m^2 \rangle = 12/m_s^2, \quad (6)$$

However, there is no clear link of all equations stated earlier to Eq. (6) *vis-à-vis* the fact the dipole parameter is a direct experimental variable if assumed to be so. Whatever be the case, it is necessary to know if Eq. (6) is derivable on the basis of QCD framework with stepwise justifiable evidence. Besides, it is once again most appropriate to state

$$\langle R_m^2 \rangle = \frac{12\zeta}{m_s^2}, \quad (7)$$

where  $\zeta$  is a proportionality constant whose unit is  $J^2m^2$ . If all except  $M$  are to be experimentally known, then solving for  $m_s^2$  in Eq. (5) and substituting into Eq. (7) gives,

$$\langle R_m^2 \rangle = \frac{12\zeta}{t} \left(1 - \frac{M}{G(t)}\right)^{1/2}, \quad (8a)$$

Ultimately,

$$R_m = \left[ \left( \frac{144\zeta^2}{t^2} \right) \left(1 - \frac{M}{G(t)}\right) \right]^{1/4}, \quad (8b)$$

However, Eq. (8b) remains limited in value if  $t$  and  $G(t)$  remain unknown. Now, is the time to determine the fundamental constant ( $\zeta$ ). First, the equation is given as follows:

$$\xi = \frac{\hbar c}{\ell}, \quad (9)$$

where  $\xi$ ,  $\hbar$ ,  $c$ , and  $\ell$  are the energy of the particle, reduced Planck constant, velocity of light in vacuum, and a one dimensional space whose status in the context of QCD framework is still in contention.

$$\ell = \frac{\hbar c}{\xi} \quad (10)$$

$$R_m = (12\ell^2)^{1/2} \quad (11)$$

$$R_m = 3.46410\ell \quad (12)$$

Thus, substituting Eq. (10) into Eq. (12) gives,

$$R_m = 3.46410 \frac{\hbar c}{\xi} \quad (13)$$

where  $\xi$ , indeed may serve as the effective mass designated usually as  $m_s$ . Therefore, one can rewrite Eq. (13) to take the form below in order to come to terms with Eq. (11) which needs to be squared to give:

$$\langle R_m^2 \rangle = \frac{12\hbar^2 c^2}{m_s^2}, \quad (14)$$

## 2.2 Variants of the equations for the calculation of a definite mass radius of the proton based on mass energy-equivalence principle

From the manuscript in preparation is the equation (a variant for the computation of the mass radius of the electron) as follows:

$$R_e = \left( \frac{e^3}{4h^2 m_p R_\infty c} \right)^2 \frac{m_e^3}{\epsilon_0^3}, \quad (15)$$

where  $R_e$ ,  $m_e$ ,  $c$ ,  $h$ ,  $R_\infty$ ,  $\epsilon_0$ , and  $m_p$  are respectively the mass radius of the electron, rest mass of the electron, velocity of light in free space, Planck constant, Rydberg constant, permittivity in free space, and mass of the proton.

$$R_m = \frac{64 h^6 m_p^4 R_\infty^4 \pi \epsilon_0^3 m_N R_e^2}{e^6 m_e^8}, \quad (16)$$

Note that while  $m_p$  remains strictly the rest mass of the proton,  $m_N$  can be the mass of any nucleon, proton and neutron inclusive, apart from higher-mass subatomic particles. Next, the equation of the mass radius (Udema, 2025) of the electron is substituted into Eq. (16) to give:

$$R_m = \frac{64 h^6 m_p^4 R_\infty^4 \pi \epsilon_0^3 m_i \left( \frac{4m_e h^2 \epsilon_0}{\pi m_p^2 e^2} \right)^2}{e^6 m_e^8}, \quad (17)$$

Here  $m_i$  replaces  $m_N$  so that one can end up with a far-reaching generalization. This implies  $m_i$  represents the mass of nucleons and higher mass subatomic particles. Simplification gives,

$$R_m = \frac{1024 h^{10} \epsilon_0^5 R_\infty^4}{\pi e^{10} m_e^6} m_i, \quad (18)$$

Can effective mass be described as an outcome of the complex interaction of a particle in a composite system in which low-energy fields have a controlling effect on the rest mass of such a particle? Whatever the answer might be, the issues are characteristically quantum, hence quantum chromodynamics (QCD). This study remains, as usual, classically oriented on the foundation of the Einstein's mass-energy equivalence principle.

### 3.0 METHODS

The methods are purely theoretical and computational. Nevertheless, the quote, below, is interesting because it raises the question of whether the QCD framework can also be used to derive the equation of the root mean square mass radius of the proton, which is  $\langle R_m^2 \rangle = 12\hbar^2 c^2 / m_s^2$ . "In quantum chromodynamics (QCD), the proton mass can be derived from the matrix elements of the energy-momentum tensor (EMT)." A natural question that arises is whether the proton mass can be decomposed, for example, into a quark and a gluon part. Several decompositions have been synthesized based on various criteria, including gauge invariance, Lorentz invariance, and locality; however, these concepts are not very familiar and do not relate to the classical framework of this study.

In contrast to the widely recognized top-notch leading scientific equation (Kharzeev, 2021), which is provided as  $\langle R_m^2 \rangle = 12/m_s^2$ , the inequality which is represented as  $\langle R_m^2 \rangle \neq 12/m_s^2$ , ought to have a basis for what an American institution may view as fringe science. If the inequality is based on the false assumption that it should be categorized as a fringe science, then  $\langle R_m^2 \rangle = 12\hbar^2 c^2 / m_s^2$ , whose solution is approximately 0.55 fm (*i.e.*, if  $m_s = 1.24$  GeV) cannot be an exception. In any event, the following inquiries must be answered: Can the QCD or QED framework be used to construct the equation that produces 0.55 fm? If  $\hbar^2 c^2$  was pertinent to the equation  $\langle R_m^2 \rangle = 12/m_s^2$ , in the literature materials (Kharzeev, 2021, Wang *et al.*, 2021, Wang *et al.*, 2023) where it was found, why then was it left out? Is the omission, or any other parameter or parameters, an attempt to hide the fact that the actual equation might serve as a template for creating a *nuclear bomb* that is ten times more deadly? CERN and perhaps the IAEA are in charge of looking into it; if not, they should increase the authors' research grant or prize if found valid. Such grants should be given directly to the recipients instead of being

channeled through the state, and *vice versa*, to prevent the typical lack of accountability that has plagued not only poor developing countries but also, surprisingly, some advanced scientific nations.

The importance of the mass-energy equivalence principle in achieving the results is not overstated; nonetheless, it should be noted that this paper makes no claims regarding knowledgeability in QCD formalism and that it had no bearing on the findings of the study. As a reminder,  $m_i$  in Eq. (18) represents the mass of any particle whose mass is either equal to or greater than the mass of any nucleon. The mistake in the previous publication (Udema, 2022), which an anonymous commentator without bias, I presume, called a paper with fringelike scientific content, is the use of another variant of Eq. (18) for the calculation of the mass radius of a lepton (muon) whose mass is less than the mass of the nucleon. The correction of such a mistake is, for now, outside the scope of this study. Prior to that, the desired parameters relocated in the tables of values are calculated using Eqs. (13) and (18). The question of statistical analysis is unnecessary because no measurements or replicate experiments were conducted.

#### 4.0 RESULTS AND DISCUSSION

By comparing their result with those of Kharzeev (2021), who explored  $J/\psi$ -photoproduction data produced by Ali *et al.* (2019) to yield 0.55 fm; Wang *et al.* (2021), who investigated or extracted the  $\phi$ -photoproduction produced by Mibe *et al.* (2005) to yield 0.67 +/- 0.1 fm; and Barth *et al.* (2003), who obtained 0.68 +/- 0.03 fm, Wang *et al.* (2023) believed that the magnitude of their experimental proton mass radius exceeded those found in the previously mentioned values in the literature. According to Wang *et al.* (2023), the value in question, 0.85 +/- 0.06 fm, is closer to the proton charge radius, the latter of which, according to Workman *et al.* (2022), is 0.8409 +/- 0.06 fm. This position contrast sharply with the result (0.55 fm) reported by Kharzeev (2021) and much more so with the values such as 1.10168 fm (Table 1) obtained in this study and in the literature (Hare & Papini, 1972, Ivanov & Kolikov, 2013, Utama, *et al.*, 2016, Udema, 2020, Udema, 2022, Udema, 2025).

**Table 1: Computed mass radii of selected baryons on the basis of mass-energy equivalence principle**

Name of particles	Sym.	Mass/exp. (-27) kg	Mass radius/fm	Effective mass/GeV
<b>Charm quark</b>	$c$	2.35311	1.54989	0.481307
<b>Tauon (tau lepton)</b>	$\tau^-$	3.17314	2.09001	0.356923
<b>Bottom quark</b>	$b$	7.55849	4.97844	0.149841
<b>Top quark</b>	$t$	307.866	202.734	0.003680**
<b>Delta</b>	$\Delta$	2.19624	1.44626	0.515794
<b>Xi</b>	$\Xi^0$	2.34395	1.55059	0.481090
	$\Xi^-$	2.35618	1.55868	0.478593
<b>Sigma</b>	$\Sigma^0$	2.12608	1.40036	0.532701
	$\Sigma^-$	2.13463	1.40599	0.530568
<b>Omega</b>	$\Omega$	2.98148	1.96377	0.379868
<b>Neutral lambda</b>	$\Lambda$	1.98154	1.30515	0.571562

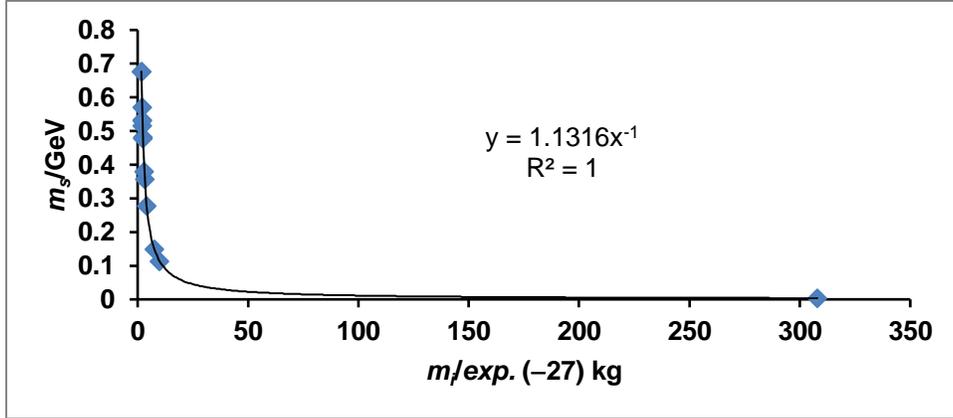
<b>Charmed lambda</b>	$(\Lambda^+c)$	4.06123	2.67495	0.278874
<b>Bottomed lambda</b>	$(\Lambda^0c)$	10.0178	6.59828	0.113056
<b>Proton*</b>	$p$	1.67262192369	1.10168	0.677123
<b>Neutron*</b>	$n$	1.67492749804	1.10320	0.676190

\*designates data from 2021 CODATA table of values (Tiesinga, *et al.*, 2021); other data are from internet based data.  $1024h^{10}\epsilon_0^5R_\infty^4/\pi e^{10}m_e^6 = 6.58656 \text{ exp. (+11) } m/kg$  (From Eq. (18));  $R_m = 6.58656 \text{ exp. (+11) } m$ ;  $3.46410 \hbar c = 1.09516 \text{ exp. (-25) } J m$  (From Eq. (13));  $R_m = 1.09516 \text{ exp. (-25) } /m_s$ ; the real absolute value in place of \*\* is 0.00367956 GeV; Sym. designates symbol. Note that whatever the value of  $m_s$ , its use requires conversion to Joules by multiplying by  $1eV$  *i.e.*  $1.602176634 \text{ exp. (-19) } J$ ;  $\hbar = 1.054571816 \text{ exp. (-34) } J s$  (Tiesinga, *et al.*, 2021).

The idea that gravitational form factors (GFFs) are useful for understanding the perturbative and nonperturbative quantum chromodynamics (QCD) effects, which provide a connection to the spatial distribution of quarks inside the proton, lends further credence to the previous position in the introduction that form factors are the key to understanding mass distribution in nucleons (Kobzarev, 1962; Ji, 1997; Pagels, 1966). The scalar GFFs are sensitive to the proton mass distribution from the QCD trace anomaly and are related to the photoproduction of a quarkonium off a proton in QCD theory (Ji, 1997; Wang, *et al.*, 2020). The proton mass radius can be derived via near-threshold photoproduction of a vector meson differential cross section, assuming a scalar form factor of dipole form (Fujii, 1999; Fujii & Kharzeem, 1999; Kharzeem, 2021). It is not an overemphasis that, unlike in this study, modern concepts epitomized in QCD dominated both theoretical and experimental studies in search of the definite mass radius of the proton. Considering modernity, one may be free to refer to the current theoretical approach as an archaic classical concept in the context of time, yet its deterministic relevance cannot be ignored by those who have the potential to see beyond their immediate convictions. Thus, on the premise that fundamental physical constants are interconnected, this study is able to show that there was a hidden physical constant whose consistency is as solid as the rest mass of any particle. Note too that while mass may be described as being thermodynamically extensive, each specific subatomic particle has a definite rest mass. Therefore, very akin to Einstein's mass-energy equivalence principle, the mass radius, which refers to the total size of the subatomic particle, is calculated based on additional derivations.

There is the view attributed to Zyla *et al.* (2020) by Wang *et al.* (2021) to the effect that only the charge and the magnetic radii of the proton are known probably as experimental values unlike the mass radius which has not been experimentally determined. However, different theoretical values of the mass radius of the nucleon (0.7 fm) in general (Hare and Papini, 1972) and specifically the proton with values such as 1.100796221 fm (Udema, 2020), 1.101171175 fm (Udema, 2022), 0.841184 fm (Ivanov & Kolikov, 2013) and 1.15 fm (Utama, *et al.*, 2016) have been reported. The term effective mass ( $m_s$ ) was explored by Kharzeev (2021). But the fundamental question is how does it relate with the mass of the proton either as rest mass or reduced mass following Einstein's theory of relativity? The value of  $m_s$  (1.24 GeV) that was extracted from GlueX collaboration data (Ali, *et al.*, 2019) in its mass equivalence is equal to  $(2.21049 \pm 0.124786) \text{ exp. (-27) } kg$ . One can also deduce that any subatomic particle has its effective mass to be determined experimentally. Another question is: Is the effective mass directly proportional to the rest mass? If the answer is yes, the derived equation hitherto containing the missing fundamental constant could yield a shorter mass radius of any particle whose rest mass is either equal to or greater than the mass of any hadron. An inverse proportionality would create opposite consequences in line with the mass-energy equivalence

principle. This inverse proportionality and its positive implication are illustrated with the plot of  $m_s$  versus  $m_i$ , which exhibited an inverse power law (Fig. 1). The resulting power law equation of regression could be useful for the estimation of the effective mass of any particle that may arise accidentally in the course of the experiment and whose mass is either equal to or greater than the rest mass of the nucleon.



**Fig. 1: Variation of dipolar parameter (effective mass,  $m_s$ ) versus rest mass ( $m_i$ )** A scenario in which power law equation relates effective mass and rest mass can be generally given as:  $m_s = \chi m_i^{-\sigma}$ . While there may be a lower limit to the masses of subatomic particles that are greater than 1000 but less than approximately 1836 times the mass of an electron, the plot may include these subatomic particles and those with even higher masses. Such a lower limit is outside the scope of this study.

Using dipole parameters from the differential cross section of  $\omega$ - and  $\phi$ -photoproduction near threshold at different photon energies, Wang et al. (2021) determined the proton's root mean squared mass radius; with the former, in contrast to the latter, there is no consistent trend in the values of the effective mass and the corresponding value of the proton's root mean squared mass radius (Table 2). It may be premature to insinuate “fringe-science-like reportage” in this regard. Thus, it appears that the proton does, in fact, manifest itself isotopically with different root mean squared mass radii. Einstein's mass-energy equivalence principle states that a subatomic particle's size must be proportionally consistent with its rest mass, and this principle aligns with the numbers displayed in Table 1, which starkly contrast with those values in Table 2.

**Table 2: Computed proton mass radii using dipole parameters from the differential cross section of  $\omega$ - and  $\phi$ -photoproduction near threshold at different photon energies**

	$\omega$ -photoproduction		$\phi$ -photoproduction		
* $\xi_\gamma/\text{GeV}$	1.1375	1.1625	1.62	1.72	1.82
* $m_s/\text{GeV}$	1.06	0.99	0.82	1.17	0.96
# $3.46410 \hbar c/m_s/\text{fm}$	0.644869	0.690465	0.83361	0.5842	0.712042

\* denotes data from the literature (Wang et al., 2021); # denotes values generated by computation in this study. Note that \* marked data are experimental values using devices best known to the authors.

As of right now, it is unclear why the author (Kharzeev, 2021) did not include  $\hbar c$  in the equation that was included in his or her publication; it is probable that the writers who referenced the article were unaware of the omission. The equation  $\langle R_m^2 \rangle = 12/m_s^2$  is contextually meaningless if the current equation ( $\langle R_m^2 \rangle = 12\hbar^2 c^2/m_s^2$ ) has nothing to do with the QCD framework, as is likely implied. Meanwhile, the average mass radius of the two extracted values is obtained to be

$\sqrt{\langle R_m^2 \rangle} = 0.68 \pm 0.03 \text{ fm}$  (Wang *et al.*, 2021). Proton mass radius as a function of the mass of the vector meson, which comes from the color dipole interacting with the proton target has been reported. The extracted values were said to be consistent with each other within the statistical uncertainties (Wang *et al.*, 2021). The combined analysis of the data of the three vector mesons gives the average proton mass radius to be  $\sqrt{\langle R_m^2 \rangle} = 0.67 \pm 0.03 \text{ fm}$ , with the average dipole parameter to be  $m_s = 1.01 \pm 0.04 \text{ GeV}$  (Wang *et al.*, 2021). On the other hand, Wang *et al.* (2023) explored center of mass (c.m.) energy concept: In short, Wang *et al.* (2023) showed that the differential cross section at near-threshold center of mass (c. m.) energy was used to compute the proton mass radius. The root mean square (RMS) mass radius *i.e.*,  $R_m$ , of all fitted results (without specifying which equation was fitted to which data) was determined to be  $0.85 \pm 0.06 \text{ fm}$  using what they called the differential cross section experimental data in c.m. energy designated as  $W$ , which ranged between 1.92 and 2.23 GeV (Wang *et al.*, 2023). This is in spite of the fact that the former appeared to have been extrapolated from a graphical plot that was mentioned. Besides, there should be several extrapolations giving closely similar values that should justify the mean and the indicated uncertainty from several plots. The endorsable equation ( $\langle R_m^2 \rangle = 12/m_s^2$ ) that was regularly cited (unlike the unknown equation given as,  $\langle R_m^2 \rangle = 12\zeta/m_s^2$  where  $\zeta$  designates  $\hbar^2 c^2$ ) did not show up as one clearly explored for the calculation of the RMS radius of the proton. This view is premised on the authors' reference to "the scalar form factor of dipole form"! Based on the assumption of a scalar form factor of *dipole form*, the value of the proton mass radius is calculated as  $(0.85 \pm 0.06) \text{ fm}$  (as stated earlier) by fitting the differential cross section of the  $\gamma p \rightarrow \rho^0 p$  reaction at near-threshold energy where  $\gamma p$  denotes the interaction of the incident photon ( $\gamma$ ) with a proton ( $p$ ) to produce an electrically neutral meson ( $\rho^0$ ) and another proton. Note that the effective mass  $m_s$  is also, known as dipole parameter. By the way, it is also necessary to consider the view that the mass radius was extracted from  $\rho^0$ -photoproduction without giving clues as to how such, for the sake of layman, align with the use of the differential cross section experimental data in c.m. energy to yield  $(0.85 \pm 0.06) \text{ fm}$  (Wang *et al.*, 2023). These intricacies run contrary to a simple and straightforward way (Eq. (18)) in the computation of the mass radius of the proton and higher subatomic particles in general (Table 1).

Other values of RMS radii of the proton reported cannot be ignored. Hence, such values can be used to gain insight on the likely effective mass. Thus, based on the differential cross sections of  $\phi$ -photoproduction at near-threshold predicted by the two-gluon exchange model, the average mass radius of the proton is derived as  $\sqrt{\langle R_m^2 \rangle} = 0.78 \pm 0.06 \text{ fm}$  since the mass radius and mechanical properties of the proton are encoded in the scalar gravitational form factor of the momentum energy tensor (Wang *et al.*, 2022). The average value of  $0.80 \pm 0.05 \text{ fm}$  was obtained by the authors (Wang *et al.*, 2022) by directly extracting the proton mass radius from the experimental data of  $\phi$ -photoproduction for comparison purposes. They found that this value was extremely similar to the outcome provided by the two-gluon exchange model. The relativistically correct definitions of the proton radius and charge density are introduced under the perturbation theory of light-front dynamics in the literature, according to Wang *et al.* (2021),

who cited Miller's (2019) work. This suggests that there may be some incorrect definitions of the proton's mass radius if this refers to proton radius. A counterargument to this is that all particles have mass, which is the amount of matter they can hold in three dimensions.

If the scalar proton radius reflects the gluon distribution (the largest component of the proton's structure) and the proton mass radius reflects the distribution of the proton's mass, which is primarily due to the energy and momentum of its constituent quarks and gluons, then the total mass of the proton due to its constituents cannot be handled in isolation of any of its constituents. In other words, because of its rigorous reliance on rest mass rather than reduced or effective mass, the proton—and in fact, any baryon—is a function of the entirety of the masses of its constituents, in accordance with Einstein's mass-energy equivalence principle as well as the classical framework devoid of the need for either reduced or effective mass.

## 5. CONCLUSION

In addition to determining that, to the best of our knowledge, the equation is unknown in the literature, the equation for a definitive identification of the basic constant missing from the disputed equation in the literature was also obtained. The values of the proton's RMS mass radii recorded in the literature seem to have been computed using the reformed equation,  $\langle R_m^2 \rangle = 12\hbar^2 c^2 / m_s^2$  and its root mean square transformation  $\sqrt{\langle R_m^2 \rangle} = 3.46410 \hbar c / m_s$ , even though the descriptive terminology used for all values obtained was dominated by the extraction of the proton's RMS mass radius. If the stated equation bears any iota of relevance to the QCD framework, then there was no basis whatsoever for the omission of the fundamental constant,  $\hbar c$  or  $\hbar^2 c^2$ , from the derived equation; thus, the omission, until proved otherwise, only served to obscure the fact that the equation does not have any bearing on the QCD framework. As usual, additional variants of the mass-energy equivalence-based equation reproduced the 1.10168 fm as the RMS mass radius of the proton. It should be premature to speculate that such is an overestimate, being  $\sim$  twice 0.55 fm in the face of other higher values in the literature. Root mean squared (RMS) mass radius of the proton is equal to 6.58656 *exp.* (+11)  $m_s$  (where  $m_s$  stands for effective mass); RMS mass radius of the proton = 1.19518 *exp.* (– 25)  $m_s$  (From Eq. (13). Note that whatever the value of  $m_s$ , its use requires conversion to Joules by multiplying by 1eV *i.e.* 1.602176634 *exp.* (–19) J. In the future, it might be necessary to theoretically calculate the effective mass of subatomic particles that are not included in this study but whose rest masses exceed those of the nucleons.

## 6. DEDICATION

This article is dedicated first to Rtd. Brig. Gen. Samuel Osaigbovo Ogbemudia (Dr. h.c.) and his commissioner, Chief Edwine Kiagbodo Clark (1927–2025), in honor of the selfless service of a former federal minister of information, specifically his time as the commissioner of education in the now-defunct Mid-West state, which later became known as Bendel state, which divided into Delta and Edo states. The military governor, Rtd. Brig. Gen. Samuel Osaigbovo Ogbemudia, approved the Headmasters' Institute that Chief E. K. Clark proposed and implemented with its well-structured curriculum for upgrading primary school teachers. He upheld moral and ethical standards, as demonstrated, among other things, by ensuring the dismissal of a former classmate (a teacher) who encouraged examination malpractice. During his tenure, each school received up

to five (5) copies of the state-owned newspaper, the Observer, which helped staff and students stay up-to-date on current events and, more importantly, improve their understanding of British English without making the newspaper outfit unprofitable. Such tradition continued in my days as a student in A. G. S. Ubulu-uku.

## 7. **DISCLAIMER (Artificial Intelligence)**

Author(s) hereby declare that no generative AI technologies such as Large Language Models (ChatGPT, COPILOT, *etc.*) and text-to-image generators have been used during the writing or editing of this manuscript. However, internet-based AI provided answers to questions that were meant to either refute or support the controverted equation discussed only in the introduction section of the article. AI played no role in other sections of the article.

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There are no known competing financial interests, non-financial interests, or personal ties that could have influenced the work presented in this publication, according to the sole author's declaration. The only issue is the monthly pension, which is much less than two USD per day and may not have been recommended by the World Bank or the IMF.

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